SOL Broadening by Edge and Pedestal Turbulence: Theory and Experiment of Entrainment Dynamics

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- Experiment: Ting Long, Ting Wu (SWIP), Filipp Khabanov, Rongjie Hong, G. Mckee, Zheng Yan, G. Yu, G. Tynan (DIII-D → Frontiers Exp.), Xi Chen (GA)

Outline

- Brief Primer on the Edge and SOL
- SOL Width Problem and the Physics of the Plasma Boundary Layer
- Turbulence Production Ratio and its Implications → Some Data
- Calculating the Scale of the Spreading-Driven SOL → Some Theory and Computation
- A Closer Look at Turbulence Spreading → More Theory
- Open Issues and Future Plans

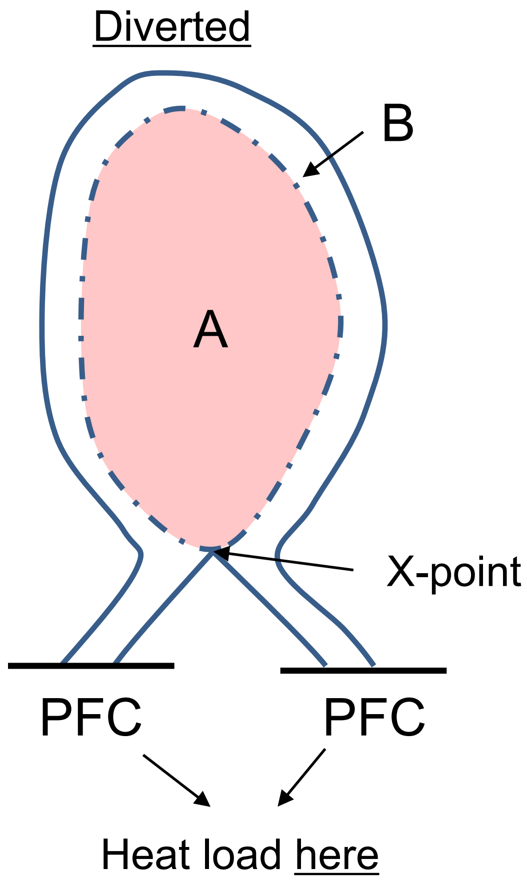
Primer (Brief)

- All confinement devices have an <u>edge</u> and SOL (scrape-off layer)

  Fueling at Edge

- Define:
  - Confined plasma boundary
  - Connection to plasma facing components
  - SOL as confined plasma 'boundary layer'

A – confined plasma
B – SOL
Dashed – separatrix



NB: Magnetic field lines are perp to plane, with slight tilt

Primer, cont'd

- SOL: $\nabla \cdot \vec{\Gamma} = \nabla \cdot \vec{Q} = 0$ (open lines)

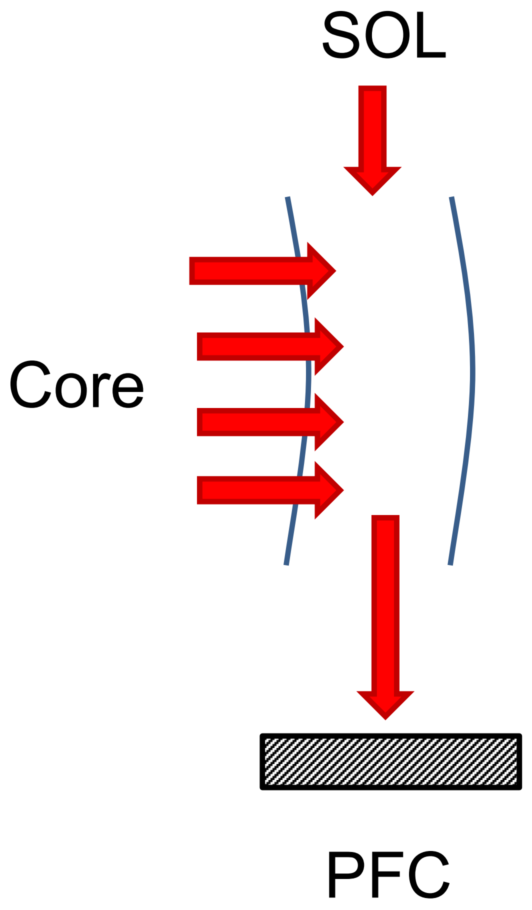
$\Gamma_\perp \approx -D\partial_r n$ (?) $\qquad \nabla_\perp \sim \partial_r \sim 1/\lambda_\perp$

$\Gamma_\parallel \approx \alpha\, c_s\, n$ $\qquad \nabla_\parallel \sim 1/L_c \sim 1/Rq$

➔ $D\, \partial_r^2 n \sim \alpha n/L_c$ $\qquad \tau_\parallel \approx Rq/c_s$

$\lambda_\perp \sim (D\tau_\parallel)^{1/2}$ ~ crude SOL width

↔ $1/\tau_\parallel \sim \chi_\parallel/L_c^2$ conduction, high density



Background

- Conventional Wisdom of SOL:

  (cf: Stangeby...)

  – Turbulent Boundary Layer, ala' Blasius, with D due turbulence

  – $\delta \sim (D\tau)^{1/2}, \tau \approx L_c/V_{th}$

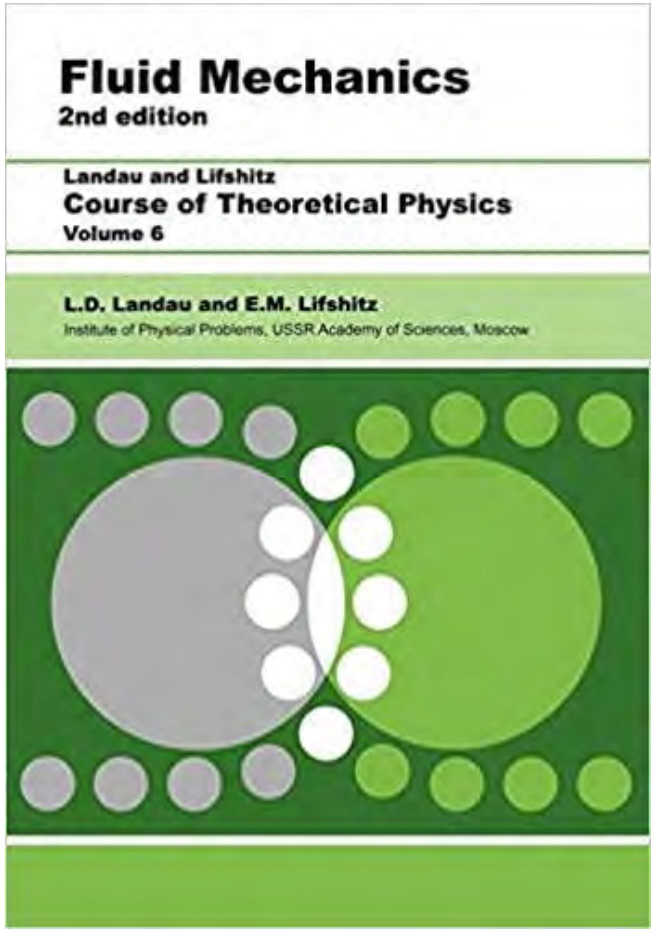
  – $D \;\; \leftrightarrow$ local production by SOL instability process

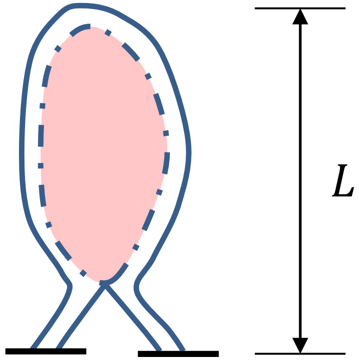
  → familiar approach, D ala' QL, …

- Features:

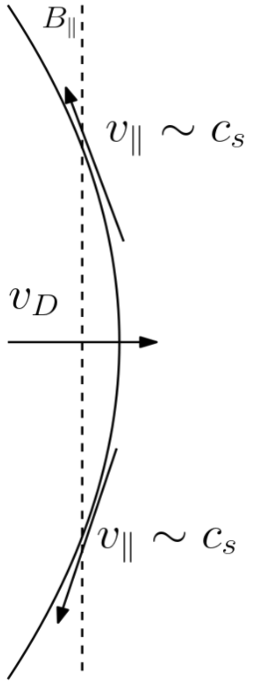
  – Open magnetic lines → dwell time τ limited by transit, conduction, ala' Blasius

  – Intermittency → "Blobs" etc. Observed. Physics?





Background, cont'd



- But... Heuristic Drift (HD) Model (Goldston +)
  - $V \sim V_{\text{curv}}$, $\tau \sim L_c/V_{thi}$, $\boxed{\lambda \sim \epsilon\, \rho_{\theta i}}$ → SOL width
  - Pathetically small
  - Pessimistic B_θ scaling, yet high I_p for confinement
  - Fits lots of data.... (Brunner '18, Silvagni '20)
- Why does neoclassical work? → ExB shear suppresses SOL modes i.e.

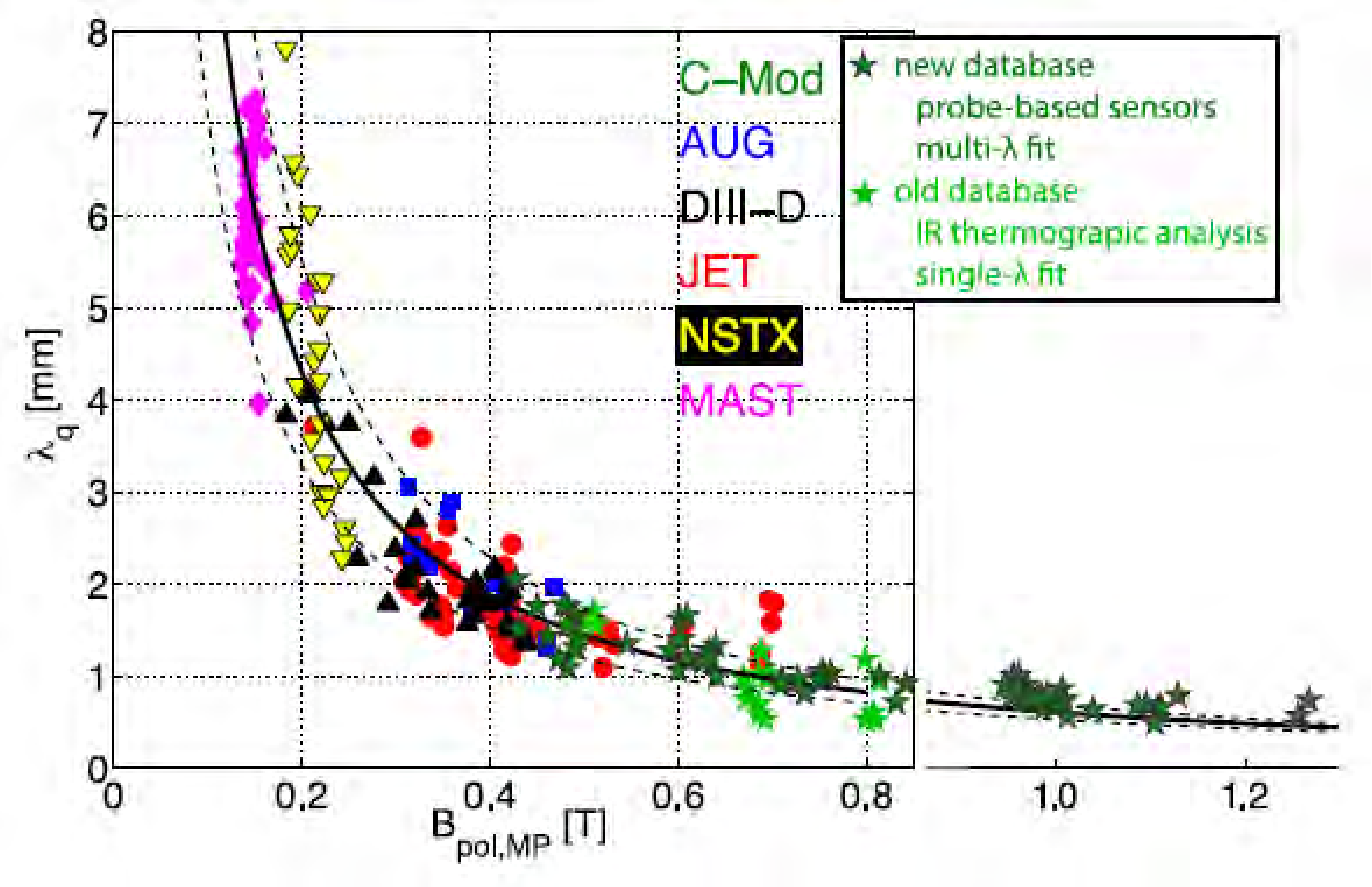
$$\gamma_{\text{interchange}} \sim \frac{c_s}{(R_c\lambda)^{\frac{1}{2}}} - \boxed{\frac{3T_{edge}}{|e|\lambda^2}}$$

shearing ←→ strong λ^{-2} scaling

from: $\frac{c_s}{(R_c\lambda)^{\frac{1}{2}}} - \langle V_E \rangle'$

Feedback Loop:
$\lambda \downarrow \;\rightarrow\; \gamma \downarrow \;\rightarrow\; D \downarrow$

Background: HD Works in H-mode



HD is Bad News…

Background, cont'd

- THE Existential Problem... (Kikuchi, Sonoma TTF):

Desire
- Confinement → H-mode ←→ ExB shear
- Power Handling → broader heat load, etc

→ Both to be good !

How reconcile? – Pay for power mgmt with confinement ?!

- Spurred:
  - Exploration of turbulent boundary states with improved confinement: Grassy ELM, WPQHM, I-mode, Neg. D ... N.B. What of ITB + L-mode edge?
  - SOL width now key part of the story
  - Simulations, Visualizations (XGC, BOUT...) ~ "Go" to ITER and all be well
- But... What's the Physics ?? How is the SOL broadened?

SOL Boundary Layer:

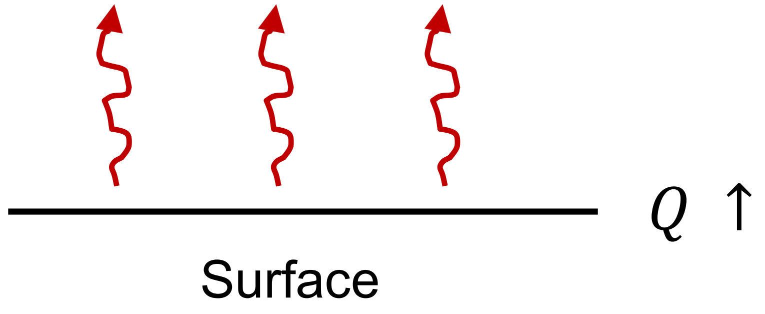
Turbulence Production Rate and the Role of Spreading

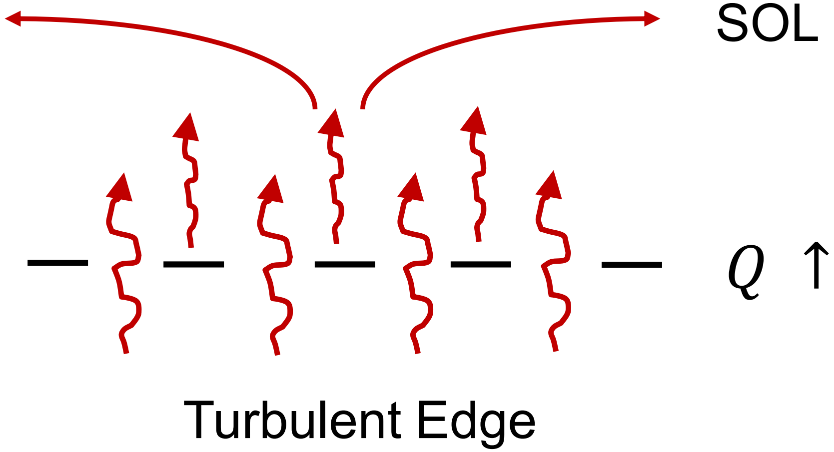
SOL BL Problem

- Classic flux-driven BL problem
  - Heat flux at surface drives
  - Production $= gQ \quad \tilde{V}_E \sim (gQz)^{1/3}$ etc
  - Plumes

  Adapt to SOL ?

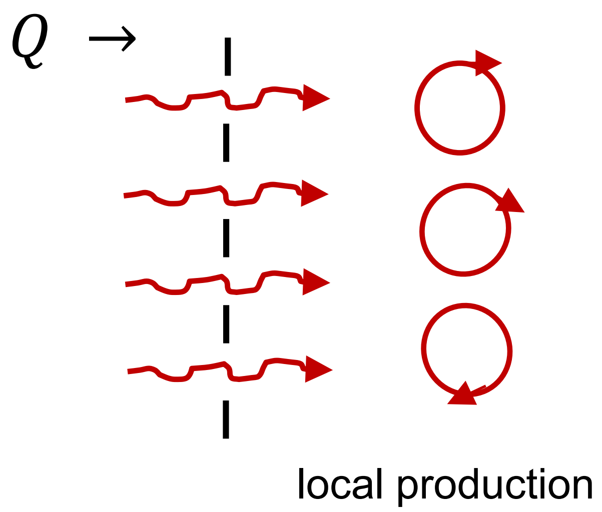
- SOL
  - Open field lines
  - Turbulent energy flux and heat flux, etc drive
  - Turbulence spreading (Garbet, P.D., Hahm, …)
  - Includes 'blobs' – c.f. Manz, 2015





SOL BL Problem

- SOL Excitation
  - Local production (SOL instabililties)
  - Turbulence energy influx from pedestal
- Key Questions:
  - Local drive vs spreading ratio → Ra
  - Is the SOL usually dominated by turbulence spreading?
  - How far can entrainment penetrate a stable SOL → SOL broadening?
  - Effects ExB shear, role structures ?



Physics Issues – Part I

- Measure and Characterize Turbulence Energy Flux at LCFS
- Determine Relative Contributions of :
  - Influx/Spreading thru LCFS
  - SOL Production

R_a → Production Ratio

- Trends in λ_q and R_a vs : ExB shear, 'Blob' Fraction...
- Question: To what extent is SOL turbulence usually spreading driven?

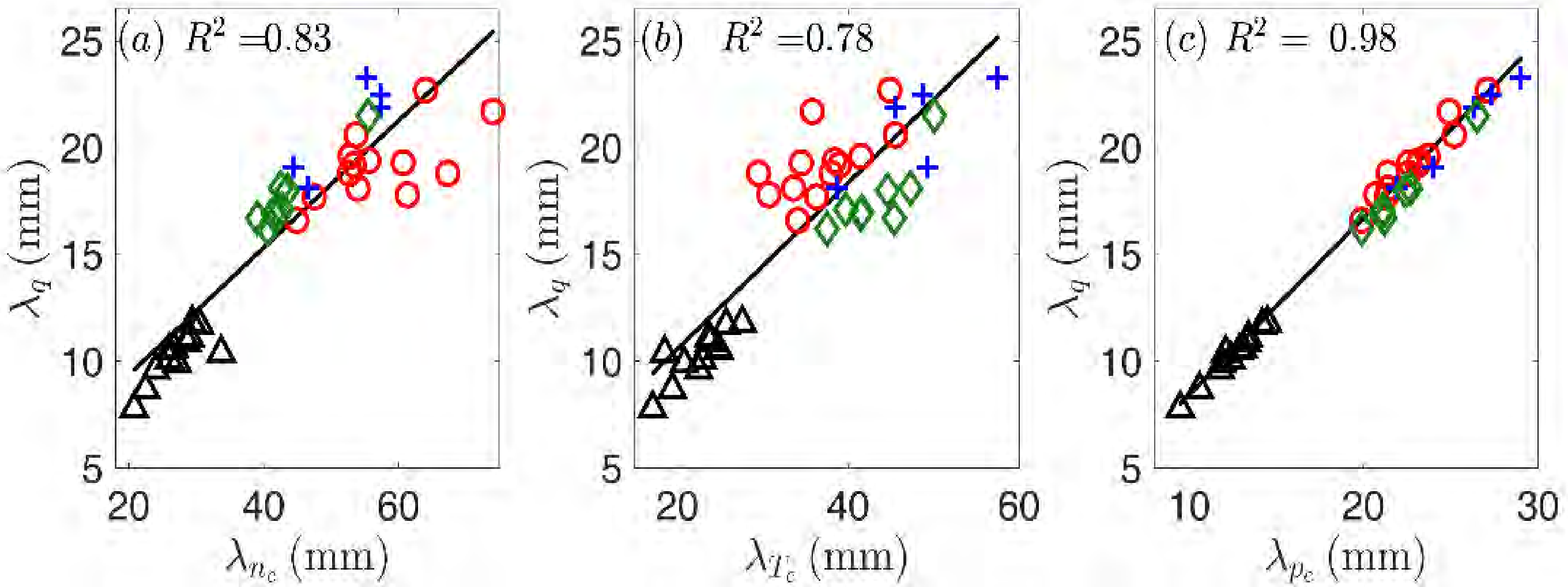
→ Phenomenology... (see Ting Wu +, NF 2023)

Experiments and Data Set

N.B.:
λ_q → SOL width

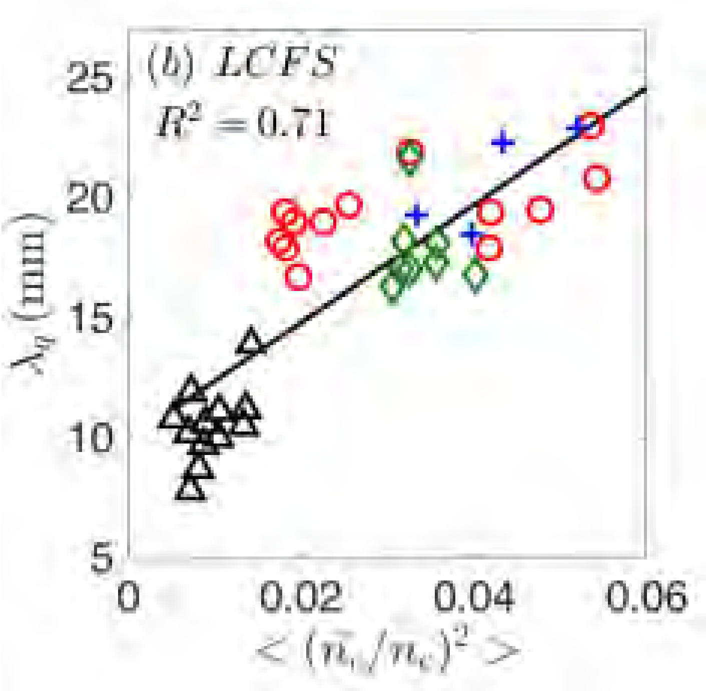
- HL-2A limited OH plasmas – classic “boring plasmas”
- Reciprocating probe array ←→ Outboard mid-plane
- $q_{\parallel} = \gamma J_{sat} T_e$, $\gamma \equiv$ sheath transmission coefficient
- Database: ‘Garden Variety OH’ ~ 150 kA, 1.4T
- 4 parameter subgroups ○ red circle + blue cross ◇ green diamond △ black triangle
- Similar, with $\lambda_q \gg \lambda_{HD}$, except: black triangles △
  - $\lambda_q > \lambda_{HD}$, not $\gg$
  - Significant GAM activity → stronger ExB shear

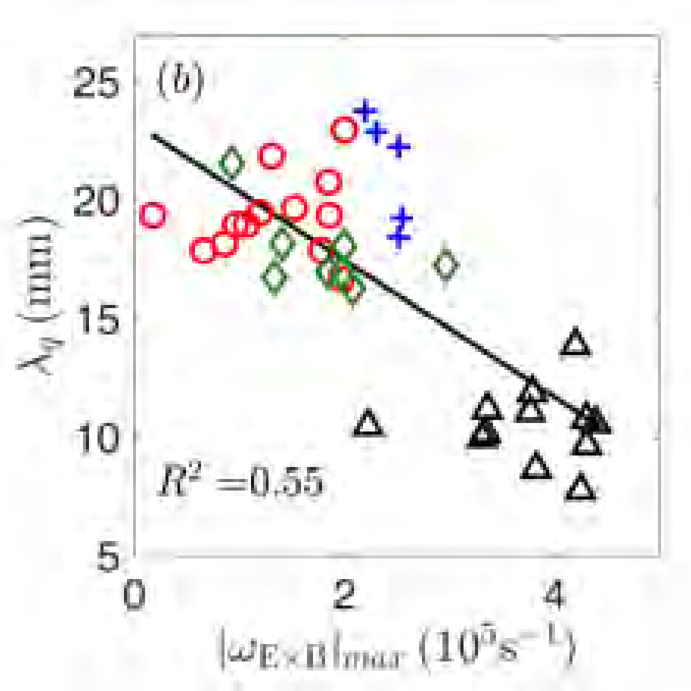
$$\lambda_{n_e} \sim \lambda_{T_e} \sim \lambda_{P_e}$$



All SOL profiles scales comparable

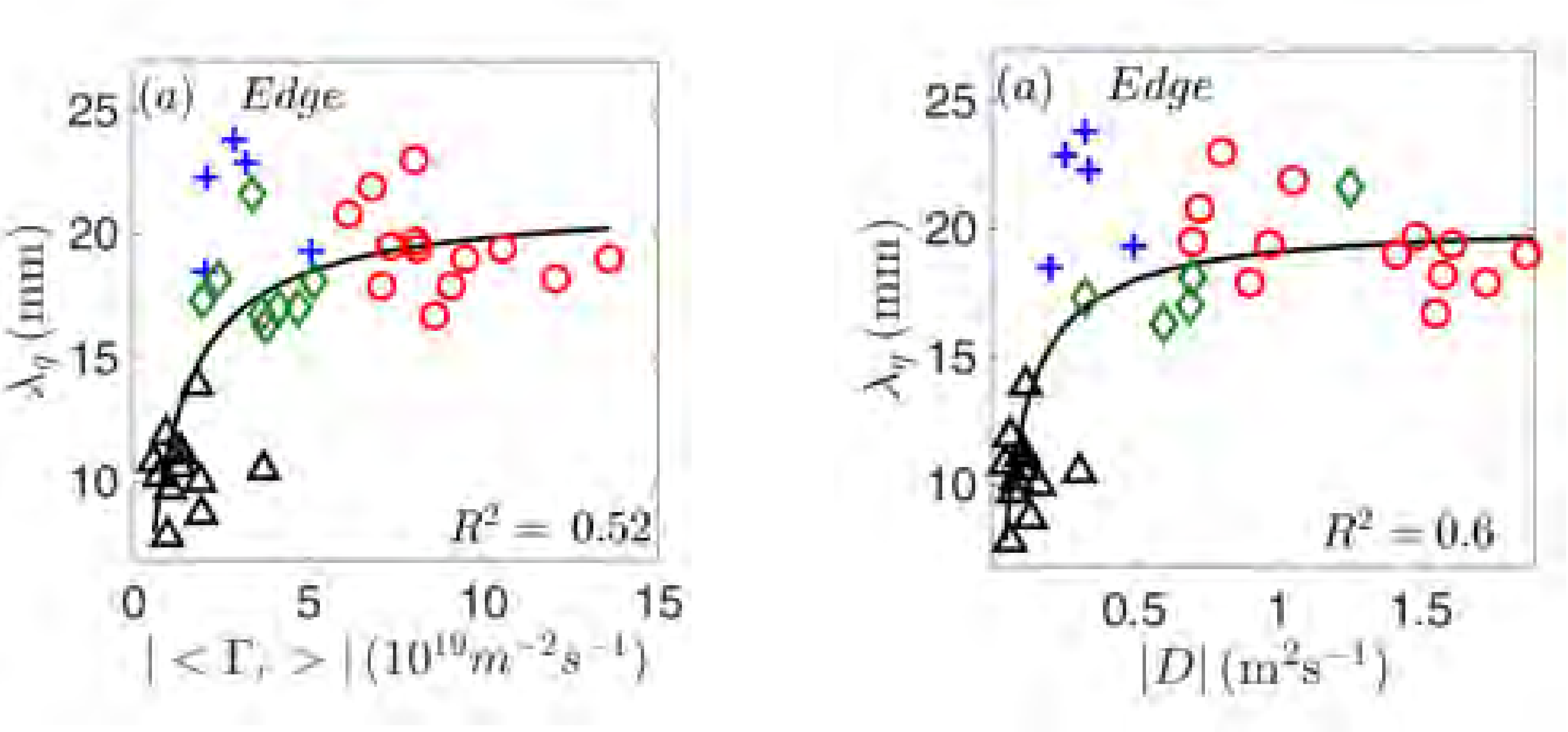
λ_q Trends 1 – Fluctuation Levels and Shearing





- λ_q increases for increasing fluctuation intensity at <u>lcfs</u>
- λ_q decreases for increasing ExB shear at <u>lcfs</u>
- Max $\omega_{E\times B}$ at shear layer ~ lcfs

λ_q Trends 2 – Particle Flux and Diffusion

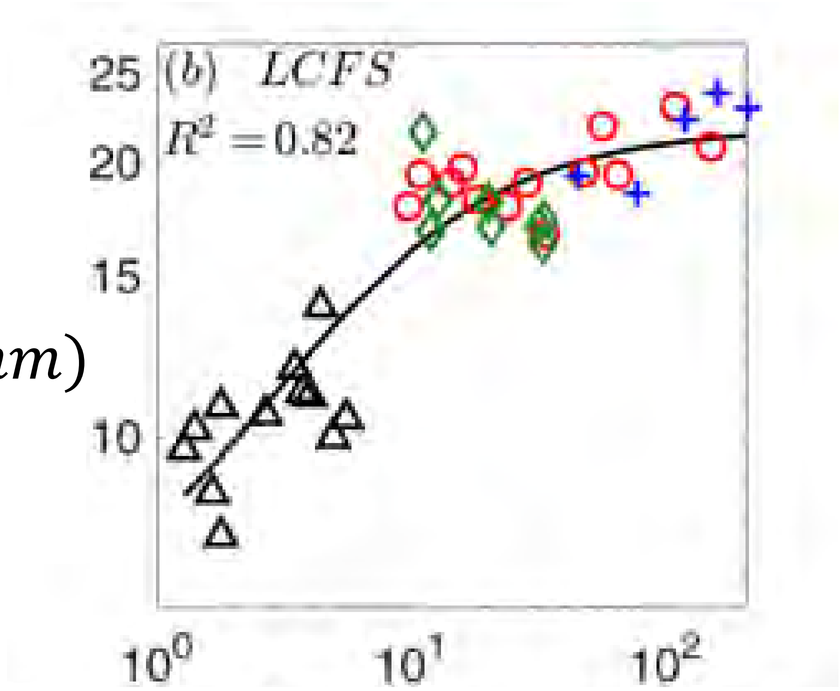


- λ_q increases for increasing <u>edge</u> Γ_n
- λ_q increases for increasing <u>edge</u> D

? Saturation – might expect $\lambda \sim (D\tau)^{1/2}$ scaling …

λ_q Trends 3 – Spreading !

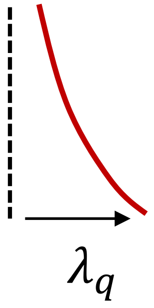
$\lambda_q(mm)$



$c_s^2 \langle \tilde{V}_r \, (\tilde{n}/n_0)^2 \rangle \, (10^8 \mathrm{m}^3 \mathrm{s}^{-3})$ → at lcfs

- $\Gamma_\varepsilon = c_s^2 \langle \tilde{V}_r \, (\tilde{n}/n_0)^2 \rangle$ → flux of turbulence internal energy thru lcfs
- Direct measurement of <u>local</u> spreading flux
- Consistent with expected trend of expanded SOL width due to increasing spreading across lcfs

SOL Fluctuation Energy – Production Ratio



1 Fluid

- $$\rho\left(\frac{\partial\vec{V}}{\partial t}+\vec{V}\cdot\nabla\vec{V}\right)=-\nabla P+\frac{1}{c}\vec{J}\times\vec{B}+\rho g\hat{r}$$

$$\nabla\cdot\vec{V}=0,\ \ \tilde{P}+\frac{\vec{B}_0\cdot\tilde{\vec{B}}}{4\pi}\approx 0$$

SOL interchange

- $$\partial_t(KE)_{SOL}=-\int_0^\lambda dr\,\nabla\cdot\Gamma_E\ +\ \int_0^\lambda dr\left[\frac{c_s^2}{R}\left\langle\frac{\tilde{V}_r\tilde{n}}{n_0}\right\rangle-\langle\tilde{V}_r\tilde{V}_\perp\rangle\frac{\partial}{\partial r}\langle V_\perp\rangle\right]$$

$$=-\Gamma_E|_{\lambda_q}+\Gamma_E|_{\text{lcfs}}+[\text{SOL Integrated local production}]$$

Fluctuation Energy Influx to SOL

- $\Gamma_E=\langle\tilde{V}_r\tilde{V}^2\rangle\approx c_s^2\left\langle\tilde{V}_r(\tilde{n}/n_0)^2\right\rangle$ → amenable to measurement

Take: KE flux ~ Int. Energy Flux (✓ for drift-interchange)

this gives ...

Aside: On Calculating the Spreading…

- Why perturbed pressure balance?
  - Else, $\langle\vec{V}\cdot\nabla P\rangle$ and $\langle\rho\nabla\cdot\vec{V}\rangle$ enter energy balance. Acoustic energy propagation irrelevant on $\tau \gg \tau_{MS}$
  - Can eliminate via vorticity eqn, $\vec{V}=\vec{E}\times\vec{B}$ etc.
- Interchange drive: $\kappa P \quad \rightarrow \quad \kappa\,\langle\tilde{V}_r\tilde{P}\rangle \;\approx g c_s^2\langle\tilde{V}_r\tilde{n}\rangle$

  as cannot measure $\tilde{P}$ fluctuations

Production Ratio, Cont'd

How important is spreading ?

$$\boxed{R_a = c_s^2 \langle \tilde{V}_r (\tilde{n}/n_0)^2 \rangle \Big|_{\mathrm{lcfs}} \Big/ \int_0^{\lambda} dr \frac{c_s^2}{R} \langle \tilde{V}_r \tilde{n}/n_0 \rangle}$$

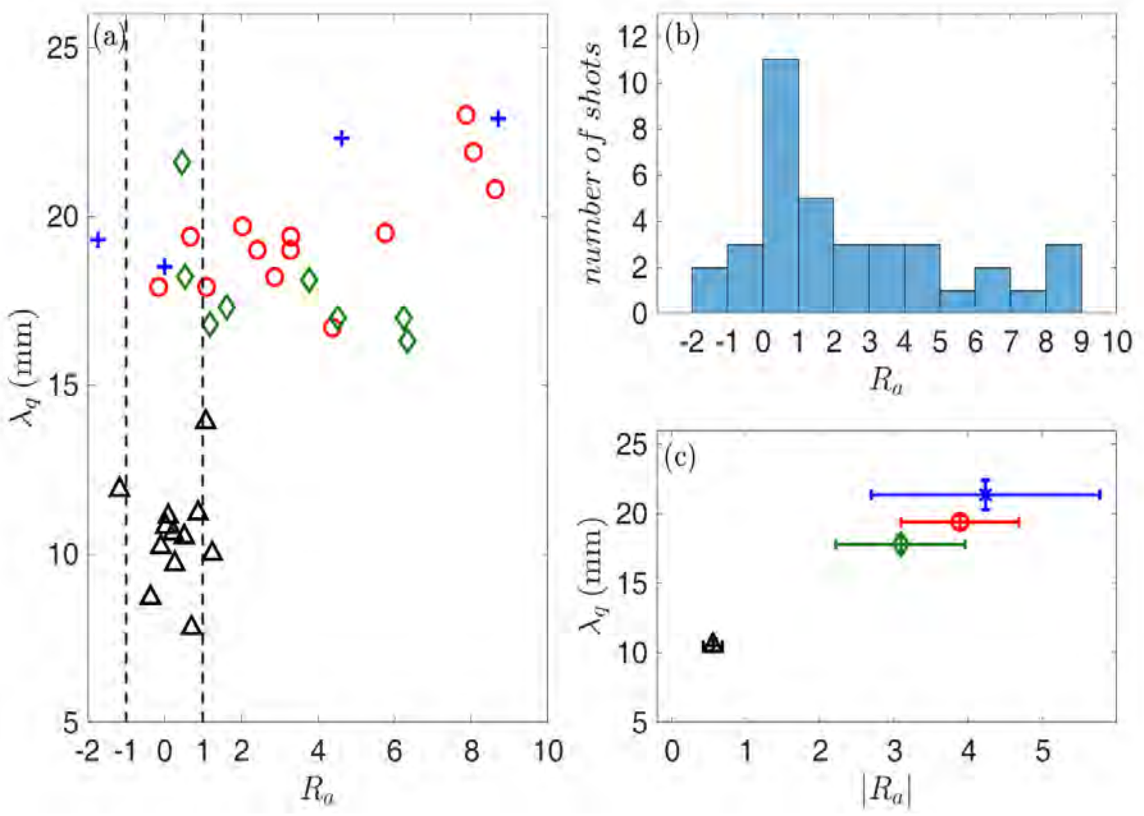
– Ratio of fluctuation energy influx from edge i.e. spreading drive - to net production in SOL

– $R_a < 1$ → SOL locally driven

– $R_a \gg 1$ → SOL is spreading driven

- Quantitative measurement by Langmuir probes
- N.B. very simple; likely lower bound, as local production smaller

Production Ratio - Measurements



$$R_a = \frac{\text{Fluctuation Energy Influx}}{\text{SOL Local Production}}$$

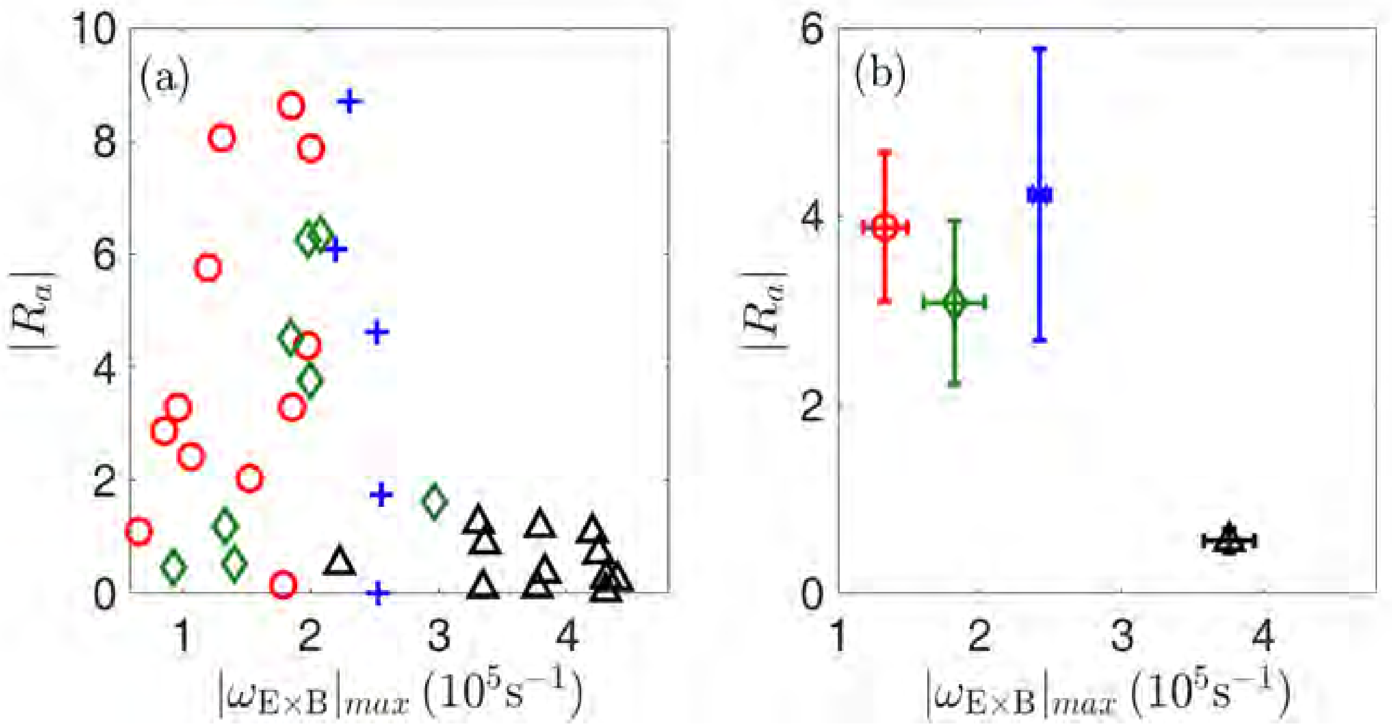
- Observe:
  - λ_q increases with R_a
  - Most cases R_a > 1
  - Broad distribution R_a values
  - Low R_a values ↔ strong ExB shear

  N.B. Non-trivial, as shear enters production, also via cross phase

- Also:
  - Some $R_a < 0$ cases → inward spreading ↔ local measurement trend outward
  - Some very large R_a values

  What is happening?

Production Ratio vs ExB Shear 1



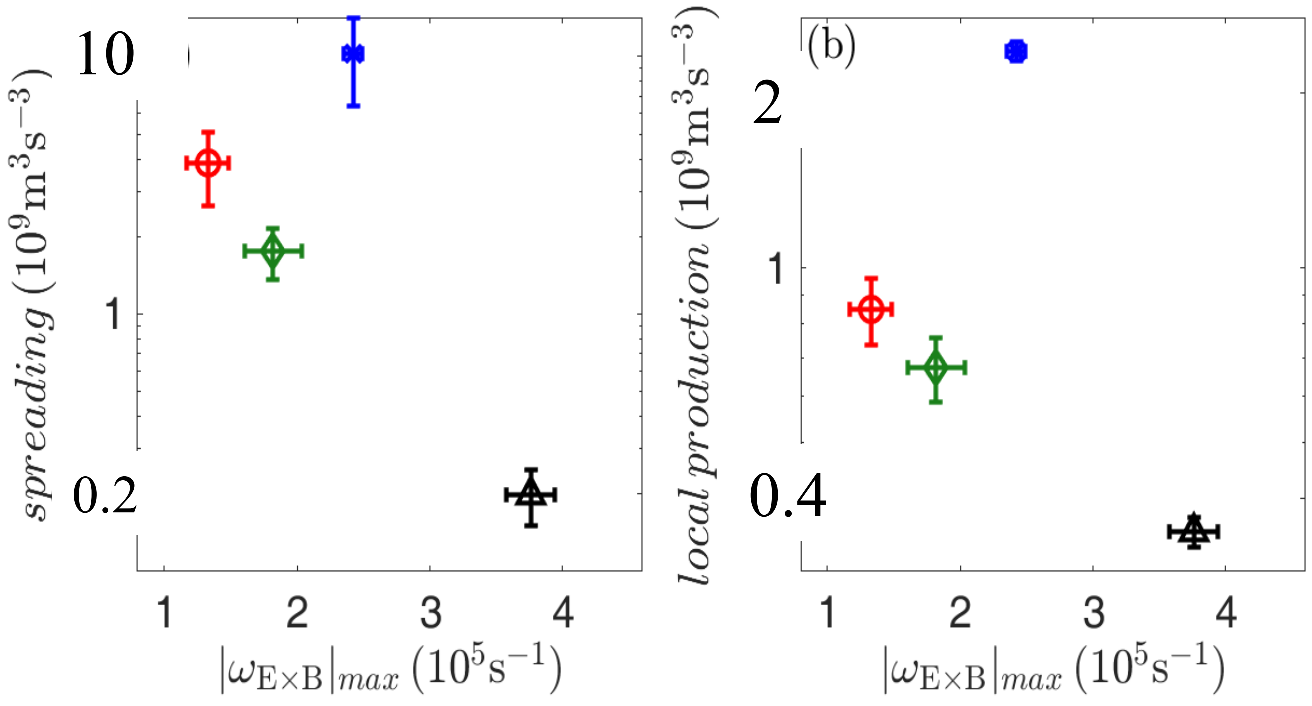
- Low values of $|R_a|$ at high V_E'
- But why?

$$R_a = c_s^2 \langle \tilde{V}_r (\tilde{n}/n_0)^2 \rangle|_{\mathrm{lcfs}} \;/\; \int_0^\lambda dr \frac{c_s^2}{R} \langle \tilde{V}_r \tilde{n}/n_0 \rangle$$

→ Expect shear inhibits both spreading and transport flux?

←→ ExB shear enters phase relation in both

Production Ratio vs ExB Shear, cont'd



- Both spreading and local production drop due high V_E'
- But spreading x (1/10) vs Production x (1/2)

➔ Spreading flux significantly more sensitive to V_E' than transport flux

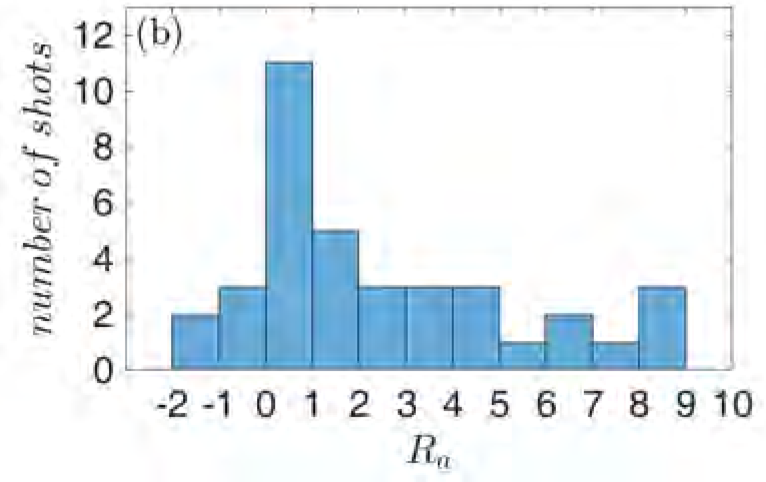
←→ Triplet vs quadratic ➔ Phases?

Large R_a → 'Blobs' ?!

- What of the large R_a values?
- Suspect – Structure Emission i.e. "blobs" !?
- Test:
  - Conditional averaging (i.e. threshold $\tilde{n} > 2\widetilde{n}_{rms}$ → "blob")
  - Threshold arbitrary → setting based upon previous studies
  - Compute R_a, Γ etc. with conditionally averaged quantities

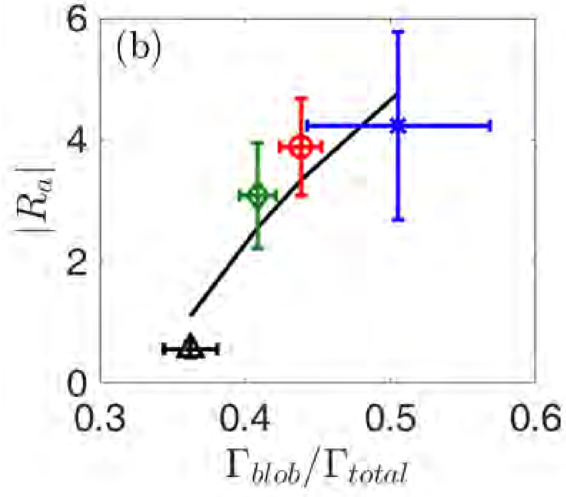
Especially: $\Gamma_{blob} \,/\, \Gamma_{total}$

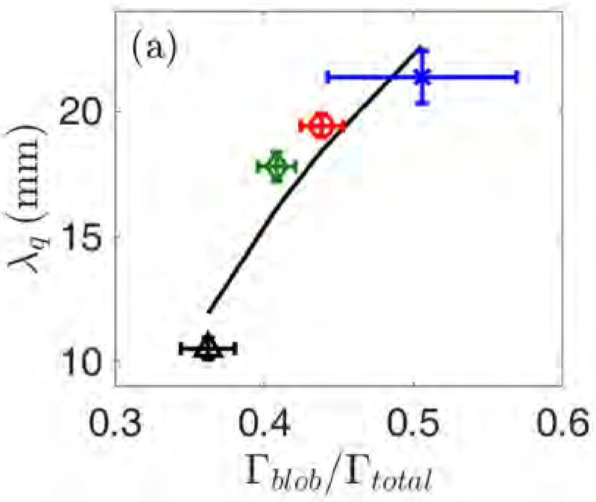
Flux carried by "blobs"



Physics of the "2" ?

Large R_a → λ_q increases with 'blob' fraction

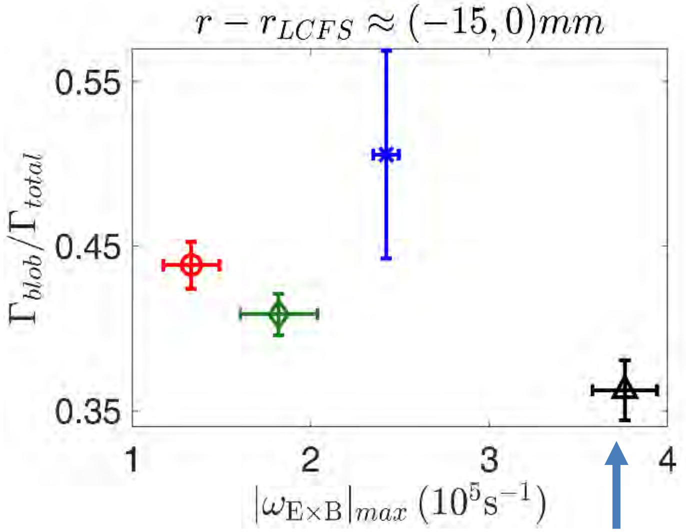




- Large R_a cases ←→ larger 'blob fraction' of flux

  ←→ spreading encompasses 'blobs' (c.f. Manz +) → $\langle \tilde{V}_r \tilde{n}^2 \rangle$

- λ_q increases with Γ_b/Γ_{Tot}



- High ExB shear cases ➔ low 'blob' fraction

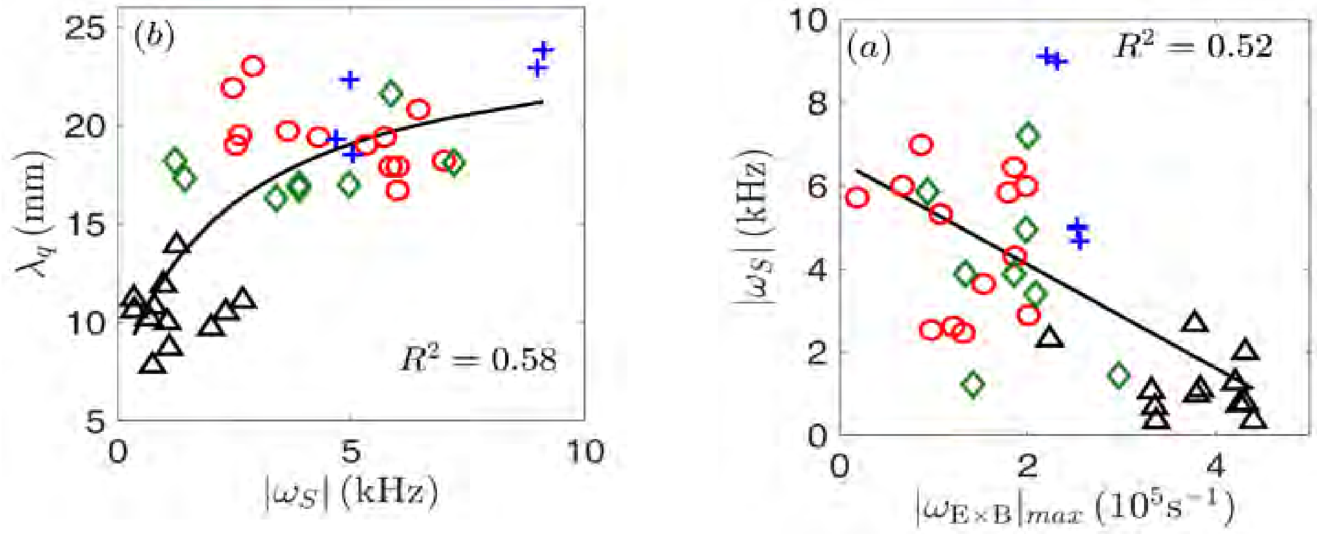
  (Consistent with Bodeo+, '03)

Time Scales

- Spreading rates: $\omega_s \approx -\partial_r \langle \tilde{V}_r \tilde{n}\tilde{n} \rangle / \langle \tilde{n}^2 \rangle$

  characteristic rate of spreading (Manz +)

- Shearing rate V_E'



- λ_q broadens for large ω_s

- Stronger shear reduces spreading rate

Partial Summary

- Significant, mostly outward, spreading measured at lcfs
- Identified and calculated production ratio

  $R_a =$ (spreading influx) / (local production)

- Most cases: $\boxed{R_a > 1 \Rightarrow \text{spreading dominant player in SOL energetics}}$
- ExB shear reduces R_a ⇔ spreading more sensitive to V_E' than transport and production – phases ?
- High R_a spreading ⇔ 'blob' dominated dynamics ➔ how calculate?

YES ➔ SOL turbulence usually spreading driven!

"The conventional wisdom is little more than convention" - JKG

N.B. No use of closure of spreading flux

Calculating the Width of the Spreading-Driven SOL

Physics Issues – Part II

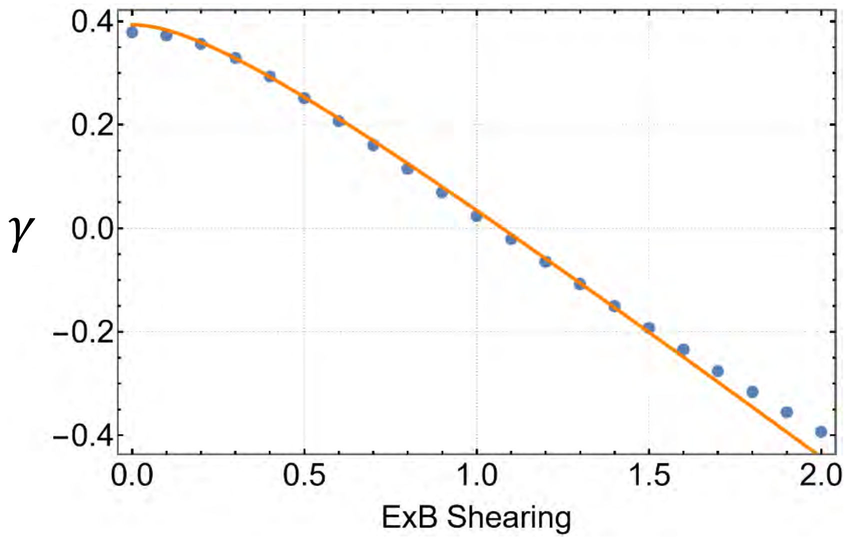
[C.f. Chu, P.D., Guo, NF 2022

P.D.+ IAEA ‘23]

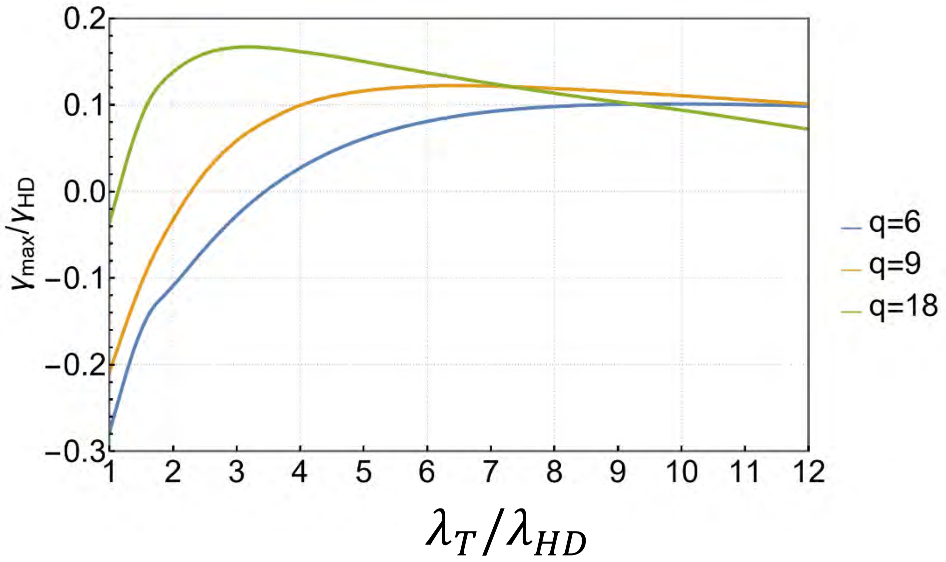
- How calculate SOL width for turbulent pedestal but a locally stable SOL?
  - spreading penetration depth
  - must recover HD in WTT limit
- Scaling and cross-over of λ_q relative HD model
- What is effect/impact of barrier on spreading mechanism?
- Can SOL broadening and good confinement be reconciled ?

Model 1 – Stable SOL – Linear Theory

- Standard drift-interchange with sheath boundary conditions + ExB shear (after Myra + Krash.)



Linear Growth Rate of a specific mode (fixed k_y) v.s. $E \times B$ shear at $q = 5, \beta = 0.001, k_y \cdot \lambda_{HD} = 1.58$.



Maximal Linear Growth Rate of Interchange Mode in the SOL v.s. normalized layer width λ_D/λ_{HD} at different SOL safety factor q (with $\beta = 0.001$)

- Relevant H-mode ExB shear strongly stabilizing $\quad \gamma_{HD} = c_s/(\lambda_{HD} R)^{1/2}$
- Need λ/λ_{HD} well above unity for SOL instability. $V_E' \approx \frac{3T_e}{|e|\lambda^2}$ → layer width sets shear

Model 2 – Two Multiple Adjacent Regions

- “Box Model” – after Z.B. Guo, P.D.

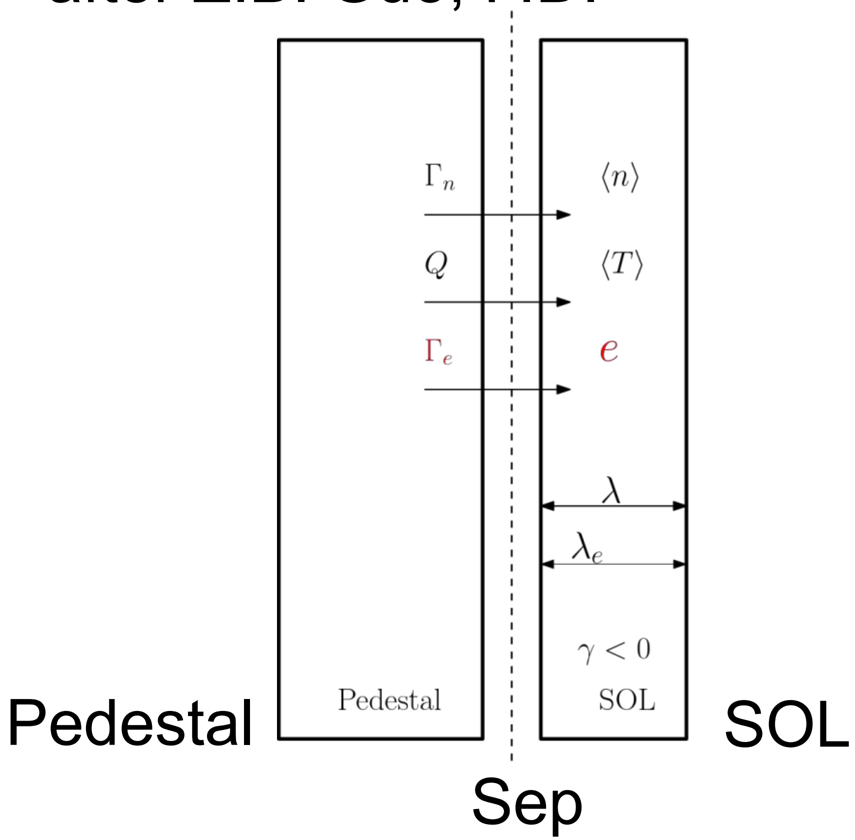


Illustration of Two Box Model: SOL driven by particle flux, heat flux and intensity flux (Γ_e) from the pedestal. The horizontal axis is the radial direction, and vertical axis is the poloidal direction.

- Key Point:
  - Spreading flux from pedestal can enter stable SOL
  - Depth of penetration ➔ extent of SOL broadening

➔ Problem in one of entrainment/penetration

Width of Stable SOL

- Fluid particle: $\frac{dr}{dt} = V_{Dr} + \tilde{V}$

  drift fluctuating velocity

- Dwell time: $\tau_{\parallel}$

Dwell time $\tau_{\parallel}$ constrains excursion

- $\delta^2 = \langle (\int (V_D + \tilde{V}) dt)(\int (V_D + \tilde{V}) dt) \rangle$

$\langle (\text{step})^2 \rangle$

$$= V_D^2 \tau_{\parallel}^2 + \langle \tilde{V}^2 \rangle \tau_c \tau_{\parallel}$$

correlation time
modest turbulence $\leftrightarrow \tau_c \geq \tau_{\parallel}$

$$= \lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2$$

turbulence energy density

See also
Fokker-Planck analysis
i.e. drift + diffusion

- So $\boxed{\lambda = \left[\lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2\right]^{1/2}}$ → <u>SOL width</u> [Effects add in quadrature]
- How compute ε ? → turbulence energy in SOL. Need relate to pedestal
- N.B. Can write: $\lambda = [\lambda_{HD}^2 + \lambda_e^2]^{1/2}$ λ_e is turbulent width

Calculating the SOL Turbulence Energy 1

- Need compute Γ_e effect on SOL levels
- $K - \epsilon$ type model, mean field approach (c.f. Gurcan, P.D. '05 et seq)
  - Can treat various NL processes via σ, κ
  - Exploit conservative form model
- $\partial_t \varepsilon = \gamma\varepsilon - \sigma\varepsilon^{1+\kappa} - \partial_x \Gamma_e$ → Spreading, turbulence energy flux

NL transfer $\gamma_{NL} \sim \sigma\varepsilon^{\kappa}$

* Growth $\gamma < 0$
here contains shear + sheath

→ • N.B.: No Fickian model of Γ_e employed, yet

- Readily extended to 2D, improved production model, etc.

Calculating the SOL Turbulence Energy 2

- Integrate ε equation $\int_0^{\lambda}$; "constant e" approximation
- Take quantities = layer average
- $\Gamma_{e,0} + \lambda_e \gamma \varepsilon = \lambda_e \, \sigma \, \varepsilon^{1+\kappa}$

Separatrix fluctuation energy flux → Single parameter characterizing spreading

So for $\gamma < 0$,

λ_e = layer width for ε

$$\boxed{\Gamma_{e,0} = \lambda_e |\gamma| \varepsilon + \sigma \lambda_e \varepsilon^{1+\kappa}}$$

$\Gamma_{e,0}$ vs linear + nonlinear damping

- Ultimately leads to recursive calculation of Γ_e

Calculating the SOL Turbulence Energy 3

[Mean Field Theory]

- Full system:

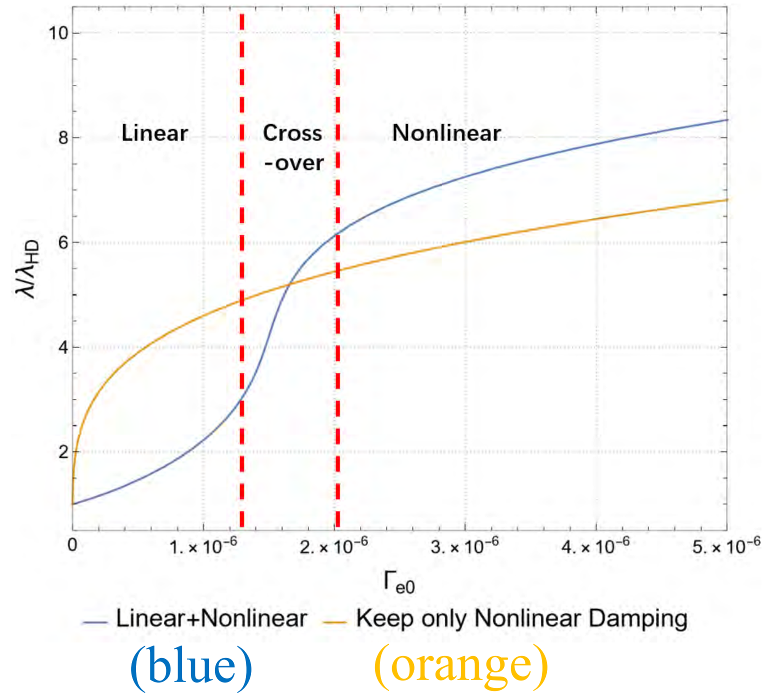
$$\boxed{\begin{gathered}\Gamma_{e,0} = \lambda_e|\gamma|\varepsilon + \sigma\lambda_e\varepsilon^{1+\kappa} \\ \lambda_e = \left[\lambda_{HD}^2 + \varepsilon\tau_{\parallel}^2\right]^{1/2}\end{gathered}}$$

Simple model of turbulent SOL broadening

- $\Gamma_{0,e}$ is single control parameter characterizing spreading
- $\tilde{\Gamma}_{0,e}$? Expect $\tilde{\Gamma}_e \sim \Gamma_0$

SOL width Broadening vs $\Gamma_{e,0}$

- SOL width broadens due spreading



λ/λ_{HD} plotted against the intensity flux Γ_{e0} from the pedestal at $q = 4, \beta = 0.001, \kappa = 0.5, \sigma = 0.6$

Variation indicates need for detailed scaling analysis

- Clear decomposition into
  - Weak broadening regime → shear dominated
  - Cross-over regime
  - Strong broadening regime

  ➔ NL damping vs spreading } relevant

- Cross-over for:

  $\langle \tilde{V}^2 \rangle \sim V_D^2$ ➔ cross-over $\Gamma_{0,e}$

- Cross-over for $\tilde{V} \sim O(\epsilon) V_*$

SOL Width: Some Analysis

Have $\Gamma_{e,0} = |\gamma| e\lambda_e + \lambda_e \sigma e^{1+\kappa}$

a) Damping dominated

$$\Gamma_e \approx |\gamma|\, \lambda_e\, e \qquad\qquad \lambda_q^2 = \lambda_e^2 + \lambda_{HD}^2$$

$$\boxed{\lambda_q = \left[\lambda_{HD}^2 + \left(\frac{\Gamma_e \tau_\parallel^2}{|\gamma|}\right)^{2/3}\right]^{1/2}}$$

- Spreading enters only via Γ_e at sep.
- Shearing via $|\gamma|$
- τ scalings → $\tau_\parallel$ vs $\tau_\parallel^{2/3}$ → current scaling of λ_e weaker

SOL Width: Some Analysis, Cont'd

b) NL dominated

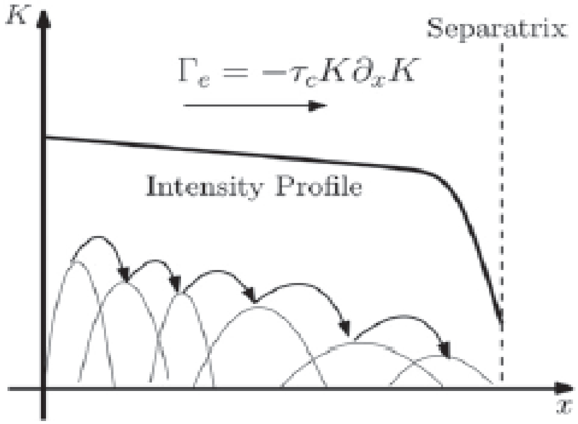
$\Gamma_e \approx \lambda_e \, \sigma \, e^{1+\kappa}$ $\qquad\qquad$ $\lambda_q^2 = \lambda_e^2 + \lambda_{HD}^2$

$$\boxed{\lambda_q = \left[\lambda_{HD}^2 + \left(\frac{\Gamma_e}{\sigma}\right)^{2/(3+4\kappa)} \tau_{\parallel}^{[4(1+\kappa)/(3+2\kappa)]}\right]^{1/2}}$$

– weaker Γ_e scaling, $\lambda_q \sim (\Gamma_e/\sigma)^{1/5}$; STT

– $\tau_{\parallel}^{3/4}$ vs $\tau_{\parallel}$ $\rightarrow$ weaker current scaling

Computing the Turbulence Energy Flux 1



- Need consider pedestal to actually compute $\Gamma_{e,0}$
- Two elements

Does another trade-off loom?

-- Pedestal Turbulence: Drift wave? Ballooning?

-- Effect of transport barrier ←→ ExB shear layer ➔ barrier permiability!?

- Key Point: shearing limits correlation in turbulent energy flux

i.e. $\Gamma_{e,0} \approx -\tau_c \, I \, \partial_x \, I \approx \tau_c \, I^2 \, / w_{\text{ped}}$ (Hahm, PD +)

ped turbulence intensity

correlation time → strongly sensitive to shearing

N.B. Caveat Emptor re: intensity flux closure !

Computing the Turbulence Energy Flux 2

- Familiar analysis for D → Kubo

$$D = \int_0^\infty d\tau \, \langle V(0)V(\tau)\rangle = \int_0^\infty d\tau \sum_k |\tilde{V}_k|^2 \exp[-k_y^2\omega_s^2 D\tau^3 - k^2 D\tau]$$

- Strong shear (relevant) $\boxed{\tau_c = \tau_t^{1/2}\omega_s^{-1/2}}$

$\tau_t \sim 1 \,/\, k\tilde{V}, \quad \omega_s \sim V_E'$

Here, via RFB ➔ $\omega_s = \partial_r \frac{\nabla P_i}{n|e|} \sim \frac{\rho^2}{w_{ped}^2}\,\Omega_{ci}$

- $\tau_c + w_{ped}$ + turbulence intensity in pedestal gives $\Gamma_{e,0} \approx \tau_c I^2 / w_{ped}$
- Need $\Gamma_{e,0} \geq \Gamma_{e,\min} \approx |\gamma|\lambda_{HD}^3\, \tau_\parallel^{-2}$

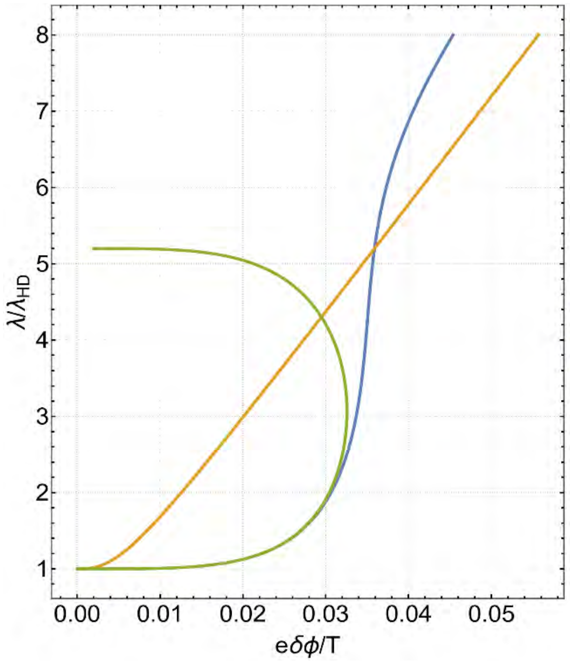
Computing the Turbulence Energy Flux 3

- Pedestal → Drift wave Turbulence
- Necessary turbulence level:
  - Weak Shear $\frac{\delta V}{c_s} \sim \left(\frac{\rho}{R}\right)^{1/2} q^{-1/4}$
  - Strong Shear $\frac{\delta V}{c_s} \sim \left(\frac{\rho}{R}\right)^{1/2} q^{-1/4} \left(\frac{w_{ped}}{\rho}\right)^{-1/8}$

blue – all damping

orange – nonlinear only

green – linear only



➔ λ/λ_{HD} vs $|e|\hat{\phi}/T_e$ in pedestal

➔ ρ/R is key parameter

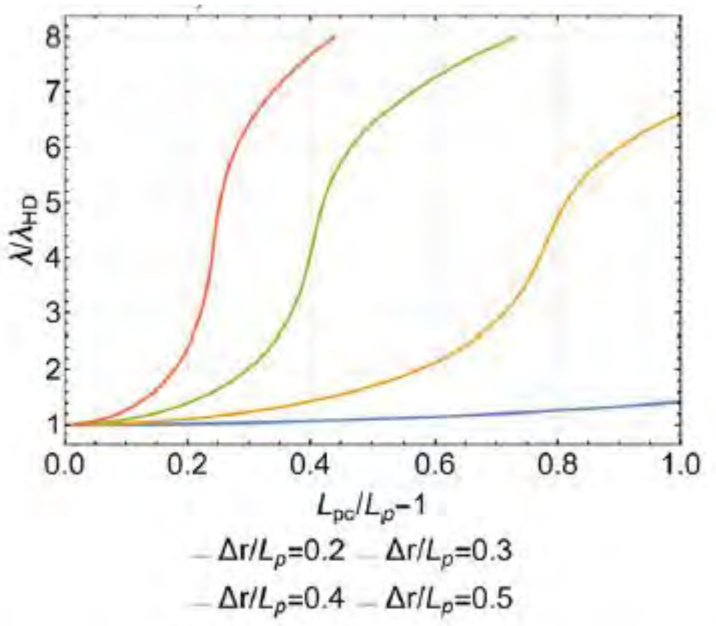
➔ Broadens layer at acceptable fluctuation level

Computing the Turbulence Energy Flux 4

- Pedestal → Ballooning modes ➔ Grassy ELMs
- Necessary relate turbulence to $L_{P,crit} \,/\, L_P \; - 1$
- Strong shear:

$$\frac{L_{P_c}}{L_P} - 1 \sim \left(\frac{q\rho}{R}\right)^{\frac{10}{7}} \left(\frac{R}{w_{ped}}\right)^{\frac{16}{7}} \left(\frac{w_{ped}}{\Delta_r}\right)^{\frac{16}{7}} \beta$$

- Supercriticality scales with $\frac{\rho}{R},\ \beta_t$



b) Strong Shear Case

Figure 10. Typical cases for ballooning. The normalized pedestal width λ/λ_{HD} is plotted against supercriticality $L_{pc}/L_p - 1$ at different mode width Δ/L_p.

Computing the Turbulence Energy Flux 5 → Bottom Line

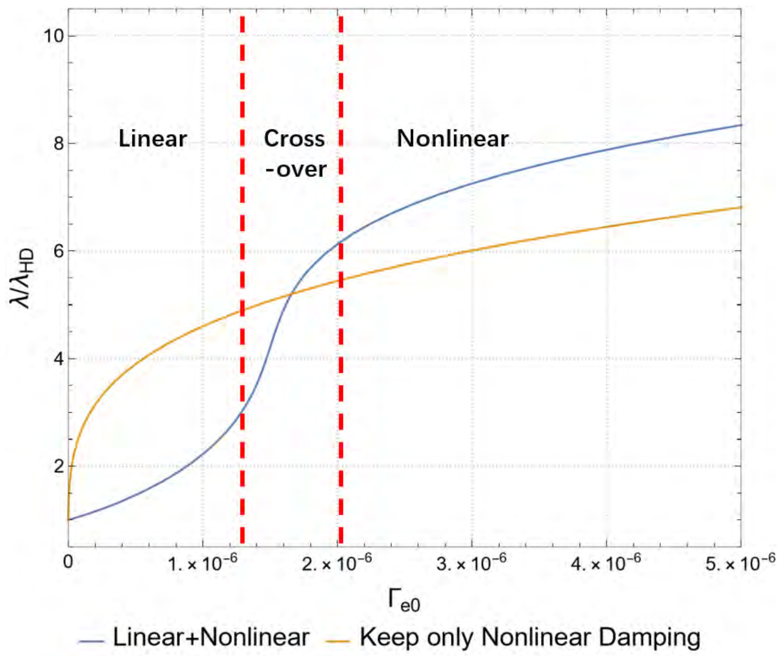
- SOL broadening to $\lambda > \lambda_{HD}$ achieveable at tolerable pedestal fluctuation levels
- DW levels scale ~ $\left(\frac{\rho}{R}\right)^{1/2}$
- Ballooning supercritical scale ~ $\left(\frac{\rho}{R}\right)^{10/7}\ \beta$
- 'Grassy ELM' state promising
- Sensitivity analysis → Cross over ε determined primarily by linear damping (shear). Conclusion ~ insensitive to NL saturation

Partial Summary

- Turbulent scattering broadens stable SOL

$$\lambda = \left(\lambda_{HD}^2 + \varepsilon\tau_{\parallel}^2\right)^{1/2}$$

- Separatrix turbulence energy flux specifies SOL turbulence drive



$$\Gamma_{0,e} = \lambda_e|\gamma|\varepsilon + \lambda\sigma\varepsilon^{1+\kappa}$$

Broadening increases with $\Gamma_{0,e}$
cross-over for $\langle\tilde{V}^2\rangle \sim V_D^2$

Non-trivial dependence

- $\Gamma_{0,e}$ must overcome shear layer barrier

Yes – can broaden SOL to $\lambda/\lambda_{MHD} > 1$ at tolerable fluctuation levels

Further analysis needed

Broader Messages

- Turbulence spreading is important – even dominant – process in setting SOL width. $\Gamma_{0,e}$ is critical element. $\lambda = \lambda(\Gamma_{0,e}, \text{parameters})$
- Production Ratio R_a merits study and characterization
- Spreading is important saturation meachanism for pedestal turbulence
- Simulation should stress calculation and characterization of turbulence energy flux over visualizations and front propagation studies.
- Critical questions include local vs FS avg, channels and barrier interaction, Turbulence 'Avalanches'
- $\boxed{\text{Turbulent pedestal states attractive for head load management}}$

Open Issues

- Quantify $\lambda = \lambda\left(\frac{|e|\hat{\phi}}{T}\Big|_{ped}\right)$ dependence
- Structure of Flux-Gradient relation for turbulence energy?
- Phase relation physics for intensity flux? – crucial to ExB shear effects
- Kinetics → $\langle \tilde{V}_r \delta f \delta f \rangle$, Local vs Flux-Surface Average, EM
- SOL Diffusive? → Intermittency('Blob'), Dwell Time ?
- SOL → Pedestal Spreading ? ←→ HDL (Goldston) ?

  i.e. Tail wags Dog ? Both wagging ? → Basic simulation, experiment ?

  Counter-propagating pulses ?

Some Simulation Results

(cf. Nami Li, X.-Q. Xu, P.D.; N.F.(Lett) ‘23)

➔ BOUT++ → pedestal + SOL

➔ 6 field model (“Braginskii for 21st century”)

➔ Focus on weak peeling mode turbulence in pedestal

→ MHD turbulence state → small/grassy ELM, also WPQHM

3D Counterpart of Brunner (λ_q vs B_θ)

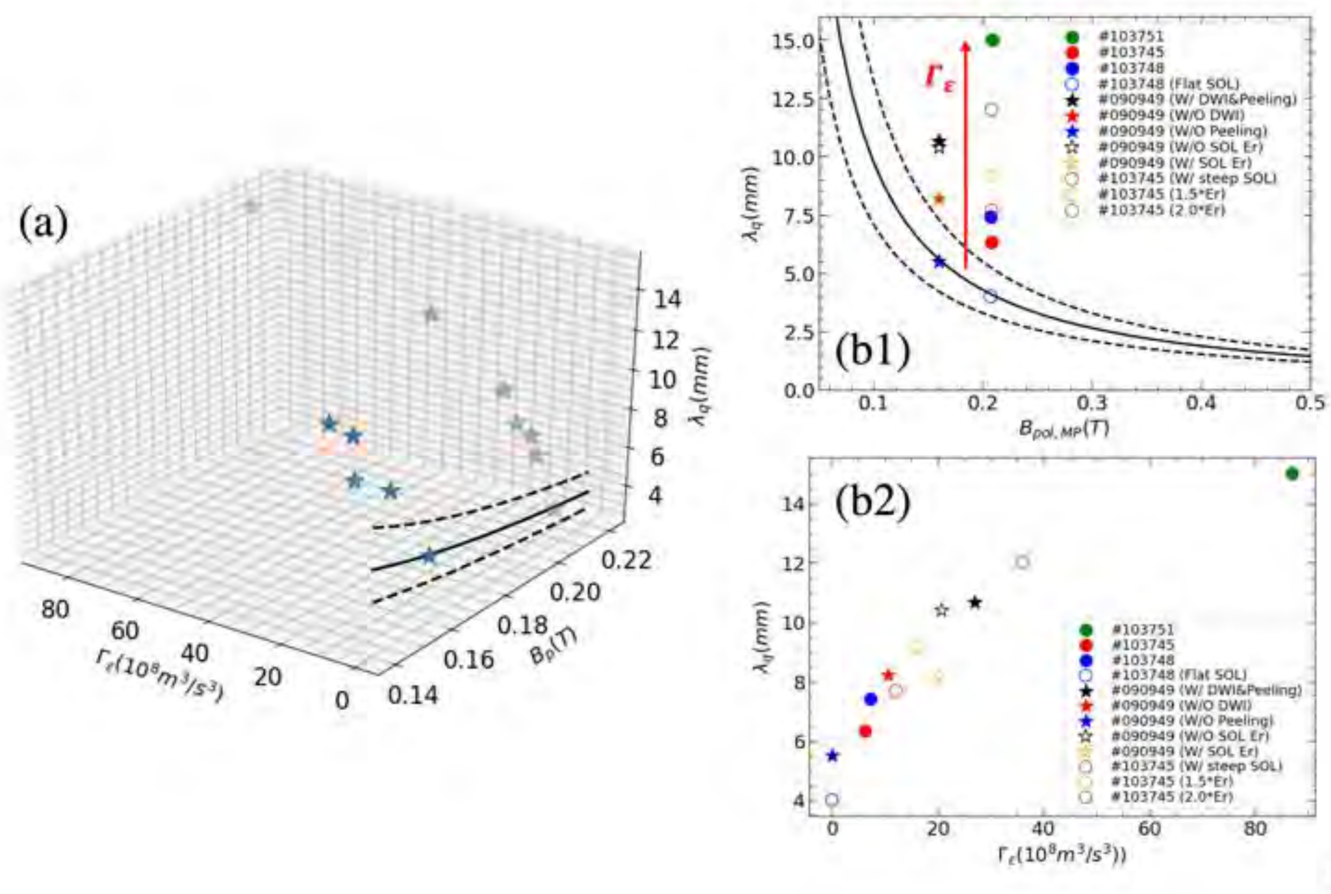


Fig. 3. (a) 3D plot of heat flux width λ_q vs poloidal magnetic field B_p and fluctuation energy density flux Γ_ε; (b) 2D plot of heat flux width λ_q vs poloidal magnetic field B_p (b1) and fluctuation energy density flux Γ_ε (b2).

3D Brunner Plot – Comments

- λ_q rises with Γ_e
- Low Γ_e , λ_q tracks hyperbola
- Large Γ_e , λ_q rises above Brunner/Goldston hyperbola
- λ_q grows with Γ_e

Spreading as Mixing Process ?

- Conjecture that λ_q should increase with <u>pedestal</u> mixing length ➔ Γ_e

- Note division into
  - drift dominated
  - cross-over (blue)
  - turbulent

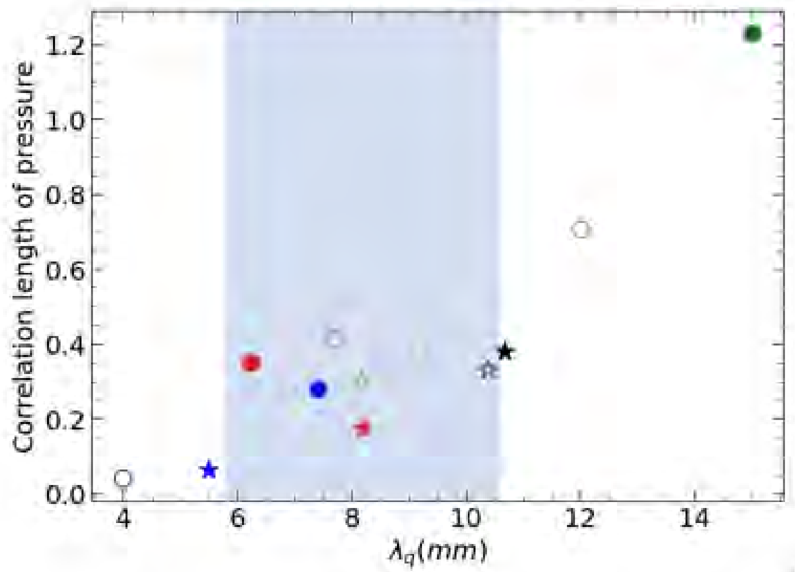


Fig 4. Radial correlation length of pressure near the separatrix vs. heat flux width λ_q.

Relate Spreading to Pedestal Conditions

N.B.

- Γ_e rises with pedestal ∇P_0 ←→ increased drive
- Collisionality dependence Γ_e:
  - high → no bootstrap current → ballooning → smaller l_{mix}
  - low → strong bootstrap → peeling → larger l_{mix}

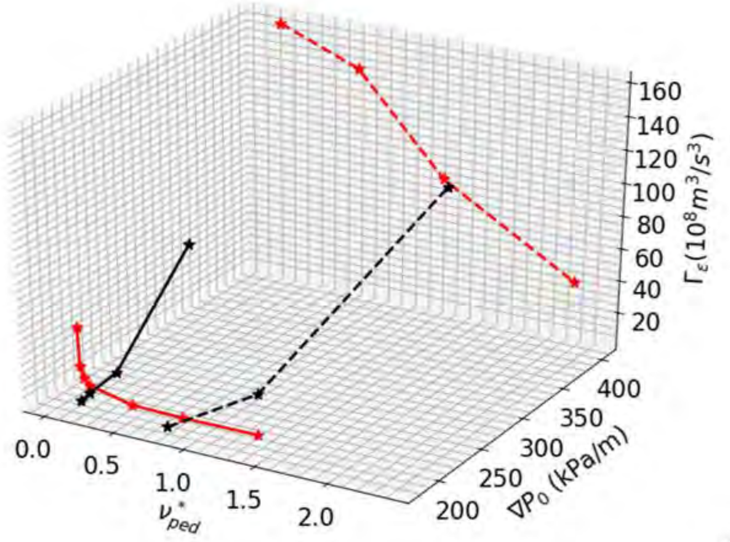


Fig. 7. 3D plot of fluctuation energy density flux Γ_ε vs pedestal peak pressure gradient ∇P_0 and υ^*_{ped}; black curves are ∇P_0 scan with low collisionality $\upsilon^*_{ped} = 0.108$ (solid curve) and high collisionality $\upsilon^*_{ped} = 1$ (dashed curve); red curves are υ^*_{ped} scan with small $\nabla P_0 \sim 200\ kPa/m$ (solid curve) and large $\nabla P_0 \sim 400\ kPa/m$ (dashed curve).

Fundamental Physics of Γ_e

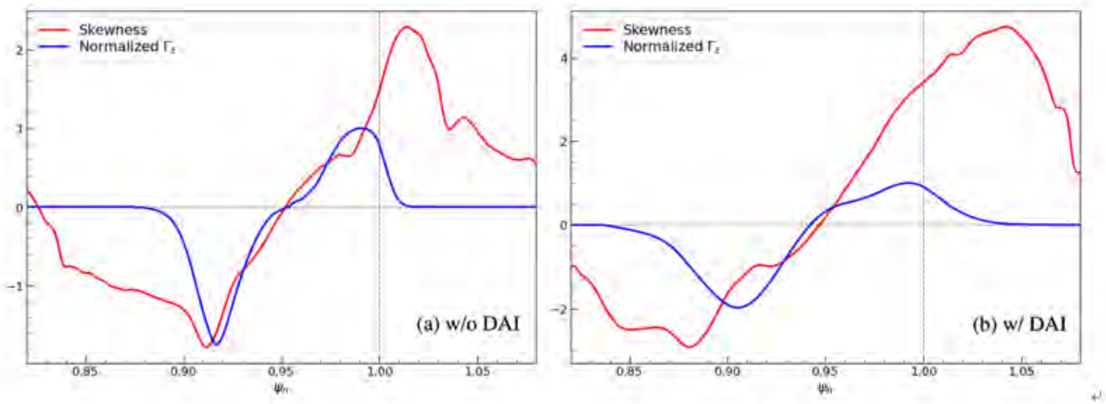


Fig. 6 Radial profiles of normalized fluctuation energy density flux Γ_ε (blue) and skewness (red) for without (a) and with (b) drift-Alfvén instability. Here fluctuation energy density flux is normalized to the max value for each case.

- Γ_e spreading tracks $\tilde{P}$ skewness
  - Outward for $s > 0$ → "blobs"
  - Inward for $s < 0$ → "voids"
- Zero-crossings Γ_e, s in excellent agreement

Fundamental Physics of Γ_e, cont'd

- Spreading appears likely linked to "coherent structures"
- Likely intermittent (skewness, kurtosis related)
- Related study (Z. Li); $Ku \sim 0.4$, so ➔ if Fokker-Planck analysis

$$\frac{\partial e}{\partial t} = -\frac{\partial}{\partial x}(Ve) + \cancel{\frac{\partial^2}{\partial x^2}(De)}$$ Convective !?

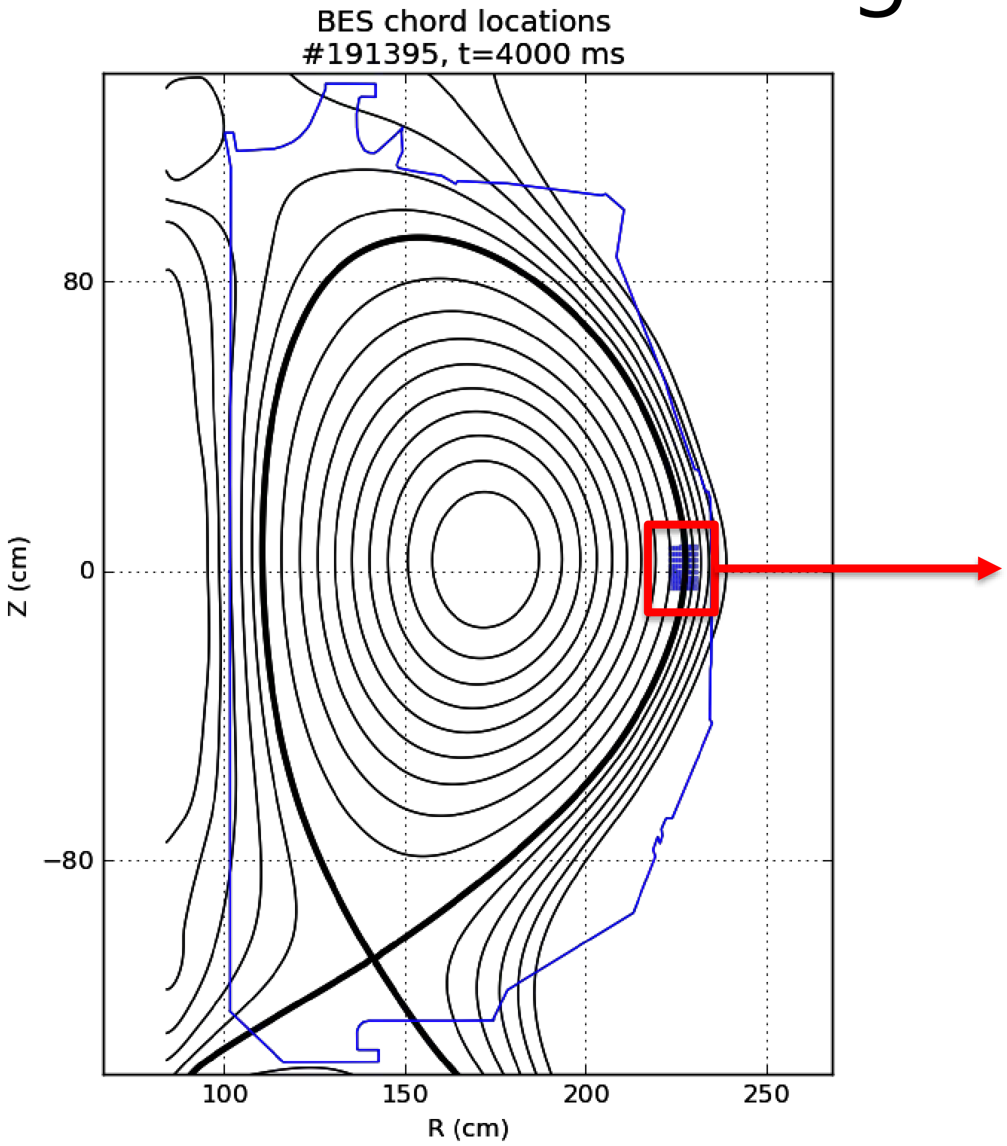
Relate V to pedestal gradient relaxation event (GRE) ?!

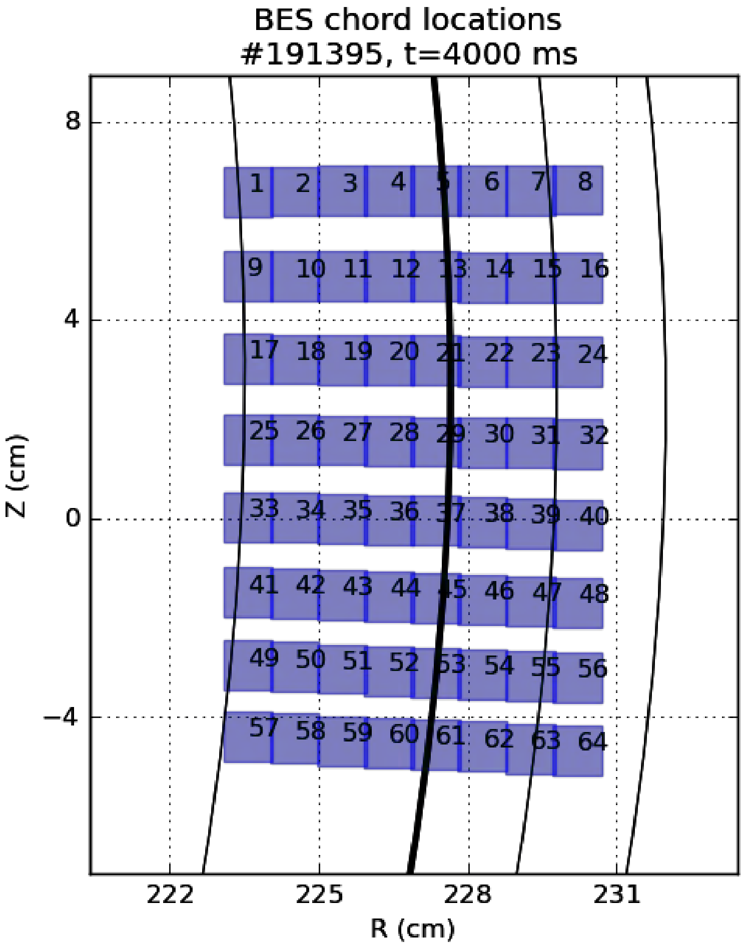
More Experimental Data

(F. Khabanov+, submitted 2024)

→ Spreading via BES Velocimetry

BES allows measuring $\boldsymbol{\delta}$n/n at the plasma edge





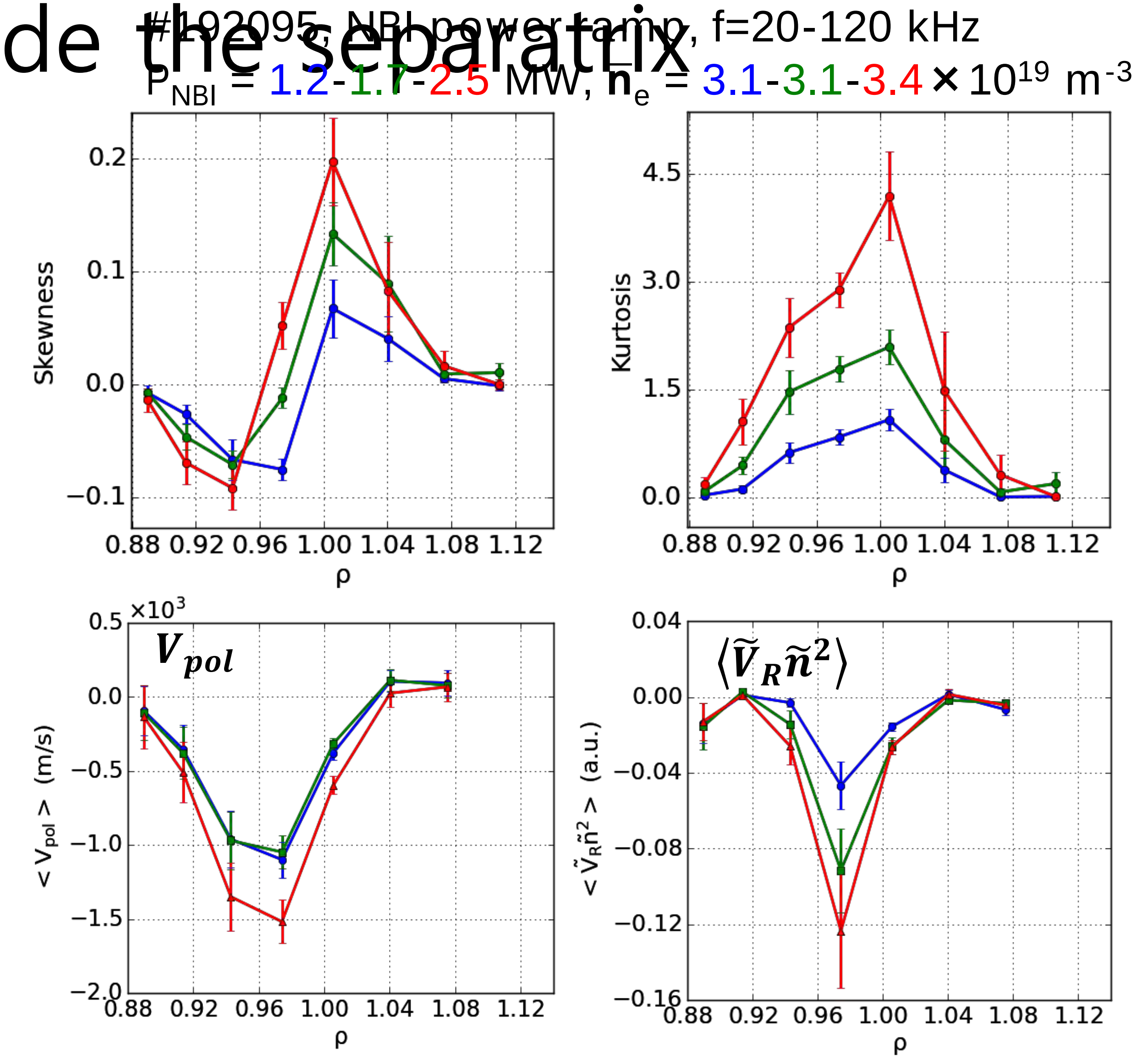
ρ = 0.87-1.1
~ 10 x 8 cm

Turbulence intensity flux $\langle \tilde{V}_R \tilde{n}^2 \rangle$ is negative inside and positive outside the separatrix

- Negative skewness of $\tilde{n}$ inside the separatrix and positive skewness outside indicate the prevalence of negative density fluctuations (voids) inside the separatrix and positive (blobs) outside.

- The formation zone of blob-void pairs (zero skewness) is located at ρ~0.96-0.98.

- Turbulence intensity flux $\langle \tilde{V}_R \tilde{n}^2 \rangle$, measured using 2D BES, shows an inward turbulence spreading inside the separatrix while outside, the turbulence spreading is outward towards the SOL.

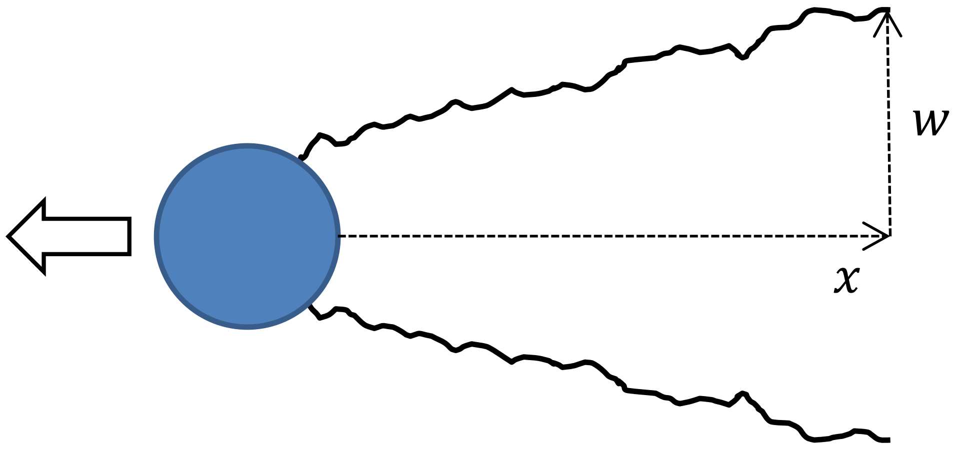


Physics of Turbulence Spreading: General Perspective

- Structure of the intensity flux-gradient relation(?)
- Avalanching into SOL

Spreading: Conventional Wisdom

- Turbulence spreading underpins turbulent wake → central example in high Re fluids



Mixing length model
Similarity theory

$\Rightarrow w \sim (F_d/\rho U^2)^{1/3} x^{1/3}$

$F_d \sim \rho U^2 S C_D$;

C_D → indep ν

- Spreading fundamental to $k-\varepsilon$ type models, as ε evolved as unresolved energy field → subgrid models

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \left(\tilde{V}\varepsilon\right) + \cdots = 0$$

How render tractable ?

Spreading: cont'd

- What you get (usually):

$$\partial_t \varepsilon + \underbrace{\vec{V}_D}_{\text{drift}} \cdot \nabla\varepsilon + \langle \vec{V}_E(\underbrace{r}_{\text{shear}}) \rangle \cdot \nabla\varepsilon - \partial_r \, D(\underbrace{\varepsilon}_{\text{turbulent mixing via closure}})\partial_r \varepsilon = P_{src}(\varepsilon) - P_{\mathrm{damp}}(\varepsilon) \rightarrow \gamma(\vec{x})\varepsilon$$

$\gamma = \gamma$(gradients, etc)

$\boxed{D(\varepsilon) \approx D_0\varepsilon \text{ , et. seq. } \rightarrow \text{ nonlinear diffusion}}$

→ ε evolution as nonlinear Reaction-Diffusion Problem!

(P.D., Garbet, Hahm, Gurcan, Sarazin, Singh, Naulin…)

- Used also in:
  - BLY-style layering models (Ashourvan)
  - 1D L→H models (Miki)

Spreading: cont'd

- Spreading as Front ➔ Fast Propagation

  i.e. $V_f \sim (\gamma D)^{1/2}$, etc [N.B. Cahn-Hillard?]

- Key component:

  $\nabla \cdot \langle \vec{V} \varepsilon \rangle \;\rightarrow\; -\nabla \cdot D(\varepsilon) \cdot \nabla \varepsilon$ [Fickian Model]

  Expectation: $D(\varepsilon) \sim \chi$, D_n etc. for electrostatic

- Copious simulations: Z. Lin, W.X. Wang, S. Yi, Jae-Min Kwon, Y. Sarazin, …

  ➔ Observations, front tracking but critical analysis of model absent ??

  No test of Fickian flux model

Experiments: Ancient

- Not exactly a new idea …   See Townsend '49 and book

Momentum and energy diffusion in the turbulent wake of a cylinder

By A. A. Townsend, *Emmanuel College, University of Cambridge*

(*Communicated by Sir Geoffrey Taylor, F.R.S.—Received 6 October* 1948)

A detailed experimental investigation of the turbulent motion in the wake of a circular cylinder, 0·953 cm. diameter placed in an air-stream of velocity 1280 cm.sec.$^{-1}$, has been carried out with particular reference to those quantities determining the transport of turbulent energy and mean stream momentum. At distances of 80, 120 and 160 diameters down-stream from the cylinder, direct measurements have been made of mean flow velocity, turbulent intensity, viscous dissipation, energy diffusion, scale, and form factors of the velocity components and their spatial derivatives. These observations show that, except close to the wake centre, the flow at a point fixed with respect to the cylinder is only intermittently turbulent, due to the passage of the point of observation through jets or billows of turbulent fluid emitted from the inner wholly turbulent core of the wake. Further consideration of the results indicates that the turbulent motion within the jets is solely responsible for the turbulent transfer of momentum, while diffusion of turbulent energy and of heat is carried out by the bulk movement of the jets. Most probably, the jets are initiated by local fluctuations of pressure inside the turbulent core, and in the later stages of their development that are slowed down by adverse pressure gradients. The existence of pressure-velocity correlations of sufficient magnitude is demonstrated by using the equation for the conservation of kinetic energy in the wake, all terms of which are known excepting the one involving the pressure-velocity correlation, which is then obtained by difference. While the conception of jets of turbulent fluid is more convenient for following the physical processes in the wake, the alternative but equivalent description that the turbulent motion consists of a motion of scale small compared with the mean flow superimposed on a slower turbulent motion whose scale is large compared with the mean flow may be used. A formal explanation of this two-stage turbulent structure in terms of the Fourier representation of the velocity field is suggested, which relates the structure to the presence of a quasi-constant source of energy of nearly fixed wave-number, and to the free boundary which allows an unlimited range of wave-numbers. It is expected that this type of motion will occur in all systems of turbulent shear flow with a free boundary, such as wakes, jets and boundary layers.

➔ Wake flow intermittently turbulent

➔ Compare transport of momentum and energy (spreading)

Experiments: Ancient, cont'd

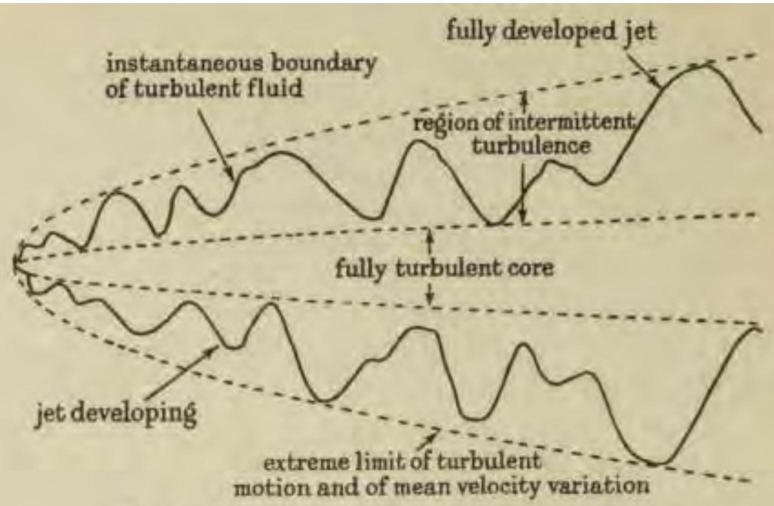


FIGURE 2. Section of hypothetical wake structure.

STRUCTURE OF THE WAKE

Let us now consider the experimental results in turn, and use them to derive information about the detailed properties of these jets of turbulent fluid. In the first place, the velocity product $\overline{uv}$, representing the Reynolds shear stress, has been measured, and, with the observed distribution of mean velocity across the wake, the effective eddy viscosity ϵ and the experimental mixing length l may be calculated, using the definitions

$$\overline{uv} = -\epsilon \frac{\partial U}{\partial y}, \quad \epsilon = l\sqrt{\overline{v^2}}.$$

Experimentally, l is found to be fairly small, approximately 0·07 of the half-width of the mean velocity wake (figure 3), and does not vary greatly over the width of the wake. The small size of l is interpreted as evidence that momentum transfer in the wake is carried out by comparatively small eddies. More significantly, ϵ/γ is not far from constant over the greater part of the wake (figure 4), but this will be discussed later.

➔ Wake expansion due jets of expanding fluid

➔ Departs mean field theory

➔ Mixing length model momentum transport

Experiments: Ancient, cont'd

The product uv may be regarded as the rate of transport of momentum (per unit mass), and similarly the rate of transport of turbulent energy is

$$\tfrac{1}{2}(\overline{u^2v}+\overline{v^3}+\overline{vw^2}),$$

and, in principle, it is possible to calculate an energy diffusion coefficient δ, analogous with ϵ, by use of the defining equation

$$\overline{u^2v}+\overline{v^3}+\overline{vw^2} = -\delta\frac{\partial}{\partial y}(\overline{u^2}+\overline{v^2}+\overline{w^2}).$$

When this is attempted (figure 5), no simple behaviour is found either for δ, or for the corresponding mixing length. Negative values occur near the wake centre, and, even where the turbulence gradient is fairly uniform, δ remains large compared with ϵ,

and decreases rapidly with distance from the wake centre. It must be concluded that the use of a diffusion coefficient to describe the transport of turbulent energy is not justified, and that energy diffusion is a process independent of momentum diffusion.

To remove this difficulty, it is not sufficient to consider the effects of intermittency. If the intermittency factor is known, then the mean intensity in the turbulent regions is

$$I_j = \frac{\overline{u^2}+\overline{v^2}+\overline{w^2}}{\gamma},$$

and I_j is found to vary only slightly over the greater part of the wake (figure 6). So a considerable transport of energy is found in the almost complete absence of a real intensity gradient, and it is difficult to see how energy flow can take place by turbulent

movements inside the jets. For the transport mechanism, there is only left the bulk movement of the jets, which is naturally outwards and away from the wake centre. The compensating inflow will consist of non-turbulent fluid transporting no turbulent energy. Consequently, the flow of energy is not dependent on the local intensity gradient (if any), but only on the mean jet velocity and the jet turbulent intensity, which in turn are determined by conditions in the turbulent core.

➔ Fickian model for turbulent energy transport

➔ "It must be concluded that the use of a diffusion coefficient to describe the transport of turbulent energy is not justified and that energy diffusion is a process independent of momentum diffusion"

Experiments: Modern (Ting Long, SWIP) 1

- HL-2A
- Aims:
  - Exploration of intensity flux – intensity gradient relation in edge turbulence (exploits spreading, shear layer collapse and density limit studies Long + NF'21)
  - Physics of "Jet Velocity" profile

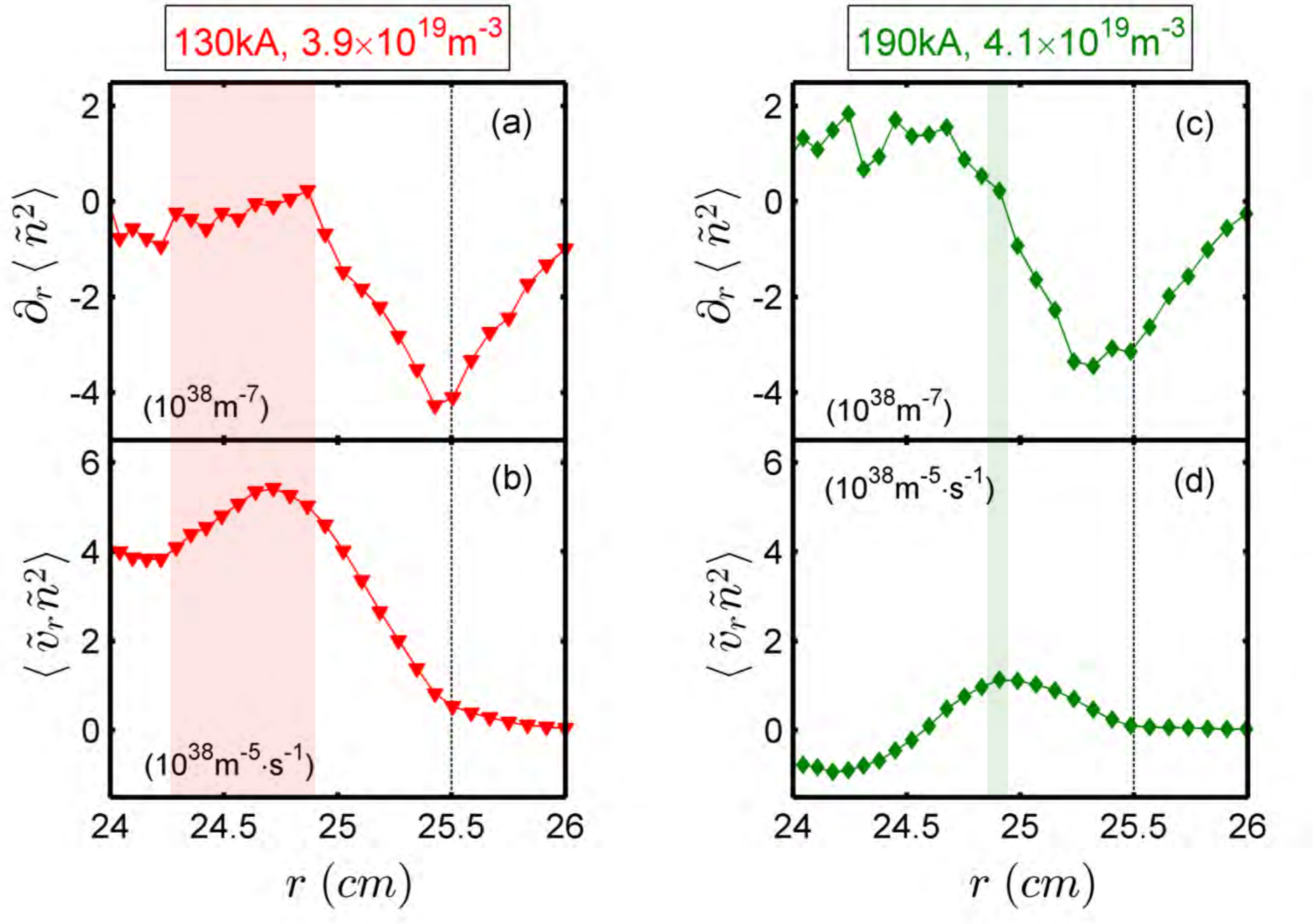
$$V_I = \langle \tilde{V}_r \tilde{n}^2 \rangle / \langle \tilde{n}^2 \rangle \rightarrow \text{effective spreading velocity}$$

N.B. Identified by Townsend

c.f. Long, P.D.+ NF (Lett), in press

Experiments: Modern 2

- There exits a region in plasma edge, where the turbulence spreading flux $\langle \tilde{v}_r \tilde{n}^2 \rangle/2$ **is large**, but the turbulence intensity gradient $\partial_r \langle \tilde{n}^2 \rangle$ **is near zero**



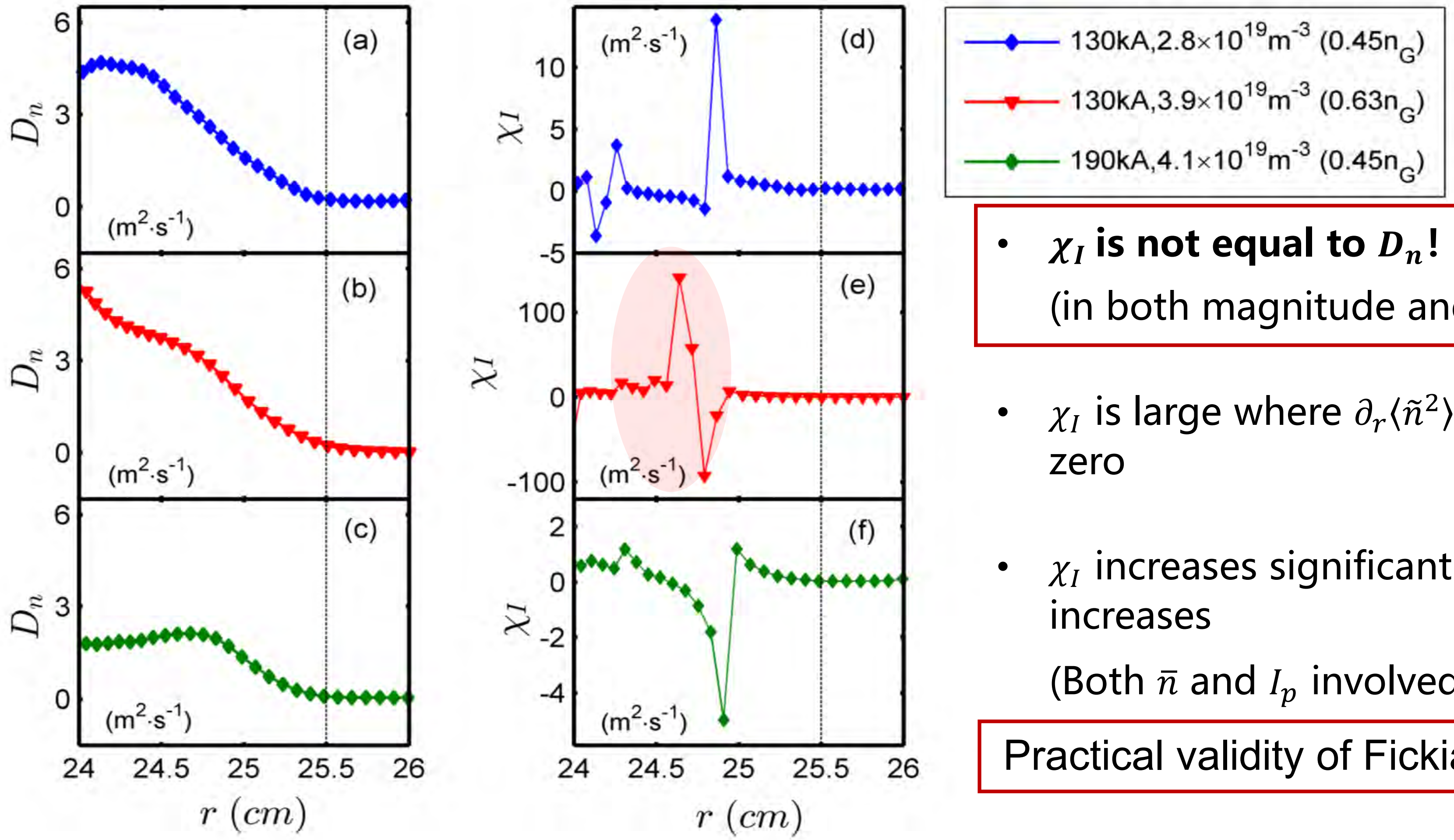
For close $\bar{n}_e$

- Lower current, width of region is $\sim 5\ mm$ $(l_{cr} \sim 4.5\ mm)$ *
- Higher current, width of region is $< 1\ mm$ $(\rho_i \sim 0.25\ mm)$
- Notice: spreading diffusivity

$$\chi_I = -\frac{\langle \tilde{v}_r \tilde{n}^2 \rangle}{\partial_r \langle \tilde{n}^2 \rangle}$$

Experiments: Modern 3

- Striking difference between particle diffusivity and energy spreading diffusivity
  - ➢ Diffusivity of turbulent particle flux $\langle \tilde{n}\tilde{v}_r \rangle = -\boxed{D_n}\,\partial_r \langle n \rangle$
  - ➢ Diffusivity of turbulence spreading $\langle \tilde{v}_r \tilde{n}^2 \rangle = -\boxed{\chi_I}\,\partial_r \langle \tilde{n}^2 \rangle$



- **χ_I is not equal to D_n!**
  (in both magnitude and sign)

- χ_I is large where $\partial_r \langle \tilde{n}^2 \rangle$ is near zero

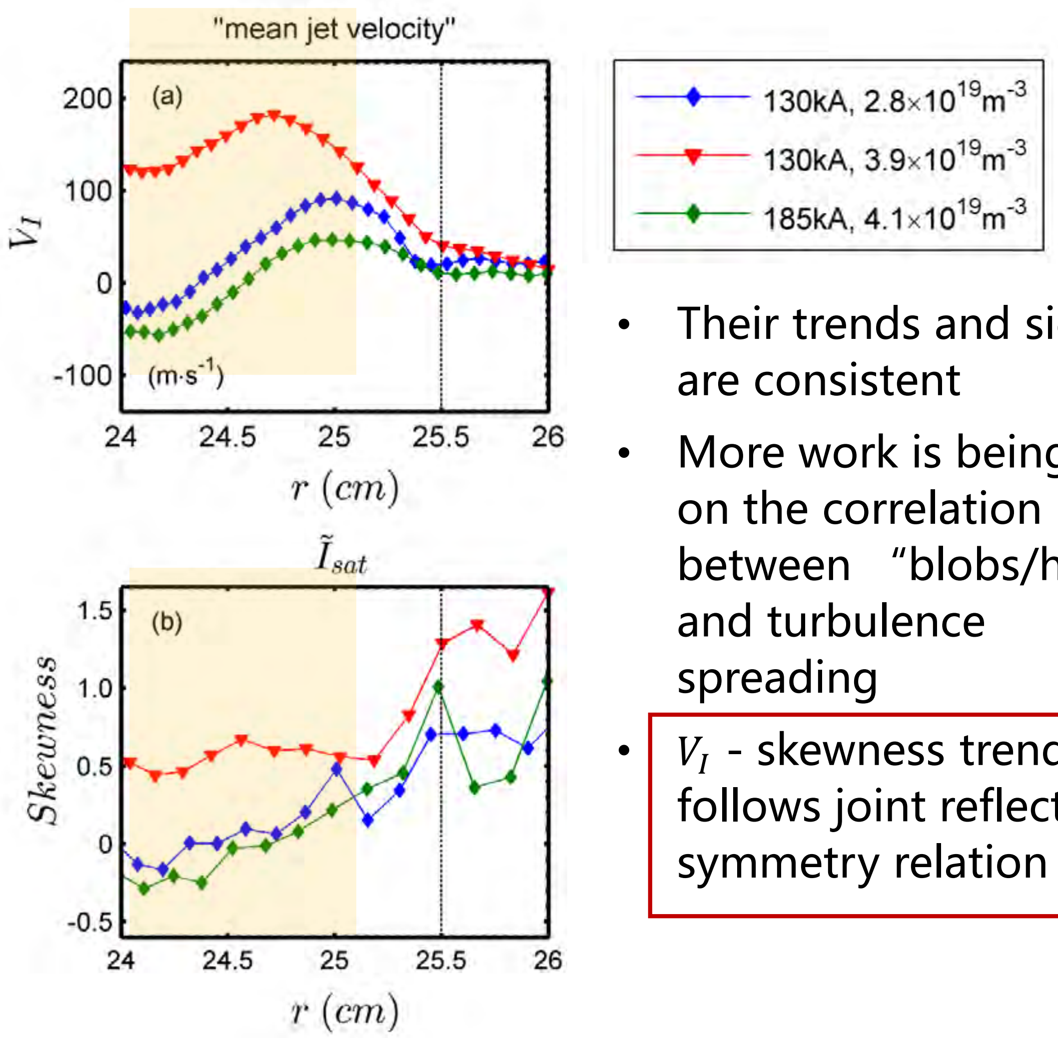
- χ_I increases significantly as $\bar{n}/n_G$ increases
  (Both $\bar{n}$ and I_p involved)

*

Practical validity of Fickian model is dubious

Experiments: Modern 4

- The "mean jet velocity" of turbulence spreading $V_I = \frac{\langle \tilde{v}_r \tilde{n}^2 \rangle}{\langle \tilde{n}^2 \rangle}$ and skewness of density fluctuations show strong correlation



- Their trends and signs are consistent
- More work is being done on the correlation between "blobs/holes" and turbulence spreading
- V_I - skewness trend follows joint reflection symmetry relation

Spreading as Fluctuation Intensity Pulses

- Edge turbulence intermittent:
  - Strong $\langle V_E \rangle'$ → ~ marginal avalanching state
  - Weaker $\langle V_E \rangle'$ → 'blobs', etc. $\Gamma_e = \langle \Gamma_e \rangle + \tilde{\Gamma}_e$
- $\boxed{\text{Pulses / Avalanches are natural description}}$

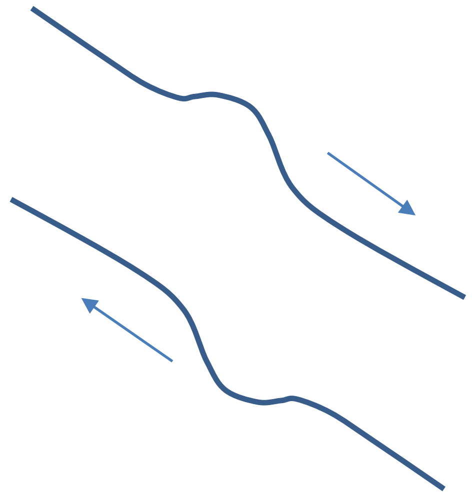
$\delta P \equiv$ deviation of profile from criticality

$$\delta P \leftrightarrow (\nabla P - \nabla P_{crit})/P$$

Naturally: $\delta P \sim \delta \varepsilon$

→ Spreading as intensity pulses

(after Hwa, Kardar, P.D., Hahm)



Pulse, void symmetry arguments etc.

Fluctuation Energy Pulses, cont'd

- Burgers is on the grill…
- New toppings:
  - $\delta P > 0$ turbulence ejected into SOL
    positive intensity fluctuation
  - $V_D > 0$ mean drift out – curvature
- * Scale independent damping
  - $(1/\tau)\delta P$ due finite dwell time in SOL → order parameter not conserved
- Noise is b.c.
  - $\tilde{\Gamma}_{0,e}|_{\text{sep}}$ drives system, space-time

Fluctuation Energy Pulses, cont'd

- Pulse model:

1 drift
2 dwell time decay
3 spreading

1 3 2

$$\partial_t \tilde{P} + V_D \partial_x \tilde{P} + \alpha \tilde{P} \partial_x \tilde{P} - D_0 \partial_x^2 \tilde{P} + \frac{\tilde{P}}{\tau} = 0$$

regularization

$$\tilde{P}(0,t) \leftrightarrow \tilde{\Gamma}_{sep}(t)$$

- Some limits:

  – $\tilde{P} \to 0$, $V_D \partial_x \tilde{P} \sim \frac{\tilde{P}}{\tau}$ $\to$ $\lambda \sim \lambda_{HD}$ scale (1 vs 2)

  – For $\tilde{P}$ to matter:

$\alpha \tilde{P} > V_D$ → amplitude vs neo drift comparison (1 vs 3)

Fluctuation Energy Pulses, cont'd

- Predictions?

Structure formation → Shock Criterion !

i.e. Recall: $\frac{d\tilde{P}}{dt} = -\frac{\tilde{P}}{\tau}$, $\frac{dx}{dt} = \alpha\tilde{P}$

- Solve via characteristics:

$$x = \alpha\left[z + \frac{\left(1 - e^{-\frac{t}{\tau}}\right)}{(1/\tau)} f(z)\right]$$

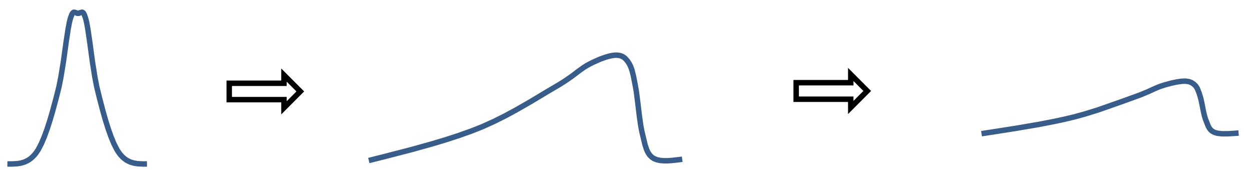
Shock for: $f'(z) < -1/\tau$

→ inital slope must be sufficiently steep to shock before damped by $1/\tau$

Spreading as Fluctuation Intensity Pulses, cont'd

- $\alpha \frac{\partial \tilde{P}}{\partial x}|_{sep} < -\frac{1}{\tau}$ → $\boxed{\text{pulse formation criterion → intensity gradient}}$

- Fate ?



$\alpha\ \varepsilon < V_D$ → defacto 'evaporation criterion'

→ defines penetration depth of pulse

- Aim to characterize <u>statistics</u> of pulses, penetration depth distribution… in terms Pdf($\tilde{\Gamma}_{0,e}$) . Challenging…

➔ Meaningful output for SOL broadening problem

Concluding Philosophy

- MFE relevant questions within reach in near future. Great attention to λ_q problem (c.f. Samuel Johnson)
- Unreasonable for tokamak experiments to probe ~ critical dynamics so as to elucidate basic questions. Simulations???
- Well diagnosed, basic experiment with some relevant features are sorely needed – akin to 'Tube' studies of flows, ala' CSDX
- How?

Thanks for Attention !

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