Ion Heat and Momentum Transport in Stochastic Magnetic Fields

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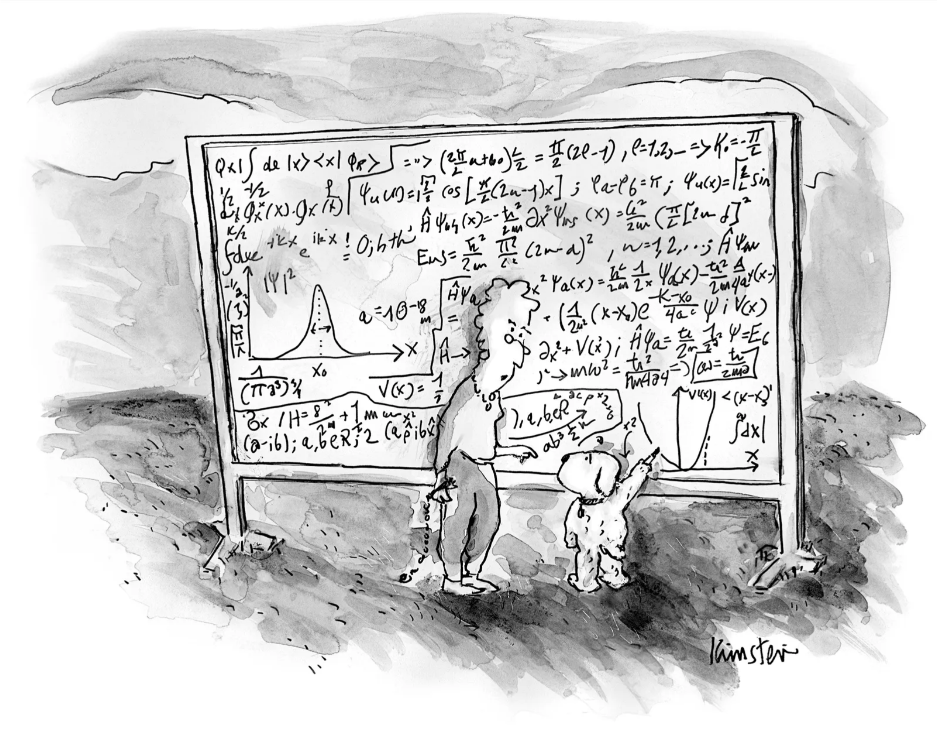
Bad dog! I said “Sit up”, not “Write Quantum Equations!”



Outline

- Short OV of Turbulence in Stochastic Magnetic Fields
- Why?
- Background: Conventional Wisdom and the Kinetic Stress
- What? : 'Dual Problem' →

  Stochastic field-induced-transport <u>in</u> Turbulence
- How? Heuristics and the Crank
- The Physics and its Implications
- Revisiting an Assumption

Overview: Turbulence in Stochastic Magnetic Field

- Deep, complex problem… c.f. Tokuzawa, this meeting as example
- Relevance increasing, due boundary control i.e. reconcile confinement and power handling…
- This session: 4 related talks by P.D., Samantha Chen, Weixin Guo, Mingyun Cao
- Questions: How do the pieces fit together?

  What is the difference?
- N.B.: Focus is on flow and ion physics! $\rightarrow \langle E_r \rangle$

Overview, cont'd

- **P.D.**
  - Parallel flows and heat
  - Stochastic field effects in presence turbulence (dual to usual problem)
- **Samantha Chen**
  - Zonal and perpendicular flows → saturation
  - Dephasing criterion for Reynolds stress due stochastic fields
- **Weixin Guo**
  - Mean field theory for $\langle V_E \rangle'$ → transitions
  - How $\langle \tilde{b}^2 \rangle$ modifies radial force balance?!
- **Mingyun Cao**
  - Interaction of stochasticity with instability mechanism
  - Development of $\langle \tilde{b} \phi \rangle \neq 0$

➔ **All interconnected**

Why? Heat, Momentum Transport meet $\langle \widetilde{B}^2 \rangle$

- Cast of thousands: Electron heat transport (c.f. Manz, 2020) — {interesting information
- S. Chen, et. al. (ApJ '20, PoP '21) Stochastic Fields → dephase

Shearing

Stress

need: $k_\perp^2 V_A D_M > 1 / \tau_c \sim \omega_*$ to quench $\langle \tilde{v}_r^c \nabla_\perp^2 \tilde{\phi} \rangle$

Inhibit jets

➔ $P_{crit}\ (n, \langle b^2 \rangle, \cdots)$ for transition

next talk

- But $\langle E_r \rangle = \frac{\nabla P_i}{nq} - \frac{1}{c} \langle v \rangle \times \langle B \rangle$

$\langle v_E \rangle'$ heat, particles $\perp, \parallel$ flows { See W.X. Guo this session

- What of ion heat and (parallel) momentum transport?

Why? Cont'd

- Relevance ➔

  Transitions: L→H with RMP; ITB (islands), Density Limits

  Intrinsic Rotation: H-mode pedestal torque with RMP

Also:

  Stochastic fields probe barrier resilience

Conventional Wisdom I

- Finn, Guzdar, Chernikov '92 (FGC) → canonical "ref.(1)"
  - n_i , $V_\parallel$ evolution in stochastic fields - motivated by rotation damping due EML - (TEXT)
  - Mean field eqns:

$$\partial_t\langle V_\parallel\rangle + \partial_r\langle \tilde{V}_r\tilde{V}_\parallel\rangle = -\frac{1}{\rho}\partial_x\langle \tilde{b}_r\tilde{P}\rangle$$ → kinetic stress

$$\partial_t\langle P\rangle + \partial_r\langle \tilde{V}_r\tilde{P}\rangle = -\rho\, c_S^2\, \partial_r\,\langle \tilde{b}_r\tilde{V}_\parallel\rangle$$

  - QL for 'acoustic wave response' for $\tilde{P}_i$, $\tilde{V}_\parallel$

➔ viscous relaxation time $\tau_l \sim [c_s\, D_M\, /\, l^2]^{-1}$

(V±P variables)

$D_M = \sum_k |b_k|^2 \pi\, \delta(k_\parallel)$, ala' RSTZ '66

i.e. 'acoustic' propagation along stochastic field

Conventional Wisdom I, Cont'd

- Nit
  - Why bother with acoustics ? ➔ static problem

    $\vec{B} \cdot \nabla \tilde{V}_\parallel + \tilde{B} \cdot \nabla \langle V_\parallel \rangle = 0$ and linear response ➔ kinetic stress

    P similarly

- Issue: Structure of fluxes? → Non-Diffusive !

  $$\langle \tilde{b}_r \tilde{P} \rangle = -D_M \frac{\partial}{\partial r} \langle P \rangle, \quad \langle \tilde{b}_r \tilde{V}_\parallel \rangle = -D_M \frac{\partial}{\partial r} \langle V_\parallel \rangle$$

  → Residual Stress, → Convection / Pinch

  drives $\langle V_\parallel \rangle$ Pinch for $\langle P \rangle$ — driven by $\langle V_\parallel \rangle'$

  kinetic stress

More Conventional Wisdom II: Kinetic Stress and Rotation

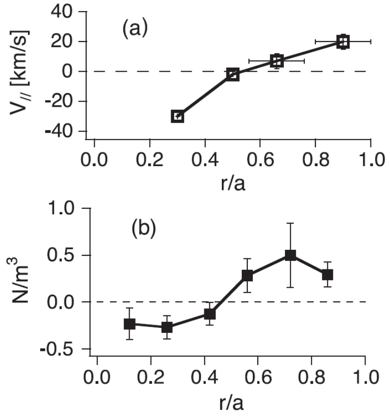
$$\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle = -\frac{c_s^2}{\rho}\, \partial_x \langle b_r P \rangle$$

"kinetic stress"

- W.X. Ding, et. al. PRL '13 – MST Rotation Studies
  - Linked plasma flows in RFP to <u>kinetic stress</u>, via <u>direct measurement</u>
  - Mean flow profile tracks profile of $\nabla \cdot$ (kinetic stress)

➔ Rare and compelling insight into the fluctuation ↔ rotation connection!

i.e. microscopic ↔ macroscopic link



What ? – the Issue

- How calculate the kinetic stress ?
- In QL approach, ala' FGC, seek:

$$\delta P \sim \tilde{b}\,\delta P/\delta b \;\Rightarrow\; \langle \tilde{b}\,\delta P\rangle \sim \langle b^2\rangle$$

But What is in $\delta P/\delta b$?

- In any relevant case, especially prior to L→H transition, turbulence will co-exist with stochastic field

So

- Need calculate kinetic stress in presence of turbulence

What ? Cont'd

- Two 'dual' analyses:
  - Reynolds stress, etc. in background $\langle b^2 \rangle$ → Chen et. al., this meeting
  - Kinetic stress, pinch in $\langle \tilde{V}_\perp^2 \rangle$ background → here
- Expect significant departure from FGC, and from standard quasilinear theory
- Implicit: Statistics $\tilde{b},\ \tilde{V}_\perp$ assumed independent

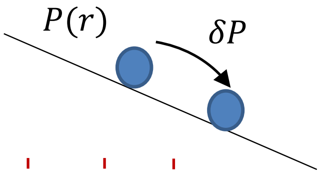
  $\tilde{b}$ $\rightarrow$ RMP induced

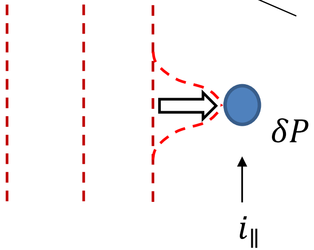
  $\tilde{V}$ $\rightarrow$ drift waves

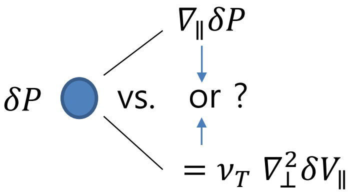
TBC later → see Mingyun Cao, this meeting

- In spirit of resonance broadening, but juicier…

Heuristics







- $c_s \tilde{b}_r\, \partial\langle P\rangle/\partial r \rightarrow \delta P$ – localized slug of pressure

  Tweaking field line produces localized pressure perturbation

- How is pressure balanced along field line? – two possibilities

  i) Build parallel pressure gradient

  $$\nabla_{\parallel}\delta P \sim -\tilde{b}_r\, \partial_r\langle P\rangle \rightarrow \text{FGC}$$

  or

  ii) Drive parallel flow, damped by turbulent mixing/viscosity due $\langle \tilde{V}_{\perp}^2\rangle$

  $$-\nu_T \nabla_{\perp}^2 \delta\tilde{V}_{\parallel} \sim -b_r\, \partial_r\langle P\rangle$$

  ν_T is to be calculated

Different form of fluxes

Critical comparison:

$c_s k_{\parallel}$ vs $k_{\perp}^2 D_T$

The Crank

- Start from $\partial_t V_\parallel$, $\partial_t P$ equations
- Seek $\langle \tilde{b}_r \tilde{P} \rangle$, $\langle \tilde{b}_r \tilde{V}_\parallel \rangle$
- Follow 'quasilinear' approach, BUT
- Posit an ambient ensemble of drift waves, so $\langle \tilde{V}_\perp^2 \rangle$ specified

  Assume $\langle \tilde{V}_\perp^2 \rangle$, $\langle \tilde{b}_r^2 \rangle$ quasi-Gaussian and statistically independent
- Calculate responses $\delta P = (\delta P/\delta b_r)\tilde{b}_r$ and $\delta V_\parallel = (\delta V_\parallel/\delta b_r)\tilde{b}_r$ (to close fluxes),

  by integration over perturbed trajectories, ala' Dupree '66
- $\delta P/\delta b_r$ is statistically averaged, nonlinear response

The Answer: Note turbulence-induced gradient couplings !

- (kinetic stress) $\langle \tilde{b}_r \, \delta P \rangle = -\sum_k |b_{r,k}|^2 \left[\frac{1}{(k_\perp^2 D_T)^2 + k_\parallel^2 c_s^2} \right] \left\{ \rho c_s^2 k_\perp^2 D_T \frac{\partial}{\partial r} \langle V_\parallel \rangle - i k_\parallel c_s^2 \frac{\partial}{\partial r} \langle P \rangle \right\}$

- (convection) $\langle \tilde{b}_r \delta V_\parallel \rangle = -\sum_k |b_{r,k}|^2 \left[\frac{1}{(k_\perp^2 D_T)^2 + k_\parallel^2 c_s^2} \right] \left\{ c_s^2 k_\perp^2 D_T \frac{\partial}{\partial r} \langle P \rangle - i k_\parallel c_s b_{r,k} c_s \frac{\partial}{\partial r} \langle V_\parallel \rangle \right\}$

- $D_T \equiv \int \langle \tilde{V}_r \tilde{V}_r \rangle dt$ ➔ <u>electrostatic</u> turbulent diffusivity

- Response Function: $1 / \left[k_\parallel^2 c_S^2 + (k_\perp^2 D_T)^2 \right]$

- Order of limits important to recover QL results

The Physics

- Limits

  $k_{\parallel} c_s > k_{\perp}^2 D_T$ → weak e.s. turbulence -- narrow regime validity
  n.b. role of anisotropy ! – contrast micro-instability c.f. Lu Wang

  $\langle \tilde{b}_r \delta P \rangle \approx -D_M \, \partial \langle P \rangle / \partial r, \; \langle \tilde{b}_r \delta V_{\parallel} \rangle \approx -D_M \partial \langle V_{\parallel} \rangle / \partial r$

  Recovers FGC. Relevance limited

- $k_{\perp}^2 D_T > k_{\parallel} c_S$ → robust electrostatic turbulence (as for pre-transition)

  $\langle \tilde{b}_r \delta P \rangle \approx -D_{st} \, \partial \langle V_{\parallel} \rangle / \partial r$, $\langle \tilde{b}_r \delta V_{\parallel} \rangle \approx -D_{st} \partial \langle P \rangle / \partial r$

  → $\boxed{\text{Viscosity!}}$ → Thermal diffusivity

  $$\boxed{D_{ST} = \sum_k c_S^2 \left| b_{r,k} \right|^2 / \, k_{\perp}^2 D_T}$$

  diffusivity

- Structure of $\left\{ \begin{array}{l} \text{correlator} \\ \text{fluxes} \end{array} \right\}$ change ! *

The Physics, Cont'd

- Stochastic viscosity/diffusivity is hybrid

$$D_T = \sum_k c_s^2 \left|b_{r,k}\right|^2 / \, k_\perp^2 D_T$$

Magnetic scattering,
with τ_{ck} set
by electrostatics

!

- Pure 'stochastic field' analysis irrelevant to any state with finite ambient electrostatic turbulence, c.f. $k_\parallel c_s$ vs $k_\perp^2 D_T$

- Easily extended to sheared magnetic geometry, etc

i.e. key: w_k vs $X_s = 1/k_\parallel' c_s \tau_{ck}$

$w_k > X_s$ → weak scattering

$w_k < X_s$ → strong scattering

Spatial spectral width

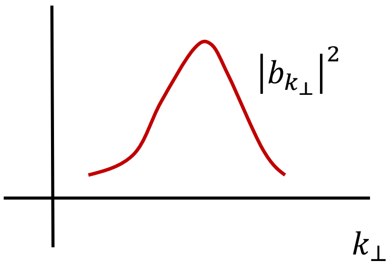
Acoustic point (analogous X_i)

Comments re: Theory

- Yes, resonance broadening, but no – not 'the usual'

  ➔ structure of flux modified – residual stress to viscosity

- Infrared behavior of wave number spectrum important !
  - Low k cut-off $|b_{r_k}|^2$?
  - Not resolved trivially, by geometry
  - Similar: Taylor, McNamara '72 → cut-off and 'locality' ?!
  - ExB shear, even if sub-BDT, can set cut-off ➔ ZF generation will enter….



N.B.:

- For ZF case, comparison is $k_\perp^2 D_T$ vs $k_\parallel V_A$ ➔ W.T. regime relatively more robust
- See Samantha Chen, next talk

Implications → Conclusions

- Pure stochastic models of limited utility for momentum, ion heat, etc.
- Need analyze stochastic field effects in presence of turbulence
- In practice, kinetic stress is stochastic field - induced viscous stress → significant drag on rotation
- $D_{ST} = c_s^2 \sum_k \left|b_{r,k}\right|^2 / \, k_\perp^2 D_T$ → (hybrid) stochastic field viscosity
- See Beyer, et. al. (2000) for hints from simulations

Open Issue

- Development of Correlation? (see Mingyun Cao)
  - are $\tilde{b}$, turbulence uncorrelated ? as assumed ....
  - $\boxed{\underline{\text{No}}}$ → interaction develops $\langle \tilde{b}\tilde{\phi} \rangle \neq 0$ → electrostatics 'lock on' to $\tilde{b}$
  - ala' Kadomtsev – Pogutse, impose $\nabla \cdot \vec{J} = 0$ to all orders
  - novel small scale convection cell, related to $\tilde{b}$ structure

Ongoing …

Open Issue, Cont'd

- Elucidate kinetic stress contribution to intrinsic torque, with RMP.

  Determine flux-gradient relation

- Beyond diffusion – Fractional kinetics with Pdf$(\tilde{V}, \tilde{b})$?

  How formulate?

i.e. avalanches in stochastic field !? (esp. electron thermal)

— testable via simulation ...

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