An Overview of Staircases in Confined Magnetized Plasmas

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Mea Culpa

- Pitched for a Classical Physics audience

  ∴ extensive development

- Approach selective, not unique

  ∴ several worthy topics neglected

- Tries to convey how confinement experiments drive new theoretical problems

Outline

- Brief Primer on Confinement Physics
- Simple Models, via Potential Vorticity / Total Charge
- Mesoscopics → Staircases
- Staircase Models → What do we learned?
- Current Issues, especially Noise effects, Resiliency
- Future Directions, especially + Fast Particles, Burning Plasma

Primer on Confinement Physics

Magnetically confined plasma → tokamaks

- Nuclear fusion: option for generating large amounts of carbon-free energy – "30 years in the future and always will be… "
- Challenge: ignition -- reaction release more energy than the input energy

  Lawson criterion:

  $$n_i \tau_E T_i > 3 \times 10^{21} \mathrm{m}^{-3} \mathrm{s\,keV}$$

  → confinement

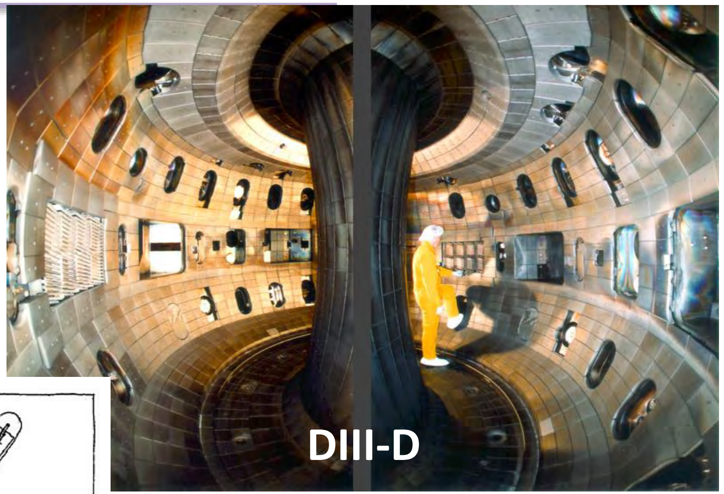
  → turbulent transport

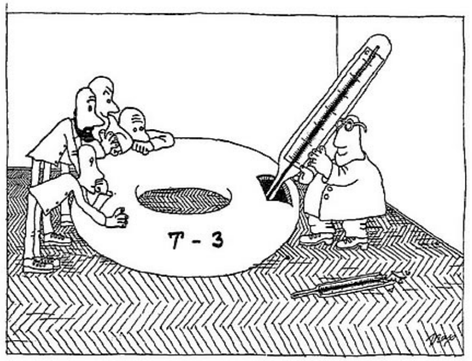
  $$\tau_E \sim \frac{W}{P_{in}}$$

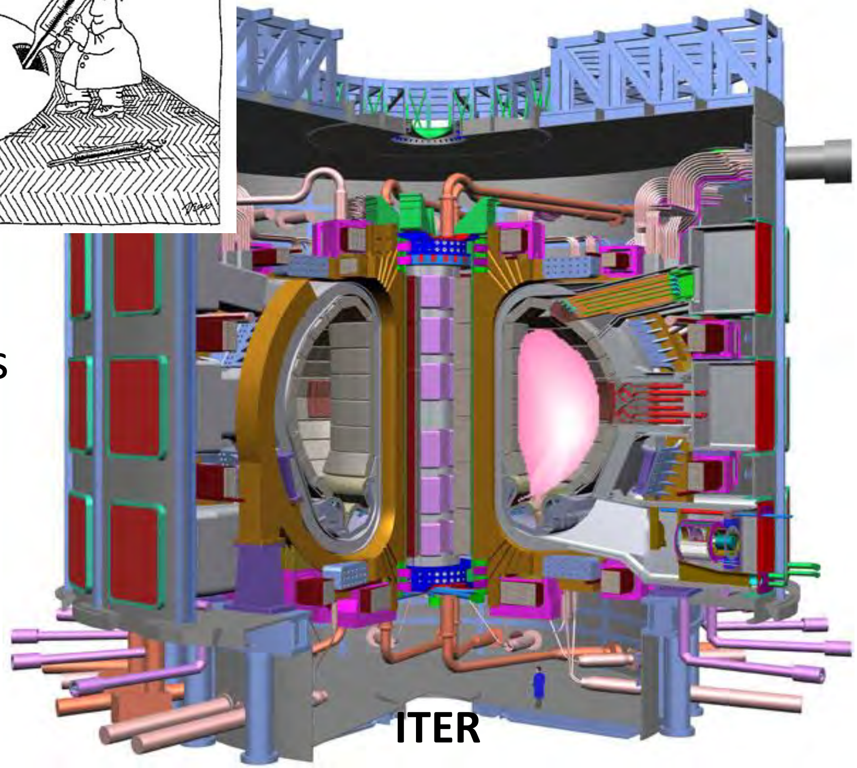
- Turbulence: instabilities and collective oscillations

  → low frequency modes dominate the transport ($\omega < \Omega_{ci}$)

- Key problem: Confinement, especially scaling
- NB: Not the only problem







- Essence of confinement problem:
  - given device, sources; what profile is achieved?
  - $\tau_E = W/P_{\rm in}$, How optimize W, stored energy
- Related problem: Pipe flow → drag ↔ momentum flux

ΔP → pressure drop

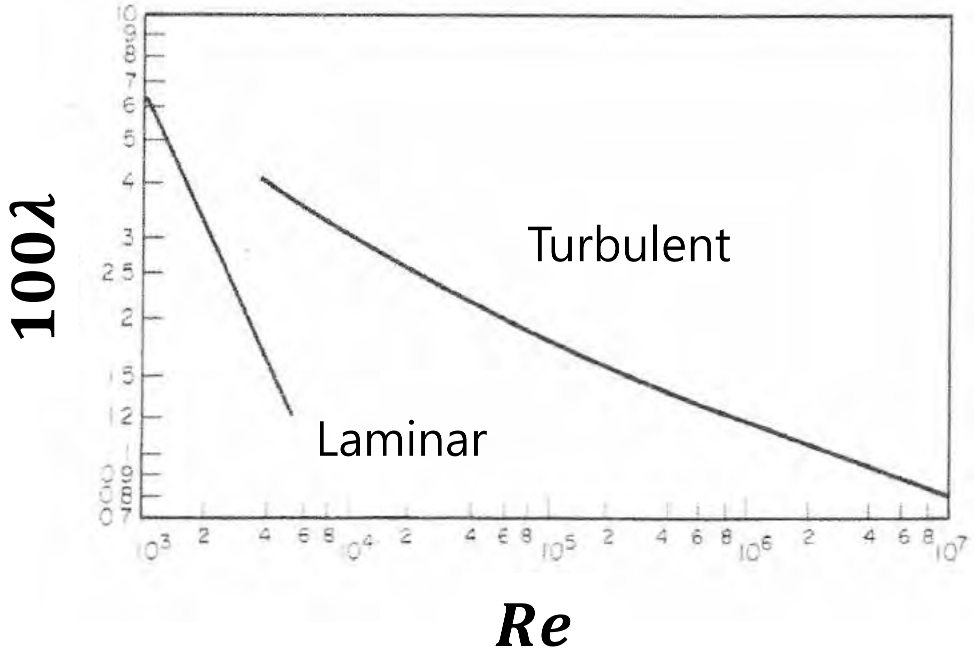
l

$$\Delta P \pi a^2 = \rho V_*^2 2\pi a l$$

→ friction velocity V_* ↔ u

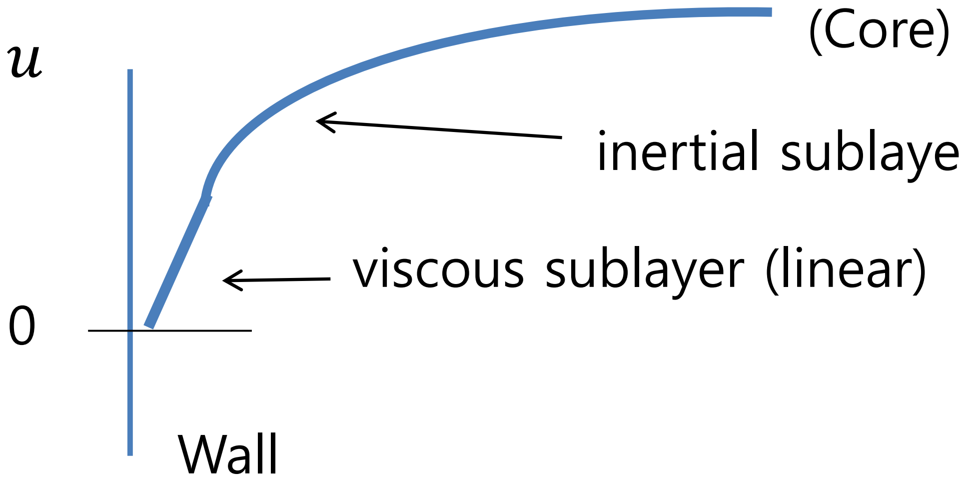
Balance: momentum transport to wall (Reynolds stress) vs ΔP

➔ Flow velocity profile



$$\lambda = \frac{2a\Delta P/l}{(1/2)\rho u^2}$$

Friction factor



Profile

- Problem: physics of ~ universal logarithmic profile?
- Universality → scale invariance

- Prandtl Mixing Length Theory (1932)
  - Wall stress = $\rho V_*^2 = -\rho \nu_T \, \partial u / \partial x$ or: $\frac{\partial u}{\partial x} \sim \frac{V_*}{x}$ ← Spatial counterpart of K41
    - eddy viscosity (ν_T)
    - Scale of velocity gradient? (x)
  - Absence of characteristic scale →

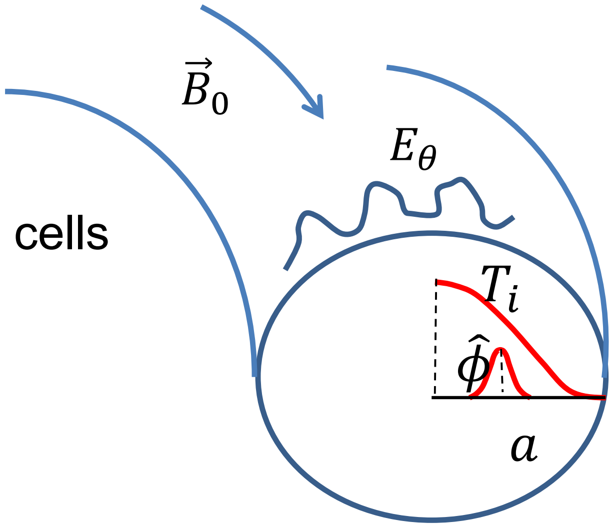
$$\nu_T \sim V_* x$$

$$u \sim V_* \ln(x/x_0)$$

$x \equiv$ mixing length, distance from wall
Analogy with kinetic theory …

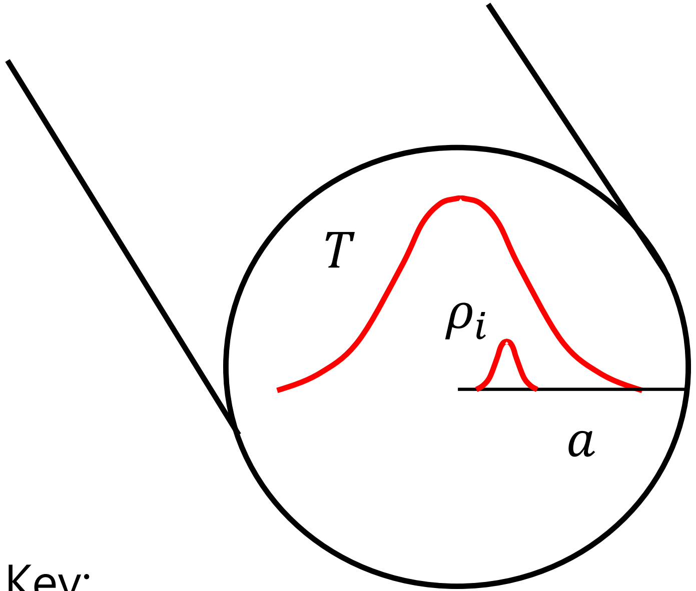
$\nu_T = \nu \to x_0$, viscous layer → $x_0 = \nu / V_*$

Primer on Turbulence in Tokamaks I



- Strongly magnetized
  - Quasi 2D cells, Low Rossby #
  - * – Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance) – pinned cells
- $\vec{V}_\perp = +\frac{c}{B}\,\vec{E} \times \hat{z}, \quad \frac{V_\perp}{l\,\Omega_{ci}} \sim R_0 \ll 1$
- $\nabla T_e, \nabla T_i, \nabla n$ driven
- Akin to thermal convection with: g → magnetic curvature
- → $\mathrm{Re} \approx VL/\nu$ ill defined, not representative of dynamics
- → Resembles 'wave turbulence', not high Re Navier-Stokes turbulence
- → $K \sim \tilde{V}\tau_c/\Delta \leq 1$ → *Kubo # near unity, Ku* is meaningful parameter
- → Broad dynamic range, due electron and ion scales, i.e. a, ρ_i, ρ_e

Primer on Turbulence in Tokamaks II



Key:

2 scales:

$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

$\rho_* \equiv \rho/a$ ➔ key ratio

$\rho_* \ll 1$

- Characteristic scale ~ few ρ_i → "mixing length"
- Characteristic velocity $v_d \sim \rho_* c_s$
- Transport scaling: $D_{GB} \sim \rho \, V_d \sim \rho_* \, D_B$ - Gyro-Bohm (optimistic)

  $D_B \sim \rho \, c_s \sim T/B$ – Bohm (pessimistic)
- i.e. Bigger is better! ➔ sets profile scale via heat balance

  (Why ITER is huge…)
- Reality: $D \sim \rho_*^{\alpha} \, D_B$, $\alpha < 1$ ➔ 'Gyro-Bohm breaking'
- 2 Scales, $\rho_* \ll 1$ ➔ key contrast to pipe flow
- Sneak preview: $\alpha \leq 1$
- related to turbulence driven zonal shear flows

Models via Potential Vorticity

Potential Vorticity

- GFD → The Fluid Dynamics of PV (R. Salmon)
- Ditto for Confined Plasmas.... (PD)
- $PV = q = \frac{\vec{\omega}+2\vec{\Omega}}{\rho} \cdot \nabla\psi$ (ala' conserved charge density)

Rotating Fluid

ψ conserved scalar

$$\frac{d}{dt}\left[\frac{\vec{\omega}+2\vec{\Omega}}{\rho} \cdot \nabla\psi\right] = 0$$

PV Conservation

- From:

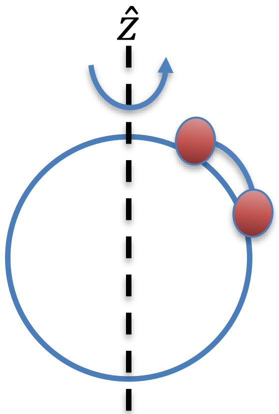
Freezing in $\frac{d}{dt}\left(\frac{\vec{\omega}+2\vec{\Omega}}{\rho}\right) = \left(\frac{\vec{\omega}+2\vec{\Omega}}{\rho}\right) \cdot \nabla\vec{\mathrm{v}}$

Conserved scalar $\frac{d}{dt}\delta\psi = 0$

Potential Vorticity, cont'd

- Displace parcel in latitude, density/thickness

➔ ω changes



$$q = \frac{(\vec{\omega}+2\vec{\Omega})\cdot\nabla\psi}{\rho}$$

- Conservation ←→ Symmetry, ala' Noether

Particle relabeling $\vec{x}(x,\tau)$ $\quad s \rightarrow s' = s + \delta s$

PV conserved when particles can be relabeled, without changing the thermodynamic state

Useful Form: β-plane Equation

- β-plane equation $\frac{d}{dt}(\omega + \beta y) = 0$

(after Charney + …)

n.b. topography

- Locally Conserved PV $q = \omega + \beta y$

parcel ↗ ↖ planetary

$q = \omega / H + \beta y$

- Latitudinal displacement → change in relative vorticity

- Linear consequence → Rossby Wave

$$\omega = -\beta k_x / k^2$$

observe: $v_{g,y} = 2\beta k_x k_y / (k^2)^2$

↳ Rossby wave intimately connected to momentum transport

➔ Reynolds stress $\langle V_x V_y \rangle$

$\omega = 0$ → zonal flow

$k_x = 0$ → azimuthal symmetry

- Latitudinal PV Flux → circulation

PV Dynamics – Plasmas

- Isn't this about plasmas, too?

- $q = (\vec{\omega} + 2\vec{\Omega}) \cdot \frac{\nabla\psi}{\rho}$

now
$$\begin{cases} 2\vec{\Omega} \rightarrow \Omega_i \hat{z} \\ \rho \rightarrow n_0(r) + \tilde{n} \\ \vec{\nabla}\psi \rightarrow \hat{z} \end{cases}$$

So $\frac{d}{dt}\left[\frac{\omega_z + \Omega_i}{n_0(r) + \tilde{n}}\right] = 0$

ala' Geostrophic balance:

$$\Rightarrow \frac{d}{dt}\widetilde{\omega}_z - \Omega_i \frac{1}{n_0}\frac{\widetilde{dn_i}}{dt} = 0$$

$$\begin{cases} \vec{V} = -\frac{c}{B}\nabla\phi \times \hat{z} \\ \omega_z = \frac{c}{B_0}\nabla^2\phi \end{cases}$$

$E \times B$ drift

with $V_{thi} \ll \frac{\omega}{k_\parallel} < V_{the}$ $\quad \frac{\tilde{n}_i}{n_0} \sim \frac{\tilde{n}_e}{n_0} \sim \frac{|e|\hat{\phi}}{T}$

$$\rightarrow \boxed{\frac{d}{dt}\left(\frac{|e|\hat{\phi}}{T} - \rho_s^2 \nabla_\perp^2 \frac{|e|\hat{\phi}}{T}\right) + V_* \partial_y \frac{|e|\hat{\phi}}{T} = 0}$$

→ Hasegawa-Mima Eqn.
PV conservation
also Sagdeev +

Linearization → drift wave

PV and Models - Plasmas

- Hasegawa-Mima, prototype:

$$\frac{d}{dt}\left(\phi - \rho_s^2 \nabla^2 \phi + \ln n_0(r)\right) = 0$$

- tip of iceberg of zoology of systems: multi-field, drift kinetics, gyrokinetics…
- captures essence ←→ minimal model
- in tokamak, zonal flows have: $k_\parallel = 0$ and $k_\theta = 0$

$\frac{d}{dt}\nabla^2\phi = 0$ { distinct evolution zonal models! ←→ electron response

➔ generation of flow → $\langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle$ → vorticity flux ➔ $\langle \tilde{V}_r \tilde{V}_\theta \rangle$ (Taylor identity)

←→ mean field of wave interactions

A bit more ↔ Hasegawa-Wakatani (life beyond CHM)

$$\frac{d}{dt}\nabla_\perp^2\phi + \chi_{\parallel e}\nabla_\parallel^2(\phi - n) = \mu\nabla_\perp^2\nabla_\perp^2\phi$$

$$\frac{d}{dt}n + \chi_{\parallel e}\nabla_\parallel^2(\phi - n) = D_0\nabla_\perp^2 n$$

$\chi_{\parallel e} = v_{the}^2/\nu_{ei}$

$\chi_{\parallel e} \to \infty$ → HM

$$\frac{d}{dt} = \partial_t + \nabla\phi \times \hat{z}\cdot\nabla \qquad n = \langle n(x)\rangle + \tilde{n} \qquad \nabla_\perp^2\phi = \langle\nabla_\perp^2\phi(x)\rangle + \nabla_\perp^2\tilde{\phi}$$

- <u>PV</u> $q = n - \nabla_\perp^2\phi$ conserved!, to μ , D_0 $\quad n \leftrightarrow \nabla_\perp^2\phi$ (→ shear) PV exchange
- $\chi_\parallel \neq 0 \to \langle\tilde{v}_r\tilde{n}\rangle \neq 0$ 'negative dissipation mechanism' → drift instability (Sagdeev, et. al., 60's)

  $\omega \leq \omega_{*e} \to \langle\tilde{v}_r\tilde{n}\rangle > 0$

  phase lag between $\tilde{n}, \tilde{v}$ → particle flux
- ZF → $k_\parallel = 0$
- ZF → $\langle\tilde{v}_r\nabla^2\tilde{\phi}\rangle$ → Reynolds force

  Corrugation → $\langle\tilde{v}\tilde{n}\rangle$ → particle flux

$\langle\tilde{n}\nabla^2\tilde{\phi}\rangle$?

c.f. Singh, P.D. 2021

Mesoscopics → Staircases

Mesoscales

- MFE plasma combine:
  - broad dynamic range
  - modest excitation ($Ku \leq 1$)

- [few ρ_i] < l < L_p : mesoscales

Δ_c (micro) — (meso) — system size (macro)

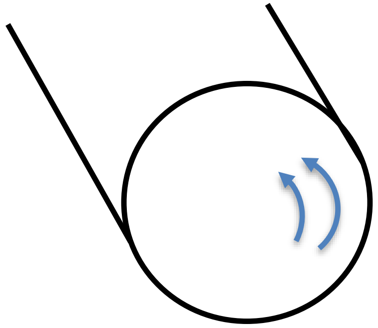
recall: $\rho_* \sim \rho_i / L_p \ll 1$

- Mesoscopic: Zonal Flows, Avalanches – see Minjun Choi, and ... Staircases ...

(PPCF accepted paper)

Plasma Zonal Flows I

- What is a Zonal Flow? – Description?
  - $n = 0$ potential mode; $m = 0$ (ZF)
  - toroidally, poloidally symmetric *ExB* shear flow



- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport ($n = 0$)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

i.e. ZF's soak up turbulence energy

Plasma Zonal Flows II

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation — cf: McIntyre and Wood

- Charge Balance → polarization charge flux → Reynolds force
  - Polarization charge ➡ $-\rho^2\nabla^2\phi = n_{i,GC}(\phi) - n_e(\phi)$
    - ρ: *polarization length scale*
    - $n_{i,GC}$: *ion GC*
    - n_e: *electron density*
  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle \neq 0$ ⬌ 'PV transport'
    - *polarization flux* → What sets coherence?
  - If 1 direction of symmetry (or near symmetry):

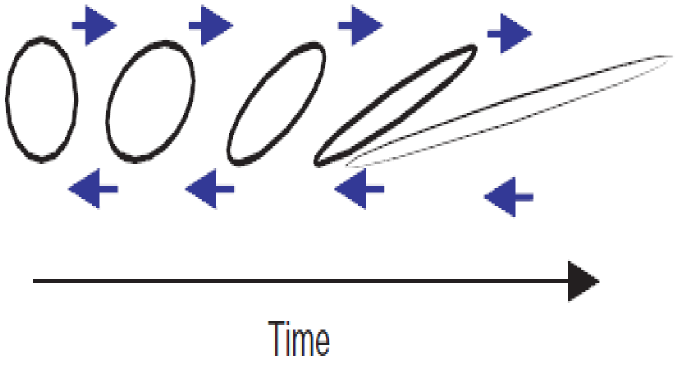
$$-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle \quad \text{(Taylor, 1915)}$$

$-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ ➡ Reynolds force ➡ Flow — Recall $\langle \omega_Z \rangle$ evolution!

Zonal Flows Shear Eddys I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree'66, BDT'90)
  - radial scattering + $\langle V_E \rangle'$ $\rightarrow$ hybrid enhanced decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$

➔ shearing restricts mixing scale!



- Other shearing effects (linear):

Response shift and dispersion

  - spatial resonance dispersion: $\omega - k_\parallel v_\parallel \Rightarrow \omega - k_\parallel v_\parallel - k_\theta \langle V_E \rangle' (r - r_0)$
  - differential response rotation $\rightarrow$ especially for kinetic curvature effects

Quasi-Particle Model – Eddy Population Evolution

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. '98, et. seq.)

  Coherent interaction approach (L. Chen et. al.)

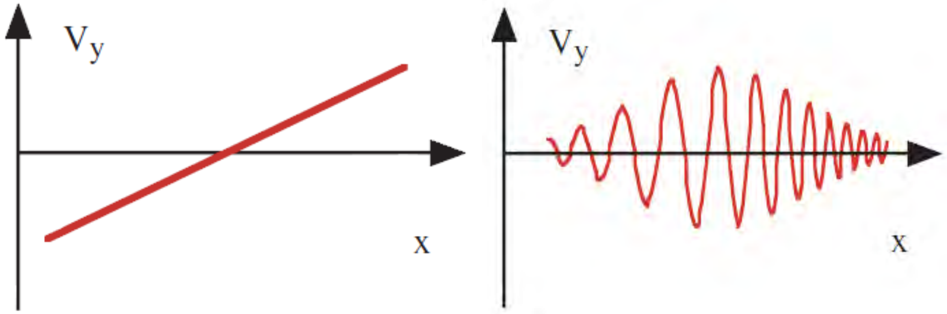
Adiabatic Theory

- $dk_r / dt = -\partial(\omega + k_\theta V_E)/\partial r \,;\; V_E = \langle V_E \rangle + \tilde{V}_E$

  Mean shearing $: k_r = k_r^{(0)} - k_\theta V_E' \tau$

  Zonal Random shearing $: \langle \delta k_r^2 \rangle = D_k \tau$

  $$D_k = \sum_q k_\theta^2 \left| \tilde{V}'_{E,q} \right|^2 \tau_{k,q}$$



- Wave ray chaos (not shear RPA) underlies $D_k \rightarrow$ induced diffusion
- Induces wave packet dispersion
- Applicable to ZFs and GAMs

- Mean Field Wave Kinetics

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

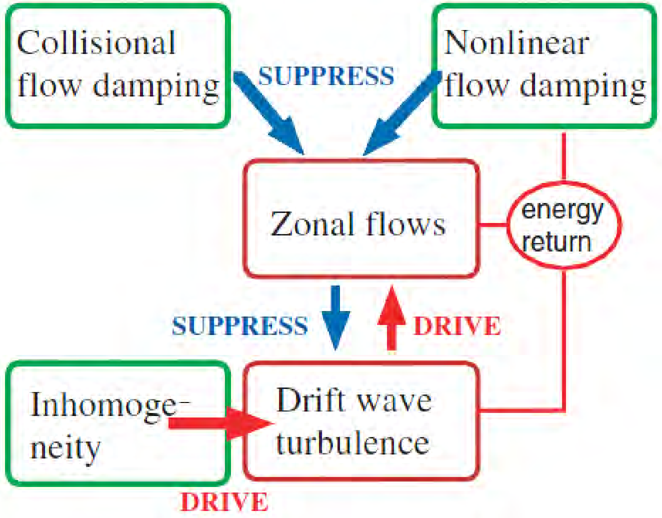
$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

← Zonal shearing via D_k

→ Evolves population in response to shearing

Feedback Loops

- Closing the loop of shearing and Reynolds work
- Spectral 'Predator-Prey' Model



Prey → Drift waves, $\langle N \rangle$

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k \langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

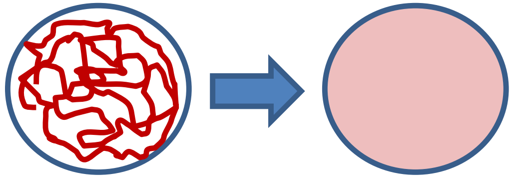
→ Self-regulating system → "ecology"

→ Transport regulated

Another Aspect: Dynamics in Real Space – What of the Configuration?

- Conventional Wisdom → Homogenization ?!
  - Prandtl, Batchelor, Rhines:

(2D fluid)



  - PV homogenized:
    Shear + Diffusion

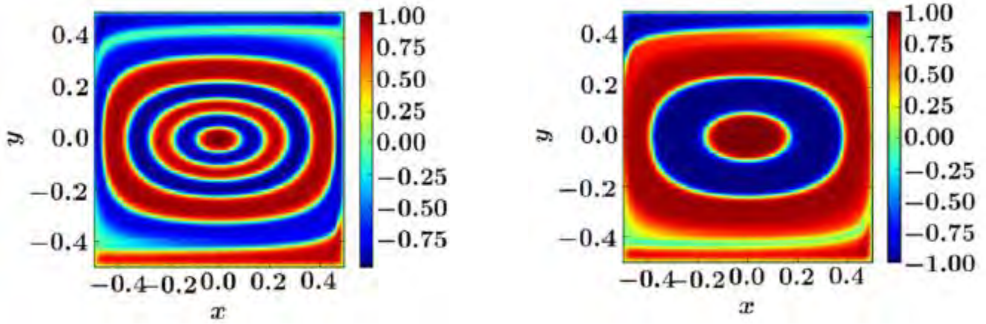
$\nabla q \to 0$

2 scales: $a, \ a/Re^{1/3}$
BL → "emergent"

  - Mechanism: - Shear dispersion $\tau \sim \tau_{rot}\,(Re)^{1/3}$ → $\tau_{rot}\, Re$

    - 'PV Mixing'

  - Introduce Bi-stable Mixing → Layers



coarsens

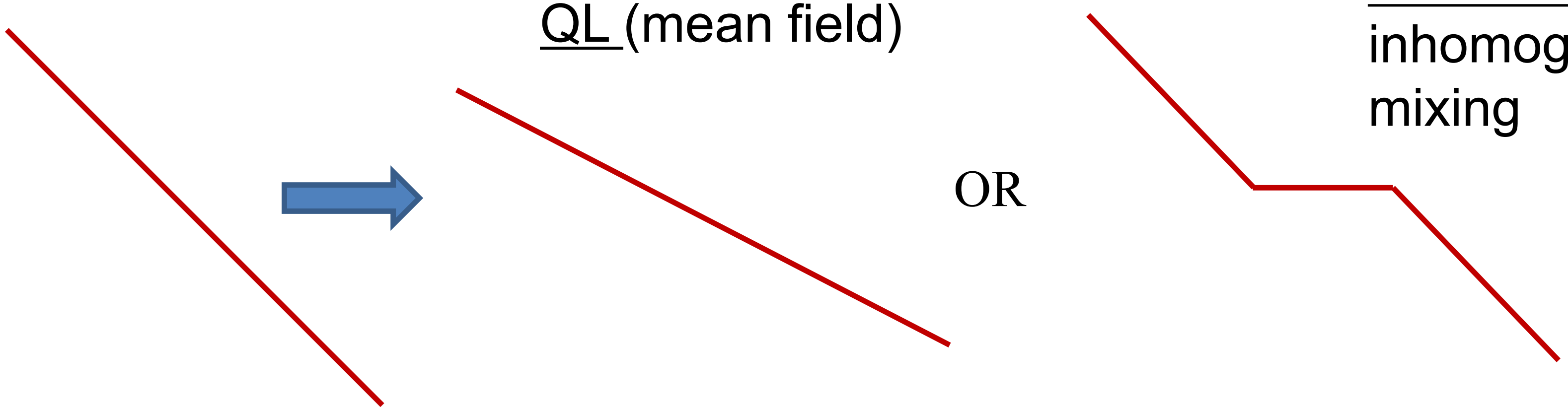
  - Cahn-Hilliard + Eddy Flow ↔ bistability

(spinodal decomposition)

→ target pattern

(Fan, P.D., Chacon,
PRE Rap. Com. '17)

Fate of Gradient?

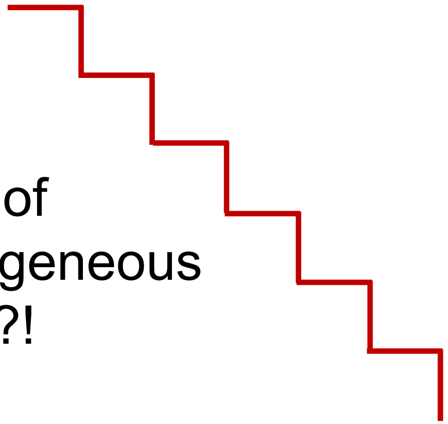


QL (mean field)

OR

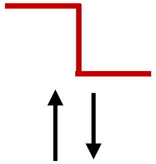
localized inhomogeneous mixing

OR - 'staircase'



pattern of inhomogeneous mixing ?!

- layers, steps, corrugations
- shear layers ←→ relation to corrugations?



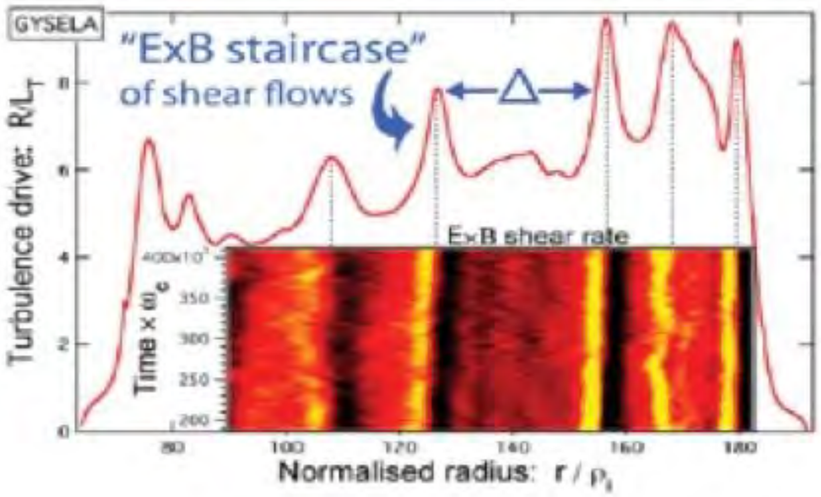
?

Zonal flows at corrugations ?

Spatial Structure: ExB staircase formation (after PV staircase Dritschel + McIntyre)

- ExB flows often observed to self-organize structured pattern in magnetized plasmas
- \`ExB staircase’ is observed to form

(G. Dif-Pradalier, P.D. et al. Phys. Rev. E. ’10)



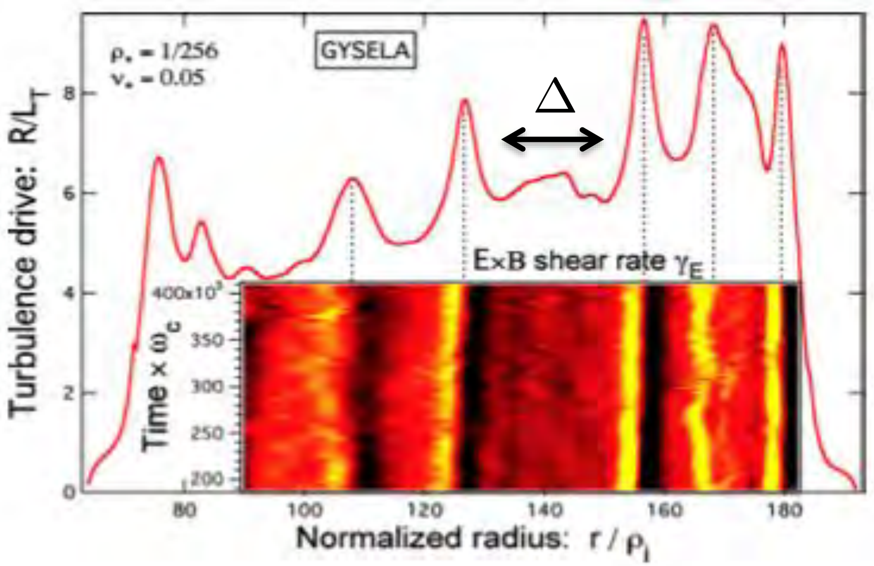
also: GK5D, Kyoto-Dalian-SWIP group, gKPSP, ... several GF codes

- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations (steps)
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases
- so-named after the analogy to PV staircases and atmospheric jets
- Step spacing → avalanche distribution outer-scale
- scale selection problem

ExB Staircase, cont'd

- Important feature: co-existence of shear flows and zones strong mixing



- Seem mutually exclusive ?

  → strong ExB shear prohibits transport

  → mesoscale scattering smooths out corrugations

- Can co-exist by separating regions into:

  1. mixing zones of the size $\Delta \gg \Delta_c$

  2. localized strong corrugations + jets

- How understand the formation of ExB staircase??

  - What is process of self-organization linking avalanche scale to ExB step scale?

  i.e. how explain the emergence of the step scale ?

- Some similarity to phase ordering in fluids – spinodal decomposition

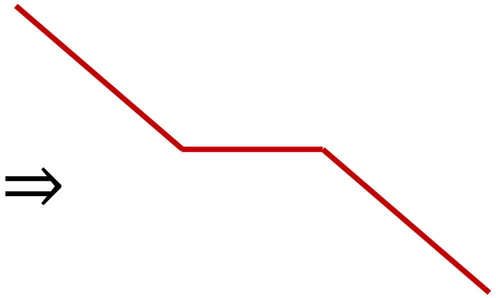
  → bistability as key

How do Staircase Form?
←→ What can be learned from (simple) models?

General Ideas on Formation

- Inhomogeneous mixing ?!
- Staircase must reconcile 2 states transport ←→ 2 types of domains

$\Gamma \Rightarrow$



strong mixing zones, shallow gradient
+
weak mixing zones, steep gradient

- Bistability is natural candidate
- Suggests 2 space/time scales. Dynamics ←→ 1 scale emergent
- (BLY): Balmforth, Llewellyn Smith, Young '98

General Ideas on Formation, cont'd

- Classic: Balmforth, Llewllyn Smith, Young '98 (BLY)
- $k - \epsilon$ model framework (TKE + scalar)
- 2 scales: l_0 → imposed

  l_{OZ} → Ozmidov scale (emergent) $\tilde{v}(l)/l \sim \omega_{bouy}$
- N.B. Emergent scale is recurring element in layering story
- i.e. Ozmidov, Rhines, Hinze … and BL in expulsion…

The Bounty of BLY, for Drift Wave Systems

∗ • A. Ashourvan, P.D. – Phys. Rev. E. Rap. Comm. (2016), PoP (2017)

➔ Hasegawa-Wakatani drift wave turbulence

• M. Malkov, P.D. – Phys. Rev. Fluids (2019)

➔ QG/β –plane

∗ • W.X. Guo, P.D., Hughes et. al. – PPCF (2019)

➔ H-W Drift Wave Turbulence

see talk by W.X. Guo, this meeting

Basic Equations ↔ Hasegawa-Wakatani (life beyond CHM)

$$\frac{d}{dt}\nabla_\perp^2\phi + \chi_{\parallel e}\nabla_\parallel^2(\phi - n) = \mu\nabla_\perp^2\nabla_\perp^2\phi$$

$$\frac{d}{dt}n + \chi_{\parallel e}\nabla_\parallel^2(\phi - n) = D_0\nabla_\perp^2 n$$

$$\frac{d}{dt} = \partial_t + \nabla\phi \times \hat{z}\cdot\nabla \qquad n = \langle n(x)\rangle + \tilde{n} \qquad \nabla_\perp^2\phi = \langle\nabla_\perp^2\phi(x)\rangle + \nabla_\perp^2\tilde{\phi}$$

zonal shear

- PV $q = n - \nabla_\perp^2\phi$ conserved! , to μ , D_0 $n \leftrightarrow \nabla_\perp^2\phi$ PV exchange
- $\chi_\parallel \neq 0 \rightarrow \langle\tilde{v}_r\tilde{n}\rangle \neq 0$ 'negative dissipation mechanism' ➔ drift instability (Sagdeev, et. al.)

  $\omega \leq \omega_{*e} \rightarrow \langle\tilde{v}_r\tilde{n}\rangle > 0$
- ZF ➔ $k_\parallel = 0$
- ZF $\rightarrow \langle\tilde{v}_r\nabla^2\tilde{\phi}\rangle \rightarrow$ Reynolds force

  Corrugation $\rightarrow \langle\tilde{v}\tilde{n}\rangle \rightarrow$ particle flux

$\langle\tilde{n}\nabla^2\tilde{\phi}\rangle$?

c.f. Singh, P.D. 2021

‘Bistable’ Mixing – A Simple Mechanism

- Mean field model with 2 mixing scales
- So, for H-W: PE, $\langle n \rangle$, $\langle \nabla^2 \phi \rangle$
- Density: $$\frac{\partial}{\partial t}\langle n\rangle = \frac{\partial}{\partial x}\left(D_n \frac{\partial \langle n\rangle}{\partial x}\right) + D_c \frac{\partial^2 \langle n\rangle}{\partial x^2}$$
- Vorticity: $$\frac{\partial}{\partial t}\langle u\rangle = \frac{\partial}{\partial x}\left[(D_n - \chi)\frac{\partial \langle n\rangle}{\partial x}\right] + \chi \frac{\partial^2 \langle u\rangle}{\partial x^2} + \mu_c \frac{\partial^2 \langle u\rangle}{\partial x^2},$$
- Potential Enstrophy(intensity): $$\frac{\partial}{\partial t}\varepsilon = \frac{\partial}{\partial x}\left(D_\varepsilon \frac{\partial \varepsilon}{\partial x}\right) + \chi\left[\frac{\partial \langle n-u\rangle}{\partial x}\right]^2 - \varepsilon_c^{-1/2}\varepsilon^{3/2} + \gamma_\varepsilon \varepsilon.$$
- $D, \chi \sim \tilde{V} l_{mix}$

$$l_{\rm mix} = \frac{l_0}{\left(1 + l_0^2[\partial_x(n-u)]^2/\varepsilon\right)^{\kappa/2}},$$

- Scale cross-over → ‘transport bifurcation’
- $l_0/l_R < 1$ → strong mixing (eddys)
- $l_0/l_R > 1$ → weak mixing (waves) → gradient sharpening feedback
- Is this ~ equivalent to ‘two-fluid’ mixing length model ala’ Ed Spiegel ?

simple mixing + 2 length scale
→ staircase

→ includes crude turbulence spreading model

l_0 → excitation scale (drive)
l_R → Rhines scale (emergent)
ω_{MM} vs $\Delta\omega$ - can be generalized

two scales!

How, Why ?

- PV is mixed → natural for 'mixing length model', exploits PV as conserved phase space density
- Potential Enstrophy is natural formulation – $\langle \delta f^2 \rangle$ for intensity → conservation
- Beyond BLY → 2 mean fields $\langle n \rangle$, $\langle \nabla^2 \phi \rangle$ + ε – fluctuation potential enstrophy

  → exchange and couplings, two channels
- Reynolds work and particle flux couple mean and fluctuations
- Nonlinear damping ↔ forward potential enstrophy cascade
- D_n, χ → turbulent transport coefficients are fundamental
- Glorified '$k - \epsilon$ model', adapted to drift wave problem

How, Why ? Cont'd

- $l_{mix} > \rho_s$ → simplifies inversion $(\nabla^2\phi \rightarrow V)$
- Dissipative DW ~ adiabatic regime: $k_\parallel^2 V_{the}^2/\nu > \omega$ $\alpha = k_\parallel^2 v_{the}^2/\omega\nu$

  $D_n \approx \tilde{v}^2/\alpha \sim \epsilon l^2/\alpha$ → $\langle v_r \tilde{n}\rangle$ phase fixed by α!

  Major simplification → <u>solid</u>, where applicable

  $\chi \sim D_n$ (non-resonant diffusion)
- $\langle \tilde{v}_r \nabla^2\phi\rangle = -\chi\partial_x\langle\nabla^2\phi\rangle + \Pi_{resid}[\nabla n]$

  $\langle\nabla^2\phi\rangle =$ <u>shear</u> [χ only in numerics]
- $\langle \tilde{v}_r \tilde{q}^2\rangle$ → $-l^2\epsilon^{1/2}\partial_x\varepsilon$ spreading, entrainment, SOFT

How, Why ? Cont'd

- D_n, χ regulate P.E. exchange between mean, fluctuations ➔ key role in model

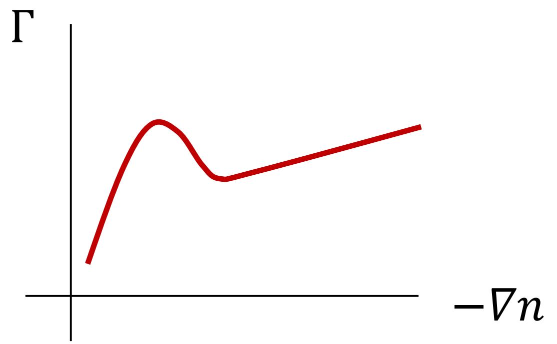
- Mixing Length: $l_{mix} = \dfrac{l_0}{\left[1+\frac{l_0^2[\partial_x(n-u)]^2}{\epsilon}\right]^{\kappa/2}} = \dfrac{l_0}{1+\left(l_0^2\,/\,l_{Rh}^2\right)^{\kappa/2}}$

  Physics: "Rossby Wave Elasticity' (ala' McIntyre)

  i.e. $D \sim \frac{\langle\tilde{v}^2\rangle}{\Delta\omega} \rightarrow \langle\tilde{v}^2\rangle \frac{\Delta\omega}{\omega_r^2+(\Delta\omega)^2} \approx \langle\tilde{v}_r^2\rangle \frac{\Delta\omega}{\omega_r^2}$ for $\Delta\omega < \omega_r$

  → waves enhance memory

  → $\omega_r \sim \nabla\langle q\rangle \rightarrow$ nonlinear Γ_{PV} vs $\langle q\rangle \rightarrow$ S-curve



- Soft point: $\kappa \rightarrow$ suppression exponent

  $\kappa = 1$ doesn't always work

  Rigorous bound on κ, from fundamental equations?

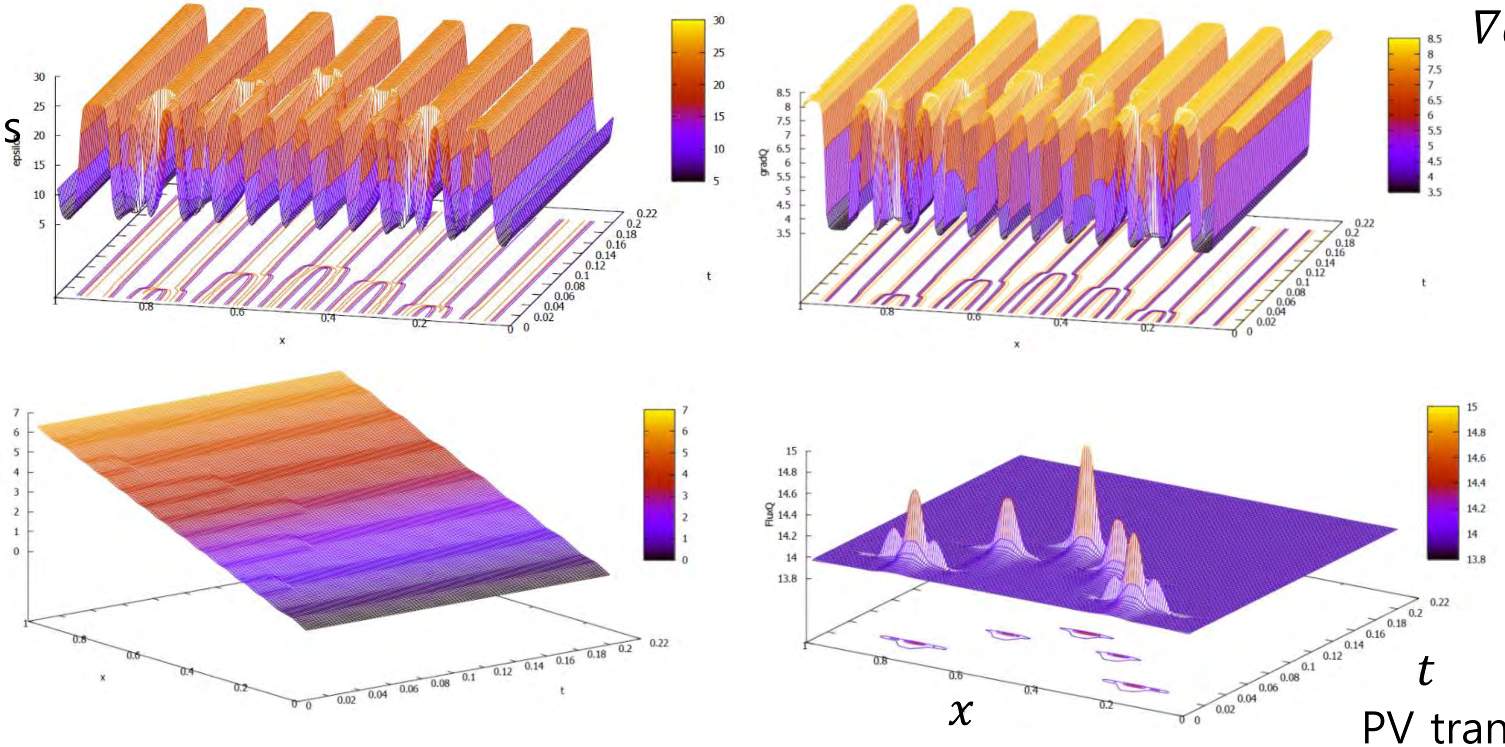
Some Results

Staircase Model – Formation and Merger (QG-HM)

Energy fluctuations

∇q

q → mergers



x

t

PV transport

- ϵ
- Q_y

} top

- Q
- Γ_q

} bottom

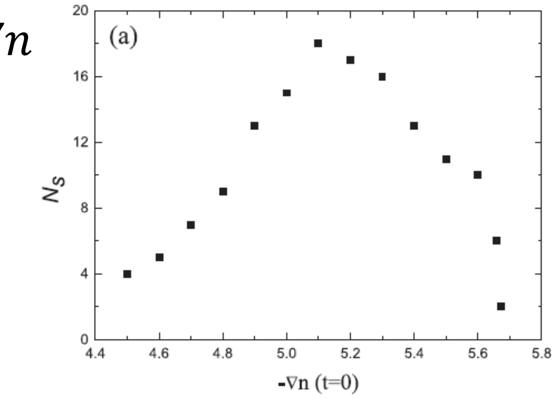
- PV mixing events

Note later staircase mergers induce strong PV flux bursts!

(Malkov, P.D.; PR Fluids 2018)

Staircase Structure?

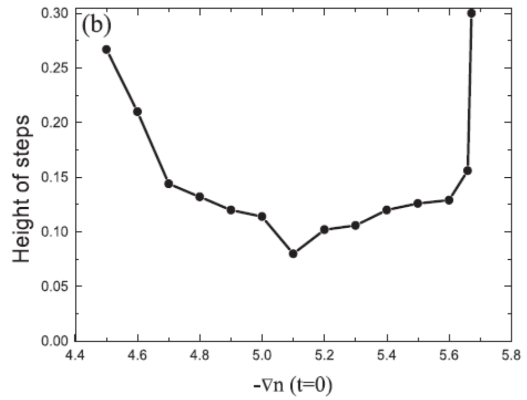
- Number of steps? - domain L → Scale Selection ?!
- Scan # steps vs ∇n at t=0 (n.b. mean gradient)
  - a maximum # steps (and minimal step size) vs ∇n
  - rise: increase in free energy as $\nabla n \uparrow$
  - drop: diffusive dissipation limits N_s



- Height of steps?
  - minimal height at maximal #

  ➔ system has a ∇n 'sweet spot' for many,

  small steps and zonal layers



W.X. Guo + (2019)

Beyond BLY

- Issues, Buried Bodies and Flux-Driven Systems

N.B. In some cases, body parts visible above ground…

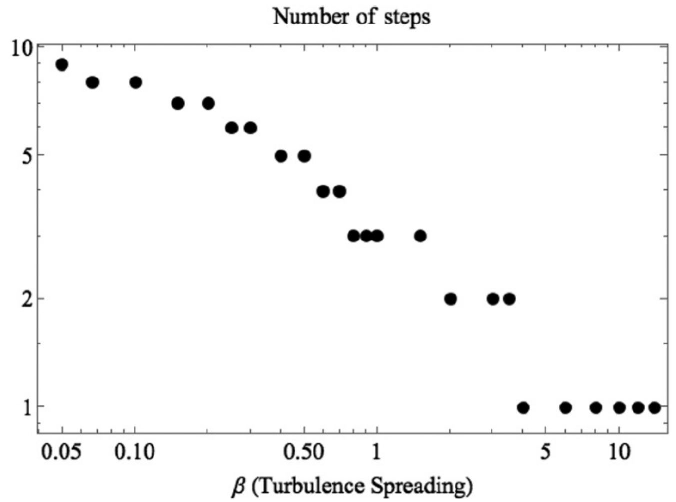
Spreading/Entrainment

- Spreading/entrainment effect on P.E. is unconstrained, beyond $\nabla \cdot \Gamma_q$ structure

  Contrast: $D_n,\ \chi$ Following standard $k - \epsilon$ model crude!

- How robust is staircase to effects of entrainment, avalanching… ? Model ??

- $D_\varepsilon \ \rightarrow \ \beta\, l^2\, \epsilon^{1/2}$

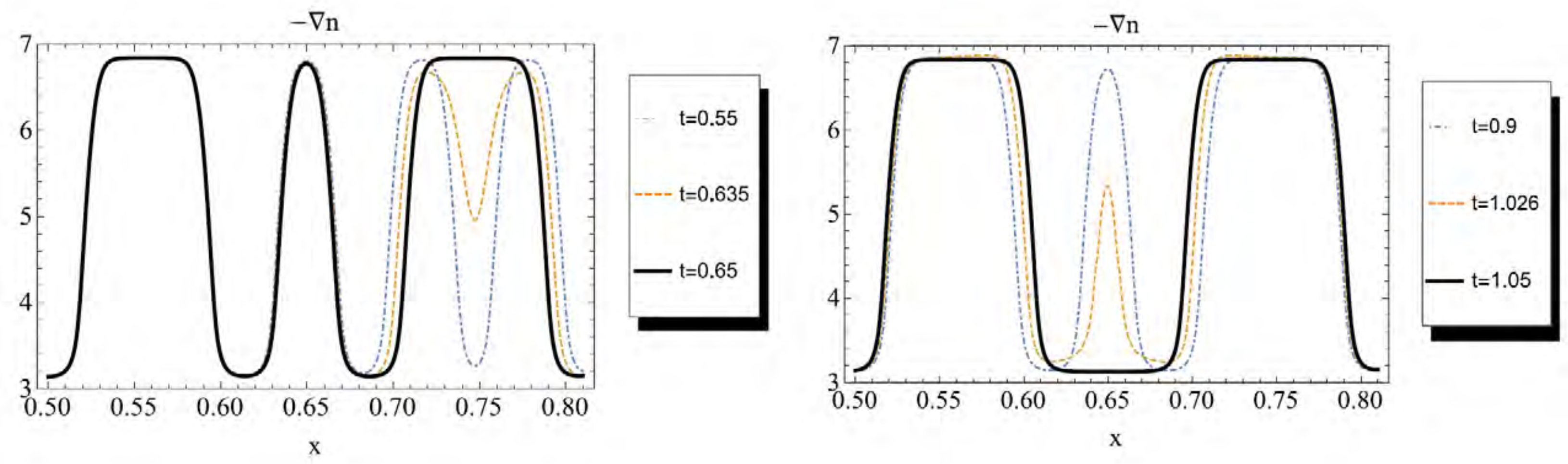


Entrainment has significant effect on S.C. structure

Large $\beta \ \rightarrow$ wash out S.C.

- Spreading model is important model constituent

Mergers Happen



- ‘Type-II’ merger (c.f. Balmforth, KITP’21)
- ‘Type-I’ (motion) mergers also observed

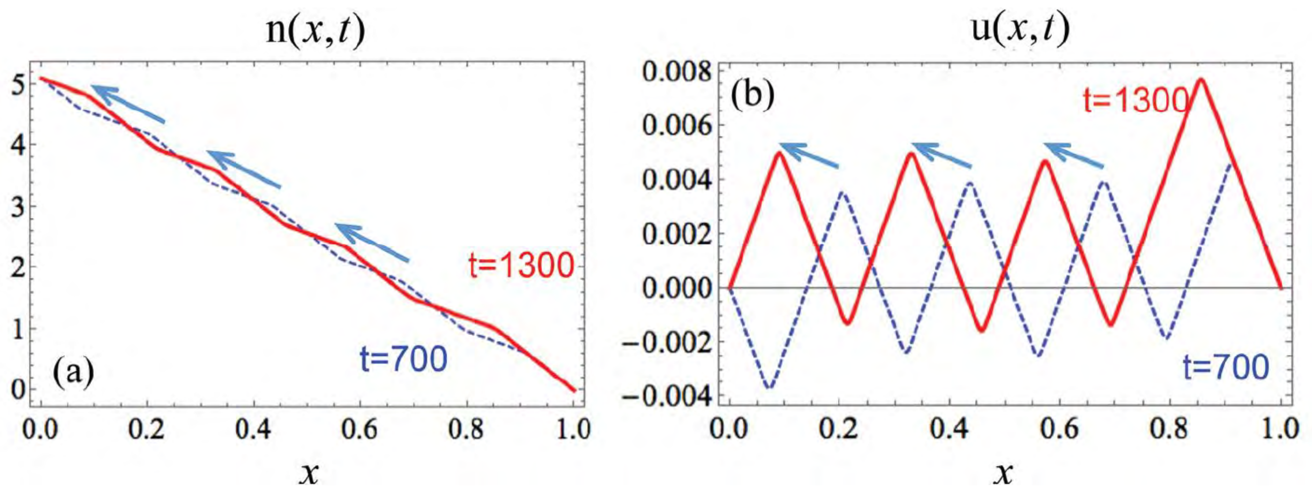
➔ Staircase coarsens….

➔ Obvious TBD:

  – Interplay/Competition of Spreading and Mergers?

  – Scan coarsening time vs β, merger rate vs increments in β

Staircases and Dynamics ! (Global)



- B.C. Neumann LHS, Dirichlet RHS.. (ala' sandpile) → asymmetry
- 'Escalator Modes'
- 'Shear Migration'

} appear. Cause, Consequence?

➔ "Non-locality" → c.f. Yan, P.D. 2022

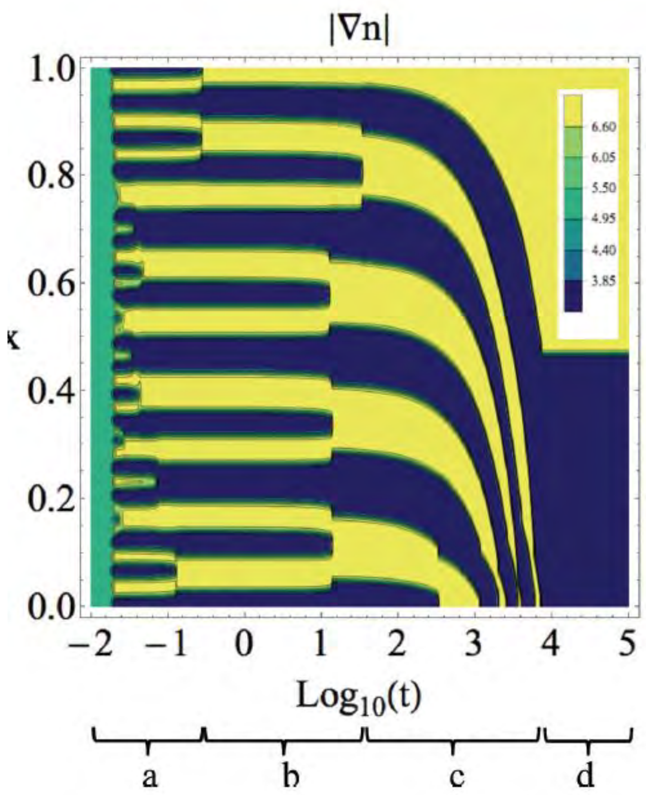
- Needs further study…

➔ Credible model must address staircase dynamics

Dynamics is both local (mergers) and global

Dynamic Staircases, Cont'd

- Steps and barriers observed to condense to outer boundary

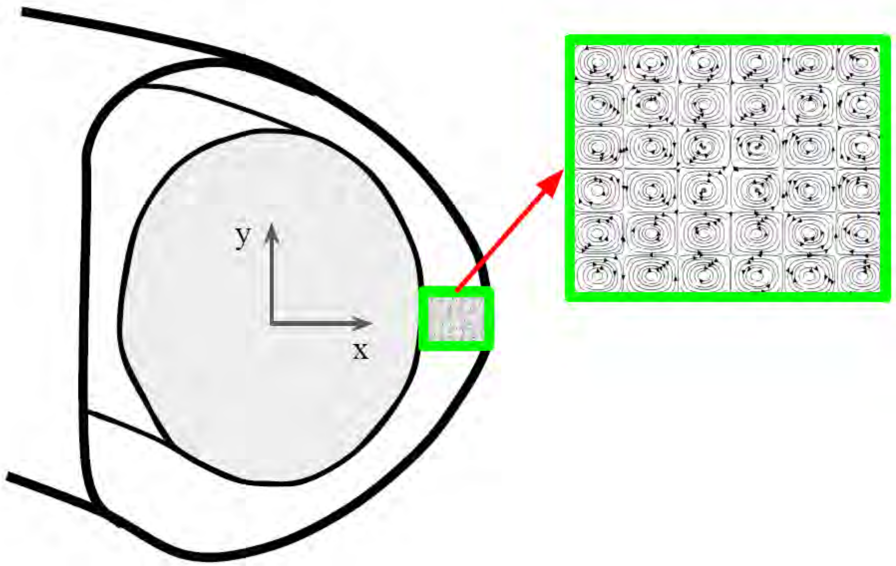


Is this a way to understand
L→H transition?, barrier formation?

Ashourvan, P.D. (2016)

- Collapse of staircase into macroscopic barriers?

- Need quantify!



(Fixed) Cellular Array Problem

→ Test bed for Resiliency Studies

$$Pe = \frac{\tau_D}{\tau_H}$$

Fixed Cellular Array

Consider a **general** case of a system of eddies not overlapping but tangent → **Staircase**

Transport? Deff ~ D Pe½

→ Two time rates: v / ℓ, D / ℓ²

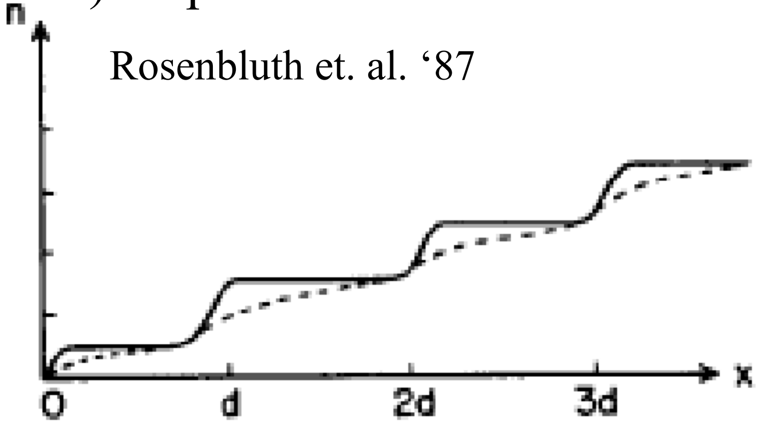
→ Pe = v ℓ / D >> 1

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D\nabla^2 n,$$

Profile?

Consider concentration of injected dye (passive scalar transport in eddy s) → profile

2 scales l vs $\sqrt{l\,D/V}$



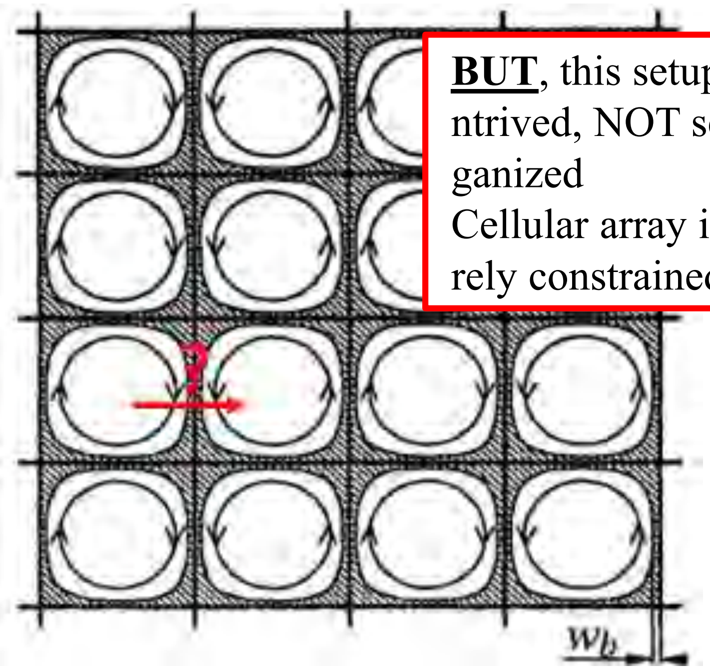
Rosenbluth et. al. '87

→ Layering!

→ **Simple** consequence of **two rates**

"Steep transitions in the density exist between each cell."

Relevant to key question of "near marginal stability"



BUT, this setup is contrived, NOT self-organized

Cellular array is severely constrained

Staircase arises in an array of stationary eddies!

Important:

- **Staircase** arises in stationary array of passive eddies (Note that there is no FEEDBACK)
- Global transport hybrid:
  - → fast rotation in cell
  - → slow diffusion in boundary layer
- Irreversibility localized to inter-cell boundary.

Fluctuating Vortex Array

Why are we doing this? We know that a system with two disparate time scales forms a staircase!

- Now consider fluctuations… → Will staircase survive?

→ We begin with the 2D NS equation that can be written in nondimensional form (Perlekar and Pandit 2010),

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

→ The "vortex array" is simply the array of cells and "fluctuation" is related to turbulence induced variability in the structure. The fluctuating vortex array (FVA) allows us to study a **less constrained** version of the array!

→ The fluctuating flow structure is created by **slowly increasing the Reynolds number** in the NS equation

$$\Omega = \frac{\tau_\nu}{\tau_H}$$

→ By increasing the Reynolds number this modifies the forcing and drag term, thus, **scattering** the vortex array. The **resilience** of the staircase is studied by **increasing disorder** in the vortex crystal through F_ω

$$F_\omega \equiv -n^3\left[\cos(nx) + \cos(ny)\right]/\Omega$$

The streamfunction, ψ, at different evolutionary stages of the "fluctuating" vortex array is inserted into the passive scalar equation to study the resilience of the staircase structure.

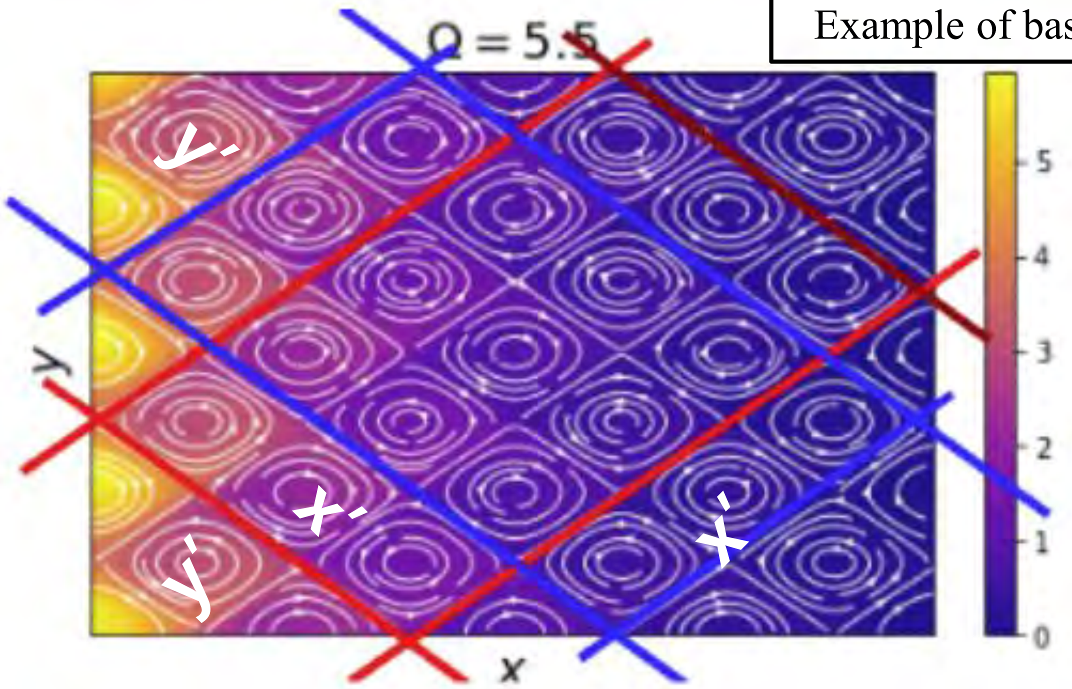
What Happens to Staircase?

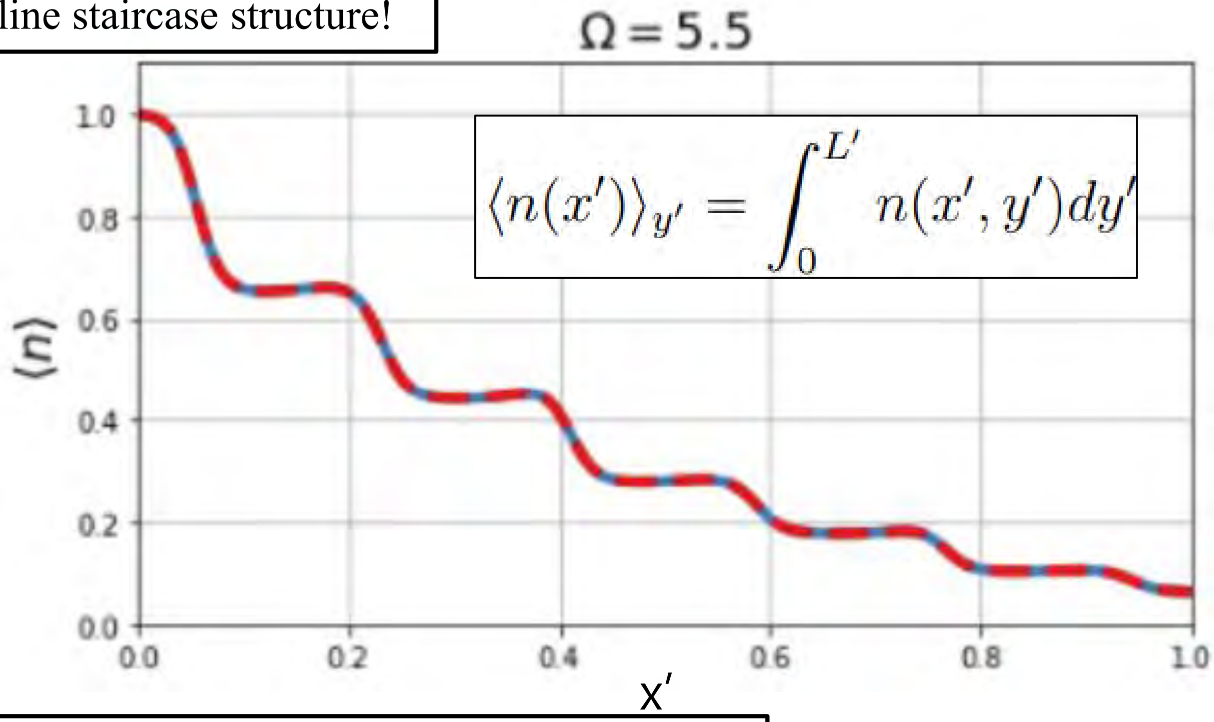
$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D \nabla^2 n,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \right) \omega = \frac{1}{\Omega} \nabla^2 \omega + F_\omega - \alpha \omega, \qquad \nabla^2 \psi = \omega.$$

The Staircase

Example of baseline staircase structure!

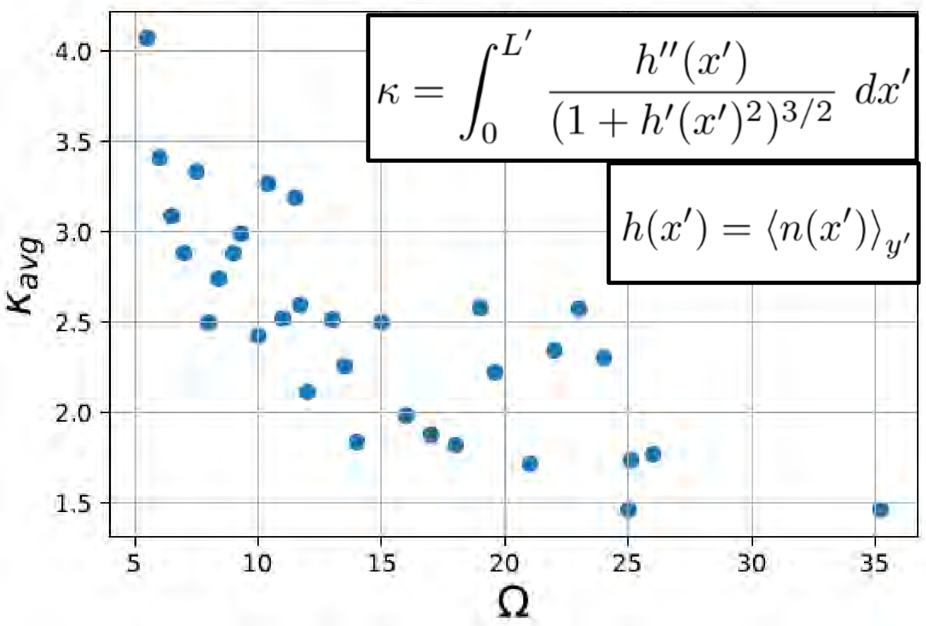


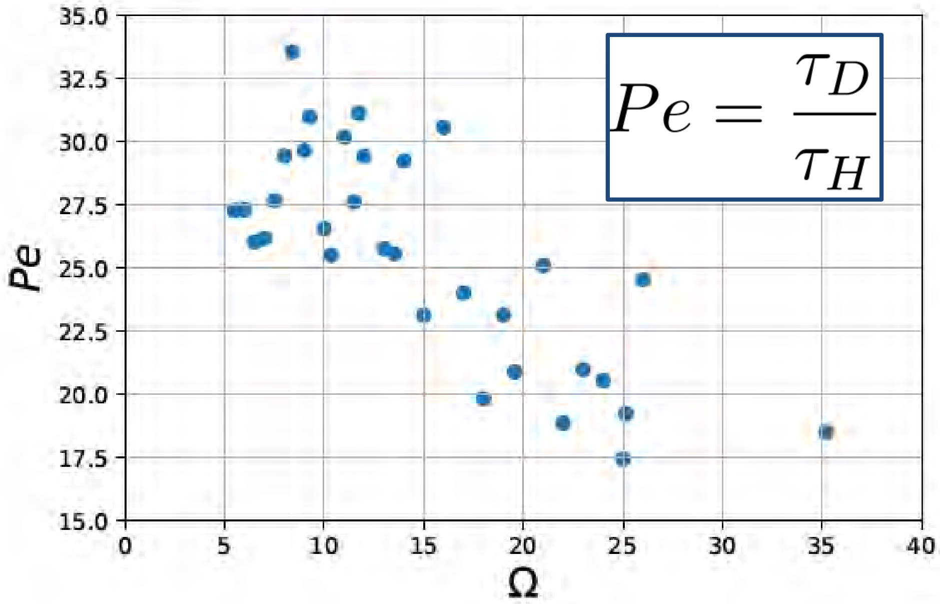


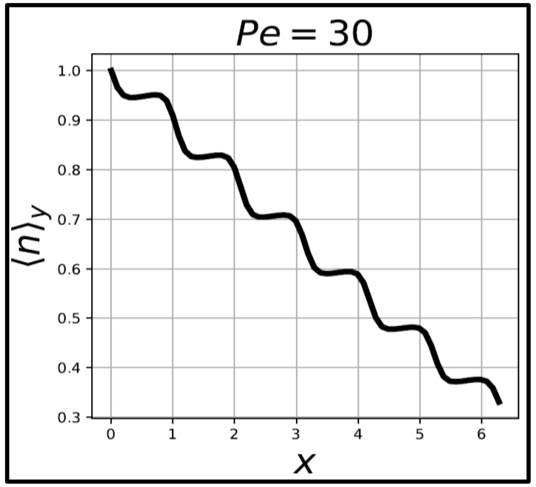
- For a weakly FVA we get a **baseline staircase** structure.
- On the left figure the blue and red box correspond to the blue and re d plot line on the right.
  - Both blue and red average scalar concentration have the same p rofile in stable stage.

So what happens to the staircase if we increase the Reynolds number in the VA?

Criteria for Staircase Resiliency







We establish a **set of criteria** to give a meaning to the statement of "**resiliency**":

1) $Pe \gg 1$ is a **necessary** condition for the **formation of transport barriers** in the process of scalar mixing (**First principles**). $Pe \gg 1$ criterion is satisfied for the range of $0 < \Omega < 40$.
2) A staircase should **maintain a sufficiently high curvature** (equivalent to sustaining a sufficient number of steps). Our studies suggest that $\kappa \gtrsim 1.5$ is an adequate value for a staircase.

N.B. Increasing Re, Ω → increasing cell excursion
→ overlap + mergers

What Next ?

Layering in Burning Plasmas !?

- Current Picture: Energetic Particles – dilute

↑ ϵ Thermals → DW's + ZW

↑ heating

{ Confinement controlled by thermally driven turbulence with hots as "extra"

- Burning Plasma: mix

EP's - α particles slowing down → Thermals

{ Confinement now a "soup" of EP + Thermals

- EP's and α's introduce new scales $\rho_{\theta hot} > \rho_{\theta thermal}$

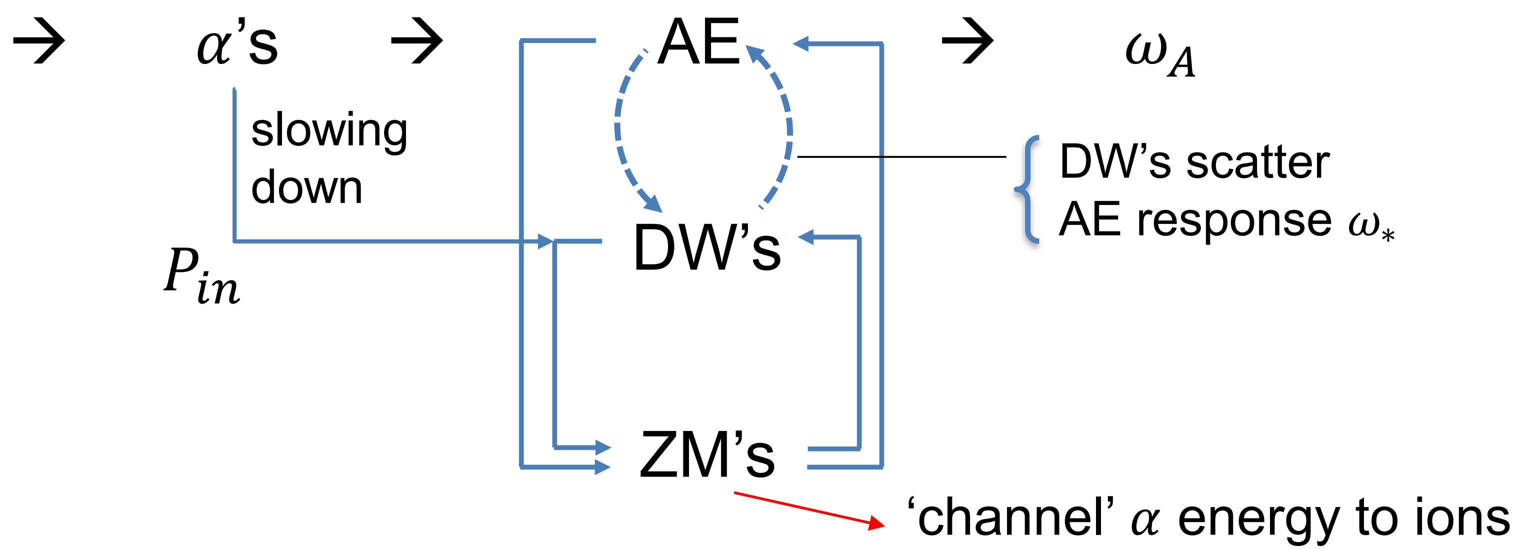
and new collective modes … → AE's (Alfven Eigenmode)

Burning Plasmas

- EP/α → AE
  - { Alfven eigenmodes (zoology); $\omega \sim \omega_A$, resonance with EP's
- Thermals → DW
  - { Drift waves $\omega \sim \omega_*$
- ZW
  - { $\omega \sim 0$
- Issues
  - Feedback loops much 'richer'. Staircase morphology?
  - ZF/Z-mode field now multi-scale
    → SC with multi-scale steps. SC in EP and thermal population.
  - α's slow down on <u>electrons</u>. Thermals: TEM → increased complexity
  - Zonal flow damping ←→ ion heating ?!
  - AE vs DW competition → layering ?!

Feedback Loops (Heuristic)

- Fusion burn



- Multiple, embedded loops – "3 Animals Problem" Zonal structures connect AE, DW

  – Competition of populations

- Traps: i.e. – separate ZF population by injection + ECH ?!

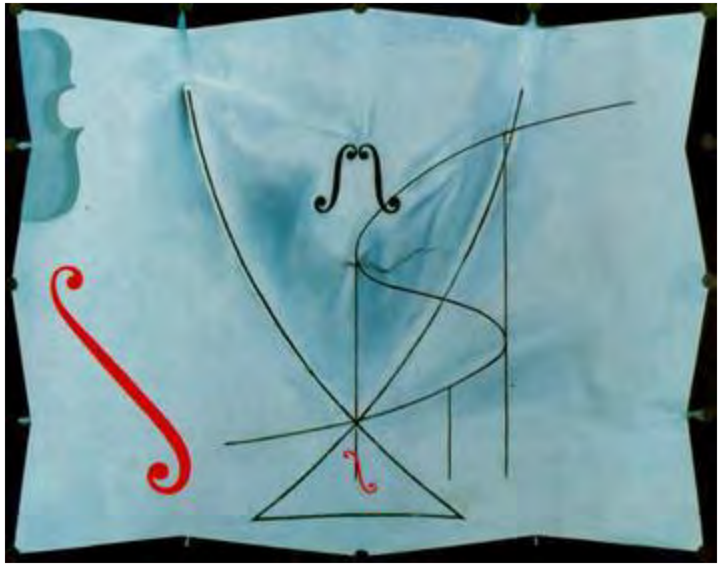
  <u>but</u> DW scattering can quench AE's which drive ZF! so?

- Adventures ahead… c.f. GJ Choi, PD, Hahm NF'23 - dilution

  → Significant effect on couplings in HM

Concluding Thoughts

- Problem of layering evolves along a winding road, with many bifurcations



Salvador Dali

- Stay tuned…

Back Up

Some Details of Model

$\rightarrow$ 2 Simple Models

a.) Hasegawa-Wakatani (collisional drift inst.)

b.) Hasegawa-Mima (DW)

a.) $\mathbf{V} = \frac{c}{B}\hat{z} \times \nabla\phi + \mathbf{V}_{pol}$

$\rightarrow m_s$

$L > \lambda_D \rightarrow \nabla \cdot \mathbf{J} = 0 \rightarrow \nabla_\perp \cdot \mathbf{J}_\perp = -\nabla_\parallel J_\parallel$

$J_\perp = n|e|V_{pol}^{(i)}$

$J_\parallel : \eta J_\parallel = -(1/c)\partial_t A_\parallel - \nabla_\parallel \phi + \nabla_\parallel p_e$

e.s.

b.) $dn_e/dt = 0$

$$\rightarrow \quad \frac{dn_e}{dt} + \frac{\nabla_\parallel J_\parallel}{-n_0|e|} = 0$$

n.b.

MHD: $\partial_t A_\parallel$ v.s. $\nabla_\parallel \phi$

DW: $\nabla_\parallel p_e$ v.s. $\nabla_\parallel \phi$

Some Details of Model, cont'd

So H-W $\rho_s^2 \frac{d}{dt}\nabla^2\hat{\phi} = -D_\parallel \nabla_\parallel^2(\hat{\phi} - \hat{n}/n_0) + \nu\nabla^2\nabla^2\hat{\phi}$

$$\frac{d}{dt}n - D_0\nabla^2\hat{n} = -D_\parallel \nabla_\parallel^2(\hat{\phi} - \hat{n}/n_0)$$

$D_\parallel k_\parallel^2/\omega$

is key parameter

$\rightarrow \langle \tilde{v}_r \tilde{n} \rangle \neq 0$

and instability

b.) $D_\parallel k_\parallel^2/\omega \gg 1 \rightarrow \hat{n}/n_0 \sim e\hat{\phi}/T_e \qquad (m, n \neq 0)$

$$\frac{d}{dt}(\phi - \rho_s^2\nabla^2\phi) + v_*\partial_y\phi = 0 \qquad \rightarrow \text{H-M}$$

n.b. $\mathrm{PV} = \phi - \rho_s^2\nabla^2\phi + \ln n_0(x) \qquad \frac{d}{dt}(\mathrm{PV}) = 0$

An infinity of technical models follows …

Recent Development

- Extension of PV Theory to inhomogeneous $\vec{B}_0(\vec{x})$ (Hahm+ 2023)

  ←→ analogy $H = H(\vec{r})$

  → PV evolution via incompressible advection of "magnetically weighted PV"

  → novel HM Eqn

- Analogous TEP Theory with $\vec{B}_0(\vec{x})$, n/B incompressibly advected

  $\Gamma \sim \partial_r (n/B)$ → diffusion + convection

  $$\sim \frac{\partial_r \langle n \rangle}{B} \qquad \sim \frac{\langle n \rangle}{B^2} \partial_r B$$

Flux Driven Studies

- MFE problems are almost always flux-driven, with source and sink. Not addressed in BLY ’98. Gradients not “pinned”

- For conservative drive:

Collisional transport (‘neoclassical’)

Drive (conservative)

$$\partial_t n = \partial_x D_n \partial_x n + D_c \partial_x^2 n - \partial_x \Gamma_{dr}(x)$$

$$\Gamma_{dr}(x) = \Gamma_0 \exp[-x/\Delta_{dr}]$$

strength

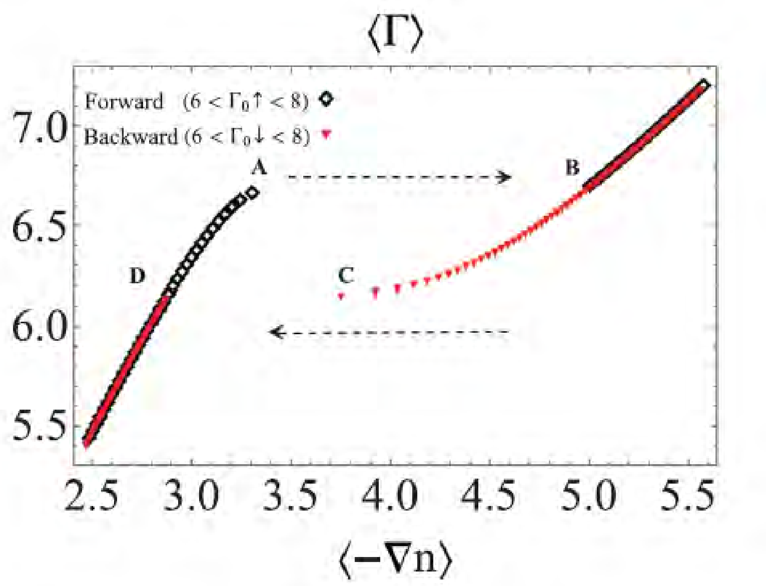
Profile of deposition

$D_n = l^2 \varepsilon / \alpha$ as before

- Now address global confinement dynamics

Global Bifurcation in Staircase

- Average $\langle\Gamma\rangle$ vs $\langle\nabla n\rangle$ plot shows GLOBAL transport bifurcation and hysteresis

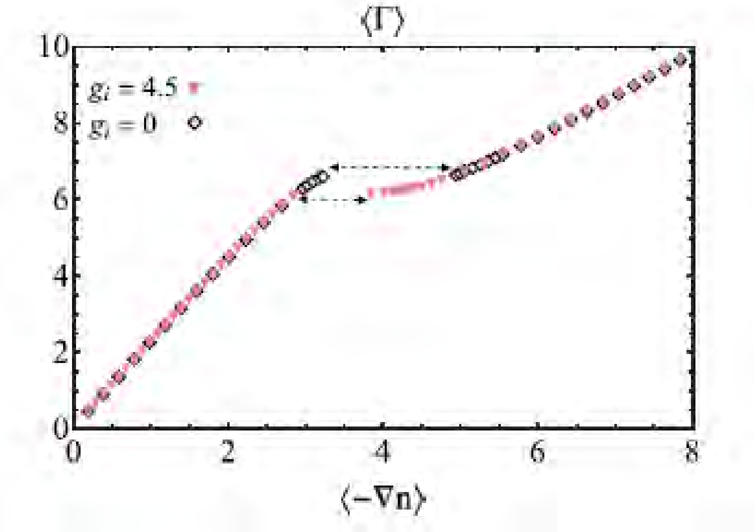


S-curve once more,
with feeling !

- Global confinement bifurcation, in staircase state
- Regional weightings l_0 , l_{Rh}. Good confinement, l_{Rh} dominates
- Merits of staircase state ?! Compare to single barrier ?!

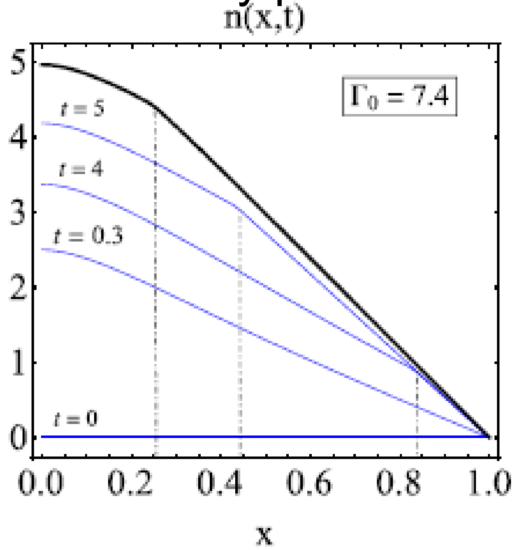
Global Bifurcation, Cont'd

~ Steady State



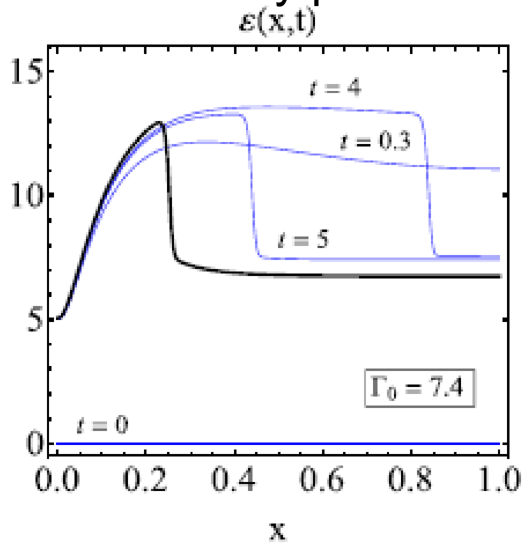
Final state $\langle \Gamma \rangle$ vs $\langle \nabla n \rangle$

Density profile



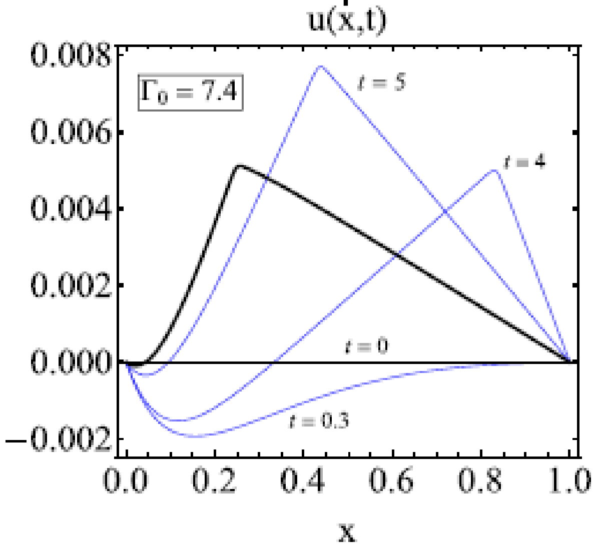
Profile steepens

Intensity profile



Intensity drops

Shear profile



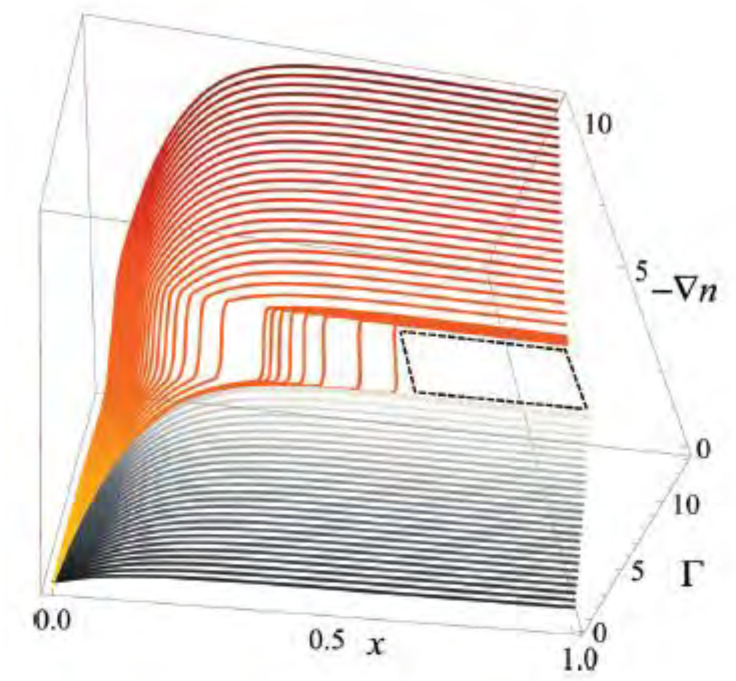
Shear broadens

Global and Local ↔ Flux Landscape

<u>Flux Landscape</u> ↔ family of S-curve

Red → enhanced confinement

Grey → normal confinement



- See also
  - P.D., V.B. Lebedev, el. al., PRL '97
  - Lebedev, P.D., Phys. Plasmas '98 (barrier propagation)

Current Issues

Ongoing Studies

- "Jamming" in Avalanches as SC mechanism

  {Kosuga, PD, Gurcan'13, also Qi + }

  Phenomenology → c.f. Minjun Choi, this meeting

* - Resiliency – how robust is S.C. ? (F. Ramirez, PD, PRE 2024)

- Physics of Spreading / Entrainment (Runlai Xu, PD) – address weakest link in model

Where to next?

N.B. Recall –

“Some models are too good to be true.

Other models are too true to be good.”

New Applications – 'Stress Test' the Model

N.B. BLY already 'flogged thru the fleet', but…

- Thermal Rossby / ITG → PV conservation broken (buoyancy)

  …. → $\langle \tilde{v}_r \tilde{T} \rangle$ - dynamic coherence in flux ➔ New Twist

* - Multi-scale: DW + ETG, AE + DW + ZF

Theory-Enhanced Model (but not too complicated!)

- NL noise – incoherent mode coupling. How represent in M.L.T. ?

  – entrainment, as above

n.b. inhomogeneous mixing – inhomogeneous noise !?

c.f.: R. Singh, P.D. – PPCF 2021

includes $\langle n \nabla^2 \phi \rangle$ coherence

- Dressed parcels – two component model (E. Spiegel, D. Gough "On taking mixing length theory seriously")

  i.e. 'slug' + waves

  ➔ akin dressed test particle model (plasma) !?

But what is the gain ?

- Exploit Relation to Wave Kinetics (Vlasov Eqn. for wave packet)

$$N = \omega E_W \approx \Omega$$ for zonal symmetry

Ω ← Potential enstrophy

WKE — stochastic: PD et. al. '05

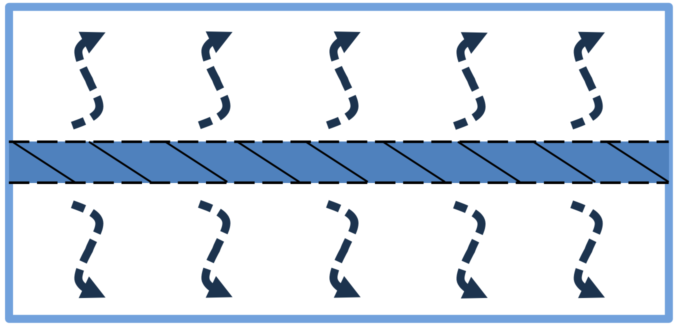
WKE — coherent: Kaw, Garbet

- Easy to propose extensions, but may jeopardize the simplicity and clarity of BLY '98

A Closer Look at Turbulence Spreading

2D Fluid: Simplest Incarnation of Spreading

⇨ Realize:



→ Forcing layer, localized

- Most of system in state of Selective Decay !
- Need Consider / Compare :

$\langle V_y(\nabla^2\varphi)^2/2\rangle$ → Enstrophy Flux

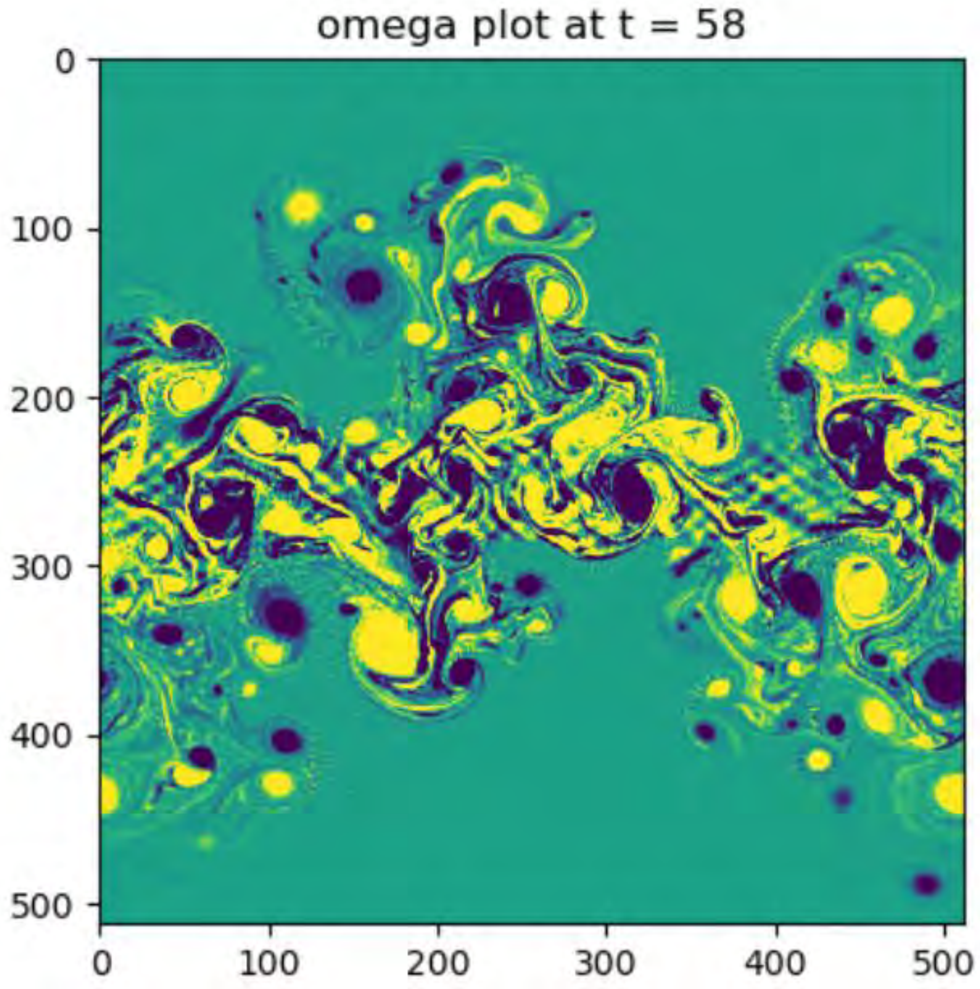
⇨ Physical Measures of Spreading

$\langle V_y(\nabla\varphi)^2/2\rangle$ → Energy Flux

as diagnostic of "intensity spreading".

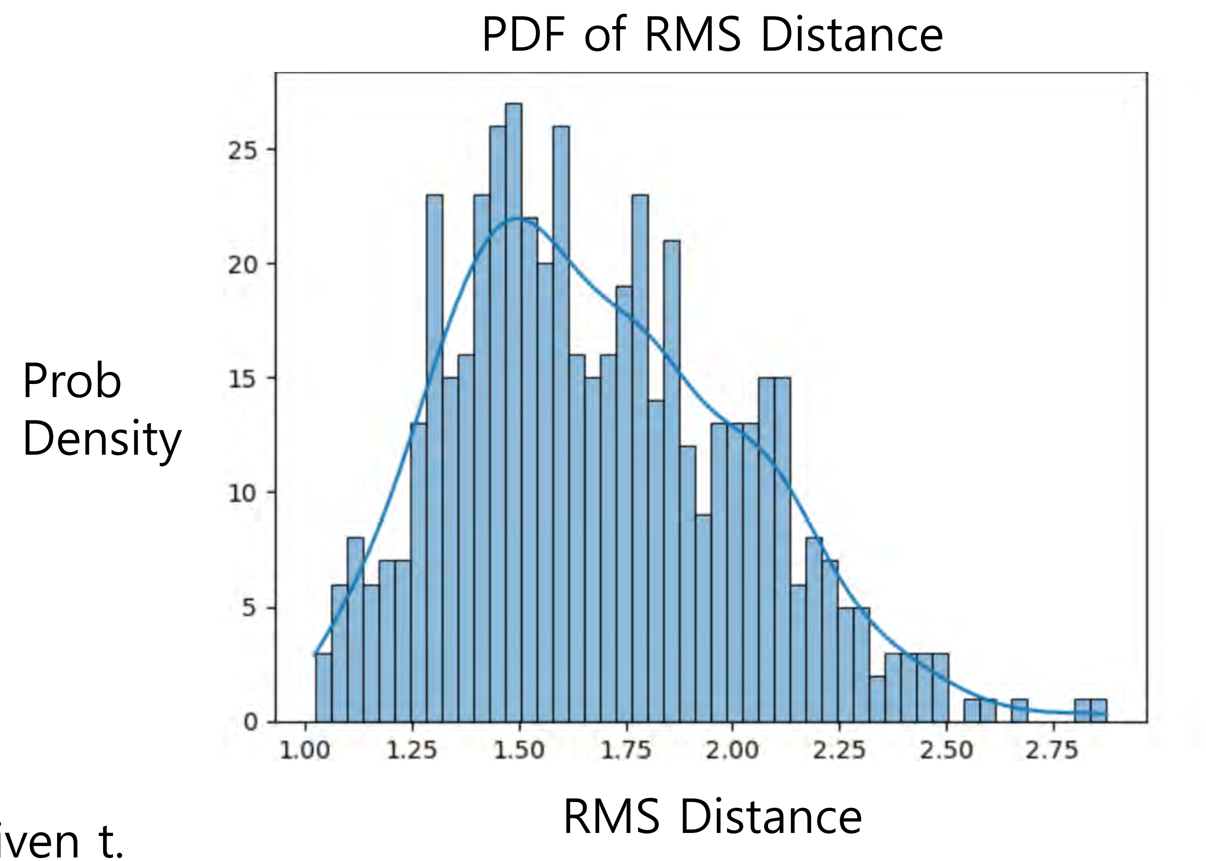
⇒ What Happens

At Re ~ 2000 (marginal resolution):



- Dipoles, Filaments cluster
- Fractalized spreading front?!

Results, cont'd



- ⇨ PDF of spreading (vorticity) at given t.
- ⇨ Calculate enstrophy-weighted rms distance for each position X; plot histogram
- ⇨ Note skewed structure.

Summary - 2D Fluid

— Coherent structures - Dipole vortices - mediate spreading of turbulent region

— Mixed region expands as $w{\sim}t$, consistent with role of dipole.

— No discernable "Front", spreading is strongly intermittent. (space+time)

— Spreading PDF is non-trivial, exhibits tail.

⇨

— Turbulence spreading strongly non-diffusive.

— More at York Fest: Comparison 2D Hydro, 2D MHD, HM+ZF