Interfaces as Transport Barriers in 2D MHD

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KITP: Interfaces Working Group

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Ref: Phys. Rev. E Rap. Comm. 99, 041201 (2019)
+ references therein

→ Continues Wednesday discussion…

Physics: Active Scalar Transport

- Magnetic diffusion, ψ transport are cases of active scalar transport
- (Focus: 2D MHD) (Cattaneo, Vainshtein ’92, Gruzinov, P. D. ’94, ’95)

scalar mixing – the usual

$$\partial_t A + \nabla\phi \times \hat{z} \cdot \nabla A = \eta \nabla^2 A$$

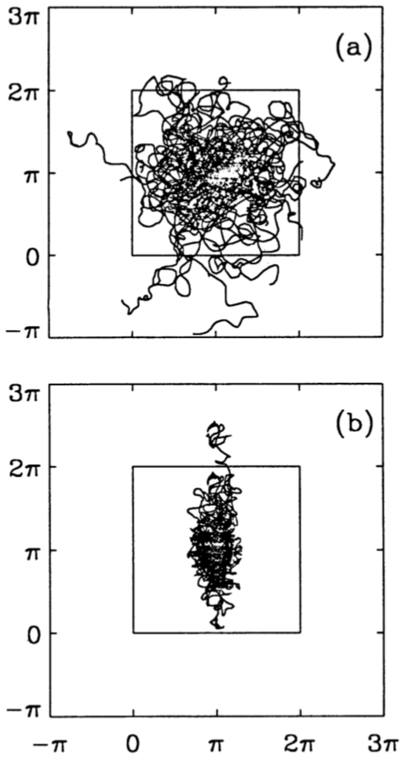
$$\partial_t \nabla^2 \phi + \nabla\phi \times \hat{z} \cdot \nabla\nabla^2 \phi = \nabla A \times \hat{z} \cdot \nabla\nabla^2 A + \nu \nabla^2 \nabla^2 \phi$$

turbulent resistivity

back-reaction

- Seek $\langle v_x A \rangle = -D_T \frac{\partial \langle A \rangle}{\partial x} - \eta \frac{\partial \langle A \rangle}{\partial x}$
- Point: $D_T \neq \sum_{\vec{k}} |v_{\vec{k}}|^2 \, \tau^K_{\vec{k}}$, often substantially less
- Why: Memory! ↔ Freezing-in
- Cross Phase

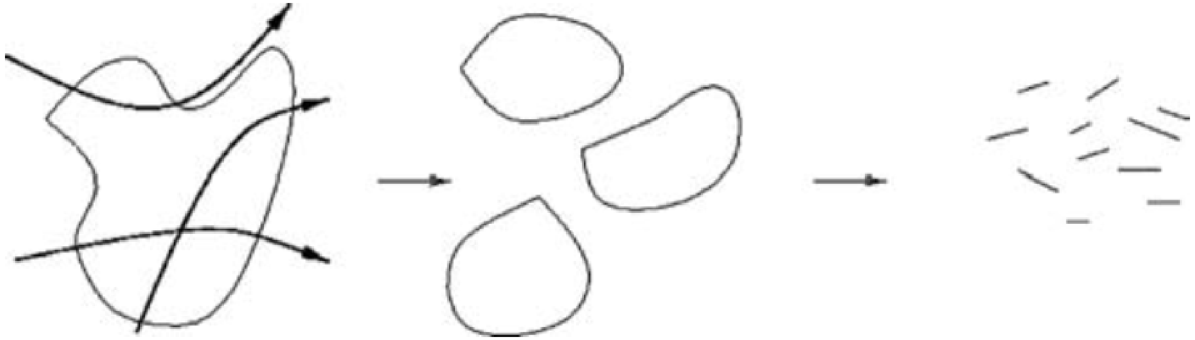
Conventional Wisdom



- [Cattaneo and Vainshtein 1991]: turbulent transport is suppressed even for a <u>weak</u> large scale magnetic field is present.
- Starting point: $\partial_t \langle A^2 \rangle = -2\eta \langle B^2 \rangle$
- Assumptions:
  - Energy equipartition: $\frac{1}{\mu_0 \rho} \langle B^2 \rangle \sim \langle v^2 \rangle$
  - Average B can be estimated by: $|\langle \mathbf{B} \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$
- Define Mach number as: $M^2 = \langle v_A \rangle^2 / \langle \tilde{v}^2 \rangle = \langle v^2 \rangle / v_A^2 = \langle v^2 \rangle / \frac{1}{\mu_0 \rho} \langle B^2 \rangle$
- Result for suppression stage: $\eta_T \sim \eta M^2$
- Fit together with kinematic stage result: $\boxed{\eta_T \sim \frac{ul}{1 + \mathrm{Rm}/M^2}}$
- Lack physics interpretation of η_T !

Origin of Memory?

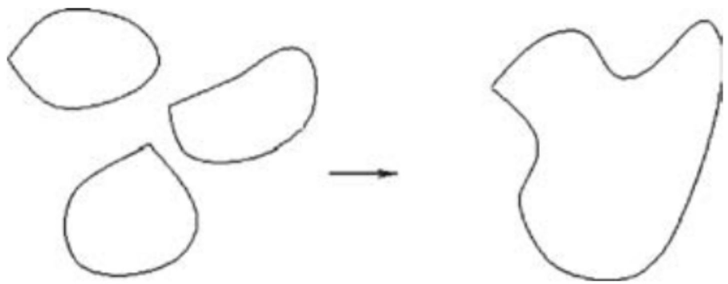
- (a) flux advection vs flux coalescence
  - intrinsic to 2D MHD (and CHNS)
  - rooted in <u>inverse cascade</u> of $\langle A^2 \rangle$ - dual cascades
- (b) tendency of (even weak) <u>*mean*</u> magnetic field to "Alfvenize" turbulence [cf: vortex disruption feedback threshold!]
- Re (a): Basic physics of 2D MHD



Forward transfer: fluid eddies chop up scalar A.

Memory Cont'd

- v.s.



Inverse transfer: current filaments and A-blobs attract and coagulate.

- Obvious analogy: straining vs coalescence; CHNS
- Upshot: closure calculation yields:

$$\Gamma_A = -\sum_{\vec{k}'}[\tau_c^{\phi}\langle v^2\rangle_{\vec{k}'} - \tau_c^{A}\langle B^2\rangle_{\vec{k}'}]\frac{\partial\langle A\rangle}{\partial x} - \sum_{\vec{k}'}\tau_c^{A}\langle A^2\rangle_{\vec{k}'}\frac{\partial\langle J\rangle}{\partial x}$$

flux of potential

competition

hyper-resistivity

scalar advection vs. coalescence ("negative resistivity")

(+) (-)

N.B.: Coalescence →Negative diffusion

Hyper-resistivity → $\langle A^2\rangle$ conservation

cf: Pouquet '78, DHK Durham Volume 2005

Conventional Wisdom, Cont'd

- Then calculate $\langle B^2\rangle$ in terms of $\langle v^2\rangle$. From:

$$\partial_t A + \mathbf{v}\cdot\nabla A = -v_x \frac{\partial\langle A\rangle}{\partial x} + \eta\nabla^2 A$$

- Multiplying by A and sum over all modes:

$$\frac{1}{2}[\cancel{\partial_t\langle A^2\rangle} + \cancel{\langle\nabla\cdot(\mathbf{v}A^2)\rangle}] = -\Gamma_A \frac{\partial\langle A\rangle}{\partial x} - \eta\langle B^2\rangle$$

Dropped stationary case

Dropped periodic boundary → introduce nonlocality?!

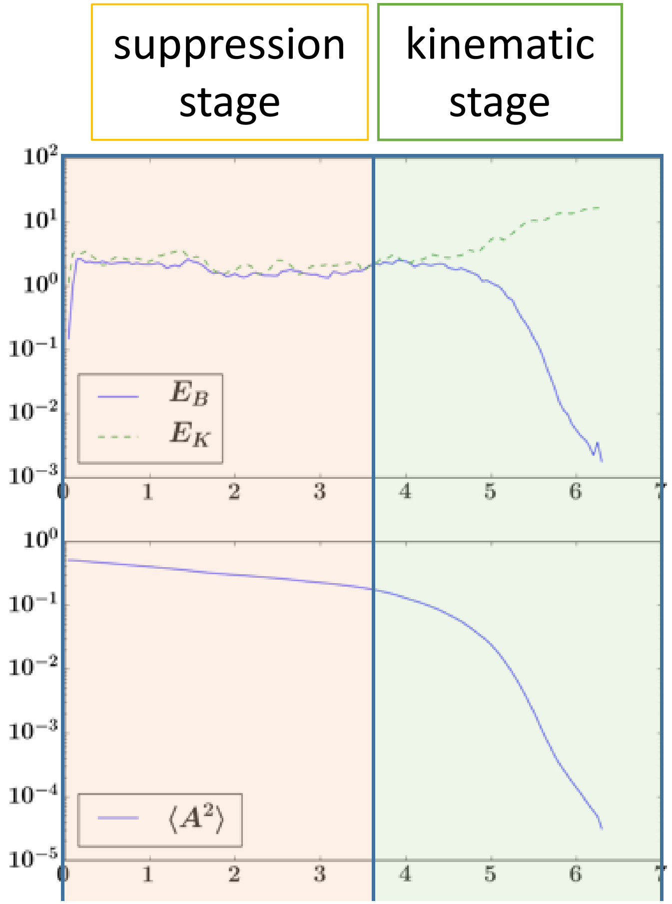
- Therefore: $$\langle B^2\rangle = -\frac{\Gamma_A}{\eta}\frac{\partial\langle A\rangle}{\partial x} = \frac{\eta_T}{\eta}B_0^2$$
- Define Mach number as: $$M^2 \equiv \langle v^2\rangle/v_{A0}^2 = \langle v^2\rangle/(\frac{1}{\mu_0\rho}B_0^2)$$
- Result: $$\boxed{\eta_T = \frac{\sum_{\mathbf{k}}\tau_c\langle v^2\rangle_{\mathbf{k}}}{1+\mathrm{Rm}/M^2} = \frac{ul}{1+\mathrm{Rm}/M^2}}$$
- This theory is not able to describe $B_0 \to 0$, though may be extended (?!)

Is this story “the truth, the whole truth and nothing but the truth’?

➔ A Closer Look

Two Stage Evolution:

- 1. The suppression stage: the (large scale) magnetic field is sufficiently strong so that the diffusion is suppressed.
- 2. The kinematic decay stage: the magnetic field is dissipated so the diffusion rate returns to the kinematic rate.
- Suppression is due to the memory induced by the magnetic field.

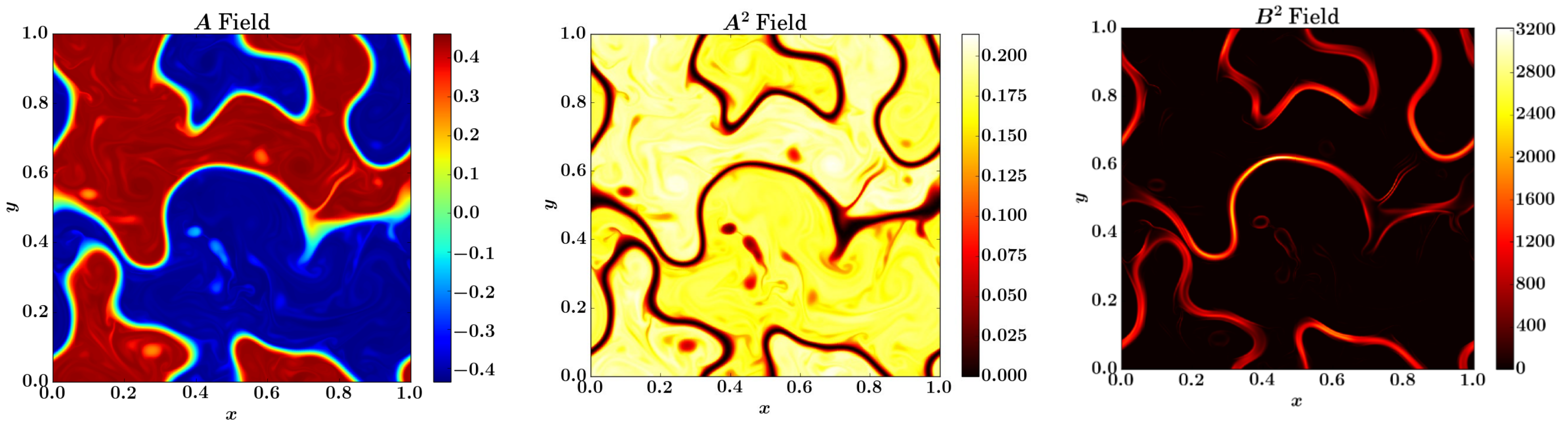


New Observations

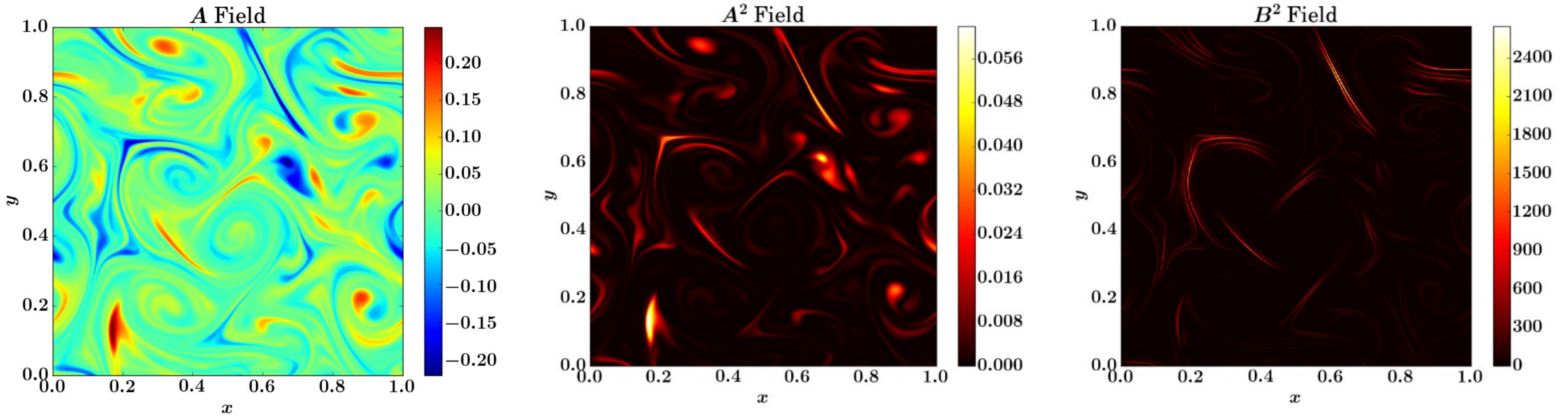
- With no imposed B_0, in suppression stage:

Field

Concentrated!



- Same run, in kinematic stage (trivial):



New Observations Cont’d

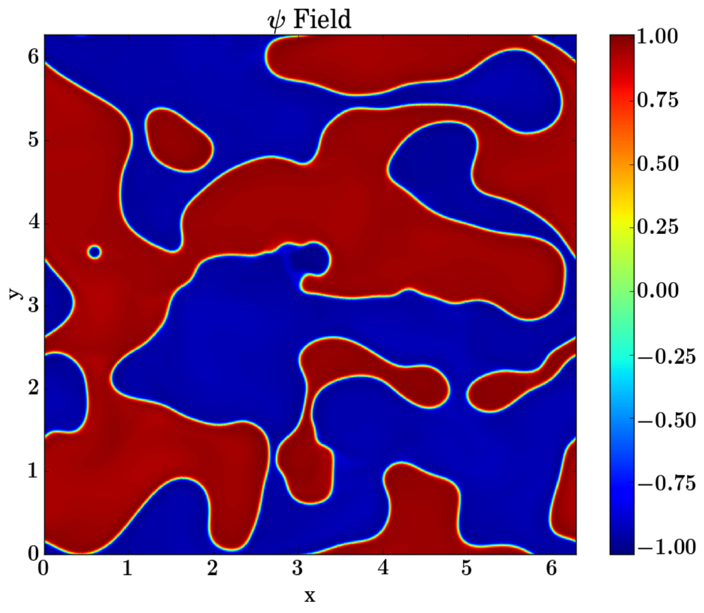
- Nontrivial structure formed in real space during the suppression stage.
- A field is evidently composed of “<u>blobs</u>”.
- The low A^2 regions are 1-dimensional.
- The high B^2 regions are strongly correlated with low A^2 regions, and also are 1-dimensional.
- We call these 1-dimensional high B^2 regions ``<u>barriers</u>'', because these are the regions where mixing is reduced, relative to η_K.

➔ Story one of ‘blobs and barriers’

2D CHNS and 2D MHD

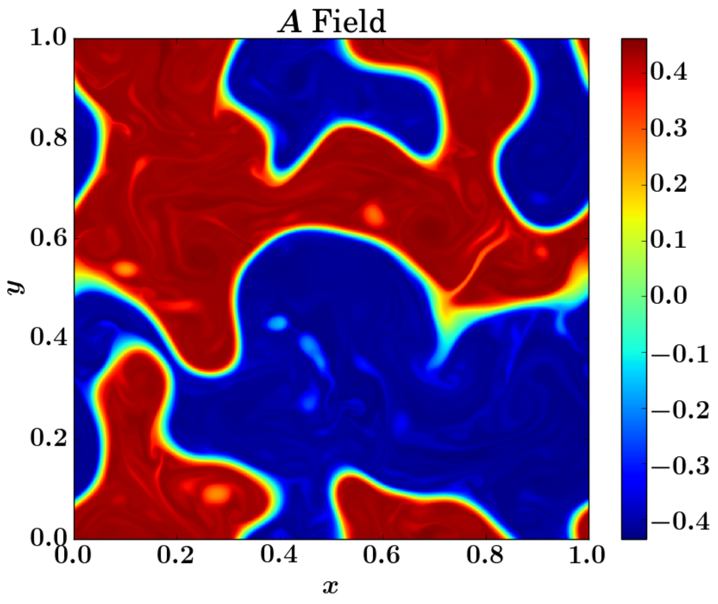
- The A field in 2D MHD in suppression stage is strikingly similar to the ψ field in 2D CHNS (Cahn-Hilliard Navier-Stokes) system:

ψ field in 2D CHNS



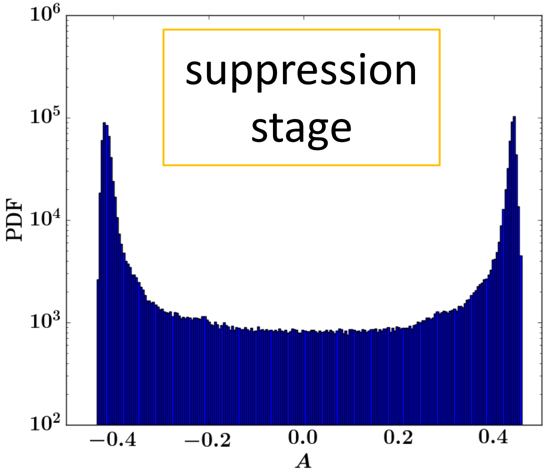
v.s.

A field in 2D MHD

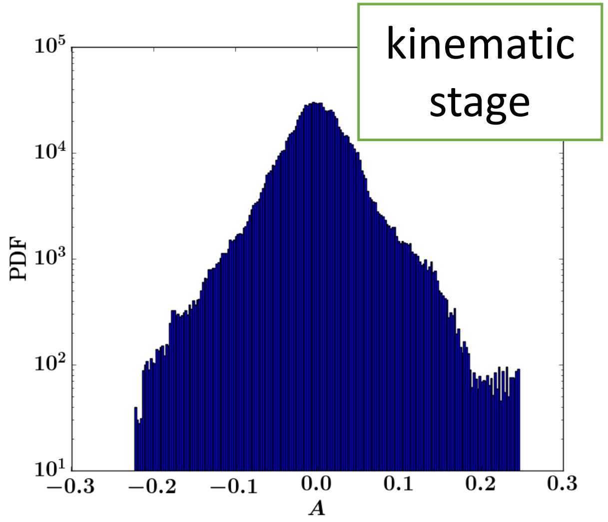


Evolution of PDF of A

- Probability Density Function (PDF) in two stage:

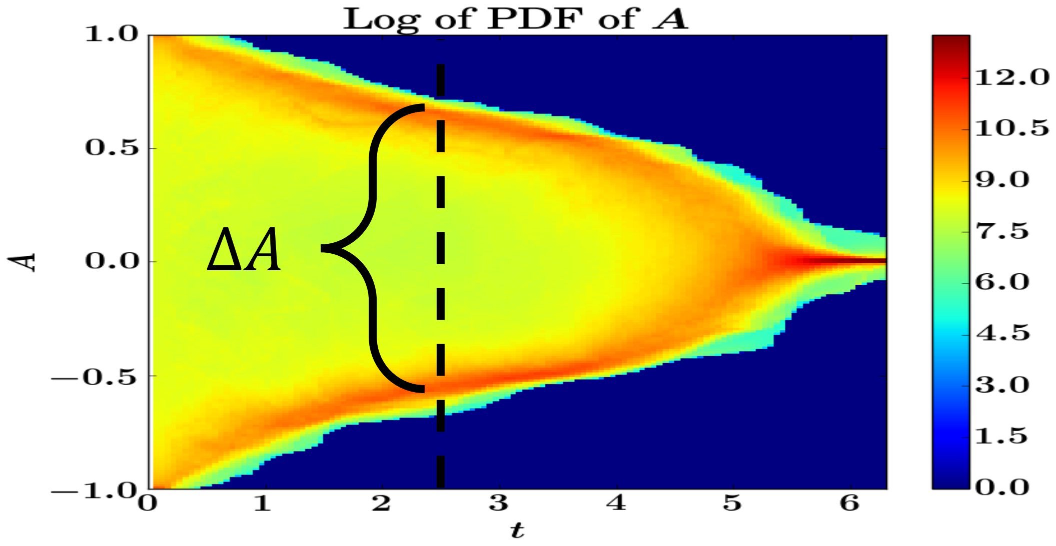


suppression stage



kinematic stage

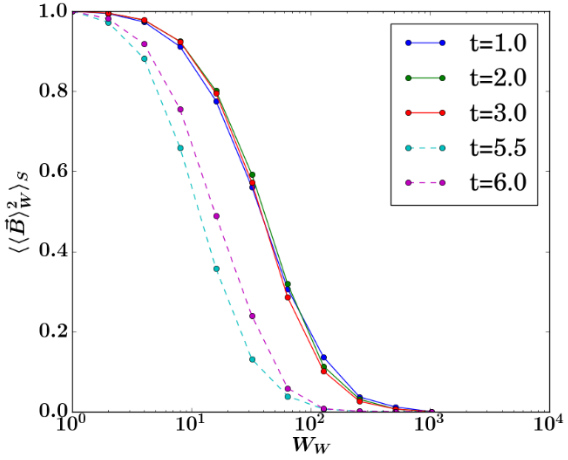
- Time evolution: horizontal “Y”.
  - The PDF changes from double peak to single peak as the system evolves from the suppression stage to the kinematic stage.

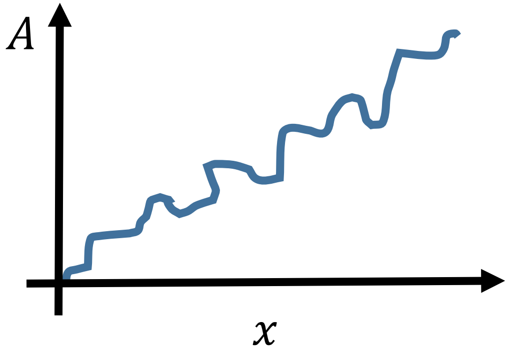


Revisiting Quenching

The problem of the mean field $\langle \boldsymbol{B} \rangle$
→ What does mean mean? – Question of the ages…

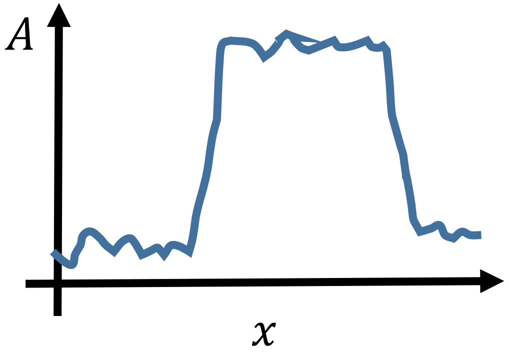
- $\langle B \rangle$ depends on the averaging window.
- With no imposed external field, B is highly intermittent, therefore the $\langle B \rangle$ is not well defined.





$|\langle B \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$ ✓

v.s.



$\langle B \rangle$ not well defined

Reality

New Understanding

- Summary of important length scales: $l < L_{stir} < L_{env} < L_0$
  - System size L_0
  - Envelope size L_{env} → emergent (blob)
  - Stirring length scale L_{stir}
  - Turbulence length scale l, here we use Taylor microscale λ
  - Barrier width W → emergent
- Quench is not uniform. Transport coefficients differ in different regions. Differs from uniform B_0 studies
- In the regions where magnetic fields are strong, Rm/M^2 is dominant. They are regions of ***barriers***.
- In other regions, i.e. Inside blobs, Rm/M'^2 is what remains. $M'^2 \equiv \langle V^2 \rangle / \left(\frac{1}{\rho} \langle A^2 \rangle / L_{env}^2 \right)$

New Understanding, cont’d

- From $\partial_t\langle A^2\rangle = -\langle \mathbf{v}A\rangle\cdot\nabla\langle A\rangle - \nabla\cdot\langle \mathbf{v}A^2\rangle - \eta\langle B^2\rangle$
- Retain 2nd term on RHS. Average taken over an envelope/blob scale.
- Define diffusion (closure):

$$\langle \mathbf{v}A\rangle = -\eta_{T1}\nabla\langle A\rangle$$
$$\langle \mathbf{v}A^2\rangle = -\eta_{T2}\nabla\langle A^2\rangle$$

- Plugging in: $\partial_t\langle A^2\rangle = \eta_{T1}(\nabla\langle A\rangle)^2 + \nabla\eta_{T2}\cdot\nabla\langle A^2\rangle - \eta\langle B^2\rangle$
- For simplicity: $\langle B^2\rangle \sim \frac{\eta_T}{\eta}(\langle B\rangle^2 + \langle A^2\rangle/L_{env}^2)$
- where L_{env} is the envelope size. Scale of $\nabla^2\langle A^2\rangle$.
- Define new strength parameter: $M'^2 \equiv \langle v^2\rangle/(\frac{1}{\mu_0\rho}\langle A^2\rangle/L_{env}^2)$
- Result: $\boxed{\eta_T = \frac{ul}{1+\mathrm{Rm}/M^2+\mathrm{Rm}/M'^2} = \frac{ul}{1+\mathrm{Rm}\frac{1}{\mu_0\rho}\langle \mathbf{B}\rangle^2/\langle v^2\rangle + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle A^2\rangle/L_{env}^2\langle v^2\rangle}}$

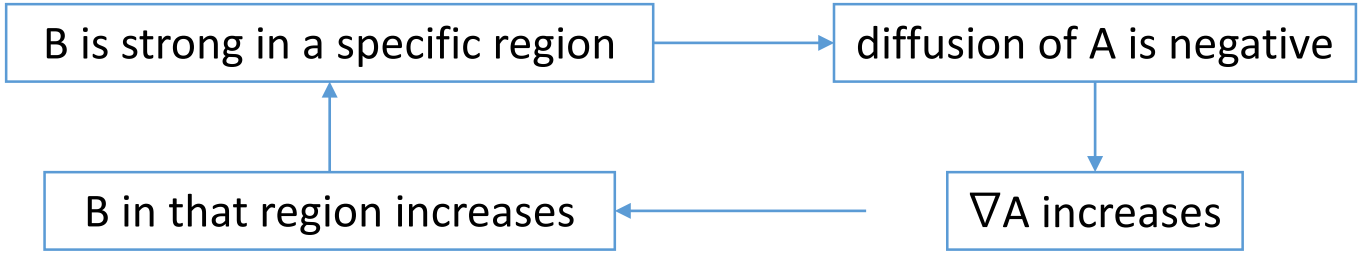
Formation of Barriers/Interfaces

- How do the barriers form?

flux coalescence

$$\eta_T = \sum_{\mathbf{k}} \tau_c[\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\langle B^2\rangle_{\mathbf{k}}]$$

- From above, strong B regions can support negative incremental $\eta_T = \delta\Gamma_A/\delta(-\nabla A) < 0$, suggesting clustering
- $\langle\eta_T\rangle > 0$
- Positive feedback: a twist on a familiar theme



B is strong in a specific region

diffusion of A is negative

∇A increases

B in that region increases

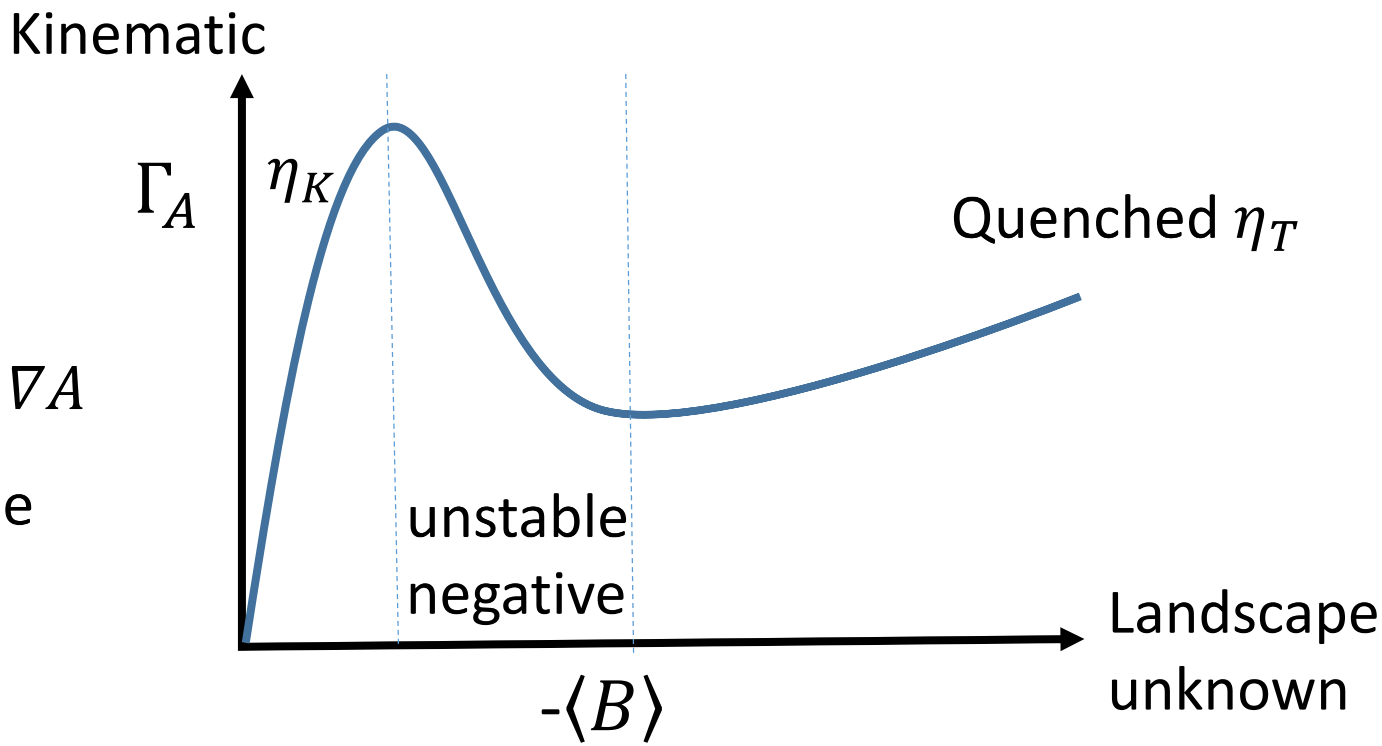
Formation of Barriers, Cont'd

- Negative resistivity leads to barrier formation.
- The S-curve reflects the dependence of Γ_A on B.
- When slope is negative → negative (incremental) resistivity.

Barriers ←→

Bistability of Γ_A vs ∇A

→ a familiar theme



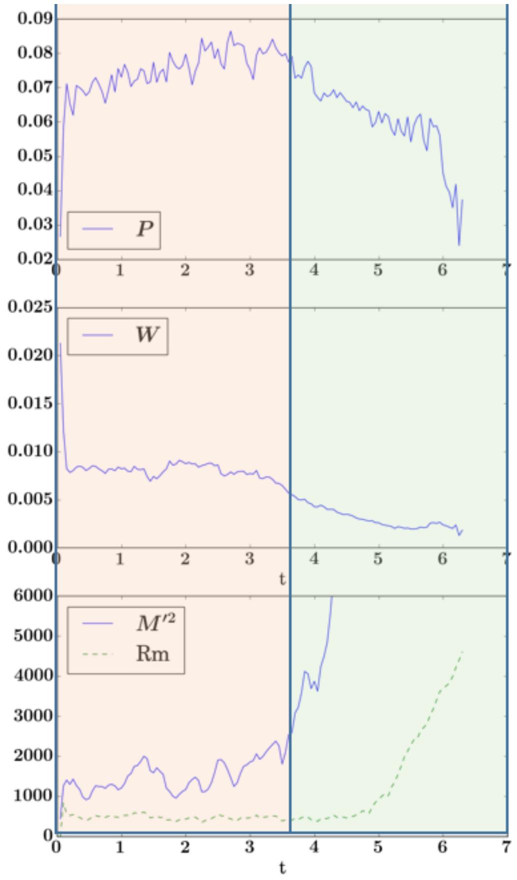
Describing the Barriers

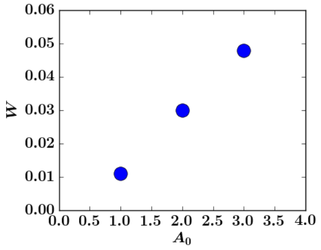
- How to measure the barrier width W ?
- Starting point: $W \sim \Delta A / B_b$
- Use $\sqrt{\langle A^2 \rangle}$ to calculate ΔA
- Define the barrier regions as: $B(x,y) > \sqrt{\langle B^2 \rangle} * \overset{\text{threshold}}{\overset{\downarrow}{2}}$
- Define barrier packing fraction: $P \equiv \dfrac{\#\text{ of grid points for barrier regions}}{\#\text{ of total grid points}}$
- Use use the magnetic fields in the barrier regions to calculate the magnetic energy: $\boxed{\sum_{\text{barriers}} B_b^2 \sim \sum_{\text{system}} B^2}$
- Thus $\langle B_b^2 \rangle \sim \langle B^2 \rangle / P$
- So barrier width can be estimated by: $\boxed{W^2 \equiv \langle A^2 \rangle / (\langle B^2 \rangle / P)}$

→ N.B. All magnetic energy in the barriers

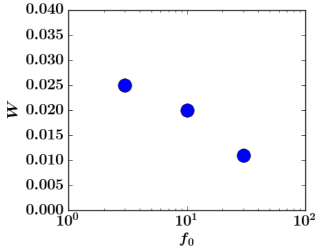
Describing the Barriers

- Time evolution of P and W:
  - P, W collapse in decay phase
  - M' rises
- Sensitivity of W:
  - A_0 or $1/\mu_0\rho$ greater → W greater;
  - f_0 greater, W smaller; (ala' Hinze)
  - W not sensitive to η or ν.





(a)



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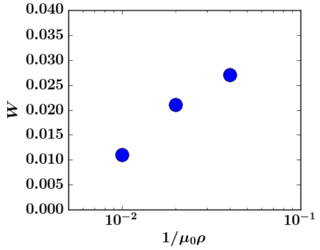

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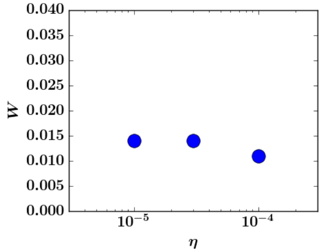

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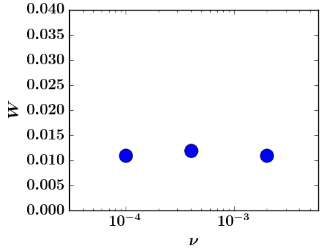
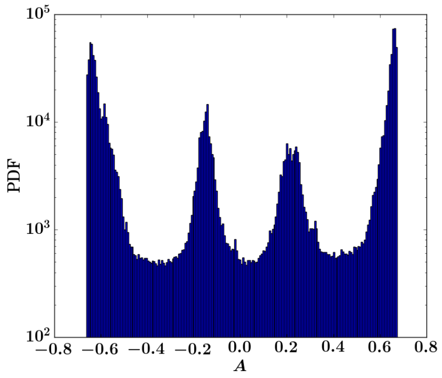


(e)

‘Staircase’ (inhomogeneous Mixing, Bistability)

- ‘Staircases’ emerge spontaneously! - Barriers
- Initial condition is the usual cos function (bimodal)
- The only major sensitive parameter (from runs above) is the forcing scale is k=32 (for all runs above k=5).
- Resembles the “staircase” in MFE.



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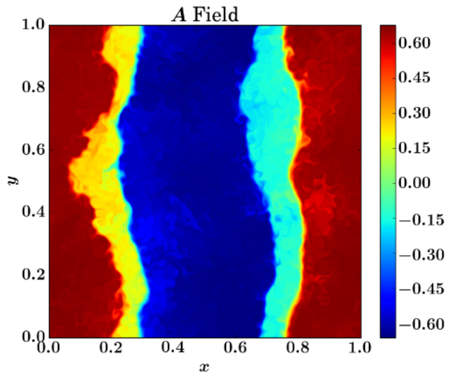

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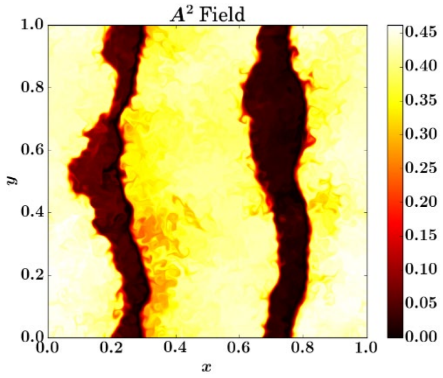

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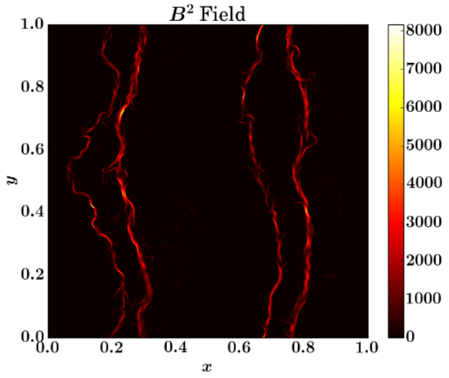


(4)

Conclusions / Summary

- Magnetic fields suppress turbulent diffusion in 2D MHD by: formation of intermittent ***<u>transport barriers</u>***.
- Magnetic structures:
  - Barriers – thin, 1D strong field regions
  - Blobs – 2D, weak field regions

ala' CHNS

- Quench not uniform:

"Inhomogeneous Mixing"

$$\eta_T = \frac{ul}{1+\mathrm{Rm}\frac{1}{\mu_0\rho}\langle\mathbf{B}\rangle^2/\langle v^2\rangle + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle A^2\rangle/L_{env}^2\langle v^2\rangle}$$

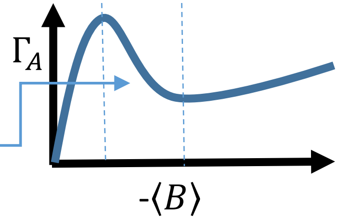
barriers, strong B

blobs, weak B, $\nabla^2\langle A^2\rangle$ remains

- Barriers form due to negative resistivity:

$$\eta_T = \sum_{\mathbf{k}} \tau_c[\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\langle B^2\rangle_{\mathbf{k}}]$$

flux coalescence



- Formation of "magnetic staircases" observed for some stirring scale

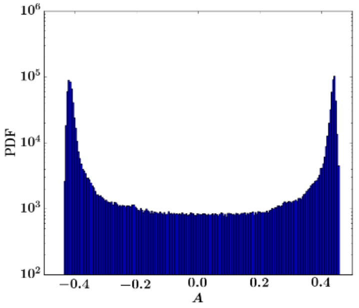
Possible Future Work

- Extension of the transport study in MHD:
  - Numerical tests of the new η_T expression ?
  - What determines the barrier width and packing fraction ?
  - Why does layering appear when the forcing scale is small ?
  - What determines the step width, in the case of layering
  - The transport study may also be extended to 3D MHD ($\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$ important instead of $\langle A^2 \rangle$)
- Other similar systems can also be studied in this spirit. e.g. Oldroyd-B model for polymer solutions. (drag reduction)
- Reduced Model of Magnetic Staircase

Back-Up

Unimodal Initial Condition

- One may question whether the bimodal PDF feature is purely due to the initial condition. The answer is <u>No</u>.
- Two non-zero peaks in PDF of A still arise, even if the initial condition is unimodal.



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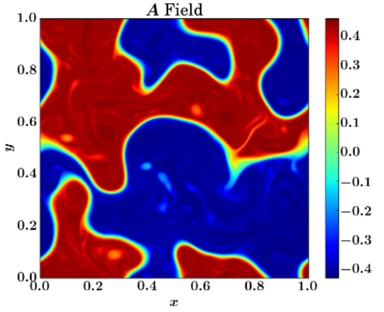
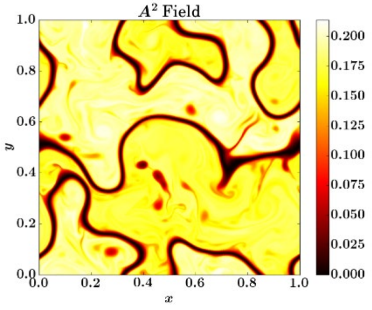
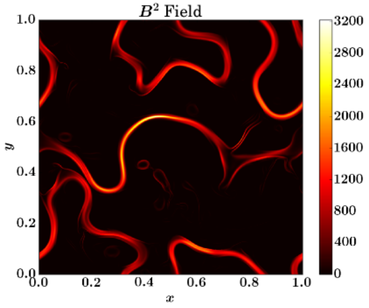


(a2)



(a3)



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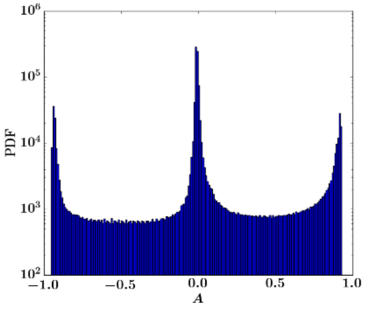
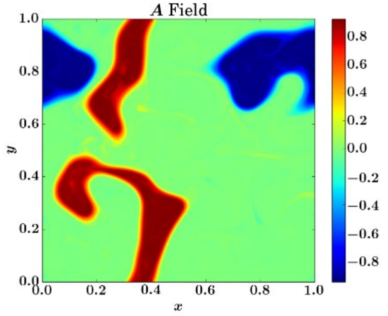


(b1)



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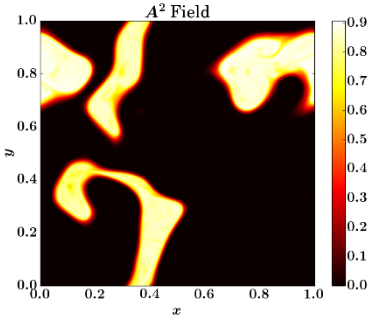
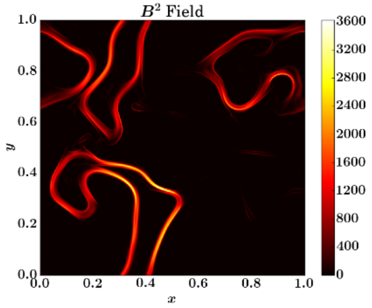


(b3)



(b4)

$$\eta_T \; = V\, l \,/\left[1 + \frac{R_m}{M^2} + \frac{R_m}{M'^2}\right]$$

- Barriers:

Strong field

$$\eta_T \approx V\, l \,/\left[1 + R_m \frac{\langle B \rangle^2}{\rho \langle \tilde{V}^2 \rangle}\right]$$

- Blobs:

Weak effective field

$$\eta_T \approx V\, l \,/\left[1 + R_m \frac{\langle A^2 \rangle}{\rho L_{env}^2 \, \langle \tilde{V}^2 \rangle}\right]$$

- Quench stronger in barriers, ,non-uniform