# How Flux Jams and Layering, Re-visited

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Dewar Session

AAPPS-DPP (菊池祭) 2024

This research was supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DEFG02-04ER54738.

# Isaac Newton Institute, 2024 Jan. → June

"Anti-Diffusion: From Sub-Celluar to Astrophysical Scales"

Focus on: Layering, Staircases etc.

- Bob Dewar participated as long stay member
- See INI program web-pages for Dewar Memorial Session
- This talk → developing idea from INI

# Origins of Layering and Staircases

- Bistable Mixing ←→ 2 Mixing Lengths
  - emergent scale (Rhines, Ozmidov etc.)
  - nonlinear flux-gradient curve
  - MFE – barriers, ZF etc.
- Phase Separation ←→ ala' spinodal decomposition
  - cf Pandit '23 plenary – CHNS
  - jamming as trigger
  - MFE: heat flux jams – Kosuga, P.D., Gurcan PRL 2013
- Homogenization
  - Cells, etc. which don't overlap
  - sharpening, inhomogeneous mixing

# Focus: Jamming as Layering Trigger. How to Jam?

- MIPS: "Mobility-Induced Phase Separation"
  - M. Cates, J. Tailleur 2013, 2015 et. seq.
  - also M. Cates, Inaugural Lecture, Lucasian Professorship
- Active Fluids
- "Self-propelled particles tend to accumulate where they move more slowly. This creates positive feedback, which can lead to MIPS between dense and dilute phases."

# How to MIPS?

- C + T: Speed $V(\rho)$, Density $\rho$

s/t
$$\frac{dV/d\rho}{V} < -\frac{1}{\rho}$$
i.e. speed should decrease as density rises

- Reminiscent of Lighthill + Whitham criterion for <u>Backward Shock</u> in kinematic wave (Traffic):

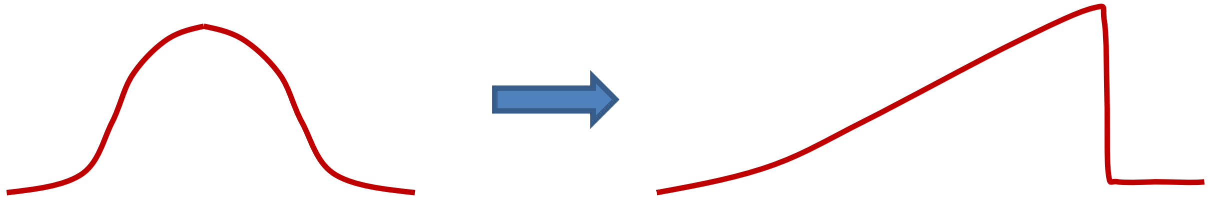
$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}[\rho V(\rho)] = 0\,, \qquad V > 0$$

$$\frac{\partial\rho}{\partial t} + c(\rho)\frac{\partial\rho}{\partial x} = 0 \qquad c(\rho) = \delta\big(\rho V(\rho)\big)/\delta\rho < 0$$

$$\frac{dV/d\rho}{V} < -\frac{1}{\rho}$$

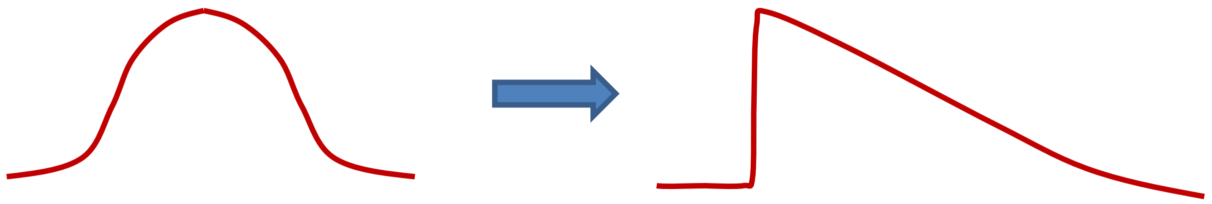
# 2 Types of Shocks – Sign of $\delta\Gamma(\rho)/\delta\rho$

- Forward (usual)
  - $dC_s(\rho)/d\rho > 0$
  - $\delta\Gamma\ /\delta\rho > 0$



- Backward
  - traffic bottleneck/jam

  → Jams, MIPS $\delta\Gamma\ /\delta\rho < 0$



- Whitham: "Individual cars can move faster than the waves, so that a driver enters such a local density increase from behind; he must decelerate rapidly though the shock…"

# Approaches to Jams

Kinematic Wave Theory

relax kinematics,
$\rho$ vs $V(\rho)$ time delay enters

Jam if $\tau_d > \tau_{d,crit}$

correspondence to heat

$Q\ ,\delta T$

Kosuga + (2013)

Remain in pure kinematic wave approach for heat flux

How obtain roll over and satisfy joint reflection symmetry?

# Heat Flux Jams

- Conventional Picture (PD, Hahm '95, after Hwa, Kardar)

$$\partial_t \delta T + \partial_x Q(\delta T) = \tilde{s}$$

$$Q(\delta T) = \underbrace{c\delta T^2}_{Q_T(\delta T)} - D_0 \partial_x \delta T$$

satisfies $Q$ invariant under $\delta T \leftrightarrow -\delta T, \quad x \leftrightarrow -x$ , but no jams!

- $Q_T(\delta T) = \frac{c\delta T^2}{1+\sigma(\delta T)^{2\alpha}}$ will work for $\alpha > 1$
  - "perturbative bistability" → in $\delta T$, not $\langle T \rangle'$
  - no time delay necessary, no need evolve $Q$ explicitly
  - reminiscent of Hinton '91

# Heat Flux Jams, cont'd

- For $\delta Q/\delta(\delta T) < 0$

$$\frac{\delta Q}{\delta(\delta T)} = \frac{c\delta T}{1+\sigma(\delta T)^{2\alpha}}\left[\frac{1\ -(\alpha-1)\sigma\delta T^{2\alpha}}{1+\sigma(\delta T)^{2\alpha}}\right]$$

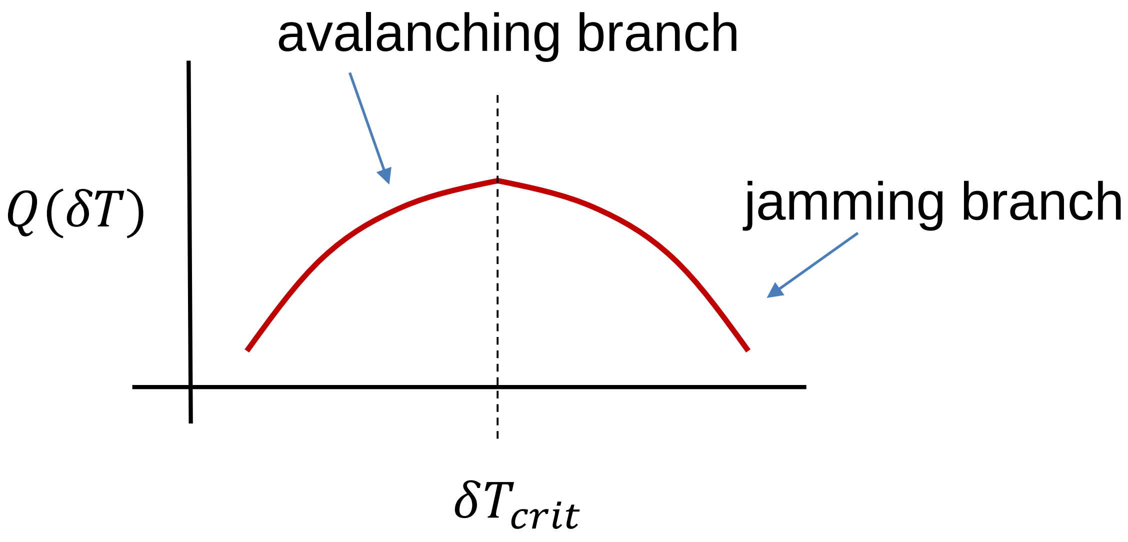
- will have $\delta Q/\delta(\delta T) < 0$ for:
  - $\alpha > 1$
  - $\delta T > \delta T_{crit} = \left(1\sigma/(\alpha-1)\right)^{1/2\alpha}$ → Jamming threshold
- Realizes MIPS in heat flux, for critical avalanche size

# Heat Flux Jams, cont'd

- 'Perturbative bistability' arguably simplest jam mechanism
- Tracks intuition from experience with transport barriers

  New twist: $\delta T_{crit}$

- Consequence of breaking of rescaling invariance of $Q(\delta T)$ – 2 branches



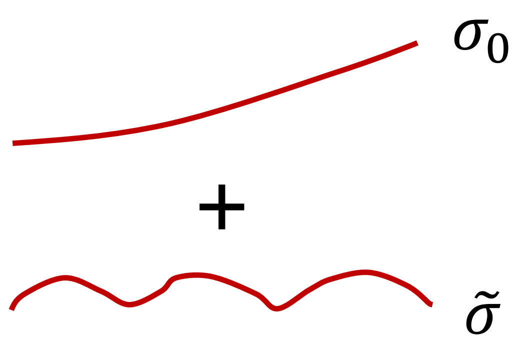

Branch Crossing ←→ Jam Formation

# Jamming Locations?

- $\sigma = \sigma(x)$ → i.e. shearing profile !?
- $\sigma = \sigma_0(x) + \tilde{\sigma}(x)$

slow variation

zonal flow
modulation envelope




- So $\delta T_{crit}$ defined by peaks in $\tilde{\sigma}$ → jamming locations
- $\tilde{\sigma}/\sigma_0$ , $\tilde{\sigma}(x)$ profile ←→ staircase ?!

# Next: Jam feedback ?!

- shearing field → jam location
- Jams nucleate barriers
- barriers ←→ gradient → shear

Does jam array ultimately lock on to $\tilde{\sigma}(x)$ i.e. ZF modulation pattern?

# Thank You !