Entrainment in Drift Wave Turbulence - A Basic Study

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Wake-Classic Example of Turbulence Spreading

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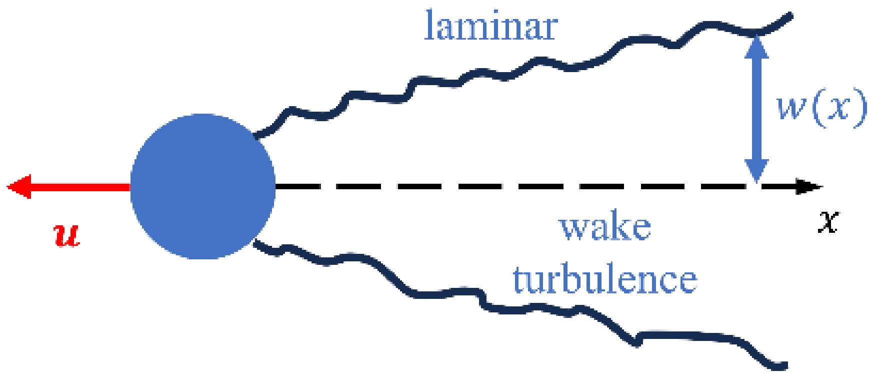


Similarity Theory
Mixing Length Theory }

$W \sim (F_d/\rho U^2)^{1/3} X^{1/3}$,

$F_d \sim C_D \rho U^2 A_s$

C_D independent of viscosity at high Re

⇨ Physics: Entrainment of laminar region by expanding turbulent region. Key is turbulent mixing. ⇨ Wake expands

⇨ Townsend '49:

— Distinction between momentum transport — eddy viscosity—and fluctuation energy transport

— Failure of eddy viscosity to parametrize spreading

— Jet Velocity: $V = \frac{<V_{perp} * V^2>}{<V^2>}$ ⇨ spreading flux FOM

See Ting Long for measurement

Spreading in MFE Theory

⇨ Numerous gyrokinetic simulations

N.B. Basic studies absent ...

$i.e.$

$$\partial_t \xi = \gamma\xi(1-\xi) + \partial_x D(\xi)\partial_x \xi + D_0 {\partial_x}^2 \xi$$

$$\gamma \sim O(\varepsilon)$$

⇨ Diagnosis primarily by:
- color VG
- tracking of “Front”

⇨ Theory
- ⇨ Nonlinear Intensity diffusion models
- ⇨ Reaction-Diffusion Equations - especially Fisher + NL diffusion
- ⇨ Continuum DP Models - Later......

Recently:

⇨ Renewed interest in context of λ_q broadening problem, cf. P.D., Z. Li, Xu Chu

⇨ Simulations measure correlation of spreading $\langle \tilde{V}_r \tilde{p}\tilde{p} \rangle$ with λ_q broadening, cf. N. Li +

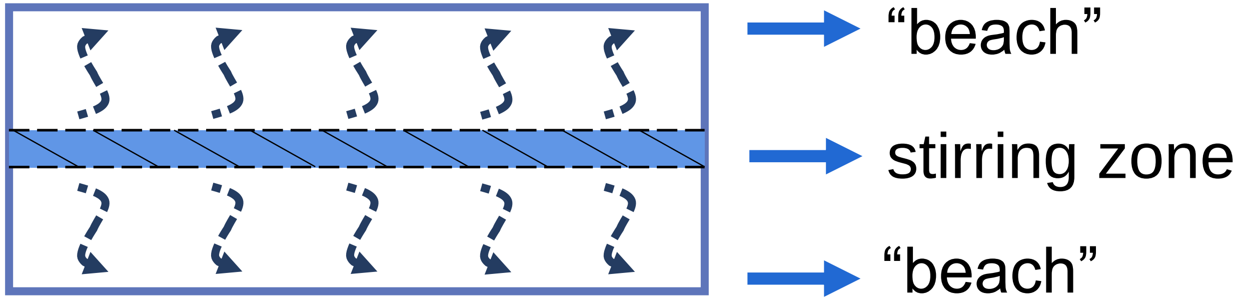
⇨ Intermittency effects T. Wu, P. D. + 2023, A. Sladkomedova 2024

⇕

Especially blobs, voids

Spreading Studies - Numerical Experiments

⇨ 2D Box, Localized Stirring Zone



⇨ Comparison of:

System	Features
2D Fluid	Selective Decay, Vortices How to Measure Spreading?
2D MHD with weak $\underline{B_0}$ perp.	Alfvenization, Vortex Bursting, Zeldovich number
Forced Hasegawa-Mima with Zonal Flow	Waves + Eddies + ZF Multiple regimes and Mechanisms

N.B. Clear distinction between "spreading" and "avalanching"

Numerics: 2D Dedalus simulation

Box Characteristics:

- Dedalus Framework analogous to BOUT++

- Grid Size: 512×512
- Doubly Periodic boundary condition, beach regulates expansion

Forcing Characteristics:

- Superposition of Sinusoidal Forcing, vorticity
- Spectrum: Constant $E(k)$, ensuring uniform energy distribution across wave numbers.
- Correlation Length**:** Approximately $1/10$ of the box scale, some room for dual cascade.
- Localized through a Heaviside step function.
- Phase of forcing randomized every typical eddy turnover time

Summary: 2D Fluid + 2D MHD Studies (R.X., P.D. submitted ‘24)

- Spreading mediated by **dipole vortices**

  2 components

  - Free flyer vortices
  - Turbulent gas/patch
- $W_{FF} \sim t$ (ballistic) – expansion of turbulent layer by dipoles
- FOM – Enstrophy distribution
- No clear ‘front’ - fractalization
- Weak transverse B-field can disrupt vortices, terminate ballistic spreading
- Zeldovich # is good FOM $Z = R_m V_{A,0}^2 / V_{eddy}^2$, $Z > 1$ for disruption
- Disrupted vortices → Alfven waves

Forced Hasegawa – Mima + Zonal Flows

H-M + Zonal Flow System

– System:

PV forced ⇓

$$\frac{d}{dt}\left(\tilde{\phi} - \rho_s^2 \nabla_\perp^2 \tilde{\phi}\right) + v_* \frac{\partial \tilde{\phi}}{\partial y} + v_{*u} \frac{\partial \tilde{\phi}}{\partial y} = \frac{\partial}{\partial r} \rho_s^2 \left\langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \right\rangle + \nu \nabla^2 \nabla^2 (\widetilde{\phi}) + \tilde{F}$$ -Waves, Eddys

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{v}_z \frac{\partial}{\partial y} - \nabla \tilde{\phi} \times \hat{\mathbf{z}} \cdot \nabla$$

$$\frac{\partial}{\partial t} \nabla_x^2 \bar{\phi}_z + \frac{\partial}{\partial r} \left\langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \right\rangle + \mu \nabla_x^2 \bar{\phi}_z = 0$$ -Zonal Flow (Axisymmetric)

N.B. $\bar{\phi}_z = \bar{\phi}_z$(x), only.

N.B. : Electrons Boltzmann for waves, not for Zonal Flow

– viscosity controls small scales

– drag controls zonal flow - μ (large scale)

– conserved: Energy ⟶ $\left\langle \tilde{\phi}^2 + \rho_s^2 \left(\nabla \tilde{\phi}\right)^2 \right\rangle + < \rho_s^2 (\nabla \phi_z)^2 >$

Potential Enstrophy ⟶ $\left\langle \left(\tilde{\phi} - \rho_s^2 \nabla^2 \tilde{\phi}\right)^2 \right\rangle + < (\rho_s^2 \nabla^2 \phi_z)^2 >$

(Waves) (ZF)

N.B. Energy, Pot Enstr. exchange between Waves and ZF possible.

H-M + Zonal Flow System, cont'd - channels

→ Now:

waves $\omega = \omega_* / (1 + k_\perp^2 \rho_s^2)$, $\underline{v_{gr}}$

eddies $\tilde{v}$

zonal mode (symmetry)

$\left\{ \begin{array}{l} \tilde{v} \text{ vs } v_* \rightarrow \\ \text{mixing length} \end{array} \right.$

i.e. ⇒

Energy Flux has two components:

$\left\{ \begin{array}{l} \sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}} \rightarrow 2^{nd} \text{ order in } e\tilde{\phi}/T \\ \langle \tilde{v}_r \xi \rangle \rightarrow 3^{rd} \text{ order in } e\tilde{\phi}/T \end{array} \right.$

N.B. 2 channels for "turbulence spreading" → Waves/Wave transport; Turbulent mixing

-Branching ratio, vs. Ku number ?

→ Multiple channels rarely discussed together

Channels, cont'd:

⇨ **Spreading in presence of fixed, externally prescribed shear layer**

⇨ **Here:** → Forcing → $\left\{\begin{matrix}\text{Waves}\\ \text{Eddies}\end{matrix}\right\}$ → Zonal flow (self-generated)

∴ forcing ($\tilde{v}_{rms}, Re$) + drag ⇒ control parameters

⇨ "weak" and "strong" Turbulence Regimes

coherency factor

v_{gr} VS v_r → $\frac{\langle \tilde{v}_r \xi \rangle}{\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}}}$ → $\frac{\tilde{v}_r \tau_c f}{\Delta_c}$ → Ku ⟺ 2nd vs 3rd order energy flux

$\Delta_c \sim v_{gr} \tau_c$

⇨ $Ku < 1$ → wave dominated spreading

$Ku > 1$ → mixing dominated spreading ⟹ ~ 2D fluid

→ Dipoles gone – density gradient

Channels, cont'd

But → Enter the ZONAL FLOW

- Multiple channels for NL interaction
- But with ZF ⟷ eddy, wave coupling to ZF dominant
- ZF is the mode of minimal inertia, damping, transport

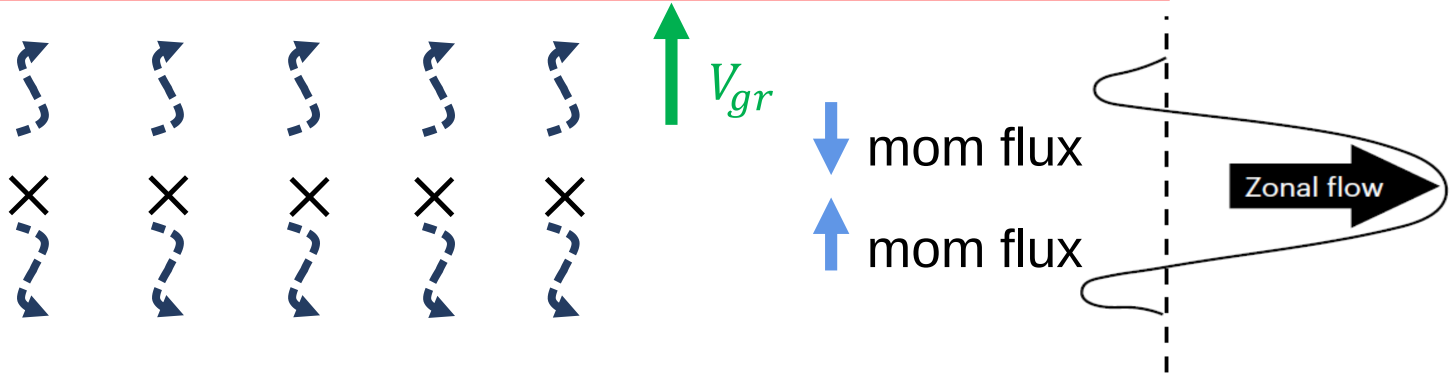
Waves:

$$\frac{\partial}{\partial t}(1 + k_\perp^2 \rho_s^2)\tilde{\phi} = \ldots\ldots$$

ZF:

$$\frac{\partial}{\partial t}(k_r^2 \rho_s^2)\bar{\phi}_z = \ldots\ldots$$

⇒ energy coupled to ZF ($\tilde{v}_r = 0$) cannot "spread", unless recoupled to waves



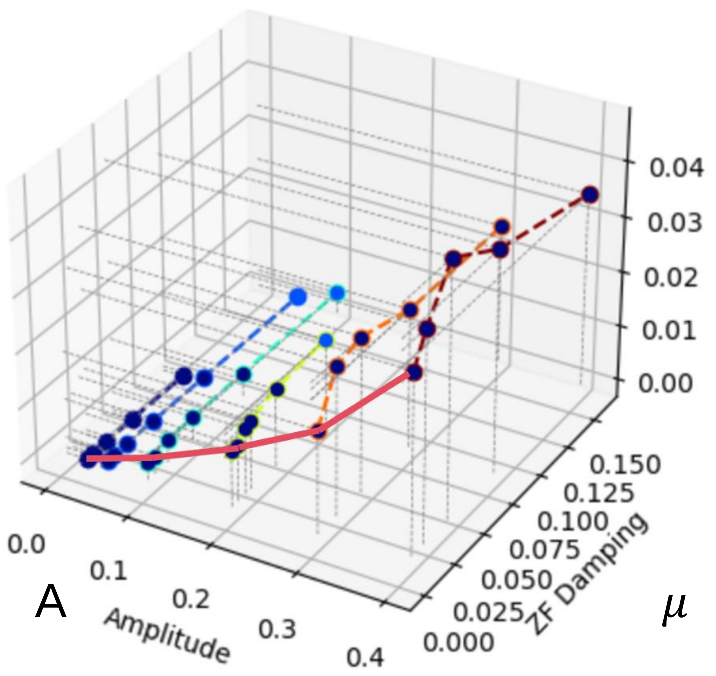
→ Degradation of ZF (back transfer) is crucial to spreading

→ ∴ μ must regulate spreading. What of $\mu \to 0$ regimes?

→ Revisit collisionless NL dissipation problem

Results

FOM – Fluctuation Potential Enstrophy Flux



“Dimits” Regime

averaged spatially and temporally

Turbulent Mixing

— Potential enstrophy flux generally <u>increases</u> as drag increases. “Dimits regime” for turbulence spreading. Spreading diminishes with power coupled to Z.F. (Fixed, spatially)

— <u>Z.F. is self-generated barrier to spreading</u>

— For A increasing, PE flux rises sharply for weak ZF damping. Fate of ZF? “KH-type” mechanism loss of Dimits regime at higher A? Characterization??

N.B. “Dimits Regime”= Condensation of energy into ZF for weaker forcing.

Results, Cont'd
Wave Energy Flux

Wave Energy Flux $< -\frac{\partial\phi}{\partial t}\nabla\phi >$ ⟷ $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k})E_{\boldsymbol{k}}$

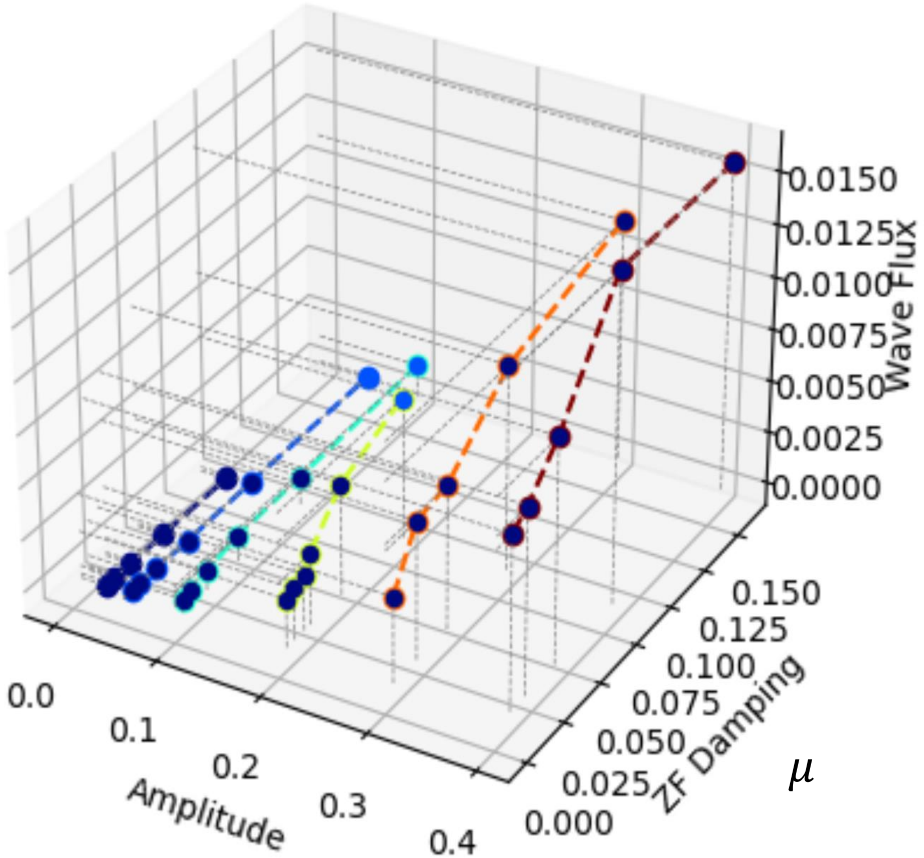
for drift waves

– Dimits regime at low forcing and ZF damping

–Increases with ZF damping and forcing amplitude

– Dominant K_x increases due ZF decorrelation

– Spectrum condensation towards low k with inverse cascade



implication for v_{gr} and $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k})E_{\boldsymbol{k}}$

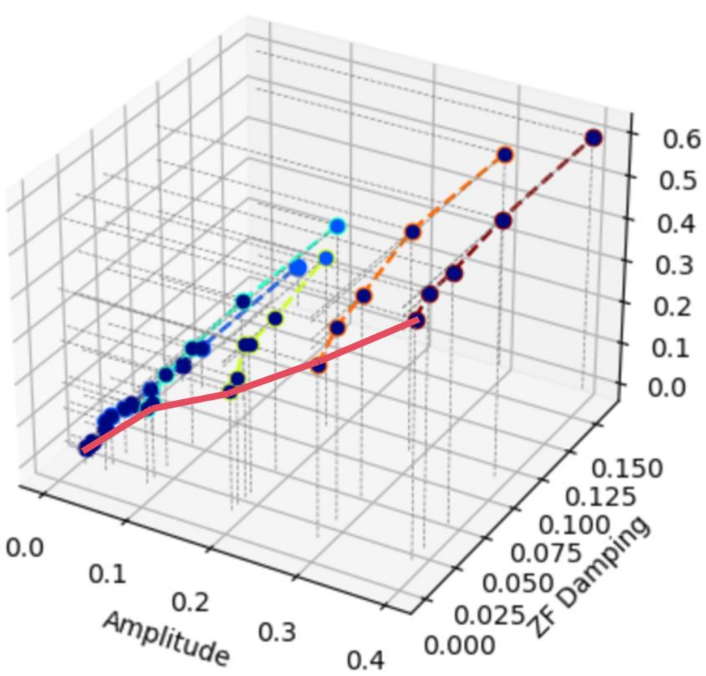
– Take note of increasing W.E.flux as $\mu \rightarrow 0$,
A increases.

Physics: ZF shears refract waves

Results, Cont'd

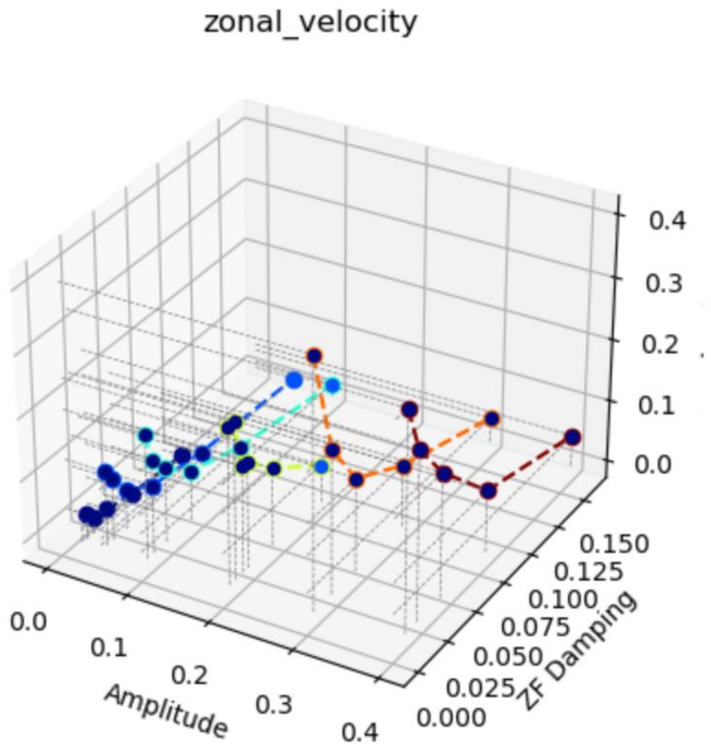
$\frac{\tilde{v}_r \tau_c f}{\Delta_{c_c}}$ where $\Delta_c \sim \langle K_x^2 \rangle^{-1/2}$

Kubo Number



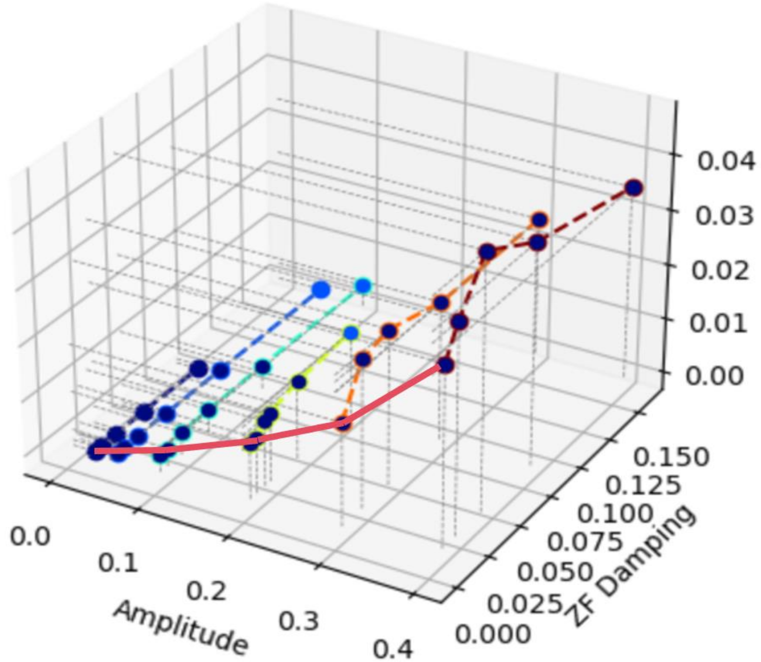
Fluctuation intensity increases as drag increases, A increases

Kubo # tracks mixing
Control parameters set Ku



Zonal velocity decreases with increasing drag (clear)

→Spreading and Fate of Zonal Flows



→ Spreading rises for increased forcing, even for $\mu \to 0$

→ Dimits regime destroyed. How?

⇒ NL back-coupling from ZF necessary for spreading in systems with ZF

→ Animal Hunt for linear instabilities(KH, Tertiary …) seems pointless in turbulence

→ Instead, $P_{\mathrm{Re}} = -\langle \widetilde{V_x}\widetilde{V_y} \rangle \cdot \dfrac{\partial \overline{V}_y}{\partial x}$ ➔ Power transfer [fluctuations → flow]

$P_{Re} < 0$: Wave → ZF transfer

$P_{Re} > 0$: ZF → Wave transfer ⇒ ZF decay

Aside:

— Of course, evokes 'happy memories' of studies of limitation of Dimits shift in G.K.

— But mere identification of 'Tertiary Instability', "R-K." etc not useful

— Seek insight to and quantification of return of energy from Z.F. to turbulence, as control parameters scanned ➔ Reynolds Power density

— Goal is nonlinear ZF decay model for improved Predator-Prey system

N.B. Reynolds power density used widely in data analysis

Quantifying Wave-ZF Power transfer

$$1/2 * \frac{\partial \overline{V}_y^2}{\partial t} = \omega_Z < \widetilde{v_x}\widetilde{v_y} > -drag * \bar{V}_y^2$$

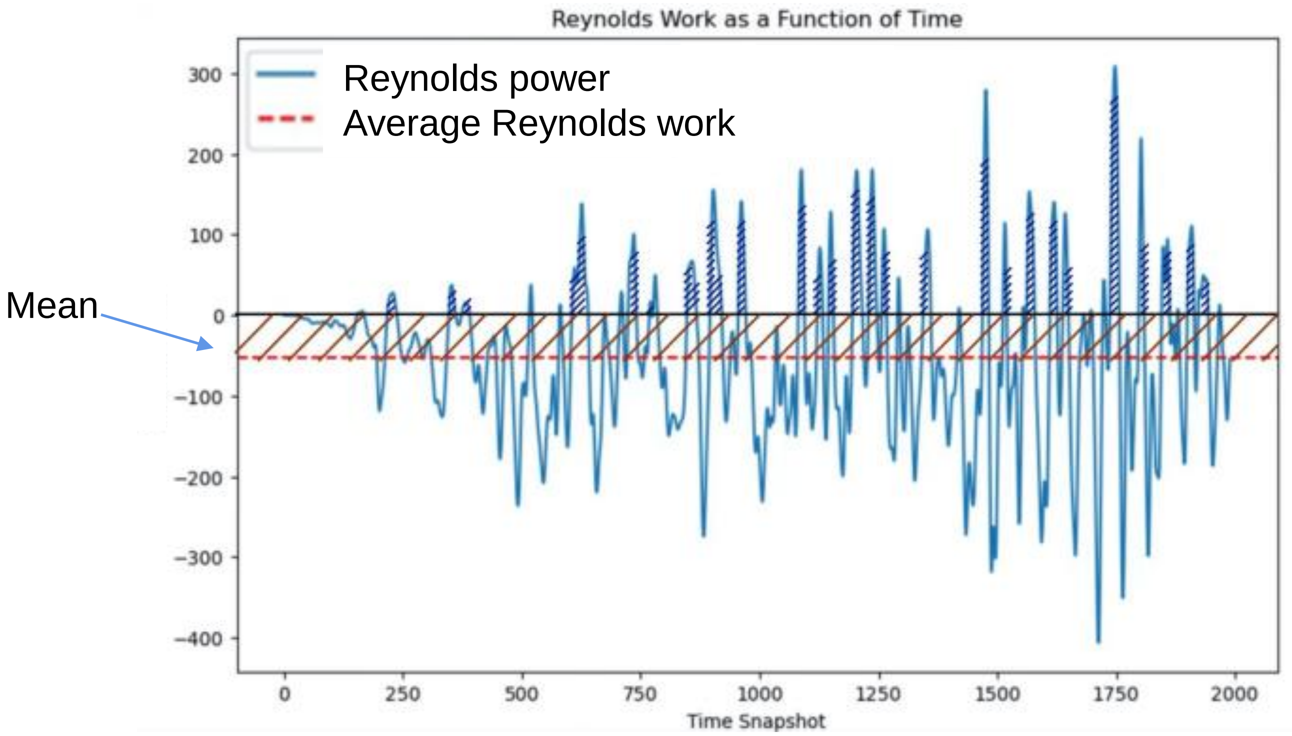
Reynolds power

We quantify ZF → Waves Power Transfer as the ratio of the area above the axis to mean work done on the zonal flow.

N.B.:

$$P_{\mathrm{Re}} = -\langle \widetilde{V_x}\widetilde{V_y} \rangle \cdot \frac{\partial \overline{V}_y}{\partial x}$$

'Turbulent viscosity' model fails capture 2 signs



Reynolds power vs time

$P_{Re} < 0 \Rightarrow$ Wave → ZF transfer

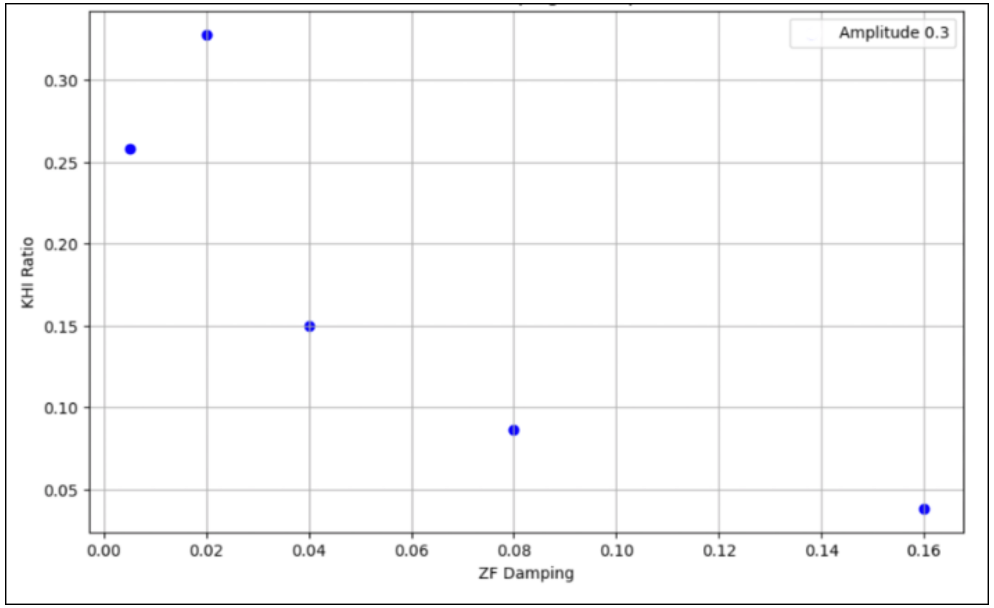
$P_{Re} > 0 \Rightarrow$ ZF → Wave transfer

Positive Reynolds power spikes → zonal flow vortex shedding events ?!

Results, Cont'd

P_{Re} *ratio vs ZF damping*

Dimits Regime



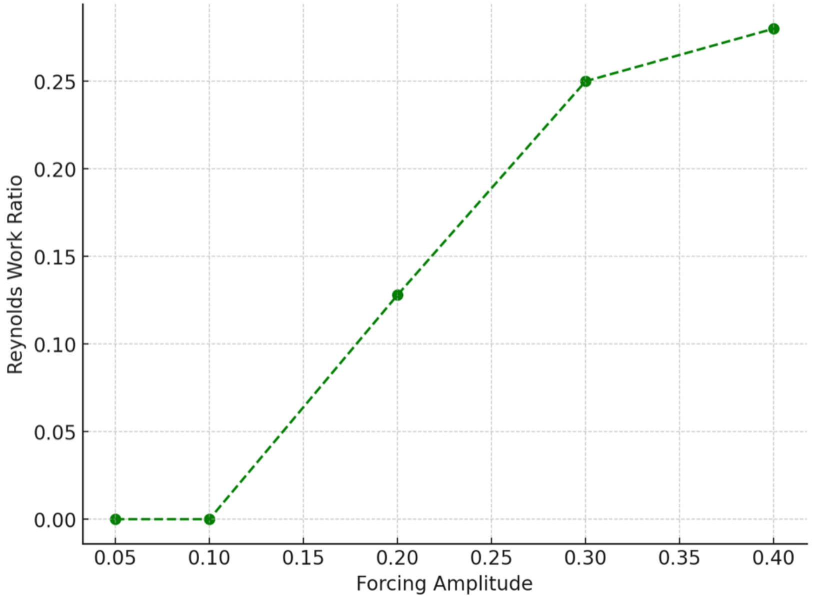
— The ratio generally decreases as a function of ZF damping

⟺ Damped Zonal Flow More Stable.

∴ Fewer Re spikes, as fewer vortex shedding events

Results, Cont'd, P_{Re} Ratio vs Forcing Strength

P_{Re} *ratio vs forcing amplitude*

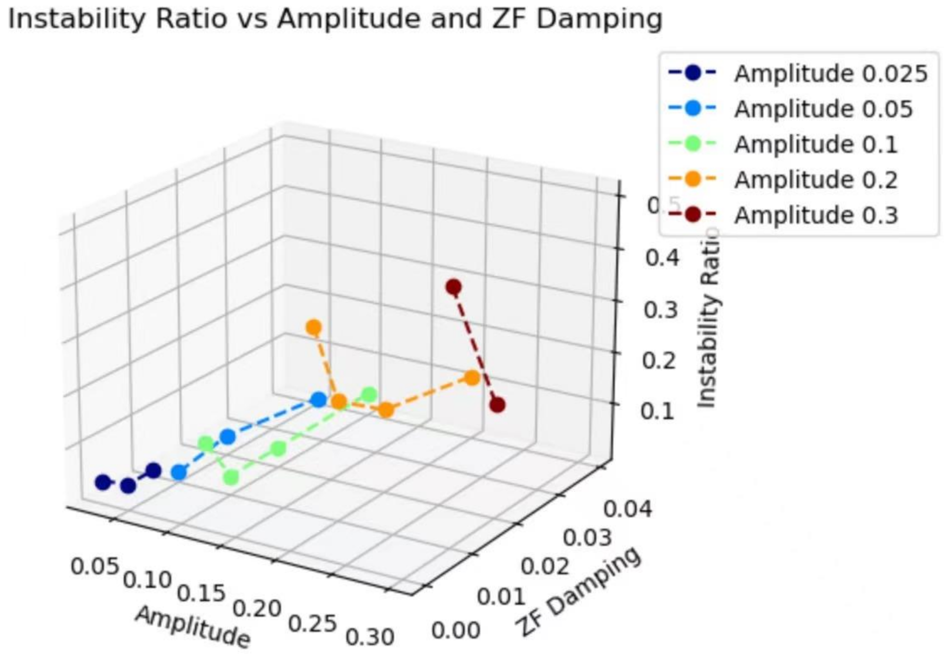


Preliminary
→ Explore other FOMs

- The ratio increases as a function of forcing strength
- Indicates that re-coupling of ZF energy to turbulence increases for stronger forcing
- This approach avoids instability morass → amenable to parametrization

➔ Significant nonlinear recoupling of energy to waves

P_{Re} Ratio vs A, μ

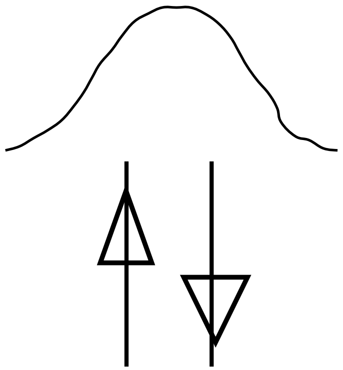


- P_{Re} back transfer increases with forcing, and as μ decreases

- Further analysis required

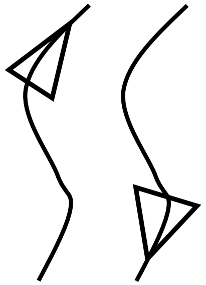
Related Problem: Jet Migration(Laura Cope)

i.e. - Here: $\mathcal{E}$



⟹ turbulence patch propagates,
drags ZF/Jet along, by generation
Near, but not at, Dimits regime?

- There:



⟹ Jet migrates
but Migration enabled by dynamics of fluctuation
field, especially zonon

⟹ Zonon → low mode # fluctuation co-located with Z.F.
→ necessary broken symmetry → propagation

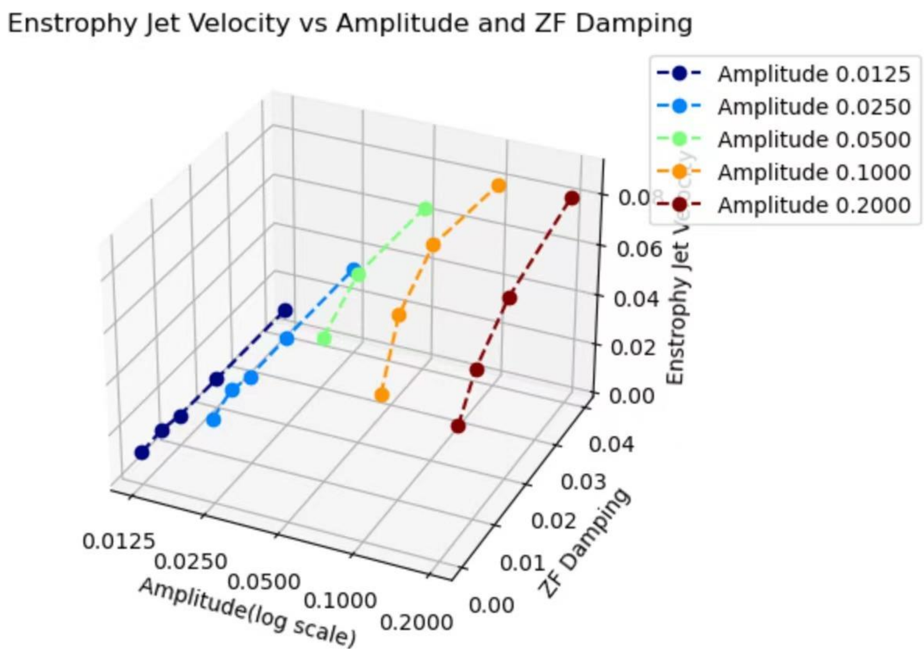
$V_{drift} \sim \mu^{0.7} \epsilon^{0.3}$ Microscopics?

So Jet Velocity !?

→ As waves/eddys drag along zonal flow, Jet velocity(ala' Townsend) is related to Jet Migration.

so

→ Enstrophy Jet Velocity?!



$V_{jet} = \langle v_r \tilde{u}^2 \rangle / \langle \tilde{u}^2 \rangle$ - not experimentally accessible

Long +

$V_{jet} = \langle v_r \tilde{n}^2 \rangle / \langle \tilde{n}^2 \rangle$

- Now familiar trends

- Seems semi-quantitatively consistent with Cope results.

Summary - Drift Wave Turbulence

→ Spreading fluxes mapped in forcing, ZF damping parameter space

→ Dominant mechanism ⟷ Ku (waves vs mixing) , Both waves and mixings in play.
What of Ku ~ 1? Interplay ?!

→ Dimits-like regime discovered. Stationary ZF pattern.

→ ZF disruption and quenching intimately linked to spreading

→ $P_{Re} > 0$ bursts track breakdown of Dimits regime and onset turbulent mixing
Spreading increases.

→ $P_{Re} > 0$ bursts likely due vortex shedding by zonal flows

→General Summary

→ Coherent structures dipoles frequently mediate spreading

←→ underpin “ballistic scaling”

→ Spreading dynamics non-diffusive; Conventional wisdom misleading, or worse.

→ In DWT, wave propagation and turbulent mixing both drive spreading

→ ZF quenching critical to spreading in DWT. Power coupling most useful to describe ZF quench.

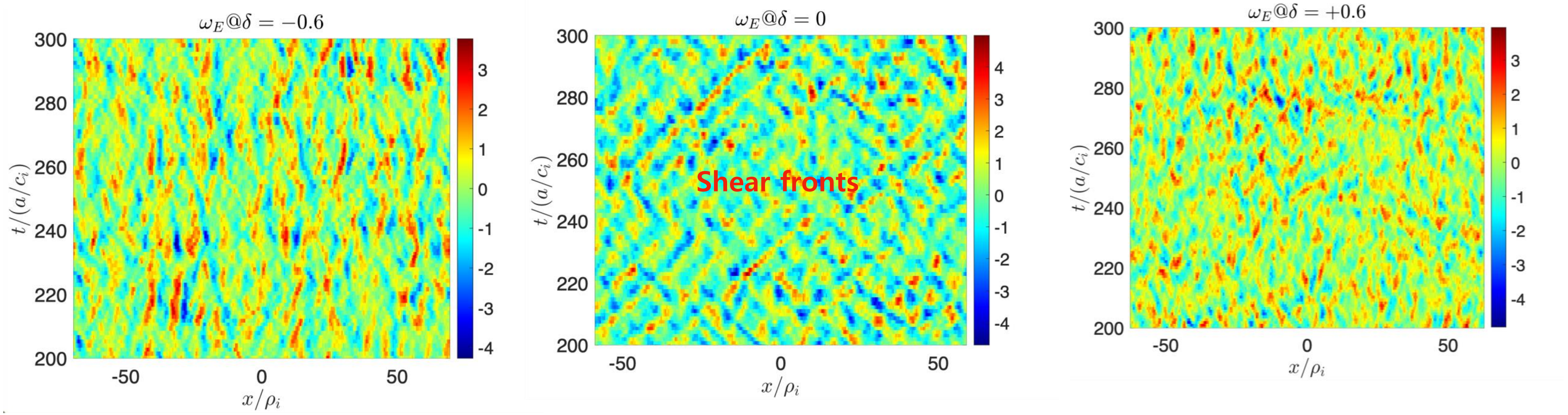
→ Closely related to jet migration.

So: " The more things change, the more they stay the same" – J.-B. Karr (1984)

- Collisionless, nonlinear damping/saturation of Z.F. remains poorly understood.
- Little progress beyond linear zoology, circa 2000. "Undead" theoretical question.
- Improved confinement in N.T. is related (R. Singh, P.D. submitted '24). Collisionless GK for ITG using GENE ➔

NT enhanced confinement ←→ ZF resiliency

Zonal ExB shearing rates: spatiotemporal features



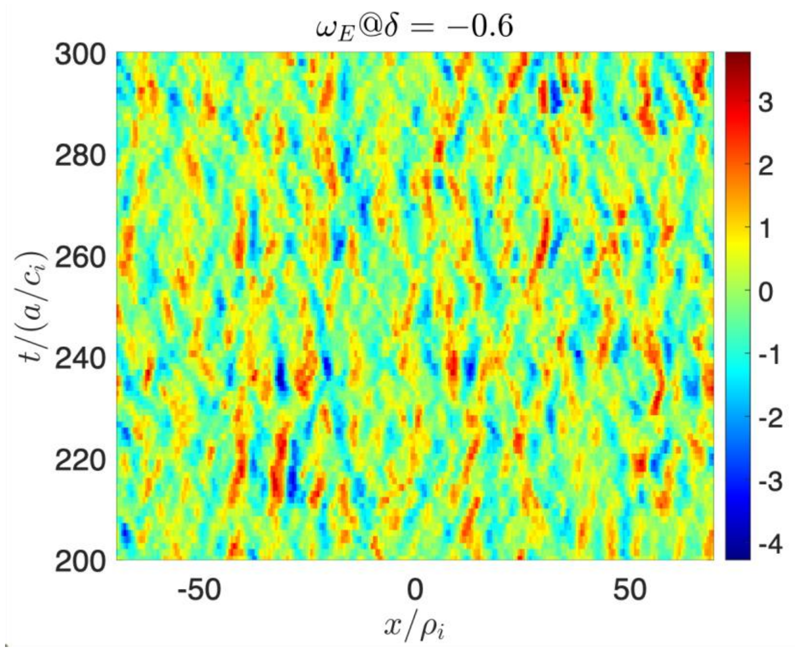
- Spatiotemporal patterns are highly sensitive to δ.
  - **Spatiotemporal shearing pattern more coherent for NT than for PT.**
  - Propagating shearing fronts → dispersive feature for $\delta = 0$! Front speed $\sim 2.25\rho_\star v_{th}$.
- **More coherent spatiotemporal shearing pattern for NT → Stronger mean shearing effect for NT.**

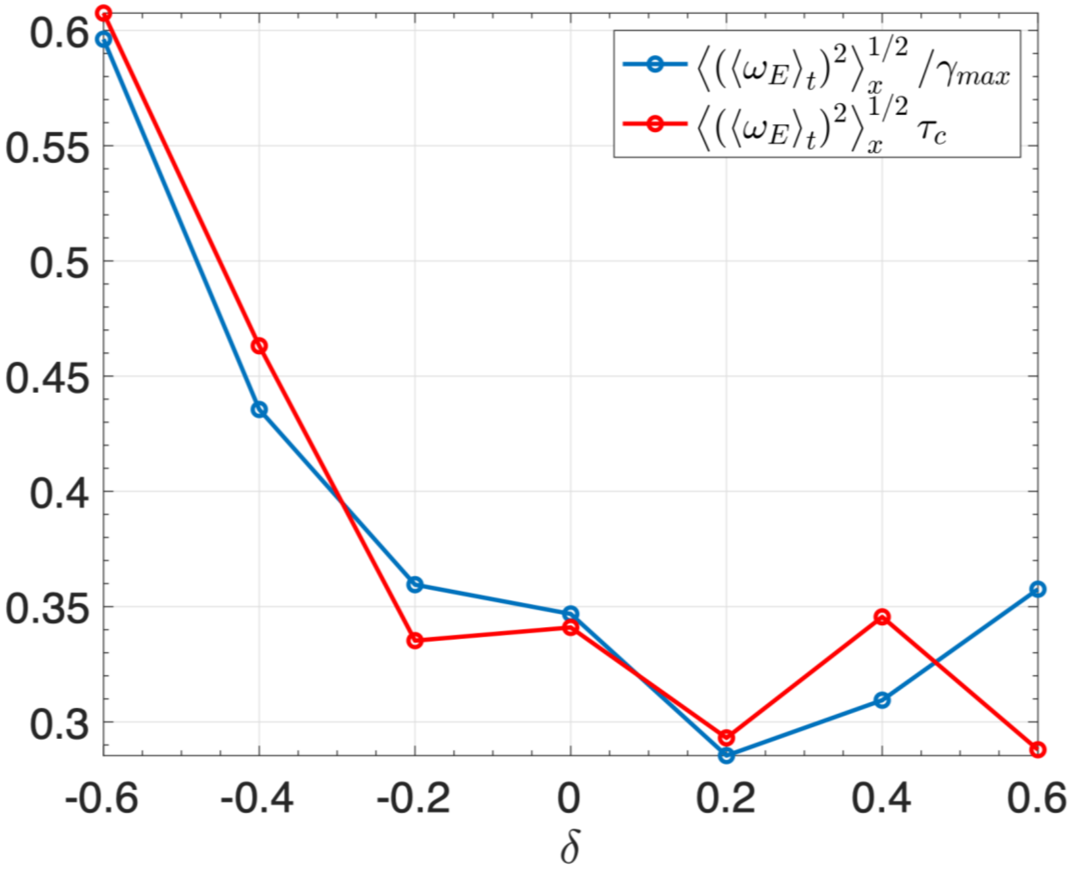
Does NT reduce power transport out of Z.F.?

Figure of Merit

- All analyses point at the dimensionless parameter $\omega_E \tau_c$ or ω_E/γ_{max} as *figure of merit*.

- $\omega_E \tau_c$ higher for NT than for PT. Nicely correlates with the δ -trend of heat diffusivity.





→Future Plans

— High resolution studies
— Understand ZF quenching physics and calculate power recoupling
— What is physics of P_{Re}>0 bursts? - shedding? – how quantify?

— Spreading vs Avalanching. Relative Efficiency? Spreading and Transport?

More general:
— Is spreading mechanism universal? Seems unlikely
— Towards a model… Ku~1 is an interesting challenge
— Relation/connection of DW+ZF spreading and Jet Migration (L. Cope)
— Is Directed Percolation of any use in this?
Details-??