SOL Broadening by Edge Turbulence: Spreading and Entrainment Dynamics

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Background

• Conventional Wisdom of SOL:

(cf: Stangeby...)

- Turbulent Boundary Layer, ala' Blasius, with D due turbulence
- $\ \delta \sim (D\tau)^{1/2}, \tau \approx L_c/V_{th}$
- $D \leftrightarrow$ local production by SOL instability process
 - \rightarrow familiar approach, D ala' QL
- Features:
 - Open magnetic lines \rightarrow dwell time τ limited by transit, conduction, ala' Blasius
 - Intermittency \rightarrow "Blobs" etc. Observed. Physics?

Fluid Mechanics

Landau and Lifshitz Course of Theoretical Physics Volume 6

L.D. Landau and E.M. Lifshitz Institute of Physical Problems, USSR Academy of Sciences, Moscow





Background, cont'd

• But... Heuristic Drift (HD) Model (Goldston +)

$$- V \sim V_{\text{curv}}$$
, $\tau \sim L_c/V_{thi}$, $\lambda \sim \epsilon \rho_{\theta i} \rightarrow \text{SOL width}$

- Pathetically small
- Pessimistic B_{θ} scaling, yet high I_p for confinement
- Fits lots of data.... (Brunner '18, Silvagni '20) → H-mode



• Why does neoclassical work? \rightarrow ExB shear suppresses SOL modes i.e.

 $\gamma_{\text{interchange}} \sim \frac{c_s}{(R_c \lambda)^{\frac{1}{2}}} - \frac{3T_{edge}}{|e|\lambda^2}$

shearing $\leftarrow \rightarrow$ strong λ^{-2} scaling

from:
$$\frac{c_s}{(R_c\lambda)^{\frac{1}{2}}} - \langle V_E \rangle'$$

Physics Issues

[C.f. Chu, P.D., Guo, NF 2022]

- How <u>calculate</u> SOL width for turbulent pedestal but a locally <u>stable</u> SOL?
 - spreading penetration depth
 - must recover HD in WTT limit
- Scaling and cross-over of λ_q relative to HD model
- What is effect/impact of barrier on spreading mechanism?
 - Can SOL broadening and good confinement be reconciled ?

Model – Stable SOL – Linear Theory

 Standard drift-interchange with sheath boundary conditions + ExB shear (after Myra + Krash.)



v.s. $E \times B$ shear at $q = 5, \beta = 0.001, k_y \cdot \lambda_{HD} = 1.58$.

- Relevant H-mode ExB shear strongly stabilizing $\gamma_{HD} = c_s / (\lambda_{HD} R)^{1/2}$
- Need λ/λ_{HD} well above unity for SOL instability. $V'_E \approx \frac{3T_e}{|e|\lambda^2} \rightarrow$ layer width sets shear

Width of Stable SOL

• Fluid particle:
$$\frac{dV}{dt} = V_D + \tilde{V}$$

drift fluctuating velocity

• Dwell time: τ_{\parallel} dwell time contrains excursion

$$\delta^{2} = \langle (\int (V_{D} + \tilde{V})dt) (\int (V_{D} + \tilde{V})dt) \rangle$$

$$= V_D^2 \tau_{\parallel}^2 + \langle \tilde{V}^2 \rangle \tau_c \tau_{\parallel}$$

 τ_c cut-off at τ_{\parallel}

 $\approx \lambda_{HD}^2 + \varepsilon \, \tau_{\parallel}^2$

• So
$$\lambda = \tau_{\parallel} [V_D^2 + \varepsilon]^{1/2}$$
 SOL width effects add in quadrature

• How compute ε ? \rightarrow turbulence energy in SOL?

Calculating the SOL Turbulence Energy 1

- $K \epsilon$ type model, mean field approach (c.f. Gurcan, P.D. '05 et seq)
 - Can treat various NL processes via σ, κ
 - Exploit conservative form model
- $\partial_t \varepsilon = \gamma \varepsilon \sigma \varepsilon^{1+\kappa} \partial_x \Gamma_e$ \longrightarrow Spreading, turbulence energy flux Growth $\gamma < 0$ NL transfer $\gamma_{NL} \sim \sigma \varepsilon^{\kappa}$ here contains shear + sheath
- N.B.: No Fickian model of Γ_e employed
- Readily extended to 2D, improved production model, etc.

Calculating the SOL Turbulence Energy 2

- Integrate ε equation \int_0^{λ}
- Take quantities = layer average

•
$$\Gamma_{e,0} + \lambda_e \gamma \varepsilon = \lambda_e \sigma \varepsilon^{1+\kappa}$$

Separatrix fluctuation energy flux \longrightarrow Single parameter characterizing spreading

So for $\gamma < 0$, $\Gamma_{e,0} = \lambda_e |\gamma| \varepsilon + \sigma \lambda_e \varepsilon^{1+\kappa}$

 $\Gamma_{e,0}$ vs linear + nonlinear damping

 λ_e = layer width for ε

Calculating the SOL Turbulence Energy 3

[Mean Field Theory]

• Full system:

$$\begin{split} \Gamma_{e,0} &= \lambda_e |\gamma| \varepsilon + \sigma \lambda_e \varepsilon^{1+\kappa} \\ \lambda_e &= \left[\lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2 \right]^{1/2} \end{split}$$

Simple model of turbulent SOL broadening

• $\Gamma_{0,e}$ is single control parameter characterizing spreading

•
$$\tilde{\Gamma}_{0,e}$$
 ? Expect $\tilde{\Gamma}_e \sim \Gamma_0$

SOL width Broadening vs $\Gamma_{e,0}$

• SOL width broadens due spreading



 λ/λ_{HD} plotted against the intensity flux Γ_{e0} from the pedestal at $q = 4, \beta = 0.001, \kappa = 0.5, \sigma = 0.6$

Variation indicates need for detailed scaling analysis

- Clear decomposition into
 - <u>Weak</u> broadening regime \rightarrow shear dominated
 - Cross-over regime
 - <u>Strong</u> broadening regime
 - → NL damping vs spreading

relevant

- Cross-over for: $\Gamma_{0,e}$ sufficient s/t $\langle \tilde{V}^2 \rangle \sim V_D^2$
- Cross-over for $\tilde{V} \sim O(\epsilon) V_*$
 - ➔ weak turbulence

Computing the Turbulence Energy Flux 1

- Need consider pedestal to actually compute $\Gamma_{e,0}$
- Two elements



Does another
trade-off loom?-- Pedestal Turbulence: Drift wave? Ballooning?-- Effect of transport barrier $\leftarrow \rightarrow$ ExB shear layer \rightarrow barrier permiability!?

• Key Point: shearing limits correlation in turbulent energy flux

i.e.
$$\Gamma_{e,0} \approx -\tau_c I \partial_x I \approx \tau_c I^2 / w_{ped}$$
 (Hahm, PD +)
ped turbulence correlation time \rightarrow strongly sensitive to shearing

N.B. Caveat Emptor re: intensity flux closure !

Computing the Turbulence Energy Flux 2

• Familiar analysis for $D \rightarrow Kubo$

•

$$D = \int_0^\infty d\tau \, \langle V(0)V(\tau) \rangle = \int_0^\infty d\tau \, \sum_k \left| \tilde{V}_k \right|^2 \exp\left[-k_y^2 \omega_s^2 D\tau^3 - k^2 D\tau \right]$$

Strong shear (relevant) $\tau_c = \tau_t^{1/2} \omega_s^{-1/2}$
 $\tau_t \sim 1 / k \tilde{V}, \quad \omega_s \sim V'_E$

Here, via RFB
$$\rightarrow \omega_s = \partial_r \frac{\nabla P_i}{n|e|} \sim \frac{\rho^2}{w_{ped}^2} \Omega_{ci}$$

- $\tau_c + w_{ped}$ + turbulence intensity in pedestal gives $\Gamma_{e,0} \approx \tau_c I^2 / w_{ped}$
- Need $\Gamma_{e,0} \ge \Gamma_{e,\min} \approx |\gamma| \lambda_{HD}^3 \tau_{\parallel}^{-2}$

Computing the Turbulence Energy Flux 3

- Pedestal → Drift wave Turbulence
- Necessary turbulence level:
 - Weak Shear $\frac{\delta V}{c_s} \sim \left(\frac{\rho}{R}\right)^{1/2} q^{-1/4}$





- → λ/λ_{HD} vs $|e|\hat{\phi}/T_e$ in pedestal
- → ρ/R is key parameter
- Broadens layer at acceptable fluctuation level

Partial Summary

Turbulent scattering broadens stable SOL

 $\lambda = \left(\lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2\right)^{1/2}$

• Separatrix turbulence energy flux specifies SOL turbulence drive



$$\Gamma_{0,e} = \lambda_e |\gamma|\varepsilon + \lambda \sigma \varepsilon^{1+\kappa}$$

Broadening increases with $\Gamma_{0,e}$

Non-trivial dependence

- $\Gamma_{0,e}$ must overcome shear layer barrier
- Yes can broaden SOL to $\lambda/\lambda_{MHD} > 1$ at tolerable fluctuation levels Further analysis needed

Broader Messages

- Turbulence spreading is important even dominant process in setting SOL width. $\Gamma_{0,e}$ is critical element. $\lambda = \lambda(\Gamma_{0,e}, \text{ parameters})$
- Production Ratio R_a merits study and characterization
- Spreading is important saturation meachanism for pedestal turbulence
 - Simulation should stress <u>calculation</u> and characterization of turbulence energy flux over
 visualizations and front propagation studies.
- Critical questions include local vs FS avg, channels and barrier interaction, Turbulence 'Avalanches'
- Turbulent pedestal states attractive for head load management

Physics of Turbulence Spreading: General

Perspective

- Structure of the intensity flux-gradient relation(?)
- Spreading as directed percolation...

Spreading: Conventional Wisdom

• Turbulence spreading underpins turbulent wake \rightarrow central example in high *Re* fluids



 Spreading fundamental to k − ε type models, as ε evolved as unresolved energy field → subgrid models

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot (\tilde{V}\varepsilon) + \dots = 0$$

How render tractable 7

Spreading: cont'd

• What you get (usually):

$$\begin{array}{ll} \partial_t \varepsilon + \vec{V}_D \cdot \nabla \varepsilon + \langle \vec{V}_E(r) \rangle \cdot \nabla \varepsilon &- \partial_r D(\varepsilon) \partial_r \varepsilon = P(\varepsilon) - P_{damp}(\varepsilon) \rightarrow \gamma(\vec{x}) \varepsilon \\ & \text{drift} & \text{shear} & \text{turbulent mixing via closure} & \gamma = \gamma(\text{gradients, etc}) \end{array}$$

 $D(\varepsilon) \approx D_0 \varepsilon$, et. seq. \rightarrow nonlinear diffusion

 $\rightarrow \varepsilon$ evolution as nonlinear Reaction-Diffusion Problem!

(P.D., Garbet, Hahm, Gurcan, Sarazin, Singh, Naulin...)

- Used also in:
 - BLY-style layering models (Ashourvan)
 - 1D L→H models (Miki)

Spreading: cont'd

Spreading as Front → Fast Propagation

i.e. $V_f \sim (\gamma D)^{1/2}$, etc [N.B. Cahn-Hilliard?]

• Key component:

 $\nabla \cdot \langle \vec{V} \varepsilon \rangle \rightarrow -\nabla \cdot D(\varepsilon) \cdot \nabla \varepsilon$ [Fickian Model]

Expectation: $D(\varepsilon) \sim \chi$, D_n etc. for electrostatic turbulence

• Copious simulations: Z. Lin, W.X. Wang, S. Yi, Jae-Min Kwon, Y. Sarazin, ...

➔ Observations, front tracking but critical analysis of model absent ?? No test of Fickian flux model !

Experiments: Ancient

• Not exactly a new idea ... See Townsend '49 and book

Momentum and energy diffusion in the turbulent wake of a cylinder

BY A. A. TOWNSEND, Emmanuel College, University of Cambridge

(Communicated by Sir Geoffrey Taylor, F.R.S.-Received 6 October 1948)

A detailed experimental investigation of the turbulent motion in the wake of a circular cylinder, 0.953 cm. diameter placed in an air-stream of velocity 1280 cm.sec.-1, has been carried out with particular reference to those quantities determining the transport of turbulent energy and mean stream momentum. At distances of 80, 120 and 160 diameters down-stream from the cylinder, direct measurements have been made of mean flow velocity, turbulent intensity, viscous dissipation, energy diffusion, scale, and form factors of the velocity components and their spatial derivatives. These observations show that, except close to the wake centre, the flow at a point fixed with respect to the cylinder is only intermittently turbulent, due to the passage of the point of observation through jets or billows of turbulent fluid emitted from the inner wholly turbulent core of the wake. Further consideration of the results indicates that the turbulent motion within the jets is solely responsible for the turbulent transfer of momentum, while diffusion of turbulent energy and of heat is carried out by the bulk movement of the jets. Most probably, the jets are initiated by local fluctuations of pressure inside the turbulent core, and in the later stages of their development that are slowed down by adverse pressure gradients. The existence of pressure-velocity correlations of sufficient magnitude is demonstrated by using the equation for the conservation of kinetic energy in the wake, all terms of which are known excepting the one involving the pressure-velocity correlation, which is then obtained by difference. While the conception of

jets of turbulent fluid is more convenient for following the physical processes in the wake, the alternative but equivalent description that the turbulent motion consists of a motion of scale small compared with the mean flow superimposed on a slower turbulent motion whose scale is large compared with the mean flow may be used. A formal explanation of this twostage turbulent structure in terms of the Fourier representation of the velocity field is suggested, which relates the structure to the presence of a quasi-constant source of energy of nearly fixed wave-number, and to the free boundary which allows an unlimited range of wave-numbers. It is expected that this type of motion will occur in all systems of turbulent shear flow with a free boundary, such as wakes, jets and boundary layers.

→ Wake flow intermittently turbulent

→ Compare transport of momentum and energy (spreading)

Experiments: Ancient, cont'd



STRUCTURE OF THE WAKE

Let us now consider the experimental results in turn, and use them to derive information about the detailed properties of these jets of turbulent fluid. In the first place, the velocity product \overline{uv} , representing the Reynolds shear stress, has been measured, and, with the observed distribution of mean velocity across the wake, the effective eddy viscosity ϵ and the experimental mixing length l may be calculated, using the definitions

$$\overline{uv} = -erac{\partial U}{\partial y}, \quad e = l\sqrt{v^2}.$$

Experimentally, l is found to be fairly small, approximately 0-07 of the half-width of the mean velocity wake (figure 3), and does not vary greatly over the width of the wake. The small size of l is interpreted as evidence that momentum transfer in the wake is carried out by comparatively small eddies. More significantly, ϵ/γ is not far from constant over the greater part of the wake (figure 4), but this will be discussed later.

→ Wake expansion due jets of expanding fluid

→ Departs mean field theory

➔ Mixing length model momentum transport

Experiments: Ancient, cont'd

The product *uv* may be regarded as the rate of transport of momentum (per unit mass), and similarly the rate of transport of turbulent energy is

 $\frac{1}{2}(\overline{u^2v}+\overline{v^3}+\overline{vw^2}),$

and, in principle, it is possible to calculate an energy diffusion coefficient δ , analogous with ϵ , by use of the defining equation

$$\overline{u^2v} + \overline{v^3} + \overline{vw^2} = -\delta \frac{\partial}{\partial y} (\overline{u^2} + \overline{v^2} + \overline{w^2}).$$

When this is attempted (figure 5), no simple behaviour is found either for δ , or for the corresponding mixing length. Negative values occur near the wake centre, and, even where the turbulence gradient is fairly uniform, δ remains large compared with ϵ ,

and decreases rapidly with distance from the wake centre. It must be concluded that the use of a diffusion coefficient to describe the transport of turbulent energy is not justified, and that energy diffusion is a process independent of momentum diffusion.

To remove this difficulty, it is not sufficient to consider the effects of intermittency. If the intermittency factor is known, then the mean intensity in the turbulent regions is

$$I_j = \frac{\overline{u^2} + \overline{v^2} + \overline{w^2}}{\gamma},$$

and I_j is found to vary only slightly over the greater part of the wake (figure 6). So a considerable transport of energy is found in the almost complete absence of a real intensity gradient, and it is difficult to see how energy flow can take place by turbulent

movements inside the jets. For the transport mechanism, there is only left the bulk movement of the jets, which is naturally outwards and away from the wake centre. The compensating inflow will consist of non-turbulent fluid transporting no turbulent energy. Consequently, the flow of energy is not dependent on the local intensity gradient (if any), but only on the mean jet velocity and the jet turbulent intensity, which in turn are determined by conditions in the turbulent core. → Fickian model for turbulent energy transport

→ "It must be concluded that the use of a
 diffusion coefficient to describe the transport of
 turbulent energy is not justified and that energy
 diffusion is a process independent of momentum
 diffusion"

Experiments: Modern (Ting Long, SWIP) 1

- HL-2A
- Aims:
 - Exploration of intensity flux intensity gradient relation in edge turbulence (exploits spreading, shear layer collapse and density limit studies Long + NF'21)
 - Physics of "Jet Velocity" profile

$$V_I = \langle \tilde{V}_r \tilde{n}^2 \rangle / \langle \tilde{n}^2 \rangle$$

N.B. Identified by Townsend

Experiments: Modern 2

• There exits a region in plasma edge, where the turbulence spreading flux $\langle \tilde{v}_r \tilde{n}^2 \rangle / 2$ is large, but the turbulence intensity gradient $\partial_r \langle \tilde{n}^2 \rangle$ is near zero



Experiments: Modern 3

- Striking difference between particle diffusivity and energy spreading diffusivity
 - > Diffusivity of turbulent particle flux $\langle \tilde{n}\tilde{v}_r \rangle = -D_n \partial_r \langle n \rangle$
 - > Diffusivity of turbulence spreading $\langle \tilde{v}_r \tilde{n}^2 \rangle = -\chi_I \partial_r \langle \tilde{n}^2 \rangle$



- $130 \text{kA}, 2.8 \times 10^{19} \text{m}^{-3} (0.45 \text{n}_{\text{G}})$ $130 \text{kA}, 3.9 \times 10^{19} \text{m}^{-3} (0.63 \text{n}_{\text{G}})$ $190 \text{kA}, 4.1 \times 10^{19} \text{m}^{-3} (0.45 \text{n}_{\text{G}})$
- *χ_I* is not equal to *D_n*!
 (in both magnitude and <u>sign</u>)
- χ_I is large where $\partial_r \langle \tilde{n}^2 \rangle$ is near zero
- χ_I increases significantly as \bar{n}/n_G increases

(Both \bar{n} and I_p involved)

Practical validity of Fickian model is dubious

*

Experiments: Modern 4

• The "mean jet velocity" of turbulence spreading $V_I = \frac{\langle \tilde{v}_r \tilde{n}^2 \rangle}{\langle \tilde{n}^2 \rangle}$ and skewness of density fluctuations show strong correlation



Spreading as Fluctuation Intensity Pulses

- Edge turbulence intermittent \rightarrow mean field theory?
 - Strong $\langle V_E \rangle' \rightarrow \sim$ marginal avalanching state
 - Weaker $\langle V_E \rangle' \rightarrow$ structures, etc. $\Gamma_e = \langle \Gamma_e \rangle + \tilde{\Gamma}_e$
- Pulses / Avalanches are natural description

 $\delta P \equiv$ deviation of profile from criticality

$$\delta P \leftrightarrow (\nabla P - \nabla P_{crit})/P$$

Naturally: $\delta P \sim \delta \varepsilon$

→ Spreading as intensity pulses (after Hwa, Kardar, P.D., Hahm)

Pulse, void symmetry arguments etc.



Fluctuation Energy Pulses, cont'd

- Burgers is on the grill...
- New toppings:
 - $-\delta \varepsilon > 0$ turbulence ejected into SOL

positive intensity fluctuation

- $-V_D > 0$ mean drift out curvature
- Scale independent damping

 $(1/\tau)\delta\varepsilon$ due finite dwell time in SOL \rightarrow order parameter not conserved

- Noise is b.c.
 - $\tilde{\Gamma}_{0,e}|_{sep}$ drives system, space-time

Fluctuation Energy Pulses, cont'd

Pulse model: \bullet



Some limits:

$$-\varepsilon \rightarrow 0$$
, $V_D \partial_x \tilde{\varepsilon} \sim \frac{\tilde{\varepsilon}}{\tau} \rightarrow \lambda \sim \lambda_{HD}$ scale (1) vs (2)

– For ε to matter:

 $\alpha \tilde{\varepsilon} > V_D \rightarrow \text{amplitude vs neo drift comparison}$ (1) VS (3)

Structure is Burgers + Krook \rightarrow Crooked Burgers ۲

Fluctuation Energy Pulses, cont'd

• Predictions?

Structure formation \rightarrow Shock Criterion !

i.e. Recall:
$$\frac{d\varepsilon}{dt} = -\frac{\varepsilon}{\tau}$$
, $\frac{dx}{dt} = \alpha\varepsilon$

• Solve via characteristics:

$$x = \alpha \left[z + \frac{\binom{1-e^{-\frac{t}{\tau}}}{1-\tau}}{(1/\tau)} f(z) \right]$$

Shock for: $f'(z) < -1/\tau$

 \rightarrow initial slope must be sufficiently steep to shock before damped by $1/\tau$

Spreading as Fluctuation Intensity Pulses, cont'd

- $\alpha \frac{\partial \varepsilon}{\partial x}|_{sep} < -\frac{1}{\tau} \rightarrow$ pulse formation criterion \rightarrow intensity gradient
- Fate? $\bigwedge \Rightarrow \checkmark \Rightarrow \checkmark$

 $\alpha \ \varepsilon < V_D \rightarrow$ defacto 'evaporation criterion'

 \rightarrow defines penetration depth of pulse

- Aim to characterize <u>statistics</u> of pulses, penetration depth distribution... in terms $Pdf(\tilde{\Gamma}_{0,e})$. Challenging...
 - ➔ Meaningful output for SOL broadening problem

Directed Percolation - Remark

• Goldenfeld Rosenbluth lecture, Festival de Theorie '17

→ Fundamentally, spreading as a directed percolation process...

D.P. as P. with sense of time's arrow
 D.P. ←→ avalanching... → pulses

e.g. Do avalanches span system?

BTW '87 interprets SOC state as percolation cluster, critical to addition of single grain

• Mean field models of DP \rightarrow reaction diffusion, hydrodynamics

But...

• Fluctuations significant near criticality !

→ R. G. ... TBC ...

Open Issues

• Quantify
$$\lambda = \lambda \left(\frac{|e|\hat{\phi}|}{T} \Big|_{ped} \right)$$
 dependence

- Structure of Flux-Gradient relation for turbulence energy?
 - Phase relation physics for intensity flux? crucial to ExB shear effects
- Kinetics $\rightarrow \langle \tilde{V}_r \delta f \delta f \rangle$, Local vs Flux-Surface Average, EM
 - SOL Diffusive? → Intermittency('Blob'), Dwell Time ?
 - SOL \rightarrow Pedestal Spreading ? $\leftarrow \rightarrow$ HDL (Goldston) ?
 - i.e. Tail wags Dog ? Both wagging ? \rightarrow Basic simulation, experiment ?

Counter-propagating pulses ?

Some Concluding Philosophy

- MFE relevant questions within reach in near future. Great attention to λ_a problem (c.f. Samuel Johnson)
- Unreasonable for tokamak experiments to probe ~ critical dynamics so as to elucidate basic questions. Simulations???
- Well diagnosed, basic experiment with some relevant features are sorely needed – akin to 'Tube' studies of flows, ala' CSDX
- How?