Turbulence Spreading and Explicit Nonlocality*

* Submitted to PPCF

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Outline

- Prologue – What is this about and Why are we doing it?
- Analysis – Exploiting PV Evolution
  - – Dupree with a difference

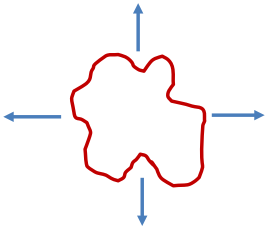
(Schematics)

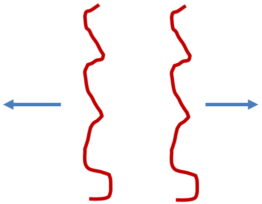
  - – PV Inversion
- Results – Intensity Equation
  - – Explicit Nonlocality – Surprise!
  - – Simulation Comparisons
- Next Step

Prologue

Turbulence Spreading

- What? – Entrainment { ‘nibbling’ ✓ ; ‘engulfment’ X }
- Advance of turbulence into region where stable or excitation absent





Turbulence

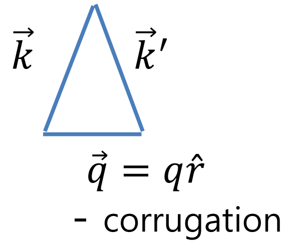
propagation

- Key component of all $k - \epsilon$ type models of inhomogeneous turbulence
- Intimately connected to avalanching

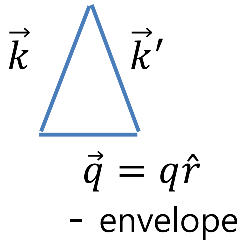
c.f. X. Garbet, et. al. ‘94

P.D., Hahm, ‘95

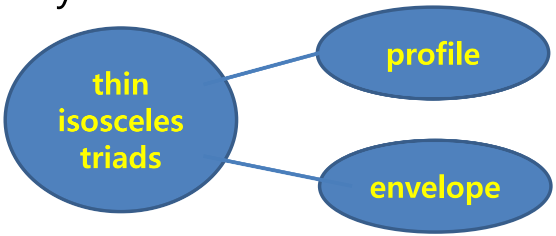
- **Turbulence Spreading vs Avalanching**
  - Both: (non-Brownian) radial propagation of excitation
  - Avalanching:
    - via overturning and mixing of neighboring cells
    - Coupling via $\nabla\langle P\rangle$
    - $\partial_t \delta P \sim \partial_x\,(\alpha \delta P^2)$ → Burgers



  - Turbulence spreading (t.s. Joint reflection symmetry)
    - via spatial scattering due nonlinear coupling
    - Couple via turbulence intensity field
    - Usually $\partial_t I \sim \partial_x(D_0 I \partial_x I)$ → $k - \epsilon$



- **Bottom Line:**
  - Very closely linked



  - ~ impossible to have one without other
  - t.s. can persist in strong driven, non-marginal regimes
  - Which effect more dramatic is variable → specifics?
  - Discussion sociological (or sociopathic)…

Review: Hahm, P.D.; J. Kor. Phys. Soc. 2018.

Model Status Quo

- Turbulence Intensity Evolution / Spreading almost always treated via $k - \epsilon$ style:

$$\frac{\partial I}{\partial t} = \frac{\partial}{\partial r}\chi(I)\frac{\partial I}{\partial r} + \gamma(r)I - \alpha I^{1+\sigma}$$

Nonlocality

- Key question: Flux-Gradient relation? Local?
- $I(x,t), \quad Q(x,t) \leftrightarrow \nabla P(x,t)$?
- Local vs "Nonlocal" dynamics

i.e. $Q = -\kappa \nabla P$ vs $Q = -\int dr' \kappa(r-r') \nabla P(r')$

$\partial_t I = \gamma I - I^2$ vs $\partial_t I = \int dr' \, \gamma(r-r') I(r')$

- Is the nonlocality <u>explicit</u> or is it due fast, local front? <u>Physics</u>?
- <u>Scale</u> of nonlocality? – i.e. $\kappa(r-r'), \gamma(r-r')$?

Nonlocality, cont'd

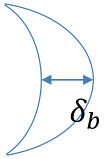
- Status Quo:
  - Dynamics are non-local, albeit only weakly so
  - Scale uncertain, physics controversial
  - No clue on explicit vs implicit

→ Confusion

Analysis

[N.B. Necessarily Schematic]

Model



- Darmet (Pellat) → Trapped Ion DW $\omega < \omega_{bi}$

$$\partial_t \bar{f} + \Omega_{\mathrm{D}}\, E\, \partial_\alpha \bar{f} - \left[J_0 \phi, \bar{f}\right] = 0$$

- Trapped ions ~ 2D (simple!)
- Precession(kinetics) resonance: $\Omega_D E \partial_\alpha$ vs ∂_t
- Poisson + Boltzmann Response

  Polarization: $\delta_b^2 \nabla^2 \phi$

- Scales: $\rho_i < \delta_b < l_r \leq l_s < L_T$

  (l_r → mode, l_s → spectral)

Moment Equations

$$\left(\partial_t + \tilde{V} \cdot \nabla + \tilde{V}_Z \cdot \nabla\right) \left[C_i \nabla^2 \tilde{\phi}\right]$$

curvature

$$= \frac{3}{2} \Omega_D \partial_\alpha \tilde{T}_i - C_e \left(\omega - \omega_E + \frac{\omega_{*n}^i}{\tau}\right) \tilde{\phi} - C_i \tilde{V}_r \partial_\chi (\nabla^2 \bar{\phi}_Z)$$

~ ala' HM, with PV conservation broken by curvature; but only <u>linearly</u>

$\partial_t (C_i \nabla^2 \bar{\phi}_Z) = C_i \delta_b^2 \partial_\chi^2 \langle \tilde{V}_\chi \tilde{V}_\alpha \rangle$ + Friction

~ N.B. Akin 2D fluid, not CHM, as usual for Z.F.

➔ ~ PV Conservation, with production via curvature ➔ generic

Where are we going?

- Seek intensity field, for spreading and flux

$$\partial_t \langle \tilde{\phi}_1 \tilde{\phi}_2 \rangle + \nu(1,2)\langle \tilde{u}_1 \tilde{u}_2 \rangle = \gamma(1,2) + \cdots$$

- But PV mixed, i.e. $\tilde{u} = C_e \tilde{\phi} - C_i \nabla^2 \tilde{\phi}$

$$(\partial_t + \tilde{v} \cdot \nabla + v_Z \cdot \nabla)\tilde{u} = -\frac{3}{2}\Omega_D \partial_y \tilde{T}_i + C_i \tilde{v}_r \partial_r (\nabla^2 \langle \phi_Z \rangle)$$

- So, form $\partial_t \langle \tilde{u}(1)\tilde{u}(2) \rangle$ $\rightarrow$ PV correlation

  and then use: $\tilde{\phi} = \int G(x,x')\tilde{u}(x')dx'$ etc.

PV Correlation

- Evolution (neglect ZF, hereafter)

$$\partial_t\langle\tilde{u}(1)\tilde{u}(2)\rangle + \partial_{r_1}\langle\tilde{v}_{r1}\tilde{u}_1\tilde{u}_2\rangle + \partial_{y_1}\langle\tilde{v}_{y_1}\tilde{u}_1\tilde{u}_2\rangle + (1 \leftrightarrow 2)$$

$$= \ -\frac{3}{2}\Omega_D\langle\tilde{u}_2\partial_{y_1}\tilde{T}(1)\rangle + C_i\partial_r(\nabla^2\langle\phi_Z\rangle)\langle\tilde{u}_2\tilde{v}_{r_1}\rangle + (1 \leftrightarrow 2)$$

(recall Ashourvan, P.D. 2016)

- Close by " 2pt" quasi-linear calculation (Dupree, '72)
- As intensity field of interest (i.e. envelope), expect take $r_- \rightarrow 0$, calculate

  $\langle\tilde{u}(1)\tilde{u}(2)\rangle = F(r_+, y_-)$. Departs Dupree '72

PV Correlation, cont'd

$$\partial_t \langle \tilde{u}_1 \tilde{u}_2 \rangle + T_r(1,2) + T_{y_-}(1,2)$$

$$= -\frac{3}{2}\Omega_D\left(\langle \tilde{u}_2 \partial_{y_1} \tilde{T}(1) \rangle\right) + C_i \partial_r (\nabla^2 \langle \phi_Z \rangle) \langle \tilde{v}_r(1) \tilde{u}_2 \rangle + (1 \leftrightarrow 2)$$

$$T_r(1,2) = -\left(\partial_{r_1} D_{r_1,r_1} \partial_{r_1} + \partial_{r_1} D_{r_1,r_2} \partial_{r_2}\right) \langle \tilde{u}(1) \tilde{u}(2) \rangle + (1 \leftrightarrow 2)$$

→ radial scattering, focus on centroid r_+ evolution

$$T_{y_-}(1,2) = -\frac{1}{4}\,\partial_{y_-}\left(D_{y_1,y_1} + D_{y_2,y_2} - D_{y_1,y_2} - D_{y_2,y_1}\right)\partial_{y_-} \langle \tilde{u}(1) \tilde{u}(2) \rangle$$

N.B. Can average over y_+ !

From PV to Potential

- To derive potential correlation function evolution,

$$\int dx_1' \int dx_2' \; G(x_1, x_1') G(x_2, x_2') \; * [PV \; eqn.]$$

$$G(x, x') = \frac{\sqrt{A}}{2} \exp\left[-\sqrt{A} |x - x'| \right]$$

$$A \;\; = \frac{C_e}{C_i} \, \delta_b^{-2} = \frac{\tau}{\sqrt{2\epsilon}} \, \delta_b^{-2}$$

→ Banana width is scale of inversion, scale of “nonlocality”

- Ultimate spectral equation will involve $G(x, x')$ with PV correlation evolution
- Banana ↔ scale → quite modest nonlocality

From PV to Potential, cont'd

- Potential

Spreading, convolved

$$\partial_t\langle\tilde{\phi}^2\rangle = \frac{\sqrt{A}}{2}\int e^{-\sqrt{A}|r-r'|}\,\partial_{r'}\left[2D_0\langle\tilde{\phi}^2\rangle\partial_{r'}\left(\langle\tilde{\phi}^2\rangle - \frac{\delta_b^2}{2}\,\partial_{r'}^2\langle\tilde{\phi}^2\rangle\right)\right]$$

$$+\frac{1}{2}\,\partial_{y\_}D_{y\_,y\_}\partial_{y\_}\langle\tilde{\phi}^2\rangle - 3\Omega_D\frac{\sqrt{A}}{2}\int e^{-\sqrt{A}|r-r'|}\,\langle\tilde{v}_r\tilde{T}\rangle dr'$$

Relative diffusion in y
→ small scale decay

de-localized growth

- $\delta_b \to 0$

$$\partial_t\langle\tilde{\phi}^2\rangle = \partial_r\,D_{r,r}\,\partial_r\langle\tilde{\phi}^2\rangle - 3\,\Omega_D\,\langle\tilde{v}_r\tilde{T}\rangle + \frac{1}{2}\,\partial_{y\_}D_{y\_,y\_}\partial_{y\_}\langle\tilde{\phi}^2\rangle$$

- Heuristics (usual)

$$\partial_t\,\varepsilon = \partial_x\left[(D_0\varepsilon)\,\partial_x\varepsilon\,\right] - \gamma_{NL}\,\varepsilon^2 + \gamma\varepsilon$$

From PV to Potential, cont'd

- The Answer, of sorts:

$$\partial_t\langle\tilde{\phi}^2\rangle = G * \left\{\frac{\partial}{\partial r}\left[2D_0\langle\tilde{\phi}^2\rangle\frac{\partial}{\partial r}\left(\langle\tilde{\phi}^2\rangle - \frac{\delta_b^2}{2}\frac{\partial^2}{\partial r^2}\langle\tilde{\phi}^2\rangle\right)\right]\right\} + G * \left\{\left(\gamma_L(r)\langle\tilde{\phi}^2\rangle\right)\right\} - \frac{D_0}{l_r^2}\langle\tilde{\phi}^2\rangle^2$$

$G * \{\ \}$ → convolution

from $\langle\tilde{v}_\theta^2\rangle$

- Ingredients:
  - Non-local Growth: $G * \{\gamma_L(r)\langle\tilde{\phi}^2\rangle\}$
  - Non-local Nonlinear Diffusion: $G * \left\{\frac{\partial}{\partial r}\left[2D_0\langle\tilde{\phi}^2\rangle\frac{\partial}{\partial r}\left(\langle\tilde{\phi}^2\rangle - \frac{\delta_b^2}{2}\frac{\partial^2}{\partial r^2}\langle\tilde{\phi}^2\rangle\right)\right]\right\}$

Results

Results – Model

- Intensity Eqn. $\leftrightarrow$ Delocalized nonlinear Fisher Eqn. (Hahm, P.D., Gurcan, …)

$$\partial_t \langle \tilde{\phi}^2 \rangle = G * [\text{NL Diffusion}] + G * \{\gamma_L(r) \langle \tilde{\phi}^2 \rangle\} - \frac{D_0}{l_r^2} \langle \tilde{\phi}^2 \rangle^2$$

Nonlocal Growth

- What's New and Important?
  - Nonlocal Growth !
  - Explicit Nonlocality
  - Range: δ_b (modest)

PV inversion is origin of explicit non-locality

Results, cont'd

- Heuristics of Waltz, Candy ('05) on utility of non-local growth model vindicated
- N.B. Here: Physics, Scale determined
- Non-local growth is dominant effect
- NLG accelerates:
  - Front propagation
  - Penetration of stable region from unstable reservoir (connects stable, unstable)
- Speed fits Fisher, with δ_{b*} correction

Bigger Picture / Conclusion

- Yet another example of utility of PV …
- Nonlocality is explicit
- Nonlocality exists, but is modest

→ Appears conventional wisdom of 'weakly non-local' dynamics on target

→ But now have Physics mechanism and scale / range

→ N.B. : Weakly non-local $\neq$ Q.L. !

Intensity evolution is crucial constituent for calculating fluxes

Ongoing and Future Plans

- Dispense with $\langle \phi\phi \rangle$, calculate flux evolution, i.e. $\partial_t \langle \tilde{v}_r \tilde{T} \rangle$ ...

  ➔ jams

  Ongoing

- Nonlocality scale of δ_b suggests that explicit nonlocality <u>much</u> stronger for energetic particles

  ➔ EPM spreading fertile territory !?

  ➔ Models for burning plasma ?!