UC San Diego

Ballooning Mode in a Stochastic Magnetic Field —A Quasi-mode Model

Mingyun Cao, P.H. Diamond

University of California, San Diego

AAPPS DPP | Nagoya, November 15, 2023

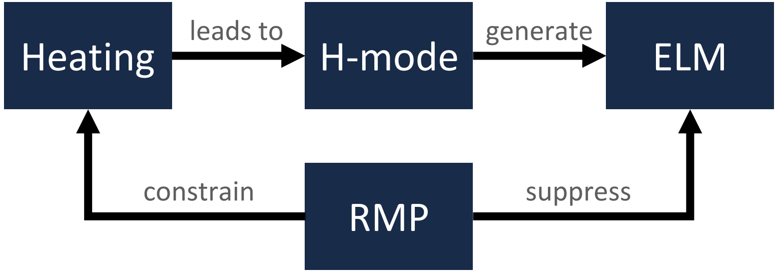
Research supported by U.S. Department of Energy under award number DE-FG02-04ER54738.

Contents

1. Background: “noisy” plasma turbulence under the application of RMP
2. Challenge: disparate geometries in the theories of ballooning mode and RMP
3. Model: quasi-mode, the counterpart of ballooning mode in a cylinder
4. Results: lessons learned for ballooning modes in a stochastic magnetic field
5. Future: suggested experimental and theoretical studies

Background: theoretical progress

UC San Diego



Kobayashi, Masahiro, et al. *Physical review letters* 128.12 (2022): 125001.

Question: how does stochastic magnetic field modify plasma turbulence & instability process?

Instability and turbulent relaxation in a stochastic magnetic field

Mingyun Cao[2,1] and P H Diamond[1]

Published 7 February 2022 • © 2022 IOP Publishing Ltd

The first model which

- maintains $\nabla \cdot \boldsymbol{J} = 0$ at all scales
- connects dynamics at micro and macro scales

Experiments are also needed…

However, spectral analysis is not enough to characterize the turbulence state

Need other ways to study the statistics of plasma turbulence

Background: a method from the information theory

— Turbulence vs. noise

- wide-band power spectrum • short autocorrelation time • random orbits

— Difference in nature: a spectral energy flux

— How to distinguish chaos from noise?

— complexity-entropy analysis

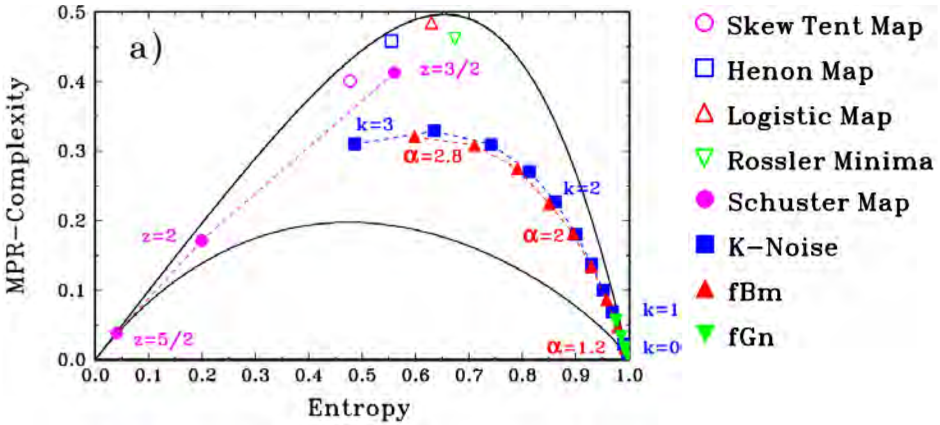
— Jensen-Shannon complexity C_{JS}:

$$C_{JS} = \underbrace{H}_{\text{Shannon entropy}} \times \underbrace{Q}_{\text{J-S divergence}}$$

— a metric of a system/signal's predictability

chaos □○△: high C_{JS}; noise ▲▼ : low C_{JS}[1]

— Application in MFE: the **chaotic** nature of the edge fluctuations in L-, H-, and I-mode are identified.[2]

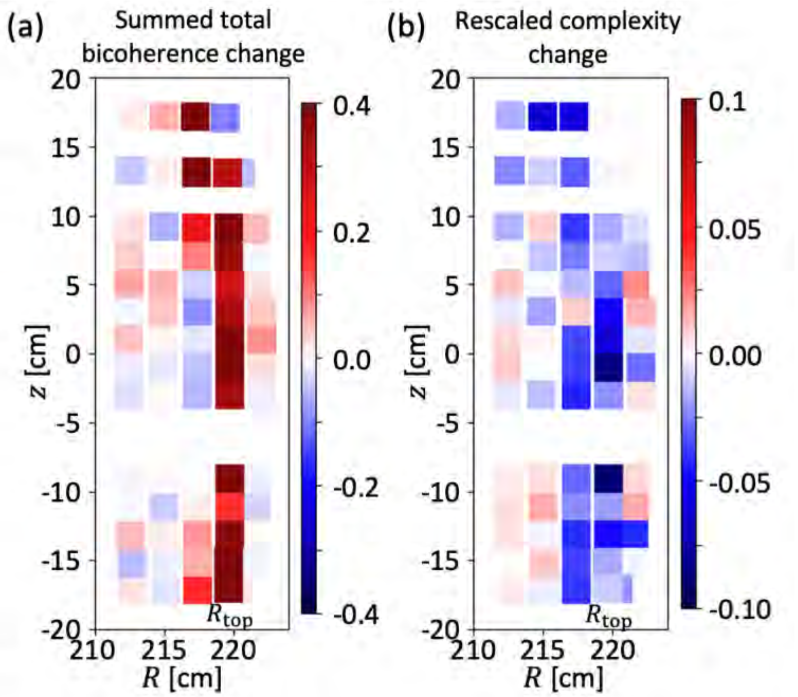


1. Rosso, Osvaldo A., et al. *Physical review letters* 99.15 (2007): 154102.
2. Zhu, Ziyan, et al. *Physics of Plasmas* 24.4 (2017).

Background: recent experiment

UC San Diego

- Complexity-entropy analysis: characterize the state of turbulence w/wo RMP.
- Experiments on KSTAR: pedestal temperature fluctuations collected from electron cyclotron emission imaging (ECEI)[1]
- An increase in the bicoherence and a reduction in complexity in the RMP ELM suppression phase. ⟶ Edge plasma turbulence becomes more “noisy” .
- Need to understand these phenomena ⟶ a step forward from resistive interchange mode



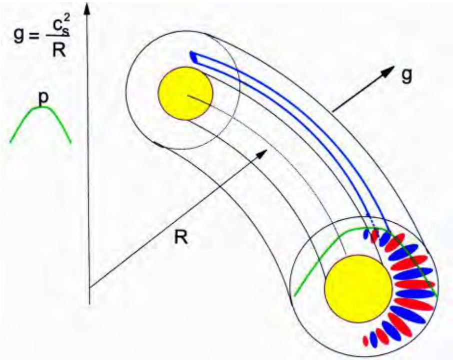
1. Choi, Minjun J., et al. *Physics of Plasmas* 29.12 (2022).

Challenge: disparate geometries

UC San Diego

— From interchange to a more **relevant** instability → ballooning mode

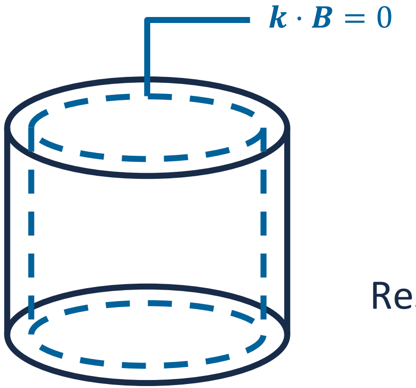
— A hard nut to crack: difference in geometries on which theories of the ballooning mode and RMP are based.



Ballooning mode

↕

Toroidicity effect



RMP

↕

Resonant surfaces in a **cylinder**

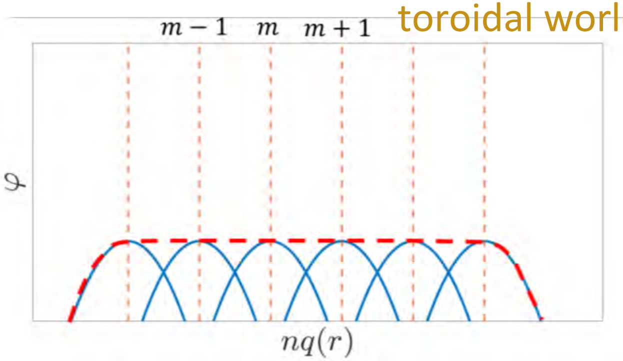
Ballooning mode in a **torus** vs. resonant magnetic perturbations in a **cylinder**

Question: Is there a way to circumvent this problem?

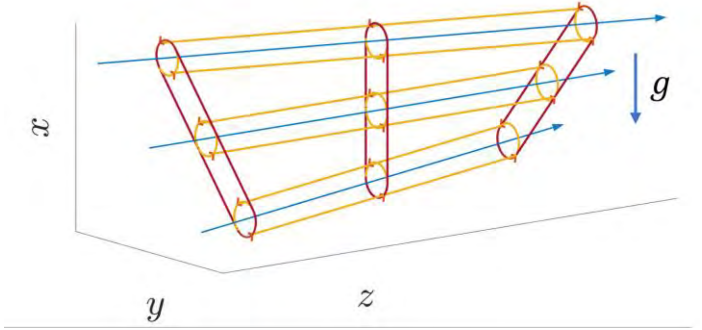
Strategy: find the counterpart of the ballooning mode

Theories of ballooning mode and RMP "reside" in different parallel universe.

toroidal world



cylindrical world



— Mode structure of the ballooning mode: a coupling of localized poloidal harmonics at different resonant surfaces

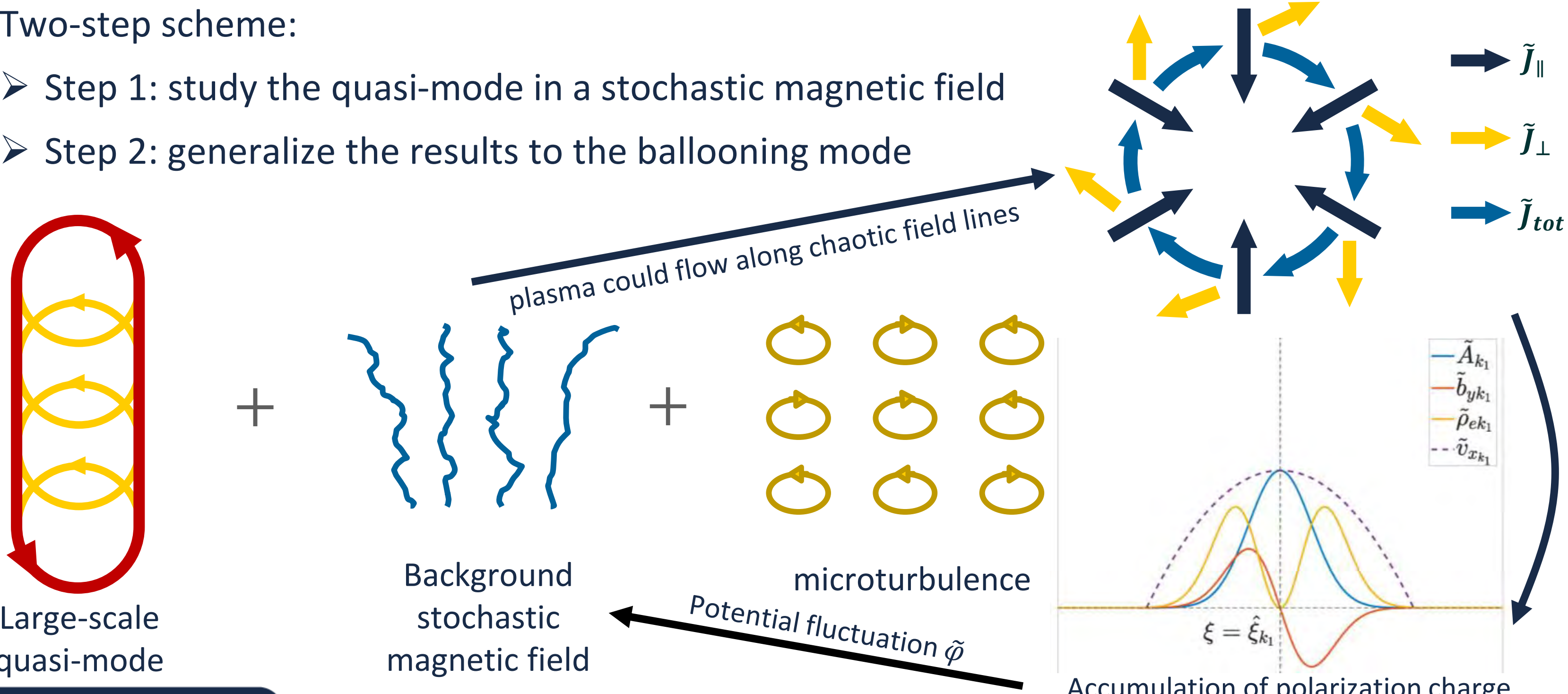
— Mode structure of the quasi-mode: a wave-packet of radially-localized interchange modes at different resonant surfaces

Takeaway: a quasi-mode in a cylinder resembles a ballooning mode in a torus

A Multi-scale Model: quasi-mode in a stochastic magnetic field

— Two-step scheme:

- Step 1: study the quasi-mode in a stochastic magnetic field
- Step 2: generalize the results to the ballooning mode



Model Development

UC San Diego

— Quasi-mode in a stochastic magnetic field:

vorticity equation

$$\underbrace{-\frac{\rho_0}{B_0^2}\frac{\partial}{\partial t}\nabla_\perp^2\varphi}_{\nabla_\perp\cdot \boldsymbol{J_{pol}}}\underbrace{-\frac{1}{\eta}(\boldsymbol{b_0}\cdot\nabla)^2\varphi}_{\nabla_\parallel\cdot J_\parallel}+\underbrace{\frac{g}{B_0}\frac{\partial}{\partial y}\rho}_{\nabla_\perp\cdot \boldsymbol{J_{PS}}}=0 \longrightarrow \boxed{\nabla\cdot\boldsymbol{J}=0}$$

continuity equation

$$\frac{\partial\rho}{\partial t}=-\boldsymbol{v}\cdot\nabla\rho_0=-v_x\alpha\rho_0$$

$\tilde{\varphi},\tilde{\rho},\tilde{v}_x$: microturbulence

$$\rho_0\left(\frac{\partial}{\partial t}-\nu_T\nabla_\perp^2\right)\nabla_\perp^2(\bar{\varphi}+\tilde{\varphi})+\frac{B_0^2}{\eta}\left(\boldsymbol{b_0}\cdot\nabla+\tilde{\boldsymbol{b}}\cdot\nabla_\perp\right)^2(\bar{\varphi}+\tilde{\varphi})-gB_0\frac{\partial(\bar{\rho}+\tilde{\rho})}{\partial y}=0$$

$$\left(\frac{\partial}{\partial t}-D_T\nabla_\perp^2\right)(\bar{\rho}+\tilde{\rho})=-(\bar{v}_x+\tilde{v}_x)\alpha\rho_0$$

turbulent viscosity
turbulent diffusivity → comes from the random convection of the quasi-mode due to microturbulence

$$\frac{1}{\bar{v}_x}\underbrace{\frac{\partial}{\partial\xi}}_{=0}\bar{\varphi}\ll\frac{1}{\bar{v}_x}\frac{\partial}{\partial\zeta}\bar{\varphi}\ll\frac{1}{\bar{v}_x}\frac{\partial}{\partial\chi}\bar{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial x}\tilde{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial\chi}\tilde{\varphi}$$

switched

$$\frac{1}{\bar{v}_x}\frac{\partial}{\partial\zeta}\bar{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial\zeta}\tilde{\varphi}\ll\frac{1}{\tilde{v}_x}\frac{\partial}{\partial\chi}\tilde{\varphi}\qquad \nu_T k_y^2\ll\gamma_{\boldsymbol{k}}\ll\gamma_{\boldsymbol{k_1}}<\nu_T k_{1y}^2$$

Model Development

UC San Diego

The full set of equations of the model is：

$$\rho_0\left(\frac{\partial}{\partial t}-\nu_T\nabla_\perp^2\right)\nabla_\perp^2\bar{\varphi}+\frac{B_0^2}{\eta}\frac{\partial^2}{\partial\zeta^2}\bar{\varphi}+\frac{B_0^2}{\eta}\left\{\underbrace{\left\langle(\tilde{b}\cdot\nabla_\perp)^2\right\rangle}_{(a)}\bar{\varphi}+\underbrace{\left\langle\frac{\partial}{\partial\zeta}(\tilde{b}\cdot\nabla_\perp)\tilde{\varphi}\right\rangle}_{(b)}+\underbrace{\left\langle(\tilde{b}\cdot\nabla_\perp)\frac{\partial}{\partial\zeta}\tilde{\varphi}\right\rangle}_{(c)}\right\}-gB_0\frac{\partial}{\partial y}\bar{\rho}=0$$

$$\rho_0\left(\frac{\partial}{\partial t}-\nu_T\nabla_\perp^2\right)\nabla_\perp^2\tilde{\varphi}+\frac{B_0^2}{\eta}\frac{\partial^2}{\partial\zeta^2}\tilde{\varphi}+\frac{B_0^2}{\eta}\left\{\frac{\partial}{\partial\zeta}(\tilde{\boldsymbol{b}}\cdot\nabla_\perp)\bar{\varphi}+(\tilde{\boldsymbol{b}}\cdot\nabla_\perp)\frac{\partial}{\partial\zeta}\bar{\varphi}\right\}-gB_0\frac{\partial}{\partial y}\tilde{\rho}=0$$

$$\left(\frac{\partial}{\partial t}-D_T\nabla_\perp^2\right)\bar{\rho}=-\bar{v}_x\alpha\rho_0 \qquad \left(\frac{\partial}{\partial t}-D_T\nabla_\perp^2\right)\tilde{\rho}=-\tilde{v}_x\alpha\rho_0 \qquad \langle A\rangle=\bar{A}=\frac{1}{L_y}\int_{-L_y/2}^{L_y/2}e^{-ik_y\chi}A\,d\chi$$

$$\begin{cases}\xi=x\\ \chi=y-sxz\\ \zeta=z\end{cases}$$

twisted coordinate system

Scale separation
- Define a proper spatial averaging.
- Split the dynamics of small and large scales.

Solve small-scale dynamics
- Get the linear response of $\tilde{v}_x$ to $\tilde{\boldsymbol{b}}$ and $\bar{v}$.
- Get a non-vanishing $\langle\tilde{v}_x\tilde{\boldsymbol{b}}\rangle$ correlation.

Solve large-scale dynamics
- Plug in the response and get the large-scale eigenmode equations.
- Calculate the corrected growth rate.

Nonlinear Closure
- Calculate the scaling of turbulent viscosity and turbulent diffusivity generated by microturbulence.

Quantitative results

UC San Diego

— The linear response of $\tilde{v}_x$ to $\tilde{\boldsymbol{b}}$

$$\tilde{v}_x = \frac{S}{\tau_A}\int G(\xi,\xi')\underbrace{\left[\partial_\zeta(\tilde{\boldsymbol{b}}\cdot\nabla)\bar{v}_x + (\tilde{\boldsymbol{b}}\cdot\nabla)\partial_\zeta\bar{v}_x\right]}_{\sim beat(\tilde{\boldsymbol{b}},\bar{v}_x)} d\xi'$$

— Correlation of $\tilde{v}_x$ with $\tilde{\boldsymbol{b}}$.

$$\langle\tilde{v}_x\tilde{b}_x\rangle = \frac{iL_yL_z}{(2\pi)^2}\int d\,k_{1y}\frac{s^2k_yS\left|\tilde{A}_{0\boldsymbol{k}_1}\right|^2}{\tau_A v_T|k_{1y}|}\frac{12\sqrt{\pi}o_{\boldsymbol{k}_1}^2}{w'}\zeta\partial_\zeta\bar{v}_x$$

$$\langle\tilde{v}_x\tilde{b}_y\rangle = -\frac{iL_yL_z}{(2\pi)^2}\int dk_{1y}\frac{s^3k_yS\left|\tilde{A}_{0\boldsymbol{k}_1}\right|^2}{\tau_A v_T|k_{1y}|}\frac{12\sqrt{\pi}o_{\boldsymbol{k}_1}^4}{w'}\zeta\bar{v}_x$$

$$\boxed{\frac{1}{\bar{v}_x}\underbrace{\frac{\partial}{\partial\xi}\bar{\varphi}}_{=0}} \ll \frac{1}{\bar{v}_x}\frac{\partial}{\partial\zeta}\bar{\varphi} \ll \boxed{\frac{1}{\bar{v}_x}\frac{\partial}{\partial\chi}\bar{\varphi}}$$

switched

No counterpart in prior study on resistive interchange mode

Quantitative results

UC San Diego

1. Rutherford, Paul Harding. *The Physics of Fluids* 16.11 (1973): 1903-1908.

— The revised eigenmode equation for quasi-mode is

$$\hat{H}_0\bar{\varphi}_{\boldsymbol{k}} = \hat{H}_1\bar{\varphi}_{\boldsymbol{k}} \qquad \hat{H}_0 = \frac{\partial^2}{\partial\zeta^2} - \frac{\gamma_{\boldsymbol{k}}\rho_0\eta}{B_0^2}s^2\zeta^2k_y^2 + \frac{\gamma_{\boldsymbol{k}}\rho_0\eta k_y^2}{B_0^2}\left(\frac{\alpha g}{\gamma_{\boldsymbol{k}}^2} - 1\right)$$

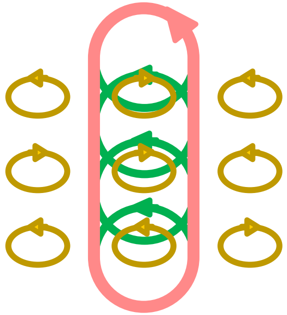
turbulent damping

$$\hat{H}_1 = \left[s^2\zeta^2k_y^2\left|\tilde{b}_x^2\right| - 2s\zeta k_y^2\left|\tilde{b}_x\tilde{b}_y\right| + k_y^2\left|\tilde{b}_y^2\right|\right] + \frac{\alpha g\rho_0\eta D_T k_y^4(1+s^2\zeta^2)}{\gamma_{\boldsymbol{k}}^2B_0^2} + \frac{\rho_0\eta}{B_0^2}\nu_T k_y^4(1+s^2\zeta^2)^2$$

$$+\frac{L_zL_y}{(2\pi)^2}\int d\,k_{1y}\frac{s^3k_y^2B_0^2\left|\tilde{A}_{0k_1}\right|^2}{\rho_0\eta\nu_T\left|k_{1y}\right|}\frac{8\sqrt{\pi}\left|o_{\boldsymbol{k}_1}\right|^2}{w'}\zeta\partial_\zeta$$

turbulent scattering

magnetic braking effect[1]



— The correction to the growth rate of the quasi-mode is

$$\gamma_{\boldsymbol{k}}^{(1)} = -\frac{5}{6}s^2\Delta^2\nu_T k_y^2\Bigg(1+\underbrace{\frac{8}{5}\frac{1}{s^2\Delta^2}}_{\text{new}}\Bigg) - \frac{1}{3}\frac{S}{\tau_A}\Bigg((1-f)\left|\tilde{b}_x^2\right|\underbrace{+\frac{2}{s^2\Delta^2}\left|\tilde{b}_y^2\right|}_{\text{new}}\Bigg)$$

— New terms arises because of the change in spatial ordering

$$\hat{H}_0 = -\overbrace{\frac{\gamma_{\boldsymbol{k}}\rho_0\eta}{B_0^2}s^2\zeta^2k_y^2}^{inertia} + \overbrace{\frac{\gamma_{\boldsymbol{k}}\rho_0\eta k_y^2}{B_0^2}\left(\frac{\alpha g}{\gamma_{\boldsymbol{k}}^2} - 1\right)}^{drive} + \cdots$$

stochastic bending → $\hat{H}_1 = \underbrace{s^2\zeta^2k_y^2\left|\tilde{b}_x^2\right|}_{enhance\ inertia} + \underbrace{k_y^2\left|\tilde{b}_y^2\right|}_{reduce\ drive}$

↕ balancing

linear bending

$$o_{\boldsymbol{k}_1} \sim \delta_{\boldsymbol{k}}\left(k_y/k_{1y}\right)^{1/2}$$ → multi-scale feature

Quantitative results

UC San Diego

— The sign of the growth rate is determined.

$$f = \frac{\langle (\tilde{\boldsymbol{b}} \cdot \nabla_\perp \partial_\zeta + \partial_\zeta \tilde{\boldsymbol{b}} \cdot \nabla_\perp)\tilde{\varphi}\rangle}{\langle (\tilde{\boldsymbol{b}} \cdot \nabla)(\tilde{\boldsymbol{b}} \cdot \nabla)\bar{\varphi}\rangle} \sim 8 \underbrace{\frac{\nu_T k_y^2}{\gamma_{\boldsymbol{k}}^{(0)}}}_{f_1} \underbrace{\frac{\alpha g}{\nu_T^2 k_{1y}^4}}_{f_2} \underbrace{\frac{|o_{\boldsymbol{k}_1}|}{w'}}_{f_3} \qquad \begin{cases} f_1 \ll 1 \\ f_2 < 1 \\ f_3 \ll 1 \end{cases}$$

$$\longrightarrow f < 1 \longrightarrow \gamma_{\boldsymbol{k}}^{(1)} < 0 \longrightarrow$$ stochastic magnetic field tend to suppress the instability

— The turbulent viscosity ν_T of quasi-mode is larger than that of resistive interchange mode!

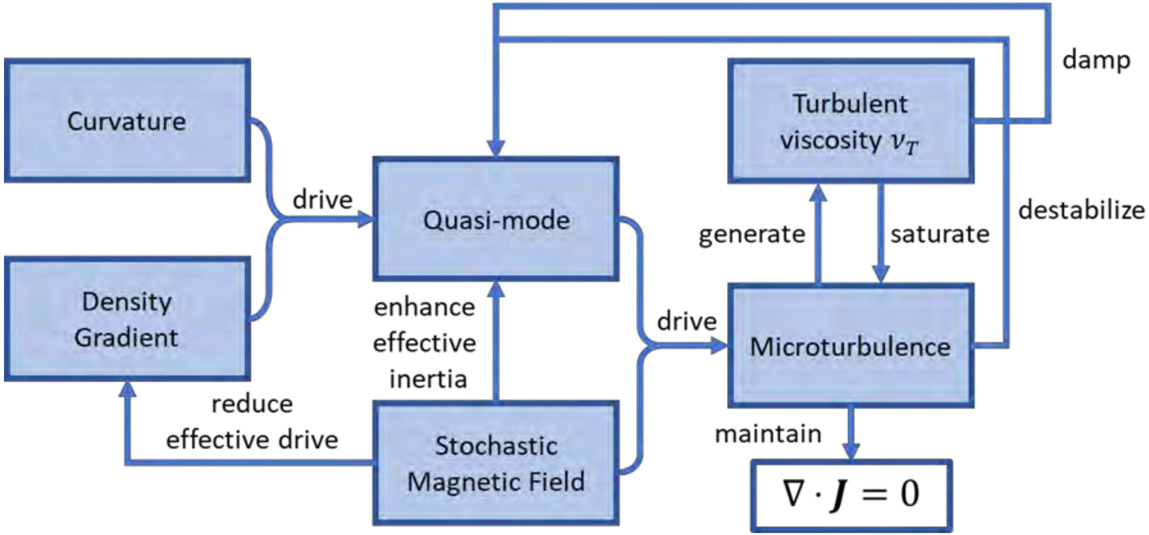
$$\nu_T = \sum_{\boldsymbol{k}_1} |\tilde{v}_{\boldsymbol{k}_1}|^2 \tau_{\boldsymbol{k}_1} \cong \left[\frac{L_z L_y}{(2\pi)^2} \int dk_{1y} \frac{s^3 S^2 |\tilde{A}_{\boldsymbol{k}_1}|^2}{\tau_A^2 |k_{1y}|^3} \frac{4\sqrt{\pi} o_{\boldsymbol{k}_1}^2 \bar{v}_{x\boldsymbol{k}}(0)^2}{w'(\alpha g)^{1/2}} \left\{ \underbrace{2}_{\text{old}} + \underbrace{\left(\frac{k_{1y} o_{\boldsymbol{k}_2}^2}{k_y w_{\boldsymbol{k}} w'} \right)^2}_{\text{new}} \right\} \right]^{1/3}$$

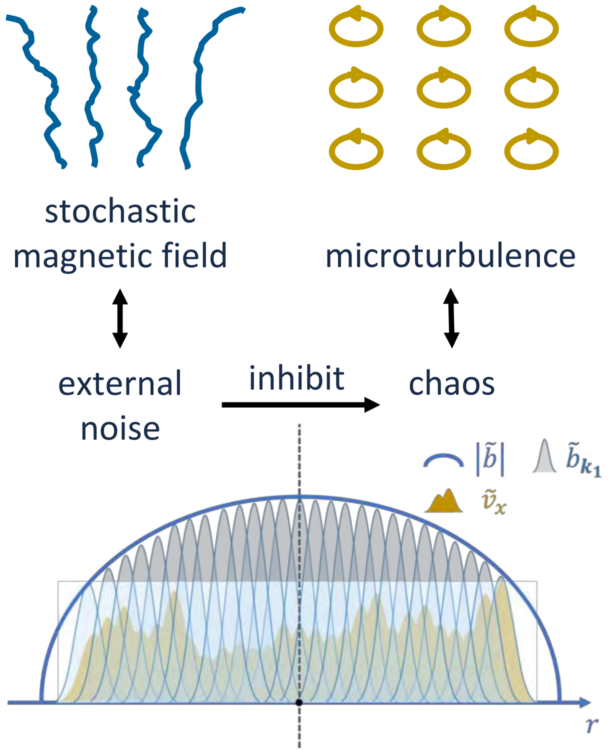
Again, new term comes from the broad radial structure of the quasi-mode

A Multi-scale Model: quasi-mode in a stochastic magnetic field

UC San Diego

Results of this model can be summarized by a flowchart





Results: lessons we learned for ballooning mode

— **microturbulence** (small-scale convective cells) is driven ⟷ $\nabla \cdot \boldsymbol{J} = 0$

→ increasing the number of triad interactions → **enhance nonlinear transfer**

→ **the increase in the bicoherence** observed in Choi's experiments

— Non-trivial correlation $\langle \tilde{\boldsymbol{b}} \tilde{v}_x \rangle$ develops

→ Not only $\langle \tilde{b}_x \tilde{v}_x \rangle$, but also $\langle \tilde{b}_y \tilde{v}_x \rangle$ → absent in prior work

→ broad mode structure

→ the microturbulence "**locks on**" to the externally prescribed $\tilde{\boldsymbol{b}}$

→ the edge plasma turbulence becomes "**noisy**"

→ the reduction in the C_{JS} in the RMP ELM suppression phase

Future: suggested experiments

—C_{JS} is somewhat abstract. Relate the reduction of the complexity to **dynamical** quantities?

Suggested experiments:

i. Use **Beam Emission Spectroscopy** (BES) velocimetry to calculate the ratio of the **turbulent heat flux** to the **total heat flux** as a function of I_{RMP}.

ii. Perform the **complexity-entropy analysis** for the data of velocity fluctuations collected from BES velocimetry during the RMP ELM suppression phase and the natural ELM-free phase.

iii. A direct examination of the presence of the correlation $\langle \tilde{v}_x \tilde{\boldsymbol{b}} \rangle$.

Directions for theoretical studies

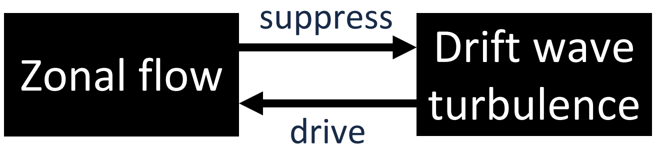
UC San Diego

— Incorporate zonal flow into our model.

velocity shear: $\frac{dk_x}{dt} = -\langle v_\theta \rangle' k_\theta$

magnetic shear: $\frac{dk_x}{dz} = -sk_\theta$

$\Bigg\} \quad \langle k_x k_\theta \rangle \neq 0, \quad \langle v_x v_\theta \rangle \neq 0$



— Study the $Ku > 1$ regime.

Quasi-linear theory is implicitly used in the calculation → $Ku > 1$. $Ku \approx (l_{ac}/l_c)^2$

Starting point → percolation theory

	medium	particle motion
percolation ($Ku > 1$)	random	deterministic
diffusion ($Ku < 1$)	fixed	stochastic

— Effects of stochastic magnetic field on blob propagation and SOL broadening.

Hint: Theory of mean E×B shear in a stochastic magnetic field[1]

1. Guo, Weixin, et al. *Plasma Physics and Controlled Fusion* 64.12 (2022): 124001.

Results: lessons we learned for ballooning mode

UC San Diego

— Stochastic magnetic field → (impede) ballooning mode

↳
1. Enhancing the effective plasma inertia (magnetic braking effect[1])
2. Reducing the effective drive → **newly** discovered
3. Promoting turbulence damping

— Microturbulence → (drive) **turbulent background**

↳ ∂_t → $\partial_t + \tilde{\boldsymbol{v}} \cdot \nabla$ → (renormalization) $\partial_t - \nu_T \nabla_\perp^2$

↳ $\nu_T^{ballooning} > \nu_T^{interchange}$ → broad mode structure

↳ electrostatic scattering → destabilizing → (weaker) magnetic braking effect

↳ opposite to conclusion for interchange mode

1. Rutherford, Paul Harding. *The Physics of Fluids* 16.11 (1973): 1903-1908.

Thank you!

Strategy: basics of the quasi-mode

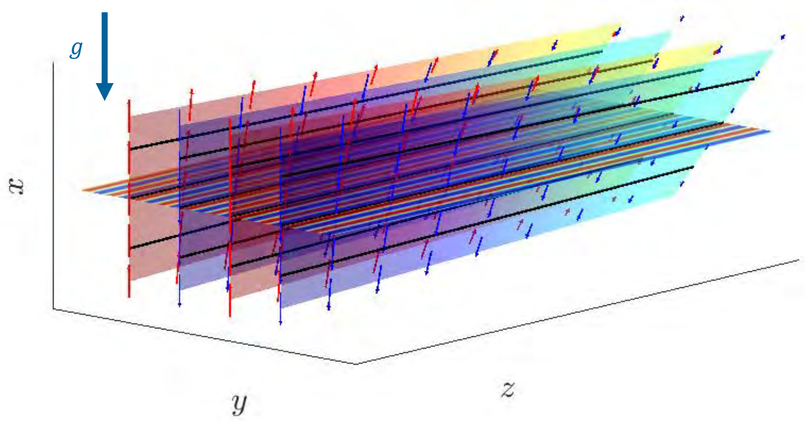
UC San Diego

1. Roberts, K. V., and J. B. Taylor. *The Physics of Fluids* 8.2 (1965): 315-322.

— Properties of the quasi-mode[1]
- ➢ Broad mode structure in the radial direction
- ➢ Finite mode length in the main field direction
- ➢ Finite, linear magnetic shear, $\boldsymbol{b_0} = (0, sx, 1)$

— Physical picture:

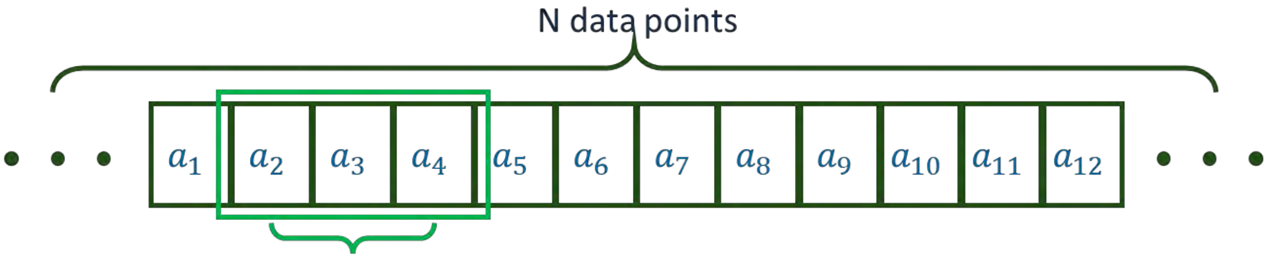
1. gravitational potential energy $\xrightarrow{\text{interchange motion}}$ kinetic energy of plasma filaments.
2. Alignment of filaments with the local magnetic field lines && finite magnetic field shear ⇒ infinite length ⇒ divergent rotational kinetic energy.
3. Mode length automatically adjusts to a finite value ⇒ resistive dissipation



— Quasi-mode: not a **true eigenmode** ⇒ a wave-packet ⇒ broad radial structure ⇒ eventually disperse

— Highly-degenerate interchange modes ⇒ Quasi-mode can maintain its shape until entering the nonlinear regime.

Jensen-Shannon Complexity



window of length d

E.g., $a_2 = 4, a_3 = 7, a_4 = 5 \Rightarrow j_2 = (1,3,2)$

$$\text{orderings} \begin{cases} j_1 = (1,2,3) \\ j_2 = (1,3,2) \\ j_3 = (2,1,3) \\ j_4 = (2,3,1) \\ j_5 = (3,1,2) \\ j_6 = (3,2,1) \end{cases}$$

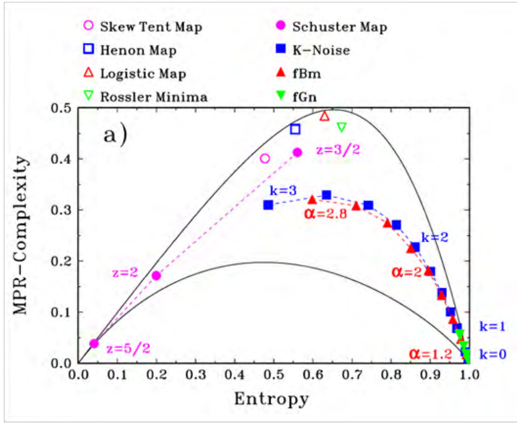
Get the probability distribution function of orderings $P = \{p_j\}_{j=1,\dots,d!}$ $(N \gg d!)$

Then [1]

$$C_{JS} = \underbrace{H}_{\text{Shannon entropy}} \times \underbrace{Q}_{\text{J-S divergence}}$$

$$H = \frac{S}{S_{max}}, S = -\sum_j p_j \ln(p_j), S_{max} = \ln(d!)$$

$$Q = Q_0 \left\{ S\left(\frac{P + P_e}{2}\right) - \frac{S(P)}{2} - \frac{S(P_e)}{2} \right\}, P_e = \left\{ p_j = \frac{1}{d!} \right\}$$



1. Rosso, O.A., et al., *Physical review letters*, *99*(15), p.154102.