Ion Heat and Momentum Transport in Stochastic Magnetic Fields

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Why? - Heat, Momentum Transport meet $\langle \widetilde{B}^2 \rangle$

- <u>Electron</u> thermal transport is usual focus

- But $\langle E_r \rangle = \frac{\nabla P_i}{nq} - \frac{1}{c}\langle v \rangle \times \langle B \rangle$

  $\langle v_E \rangle'$ heat, particles $\perp, \parallel$ flows

  → Ion heat and parallel momentum transport?

- Relevance:
  - Intrinsic rotation → pedestal torque with RMP?
  - L-H threshold with RMP
  - Island induced ITB
  - Density limits

Conventional Wisdom I

- Finn, Guzdar, Chernikov '92 (FGC) → canonical "ref.(1)"
  - n_i , $V_\parallel$ evolution in stochastic fields - motivated by rotation damping due EML - (TEXT)
  - Mean field eqns:

$$\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle = -\frac{1}{\rho} \partial_x \langle \tilde{b}_r \tilde{P} \rangle$$ → kinetic stress

$$\partial_t \langle P \rangle + \partial_r \langle \tilde{V}_r \tilde{P} \rangle = -\rho\, c_S^2\, \partial_r \langle \tilde{b}_r \tilde{V}_\parallel \rangle$$

  - QL for 'acoustic wave response' for $\tilde{P}_i$, $\tilde{V}_\parallel$

➔ viscous relaxation time $\tau_l \sim [c_s\, D_M \,/\, l^2]^{-1}$

$D_M = \sum_k |b_k|^2 \pi\, \delta(k_\parallel)$, ala' RSTZ '66

i.e. 'acoustic' propagation along stochastic field

Conventional Wisdom I, Cont'd

- Nit
  - Why bother with acoustics ? ➔ static problem

    $\vec{B} \cdot \nabla \tilde{V}_{\parallel} + \tilde{B} \cdot \nabla \langle V_{\parallel} \rangle = 0$ and linear response ➔ kinetic stress

    P similarly

- Issue: Structure of fluxes? → Non-Diffusive !

$$\langle \tilde{b}_r \tilde{P} \rangle = -D_M \frac{\partial}{\partial r} \langle P \rangle, \quad \langle \tilde{b}_r \tilde{V}_{\parallel} \rangle = -D_M \frac{\partial}{\partial r} \langle V_{\parallel} \rangle$$

→ Residual Stress, → Convection / Pinch

drives $\langle V_{\parallel} \rangle$

↓

kinetic stress

Pinch for $\langle P \rangle$ — driven by $\langle V_{\parallel} \rangle'$

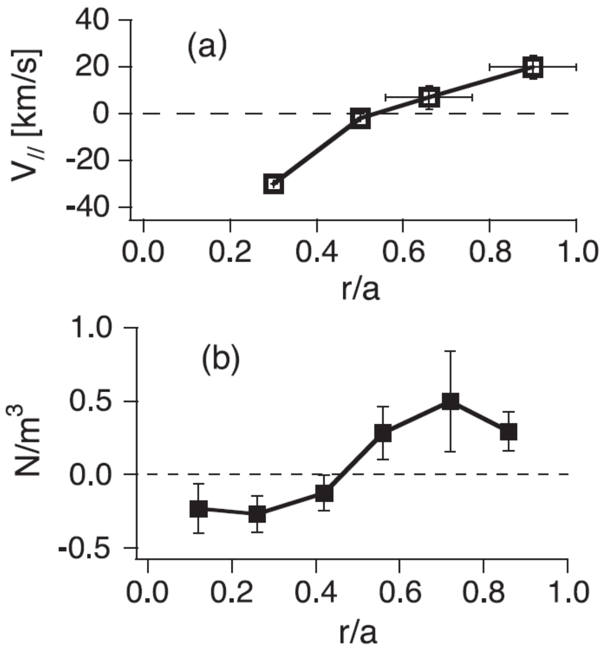
More Conventional Wisdom II: Kinetic Stress and Rotation

$$\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle = -\frac{c_s^2}{\rho}\ \partial_x \langle b_r P \rangle$$

"kinetic stress"

- W.X. Ding, et. al. PRL '13 – MST Rotation Studies
  - * – Linked plasma flows in RFP to kinetic stress, via direct measurement
  - – Mean flow profile tracks profile of $\nabla \cdot$ (kinetic stress)

➔ Rare and compelling insight into the fluctuation ↔ rotation connection!
i.e. microscopic ↔ macroscopic link



N.B. → RFP is exceptionally good example of stochasticity in fusion plasmas

What ? – the Issue

- How calculate the kinetic stress ?
- In QL approach, ala' FGC, seek:

  $\delta P \sim \tilde{b}\delta P/\delta b \Rightarrow \langle \tilde{b}\delta P\rangle \sim \langle b^2\rangle$

  But What is in $\delta P/\delta b$?

- In any relevant case, especially prior to L→H transition, turbulence will co-exist with stochastic field

So

- Need calculate kinetic stress in presence of turbulence

What ? Cont'd

- Two 'dual' analyses:
  - Reynolds stress, etc. in background $\langle b^2 \rangle$ → Chen et. al., this meeting
  - Kinetic stress, pinch in $\langle \tilde{V}_\perp^2 \rangle$ background → here
- Expect significant departure from FGC, and from standard quasilinear theory
- Implicit: Statistics $\tilde{b},\ \tilde{V}_\perp$ assumed independent

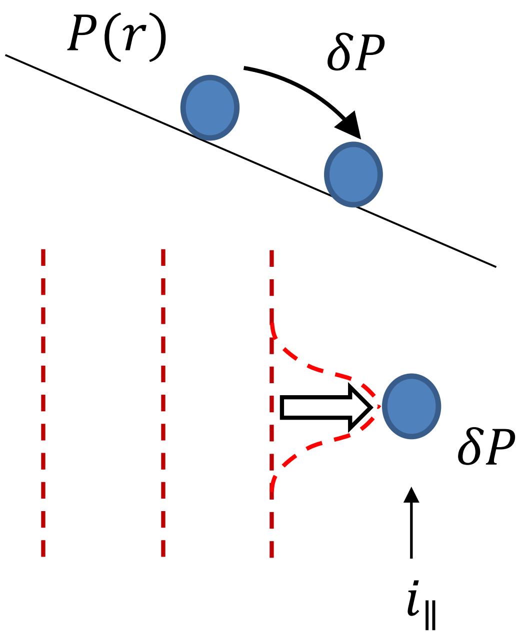
  $\tilde{b}\ \rightarrow$ RMP induced

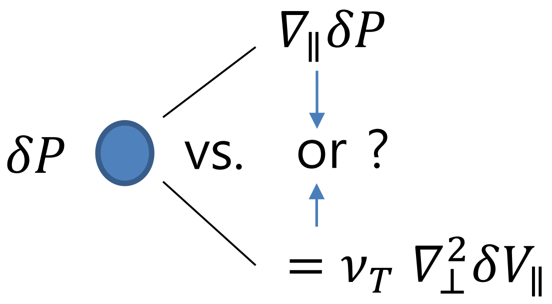
  $\tilde{V}\ \rightarrow$ drift waves

TBC later → see Mingyun Cao, this meeting

- In spirit of resonance broadening, but juicier…

The Physics





Critical comparison:

$c_s k_\parallel$ vs $k_\perp^2 D_T$

- $c_s \tilde{b}_r \, \partial \langle P \rangle / \partial r$ ➔ δP – localized slug of pressure

  Tweaking field line produces localized pressure perturbation

- How is pressure balanced along field line? – two possibilities

  i) Build parallel pressure gradient

  $\nabla_\parallel \delta P \sim -\tilde{b}_r \, \partial_r \langle P \rangle$ ➔ FGC

  or

  ii) Drive parallel flow, damped by turbulent mixing/viscosity due $\langle \tilde{V}_\perp^2 \rangle$

  $-\nu_T \nabla_\perp^2 \delta \tilde{V}_\parallel \sim -b_r \, \partial_r \langle P \rangle$

  ν_T is to be calculated

The Crank

- Start from $\partial_t V_\parallel$, $\partial_t P$ equations
- Seek $\langle \tilde{b}_r \tilde{P} \rangle$, $\langle \tilde{b}_r \tilde{V}_\parallel \rangle$
- Follow 'quasilinear' approach, BUT
- Posit an ambient ensemble of drift waves, so $\langle \tilde{V}_\perp^2 \rangle$ specified

⇨ Assume $\langle \tilde{V}_\perp^2 \rangle$, $\langle \tilde{b}_r^2 \rangle$ quasi-Gaussian and statistically independent

⇨ • Calculate responses $\delta P = (\delta P / \delta b_r) \tilde{b}_r$ and $\delta V_\parallel = (\delta V_\parallel / \delta b_r) \tilde{b}_r$ (to close fluxes), by integration over perturbed trajectories, ala' Dupree '66

- $\delta P / \delta b_r$ is statistically averaged, nonlinear response }

The Answer: Note turbulence-induced gradient couplings !

– (kinetic stress) $\langle \tilde{b}_r\, \delta P \rangle = -\sum_k |b_{r,k}|^2 \left[\frac{1}{(k_\perp^2 D_T)^2 + k_\parallel^2 c_s^2}\right] \left\{\rho c_s^2 k_\perp^2 D_T \frac{\partial}{\partial r}\langle V_\parallel \rangle - i k_\parallel c_s^2 \frac{\partial}{\partial r}\langle P \rangle\right\}$

– (convection) $\langle \tilde{b}_r \delta V_\parallel \rangle = -\sum_k |b_{r,k}|^2 \left[\frac{1}{(k_\perp^2 D_T)^2 + k_\parallel^2 c_s^2}\right] \left\{c_s^2 k_\perp^2 D_T \frac{\partial}{\partial r}\langle P \rangle - i k_\parallel c_s b_{r,k} c_s \frac{\partial}{\partial r}\langle V_\parallel \rangle\right\}$

– $D_T \equiv \int \langle \tilde{V}_r \tilde{V}_r \rangle dt$ ➔ electrostatic turbulent diffusivity

– Response Function: $1 / \left[k_\parallel^2 c_S^2 + (k_\perp^2 D_T)^2\right]$

– Order of limits important to recover QL results

The Physics, cont'd

- Limits

  $k_\parallel c_s > k_\perp^2 D_T$ → weak e.s. turbulence -- narrow regime validity

  n.b. role of anisotropy ! – contrast micro-instability c.f. Lu Wang

  $\langle \tilde{b}_r \delta P \rangle \approx -D_M \, \partial \langle P \rangle / \partial r, \ \langle \tilde{b}_r \delta V_\parallel \rangle \approx -D_M \partial \langle V_\parallel \rangle / \partial r$

  Recovers FGC. Relevance limited

- $k_\perp^2 D_T > k_\parallel c_s$ → robust electrostatic turbulence (as for pre-transition)

  $\langle \tilde{b}_r \delta P \rangle \approx -D_{st} \, \partial \langle V_\parallel \rangle / \partial r$, $\langle \tilde{b}_r \delta V_\parallel \rangle \approx -D_{st} \partial \langle P \rangle / \partial r$

  → Viscosity! → Thermal diffusivity

  $$D_{ST} = \sum_k c_s^2 \left| b_{r,k} \right|^2 / \, k_\perp^2 D_T$$

- Structure of {correlator, fluxes} change ! *

The Physics, Cont'd

- Stochastic viscosity/diffusivity is hybrid

$$D_T = \sum_k c_s^2 \left|b_{r,k}\right|^2 / \, k_\perp^2 D_T$$

Magnetic scattering,
with τ_{ck} set
by electrostatics

- Pure 'stochastic field' analysis irrelevant to any state with finite ambient

⇨ electrostatic turbulence, c.f. $k_\parallel c_s$ vs $k_\perp^2 D_T$

- Easily extended to sheared magnetic geometry, etc

i.e. key: w_k vs $X_s = 1/k_\parallel' c_s \tau_{ck}$

$w_k > X_s$ → weak scattering

$w_k < X_s$ → strong scattering

w_k: Spatial spectral width

X_s: Acoustic point (analogous X_i)

Comments re: Theory

- Yes, resonance broadening, but no – not 'the usual'

  ➔ structure of flux modified – residual stress to viscosity

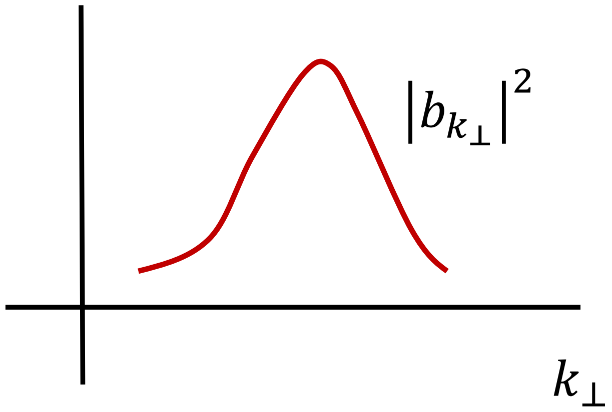
- Infrared behavior of wave number spectrum important !

  – Low k cut-off $|b_{r_k}|^2$?

  – Not resolved trivially, by geometry

  – Similar: Taylor, McNamara '72 → cut-off and 'locality' ?!

  – ExB shear, even if sub-BDT, can set cut-off ➔ ZF generation will enter….



N.B.:

– For ZF case, comparison is $k_\perp^2 D_T$ vs $k_\parallel V_A$ ➔ W.T. regime relatively more robust

– See Samantha Chen, next talk

Conclusions

- Pure stochastic models of limited utility for momentum, ion heat, etc.
- Need analyze stochastic field effects in presence of turbulence
- In practice, kinetic stress is stochastic field - induced

  viscous stress → significant drag on rotation
- $D_{ST} = c_s^2 \sum_k |b_{r,k}|^2 / k_\perp^2 D_T$ → (hybrid) stochastic field viscosity
- See Beyer, et. al. (2000) for hints from simulations

Open Issue

- Development of Correlation? (see Mingyun Cao)
  - are $\tilde{b}$, turbulence uncorrelated ? as assumed…
  - No → interaction develops $\langle \tilde{b}\tilde{\phi} \rangle \neq 0$ → electrostatics 'lock on' to $\tilde{b}$
  - ala' Kadomtsev – Pogutse, impose $\nabla \cdot \vec{J} = 0$ to all orders
  - novel small scale convection cell, related to $\tilde{b}$ structure ⇦

Ongoing …

Open Issue, Cont'd

- Elucidate kinetic stress contribution to intrinsic torque, with RMP.

  Determine flux-gradient relation

- Beyond diffusion – Fractional kinetics with Pdf($\tilde{V}, \tilde{b}$) ?

  How formulate?

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