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Theory of Heat Load Broadening by Entrainment

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Outline

1) The Problem
2) Solution:
   a) Calculating λ_q
   b) Calculating SOL Turbulence Driven by Spreading
   c) Relating Spreading to Pedestal Properties
3) Beyond Mean Field Theory

→ 4) Ongoing Work

Abstract

Developments in the theory of heat load broadening by entrainment of the stable SOL by pedestal turbulence is presented. Turbulent and neoclassical effects add in quadrature to set λ_q. The SOL intensity is determined by matching the turbulence energy flux from the pedestal to SOL. Explicit expressions are derived for λ_q for modest and strong broadening. Scalings of λ_q with R, B_θ, T_{sep}, q and spreading flux are determined. A fundamental limit on the extent of λ_q broadening is suggested. Spreading fluxes for drift wave and ballooning mode turbulence in the pedestal are derived and used to show that interesting levels of SOL broadening can be achieved for tolerable pedestal fluctuation levels. A simple treatment of the unavoidable departures from the realm of mean field theory is proposed.

1) The Problem

— Edge Transport Barriers $\Rightarrow$ $E \times B$ shear layer

— SOL $E \times B$ shear quenches SOL Turbulence

$$\boxed{v_E' \sim 3T_e/|e|\lambda^2}$$

$T_e \equiv$ separatrix temperature

$\lambda \equiv$ layer width

— Neoclassical Processes only, survive $\Rightarrow$

Goldston HD scaling

$$\boxed{\lambda_q \sim \epsilon_T \rho_\theta}$$

$\lambda \sim v_D \tau_d$

$v_D \sim \rho_s c_s/R, \tau_d \sim Rq/c_s$

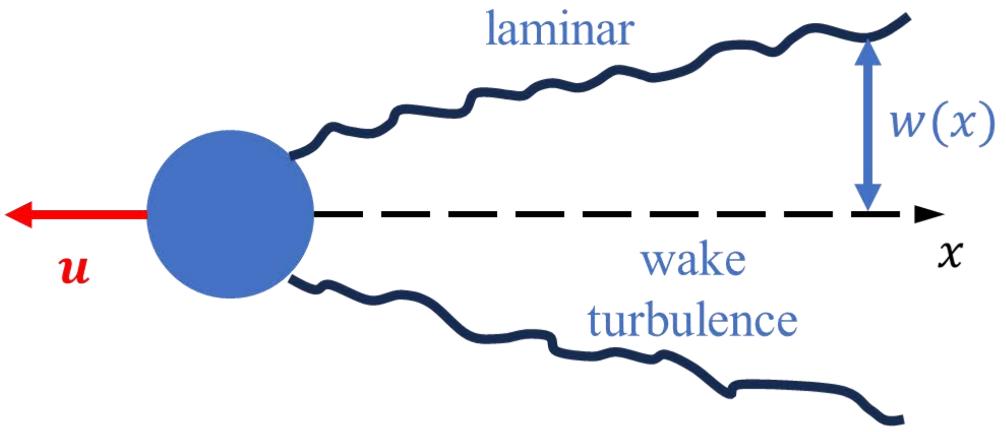
— Very Pessimistic

Remarkably Successful

2) Solution

- Spreading of turbulence from pedestal to SOL is solution of heat load problem
- Requires turbulent pedestal (c.f. Turbulent QH mode)

  Turbulence energy flux at separatrix is critical.
- Spreading: Heuristics



Turbulent wake,
a simple, familiar example
of spreading

- Calculation Required → need more than color pictures from simulations

2a) Calculating λ_q

$\rightarrow \frac{dr}{dt} = v_D + \tilde{v}_r$

$\rightarrow \delta^2 \equiv$ mean square excursion

$= v_D^2\tau_d^2 + \langle\tilde{v}_r^2\rangle\tau_c\tau_d$, $\tau_c = \int_0^\infty \tilde{v}_r(0)\tilde{v}(\tau)d\tau\,/|\tilde{v}_r(0)|^2$

$\rightarrow \tau_c \sim \tau_d$ $\longrightarrow$ $\begin{cases} \tau_c > \tau_d \text{ unphysical } \times \\ \tau_c < \tau_d \text{ strong turbulence } \times \end{cases}$

$\rightarrow \delta^2 = v_D^2\tau_d^2 + e\tau_d^2$ $e \equiv$ SOL turbulence intensity

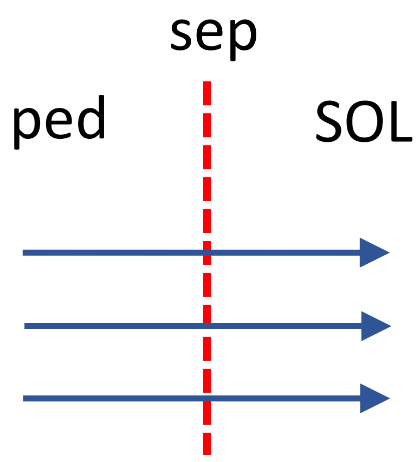
$\lambda_q^2 \equiv \lambda_{HD}^2 + \lambda_T^2$ $\lambda_T^2 = e\tau_d^2$

$\rightarrow$ turbulent width

2b) Calculating Spreading-Driven SOL Turbulence

→ $K - \epsilon$ model

$$\partial_t e = \underbrace{\gamma e}_{\text{growth}} - \underbrace{\sigma e^{1+\kappa}}_{\text{NL saturation}} - \underbrace{\partial_x \Gamma_e}_{\text{spreading flux}}$$



→ Integrating for stable SOL:

$$\Gamma_0 = \lambda_T |\gamma| e + \sigma \lambda_T e^{1+\kappa}$$

$$\lambda_q^2 = \lambda_{HD}^2 + e\tau_d^2 = \lambda_{HD}^2 + \lambda_T^2$$

$\Gamma_0 \equiv$ turbulence energy flux at separatrix

... simple, closed minimal model

→ Consider cases: 1) Linear damping
2) Nonlinear damping

dominant

2b i) Calculating Spreading-Driven SOL Turbulence Linear Damping

$\rightarrow \lambda_q = \left[\lambda_{HD}^2 + \left(\frac{\Gamma_0 \tau_d^2}{|\gamma|}\right)^{2/3}\right]^{1/2}$

N.B. $\lambda_T \sim \Gamma_0^{1/3}$

— Broadened heat load width

— $|\gamma| \rightarrow E \times B$ shearing

$|\gamma| \cong T_{sep}/|e|\lambda_{HD}^2$

$\rightarrow$ Cross-over from HD to Broadened:

$$\lambda_T^2 > \lambda_{HD}^2 \quad \Rightarrow \left[\left(\frac{\Gamma_0}{|\gamma|}\right)^{2/3} \left(\frac{B_T}{B_\theta}\right)^{4/3} \frac{r^{4/3}}{c_s^{4/3}}\right] / \epsilon_T^2 \rho_\theta^2 > 1$$

$\rightarrow \lambda_T/\lambda_{HD} \sim \Gamma_0^{1/3} R^{2/3} B_\theta^{1/3} / T_{e,sep}$

i.e., condition for turbulent width to exceed HD width

- $\rightarrow$ larger R good
- $\rightarrow$ high T_{sep} unfavorable
- $\rightarrow$ weak current dependence
- $\rightarrow$ weak Γ_0 dependence

2b ii) Calculating Spreading-Driven SOL Turbulence Nonlinear Damping

→ $\lambda_q = \left[\lambda_{HD}^2 + (\Gamma_0/\sigma)^{2/(3+4\kappa)} \tau_d^{4(1+\kappa)/(3+2\kappa)}\right]^{1/2}$

→ For $\kappa = 1/2$ (strong turbulence)

$$\lambda_q = \left[\lambda_{HD}^2 + (\Gamma_0/\sigma)^{2/5} \tau_d^{3/2}\right]^{1/2} \sim (\Gamma_0/\sigma)^{1/5} (Rq/c_s)^{3/4} \quad (*)$$

N.B.: Scaling of λ_q with Γ_0 tends to saturation!

→ Is there a maximal broadening?

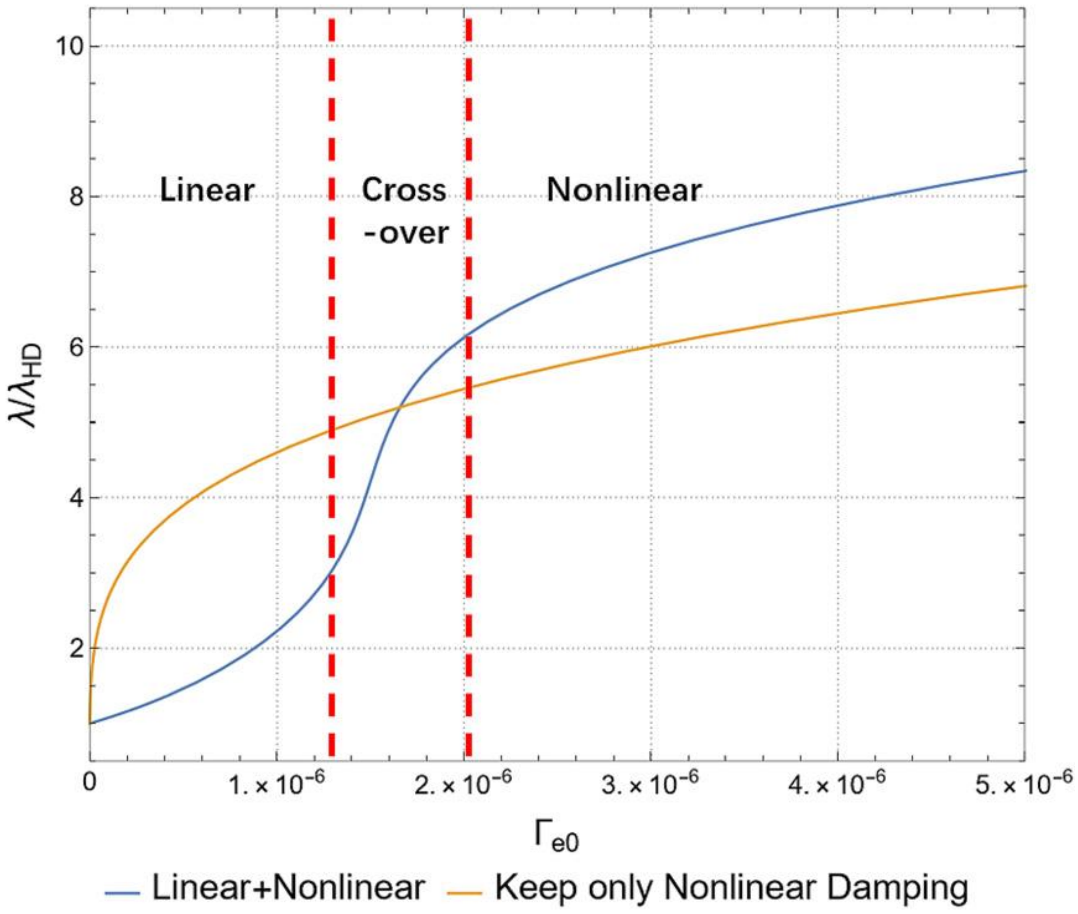
— Marginal SOL stability for $c_s/\sqrt{R\lambda} \approx T_{sep}/|e|\lambda^2$

Using $(*)$ above ⇒

$$\Gamma_{max} \cong (T_e/|e|c_s)^{10/3} R^{5/3} / (Rq/c_s)^{15/4} \sigma \cong T_{e,sep}^{85/24} / R^{15/4} q^{15/4}$$

Γ_{max} is a fundamental limit on the spreading flux to maintain stable SOL
Violation ⇒ back transition!?

2b) Calculating Spreading-Driven SOL Turbulence



2c) Relating spreading to pedestal properties

(a) Pedestal turbulence located close to the separatrix necessarily has the greatest impact on SOL broadening.

(b) The edge transport barrier (shear layer) will necessarily tend to inhibit spreading through the separatrix.

(c) Pedestal turbulence with larger mixing length will be more effective for turbulence spreading. This favors larger scale modes.

(d) Turbulence spreading into the SOL from the pedestal is almost certainly both convective and diffusive (i.e., driven by intensity gradient), and partly mediated by the dynamics of structures, such as blobs and voids. However, our understanding of how to actually calculate the non-diffusive flux is still developing. Hence this analysis is limited to a diffusive model of spreading.

2c) Relating Spreading to Pedestal Properties

— Need calculate flux of turbulence energy into SOL, from pedestal

— Challenging! — simple, nonlinear diffusion model useful

→ $\Gamma_0 = -\tau_c e \partial_r e \approx \tau_c e^2 / w_{ped}$

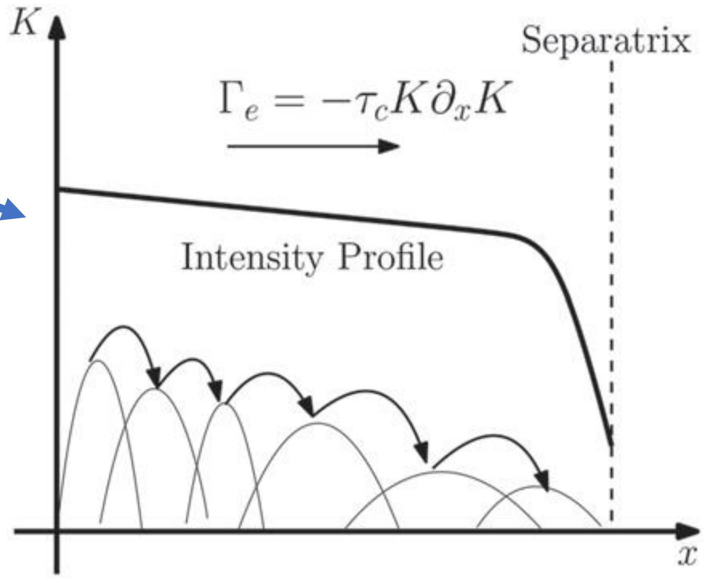
— For τ_c, D

$$D = \int_0^\infty \langle v(0) v(\tau) \rangle d\tau$$

$$= \int_0^\infty d\tau \sum_{\boldsymbol{k}} |v_{\boldsymbol{k}}|^2 \exp\left[-k_\theta^2 \omega_s^2 D \tau^3 - k^2 D \tau\right]$$

$$\tau_c^{-1} = k^2 D (1 + \omega_s^2 \tau_c^2)$$

$$D \sim |v|^{3/2} k^{-1/2} \omega_s^{-1/2}$$



→ $\Gamma_0 \cong \tau_k^{0.5} \omega_s^{-0.5} e \partial_r e \cong \tau_k^{0.5} \omega_s^{-0.5} e^2 / w_{ped}$, where

$$\omega_s = \partial_r \nabla p / n|e| \sim \left(\rho_i^2 / w_{ped}^2\right) \Omega_i$$

2c) Relating spreading to Pedestal Properties

— Calculate pedestal fluctuation intensity needed to broaden layer

$\lambda/\lambda_{HD} > 1$

— Consider:

a) Drift wave turbulence

b) Ideal ballooning turbulence

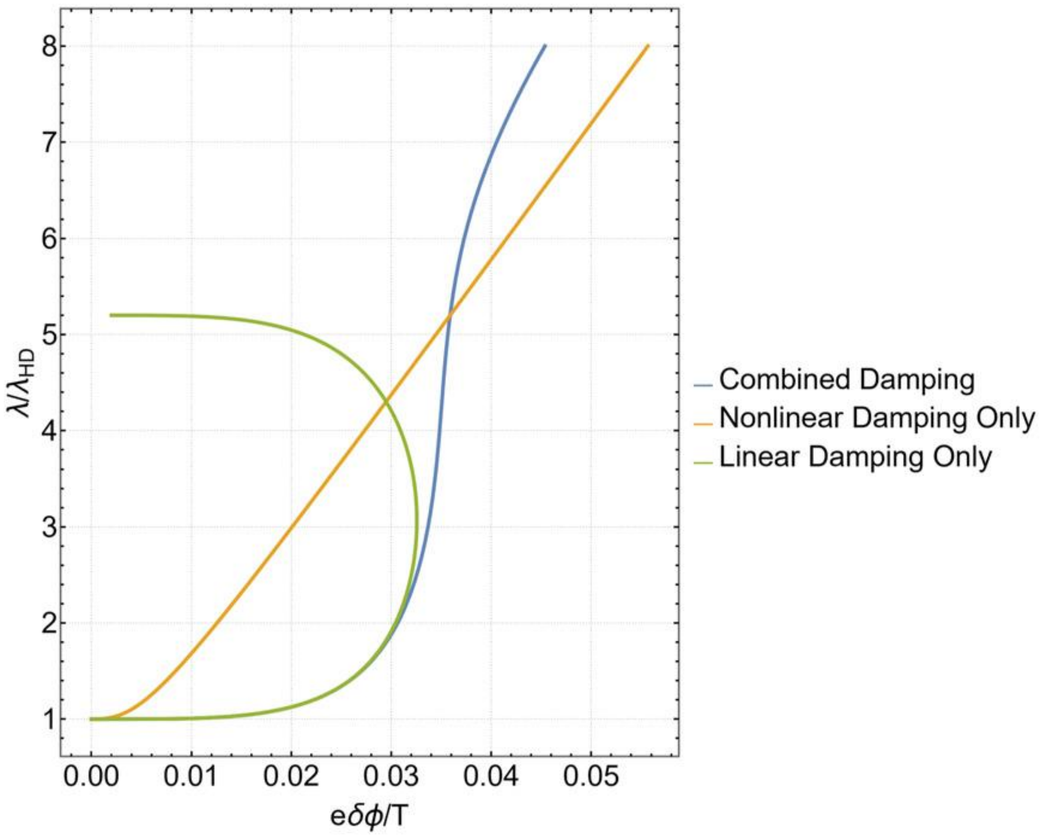
$\Rightarrow$ "grassy ELMs"

a) Drift Waves

$\lambda/\lambda_{HD} > 1$ for

key factor

$$\tilde{v}_r/c_s \sim \frac{|e|\hat{\phi}}{T} \sim \boxed{\left(\frac{\rho_i}{R}\right)^{1/2}} q^{-1/4}\left(w_{ped}/\rho_i\right)^{-1/8}$$



$\lambda/\lambda_{HD} \sim 3 \rightarrow 5$ for modest fluctuation levels at separatrix

2c) Relating spreading to Pedestal Properties

b) Ideal Ballooning/Grassy ELM

— Convenient to formulate in terms of λ/λ_{HD} vs. $L_{pc}/L_p - 1$ ($L_{pc} \equiv L_p$ critical)

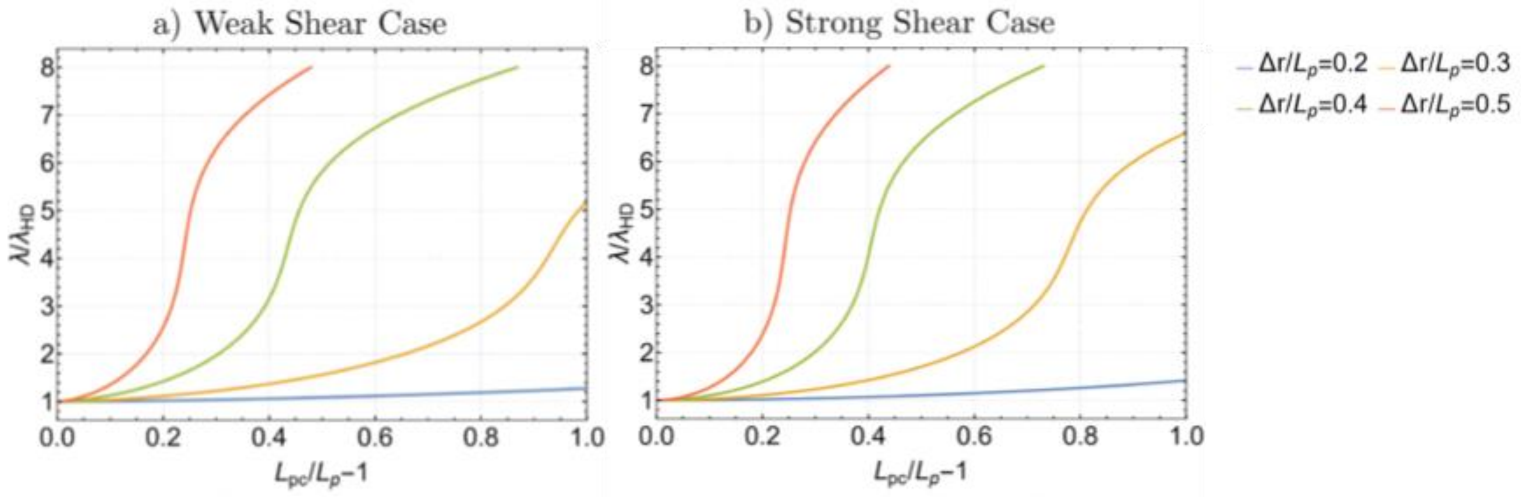
$\Rightarrow$ deviation from marginality

— Simple calculation $\Rightarrow$

i. Weak $E \times B$ shear: $\frac{L_{pc}}{L_p} - 1 \sim \left(q\frac{\rho_i}{R}\right)^{4/3} \frac{R^2}{w_{ped}^2} \left(\frac{w_{ped}}{\Delta}\right)^{8/3} \beta$

ii. Strong $E \times B$ shear: $\frac{L_{pc}}{L_p} - 1 \sim \left(q\frac{\rho_i}{R}\right)^{10/7} \left(\frac{R}{w_{ped}}\right)^{16/7} \left(\frac{w_{ped}}{\Delta_r}\right)^{16/7} \beta$

Again, ρ_i/R is a key parameter



λ/λ_{HD} vs

margin

supercriticality

Comments

a) Sensitivity analysis reveals that those results are more sensitive to linear damping in the SOL than to the details of nonlinear scattering.

b) There is little difference between cases of weak and strong $E \times B$ shear. This is due to off-setting trends in τ_c and w_{ped} in the expression for $\Gamma_{e,0}$.

c) Larger scale turbulence near the separatrix is more effective at SOL broadening.

d) The calculation of the spreading flux needs to be revisited, so as to incorporate intermittency effects. Recent simulations and experiments indicate that the spreading flux is strongly skewed, with skewness vanishing at a radius close to the separatrix. The turbulence exhibits spatio-temporal intermittency, and thus is a challenge to model. The turbulence exhibits spatio-temporal intermittency. Turbulence spreading into the SOL thus consists of positively skewed fluctuations, which may be thought of as 'blobs'. The effects of these structures are not addressed by the diffusive spreading flux model employed here. Clearly, the Fickian model of the spreading flux is inadequate. A complete model of spreading remains an unfulfilled goal, and an important one.

see Ref. 6, 7, 8

3) Beyond Mean Field Theory

— Spreading as Fluctuation Intensity Pulses

- Edge turbulence intermittent:
  - Strong $\langle v_E' \rangle \rightarrow \sim$ marginal avalanching state
  - Weak $\langle v_E' \rangle \rightarrow \sim$ 'blobs', etc. $\quad \Gamma_e = \langle \Gamma_e \rangle + \tilde{\Gamma}_e$
- Pulses/Avalanches are natural description

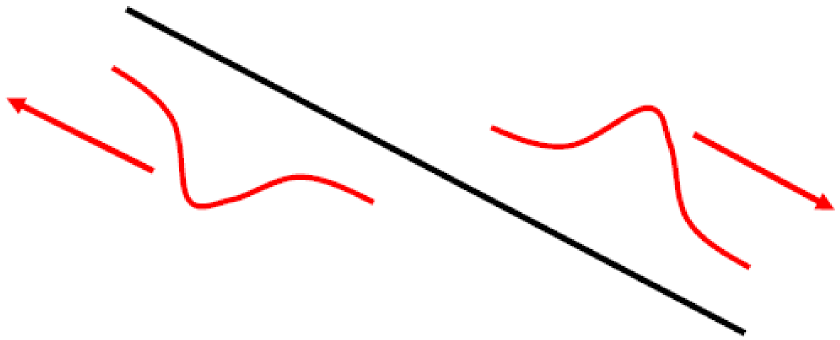
  $\delta p \equiv$ deviation of profile from criticality

  $\delta p \leftrightarrow (\nabla p - \nabla p_{crit})/p$

  Naturally: $\delta p \sim \delta \varepsilon$

$\Rightarrow$ Spreading as intensity pulses

(after Hwa, Kardar, P.D., Hahm)



Pulse, void symmetry arguments etc.

3) Spreading as Fluctuation Intensity Pulses, cont'd

- Pulse model:

① drift

② dwell time decay

③ spreading

$$\partial_t\tilde{\varepsilon} + \overbrace{v_D\partial_x\tilde{\varepsilon}}^{①} + \overbrace{\alpha\tilde{\varepsilon}\partial_x\tilde{\varepsilon}}^{③} - D_0\partial_x^2\tilde{\varepsilon} + \overbrace{\tilde{\varepsilon}/\tau}^{②} = 0$$

$$\tilde{\varepsilon}(0,t) \leftrightarrow \tilde{\Gamma}_{sep}(t)$$

- Some limits:

– $\varepsilon \to 0$, $v_D\partial_x\tilde{\varepsilon} \sim \tilde{\varepsilon}/\tau \to \lambda \sim \lambda_{HD}$ scale (① vs ②)

– For ε to matter:

$\alpha\tilde{\varepsilon} > v_D \rightarrow$ amplitude vs neo drift comparison (① vs ③)

- Structure is Burgers + Krook $\rightarrow$ "Crooked Burgers"

3) Spreading as Fluctuation Intensity Pulses, cont'd

- Predictions?

  Structure formation → Shock Criterion!

  i.e. Recall: $\frac{d\varepsilon}{dt} = -\frac{\varepsilon}{\tau}, \quad \frac{dx}{dt} = \alpha\varepsilon$

- Solve via characteristics:

$$x = \alpha\left[z + \frac{\left(1-e^{-\frac{t}{\tau}}\right)}{(1/\tau)}\right]$$

Shock for: $f'(z) < -1/\tau \Rightarrow$ defines structure formation

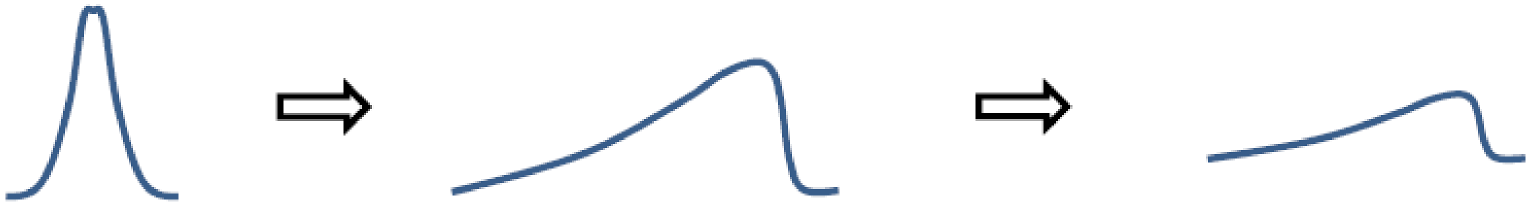
→ initial slope must be sufficiently steep to shock before damped by $1/\tau$

→ critical intensity gradient required to form shock structure

3) Spreading as Fluctuation Intensity Pulses, cont'd

- $\alpha \frac{\partial \varepsilon}{\partial x}|_{sep} < -\frac{1}{\tau}$ → pulse formation criterion → separatrix intensity gradient

- Fate?



→ defines penetration depth of pulse, scale of broadening

- Aim to characterize statistics of pulses, penetration depth distribution…
  in terms of Pdf($\tilde{\Gamma}_{0,e}$). Challenging…

  ⇒ Meaningful output for SOL broadening problem, beyond mean field theory.

N.B.: Is the heat load distribution well characterized by a single scale?

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