Transport Barrier Working Group: Summary

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KITP Staircase ‘21

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Activities:

- Formation: Dritschel, Garbet, Tobias
- Mechanisms: Guo, Frishman, Sarazin
- Symmetry Breaking: Cope, Gurcan, Knobloch
- Momentum Transport: Hughes, Galperin, Heinonen

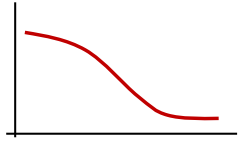
→ Thanks to all participants for stimulating and spirited discussions !

Outline

- Barriers and Layering: What and Why?
- How Barriers Form?
- Models and Scales
- Methods (How Calculate?) :
  - Quasilinear + ?
  - Quasiparticles / Wave kinetics
  - BLY / Mixing Length
  - Envelope
  - ML
- Dynamics
- Some Perspective

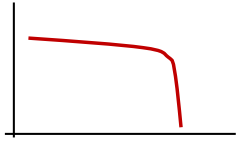
Barriers: What? , Why?

- Barrier:
  - Region, width $\Delta > \Delta_c$ where turbulent on wave transport markedly reduced
  - Usually accompanied by profile steepening
- Barrier ↔ Base state suggests:
  - Bistability – 2 States { normal / barrier } ↔ "Transport Bifurcation"



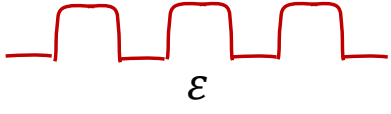
- An example: L→H Transition (Schmitz)

$$\Delta T \,/\, \Delta r \sim \text{1 kV/cm}$$



- Barriers an essential element of layering

i.e.

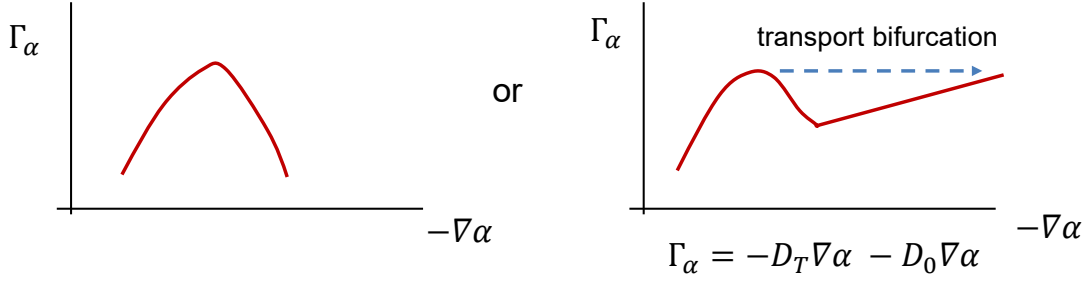


Barriers

Separate layers

Barriers: Why?

- Usually related to Γ_α vs $\nabla\alpha$ roll-over ➔ negative incremental diffusivity



- Roll-over How ? → feedback of self-sharpened order parameter on flux Γ_α i.e. $D_T \downarrow$ as $\nabla_\alpha \uparrow$
- Classic: QG turbulence + jet (cast of thousands)
  - ➔ "Predator – Prey" structure
  - ➔ Multiple fixed points, transitions

Barriers: How?

- Fundamentals (Dritschel, Frishman, Sarazin, Hughes) – mostly Jets...
- Irreversible nonlinear Rossby wave breaking for “inhomogeneous PV mixing”
- Shear induces wave breaking
  - Contour roll-up
  - ~ universal mechanism

leads to:

- Self-sharpening ➔ barrier formation
- Feedback:
  - wave breaking
  - bunching const PV contours ➔ sharper gradients
  - breaking easier

Fundamentals, cont'd

- PV gradients – following expulsion (Falkovich, Shraiman)

  – can 'protect' against shear erosion (Rossby Wave Elasticity)

  $\nabla q \geq \Lambda$ $\qquad\qquad$ $\omega = k_x \nabla \langle q \rangle \, / \, k^2$

- Processes: Scale + Space ("Inverse cascade" + beyond)

Angular momentum balance → $\frac{1}{r}\,\partial_r r\,[r\langle UV\rangle] + \alpha r U = 0$

Energy balance

+

Fluctuations:

ϵ → fluctuations → mean → disspn α

➔ spatial flux ? – yes!

$\tau_{ij}\partial_i u_j$ → Reynolds work

$\tau_{ij} = r\langle UV\rangle$ angualr momentum flux

$\frac{1}{r}\,\partial_r r[\langle VP\rangle] + \langle\frac{V(V^2+U^2)}{2}\rangle + r\langle UV\rangle\partial_r\frac{U}{r} = \epsilon$

Symmetries critical → drop in QL

spatial flux ←→ Reynolds work connection

Fundamentals - Phase Evolution

- $\langle V_r n \rangle$ → intensity, cross-phase set flux

  → Barrier via phase decorrelation at fixed fluctuation level ?!

  ➔ Connects to phase dynamics → thriving industry

- ExB flow → $\partial_r \langle UV \rangle = I\partial_x^2 \phi + I' k_y \partial_x \phi \cdots$

- Phase curvature can trigger flows, initiate barriers in turbulence with homogeneous intensity

- Observed in PPT device at PKU

(Sarazin)

Fundamentals - Phase Evolution, cont'd

- Developing interest in β −plane MHD jets and tachocline momentum transport
- $Z = R_m V_{A_0}^2 / \langle \tilde{V}^2 \rangle$ as key parameter (TDH, CD) → Magnetic field effects
- $P_m > P_{m_{crit}}$ for small scale dynamo, Reynolds stress decohere, and no large scale vortex forms (Hughes)

←→

- Small scale dynamo can suppress larger scale jet
- See also C^3, P.D. in Plasma

Models and Scales (Knobloch, Pandit, et seq)

- Poster Child Eqns, Systems?
- Cahn-Hilliard = negative diffusion, regularization + conserved order parameter

  - phase separation ↔ layering

$$\frac{\partial C}{\partial t} = \mathrm{D}\sigma^2(-C + C^3 - \gamma\nabla^2 C) \qquad F(C) = \int d^3x \left[\frac{(C^2-1)^2}{4} + \frac{\gamma(\nabla C)^2}{2}\right]$$

- Swift-Hohenberg (!?) arises in Binary Fluid Convection ↔ DDC

Cross-diffusion can arise via NL interactions

$$\frac{dV}{dt} + \cdots = P_r R_a[T + SC]\hat{z} + \cdots$$

$$\frac{dT}{dt} = \nabla^2 T \ , \qquad \frac{d}{dt}C = \tau\nabla^2 C - \nabla^2 T$$

cross-diffusion → phase separation

- C-D = Gradient driven up-gradient “pinch” → not a suppression feedback

Models and Scales, cont'd

- Rich variety of dynamic patterns

➔

- S-H Eqn. ➔ Model of DDC, without DD effects ?!

$$\partial_t U_+ = rU - (q_c^2 + \partial_x^2)^2 U + f(U)$$

intrinsic scale $2\pi/q_c$ bistable

- Variational: $\frac{\partial U}{\partial t} = -\delta F/\delta U$

➔ Scales: '<u>Emergent</u>' scales critical:

– Complement input scale for 2 mixing scales (BLY)

Rhines – Jet

Ozmidov – Mixing

Hinze-CHNS

DDC – ?

– $l_{\partial Z}^2 \sim \epsilon/N^3$, $N^2 \sim \partial\rho/\partial Z$

➔ NL gradient dependency in mixing ➔ transport bifurcation!

Methods – the T in KITP: How Calculate?

[Key Question: Irreversibility?]

(Tobias, Garbet, Guo, Gurcan, Heinonen)

- Quasilinear Theory – derivative from Sagdeev, et. al. '61

  (Tobias) $\partial_t \langle q \rangle = \partial_y \langle V \delta q \rangle + \cdots$ ← response

  evolution by mean-fluctuation interaction and linear response

  ditch inverse cascade, …

- Linear response ←→ Ku < 1 Kubo# = $\tilde{V}\tau_c/l_c$ (N.B. Predict Ku ?)

- Vlasov Plasma: Hamiltonian Chaos ←→ overlap $\frac{\omega}{k} = V$ resonances

  → physics of irreversibility clear – underpins resonant diffusion via Fokker-Plank equation

- Jet problem → absent waves / critical layers, irreversibility less clear

Need:

- Fix-up: QL + Breaking Model → recall fundamental mixing process !

  What is it?

Wave Kinetics:

$N(x,k,t)$ – wave action density
→ quasi-particles as wave-packets
→ conserved phase space density

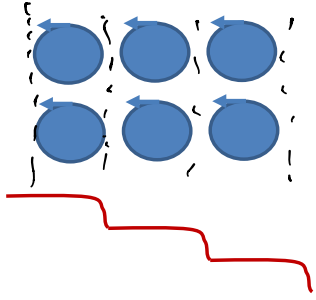
- Exploit WKE – Vlasov structural similarity
- Used for extensive prior work on DW – Zonal Flow System ←→ Quasilinear +

  ➔ Ray Chaos
- Here: "Staircase as BGK Solution" (Garbet) – NL solution of WKE

  Requires: coarse graining

  $\gamma,\ \partial_k D_k \partial_k$

  ➔ Homogenization in phase space



➔ inhomogeneous mixing in phase space

- Issue: Time evolution, mergers ...?
- Ala' Lynden-Bell, most probable BGK state ?!
- Restore time dependence ala' single plasma wave

BLY : Bistable Mixing Length Model – in $k-\epsilon$ style

→ quasi-particles as 'slugs' of density

→ here applied to Drift Wave Problem (Guo)

l_0 , l_{Rh}

→ evolve $\langle n \rangle$, $\langle \nabla^2 \phi \rangle$, ε

- l_0 / l_{Rh} > 1 → enhanced memory (also Zonostrophic, Ku < 1)

  < 1 → strong mixing

- recovers staircases in density, vorticity barrier via Rossby Wave Elasticity
- recovers several aspect of dynamics – mergers, barrier development

Issues:

- self-sharpening vs external sharpening?, staircase asymmetry
- shear vs RWE ?
- Connect to WKE via 'parcel kinetics' (DOG, EAS)

Methods, cont'd

- Envelope Theory – Eikonal + Diffusion (Gurcan)

  – NLS genre

  → Shear layer formation as 2D self-focusing phenomena

  → insight into physics of streamers / mixing (Heinonen)

- Machine Learning (2D H-W) – deduce mean field theory(glorified) regression from simulation

  – "an art form"...

  → Vorticity: Cahn-Hilliard structure

  → density: diffusion + $\nabla\langle\nabla^2\phi\rangle$ pinch / cross-diffusion

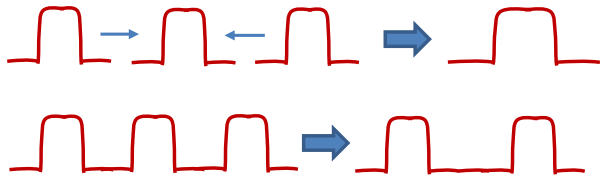
  → $\nabla(Vort)$ more significant than shear – conventional wisdom?

  → no sign of 'turbulence spreading' – conventional wisdom?

- Promising – let's see more ...

Dynamics (Cope, Galperin; Balmforth)

- Two types of mergers! (Balmforth)



- Migration (Cope)
  - Jet motion / drift ➔ relevant to turbulence spreading (c.f. GAM)
  - Migration covered by GQL only if $\Lambda \geq 1$ Zonon included among resolved modes
  - Why? → symmetry breaking by zonon

    → acts as mediator for interactions
  - $V_{mig} \sim \mu\, L_{RH}$

Dynamics, cont'd

- More on Mixing – Jet problem duality (Galperin)
  - Emergent scales – $l_R \leftrightarrow l_{oz}$
  - Zonons can contribute to jet self-sharpening and barrier formation
  - Suggests GQL – type comparison test ↔ vary cut-off

Perspective: Wave Flows '14 → BIRS Oaxaca → Staircase '21

- Interest in, citation of, BLY '98 rising, 23 years on
- Emergent scales (Rhines, Ozmidov, Hinze) appreciated

  ➔ gradient scale → transport bifurcation
- Resurgence in the ancient art of Mixing Length Theory – bistable…
- Staircase
  - Barriers
    - drift – mediators
    - mergers - variety
  - Resilience → avalanches
- CHNS identified as an instructive dynamical model (joins CHM, DDC, …)

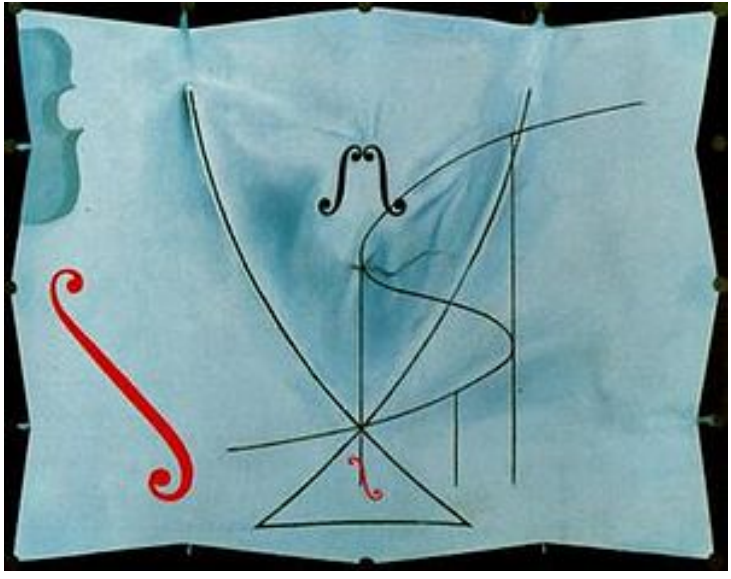
- Mean Field Theory → Quasilinear + ?

  saga continues… ? = Wave Breaking (?!)

  → “discourse” ongoing

- Arctic oceanography offers plethora of fascinating opportunities

Thanks to All !



Salvador Dali