A Critical Look at Quasilinear Theory — Primarily for Vlasov-Poisson System

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N.B. Hereafter Quasilinear Theory = QLT

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and thank: Vlasov-Poisson Workshops at W. Pauli Inst, Univ. Vienna — U. Frisch, N. Mauser, Org. 2015, 2017

Preliminary Thoughts

Some Philosophy

- “All models are wrong, but some models are useful.” — George Box
- I come neither to praise QLT nor to bury it. — apologies Shakespeare.

  I hope zealots, either pro or con, go away at least somewhat unhappy.
- Not a trivial matter, though it **seems** simple

  “If you are not confused, you don’t know what is going on”
  — Old Haitian Proverb.

Outlook

- QLT is the classic problem of nonlinear plasma theory, ~ 65 yrs old
- ‘QLT’ is frequently a catch-all for many, loosely related, ideas. Meanings vary in different fields, subfields.
- Quasilinear approaches constitute the working tool for calculating mean field evolution in plasma turbulence
- As yet, several questions re: QLT remain unanswered.

Outline — A Story, of sorts…

- → The Basics
- → Beyond QLT: Nonlinear Wave-Particle Interaction
- → Challenges to QLT: Granulations and Enhanced Growth
  - → Pesme+ Theory
- → The Quasilinear Experiment of Tsunoda+
- → The Aftermath and Recent Progress
- → Where to?

Basics

I .) Basics – from "Back in the USSR" (Landau, Vlasov, et. seq.)

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q}{m}E\frac{\partial f}{\partial v} = \cancel{c(f)}$$

$$\omega \qquad kv \qquad \omega_{NL} \qquad \gg \nu$$

Ions stationary

So $f = \langle f_0 \rangle + \delta f$

Brackets mean space, fast time avg

$$\boxed{\begin{gathered}\frac{\partial f}{\partial t} + \{H, f\} = 0 \\ \partial_x^2 \tilde{\phi} = -4\pi n_0 q \int dv \delta f\end{gathered}}$$

$\langle f \rangle$ is "close" to Maxwellian.

- Incompressible (phase space)
- $f \leftrightarrow PV$

$c = 0 \Rightarrow$ Violent Relaxation (Lynden-Bell)

Basics, cont'd

Excitations: Plasma waves + interactions with particles.
Eddies? — TBC

Waves → from linearized Vlasov-Poisson:

$$\epsilon = \epsilon(k,\omega) = 1 + \frac{\omega_p^2}{k}\int dv \frac{\partial\langle f\rangle/\partial v}{\omega - kv} = 0$$

$$\frac{1}{\omega - kv} = \frac{P}{\omega - kv} - \boxed{i\pi\delta(\omega - kv)}$$

$\epsilon(k,\omega) = 0$
$\Rightarrow \omega(k)$

resonant particle contribution

So

$$\omega(k) = \omega_k^{re} + i\gamma_k$$

$$\omega_k^{re} = \left(\omega_{pe}^2 + 3k^2 v_{the}^2\right)^{1/2}$$

compressible real space

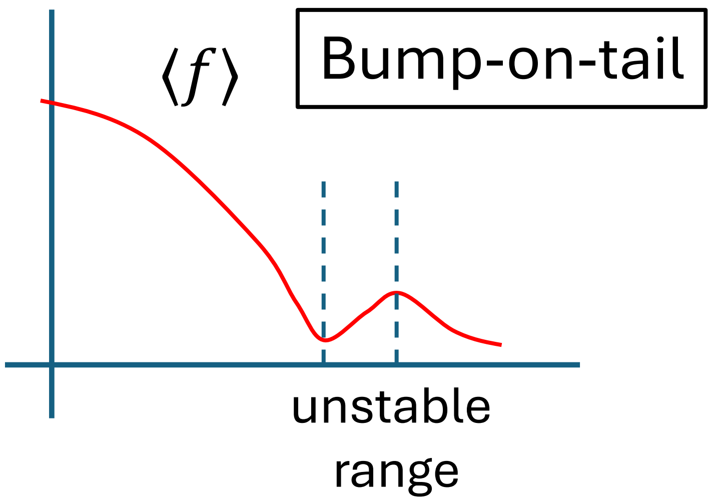
Bohm-Gross wave

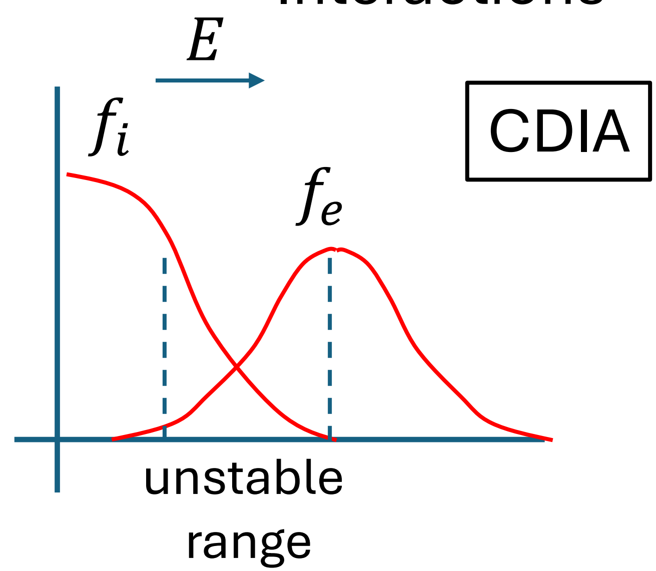
$$\gamma_k = -\frac{Im\ \epsilon}{\partial\epsilon/\partial\omega}\Big|_{\omega_k} = \frac{\pi\omega_p^2}{|k|k\partial\epsilon/\partial\omega}\frac{\partial\langle f\rangle}{\partial v}\Big|_{\omega_k}$$

Basics, cont'd

Turbulence = Plasma wave Turbulence + Wave-particle Interactions

2 classic examples:





QLT seeks to calculate $\partial\langle f \rangle/\partial t$ such that $\gamma \to 0$

Ion acoustic wave

$$\omega^2 = k^2 c_s^2 / 1 + k^2 \lambda_{De}^2$$

$$c_s^2 = T_e / m_i$$

N.B. CDIA turbulence relevant to "anomalous resistivity".

Basics, cont'd: Quasilinear Equation for $\langle f \rangle$ evolution $(q/m \to 1)$

$$\frac{\partial \langle f \rangle}{\partial t} = -\frac{\partial}{\partial v} \langle \tilde{E} \boxed{\delta f} \rangle$$

n.b. $f = f(t, \tau)$

fast ← t — i.e., wave

τ → slow — $\langle f \rangle$ evolution

Then δf = linear response

$$= -\frac{E_k \partial \langle f \rangle / \partial v}{-i(\omega - kv)}$$

brackets:

- average over x, t_{fast} (coarse grain)
- ensemble: RPA

$$\Rightarrow \boxed{\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial v} D \frac{\partial}{\partial v} \langle f \rangle}$$

→ Quasi-linear equation (Velikhov, Vedenov, Sagdeev)

$$D = Re \sum_k \frac{q^2}{m^2} |E_k|^2 \frac{i}{\omega - kv}$$

→ QL diffusion

Basics, cont'd

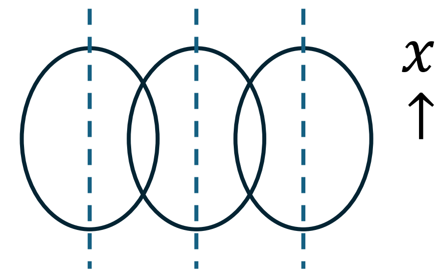
- Key properties:
  - $D = \frac{q^2}{m^2} \sum_k |E_k|^2 \frac{|\gamma_k|}{(\omega - kv)^2 + |\gamma_k|^2}$
  - Resonant → $\pi\delta(\omega - kv)$ → irreversible
  - Non-resonant → $|\gamma_k|/(\omega - kv)^2$ → reversible / 'fake'
  - Non-resonant diffusion for stationary turbulence is problematic. Energetics? — Calculate saturation?!
  - Coarse graining implicit in $\langle\ \rangle$
  - First derivation via RPA, ultimately particle stochasticity is fundamental to resonant diffusion.

Basics, cont'd

- Central elements/orderings:
  - resonant diffusion, irreversibility:
    - "chaos" ←→ coarse graining
    - Island overlap at resonances: $\frac{\omega}{k_{i+i}} - \frac{\omega}{k_i} \leq \sqrt{q\phi/m}$

      → stochasticity
  - linear response?:
  - $\boxed{\tau_{ac} < \tau_{tr},\ \tau_{decorr},\ \ \gamma_k}$ (more than "short, sudden")
  - $\tau_{ac}^{-1} = \left|\frac{d\omega}{dk} - \frac{\omega}{k}\right|\ |\Delta k|$ → correlation time of wave-particle resonant pattern
  - $\tau_{tr}^{-1} = k\sqrt{q\phi/m}$ → particle bounce time in pattern
  - $\tau_{decorr}^{-1} = (k^2 D)^{1/3}$ → particle decorrelation rate (cf. Dupree '66)

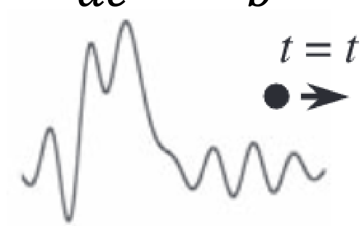
Can derive resonant D from Fokker-Planck



Comments

- No rigorous connection between phase space chaos and validity of (resonant) QLT
- $$1/\tau_{ac} = |\Delta(\omega - kv)|$$ $\rightarrow$ set by dispersion in Doppler shifted frequency
$$\approx \left|\frac{d\omega}{dk} - v\right| \Delta k$$
$$\approx \left|\frac{d\omega}{dk} - \frac{\omega}{k}\right| \Delta k,$$ for resonant particles $\rightarrow$ sensitive wave dispersion
- τ_{ac} vs τ_b

$\tau_{ac} < \tau_b$



pattern resets

or

$\tau_b < \tau_{ac}$

phase space vortex

Basics, cont'd

- QLT is Kubo # < 1 theory

$$\text{i.e., } \frac{q}{m}\tilde{E}\tau_{ac}/\Delta v_T = \Delta v_T\, k\, \tau_{ac} < 1 \longrightarrow \frac{\partial \tilde{f}}{\partial t} \text{ vs. } \frac{q}{m}\tilde{E}\frac{\partial \tilde{f}}{\partial v}$$

$$= \tau_{ac}/\tau_{tr} < 1$$

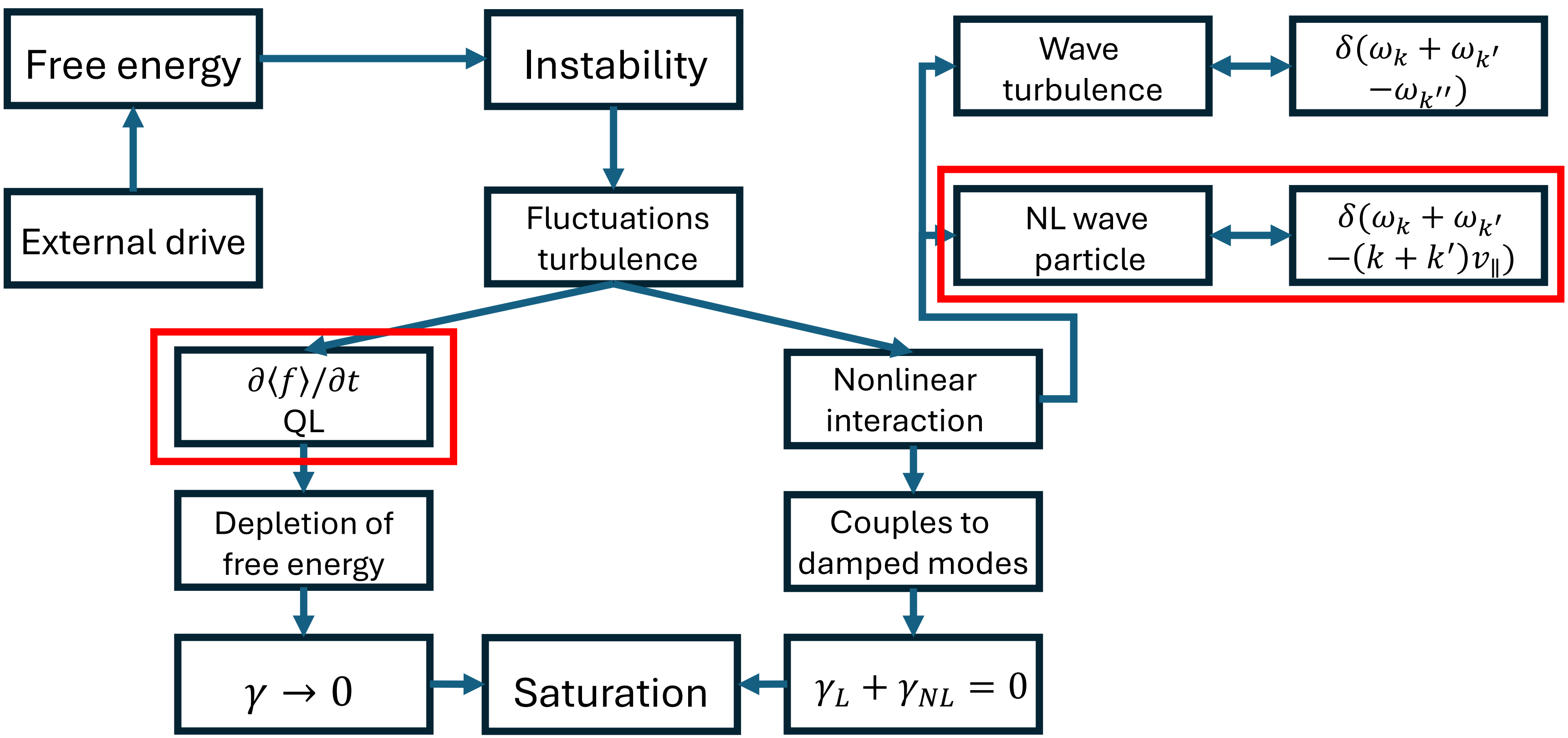
- QLT assumes:
  - all fluctuations are eigenmodes (i.e. neglect mode coupling)

$$\omega = \omega(k)$$

  - <u>all</u> $\delta f \sim \tilde{E}\ \partial\langle f\rangle/\partial v$?
    - Follow from response
    - Eddies?!

(resembles $\delta B \sim \tilde{v}\langle B\rangle$ in MF dynamo theory)

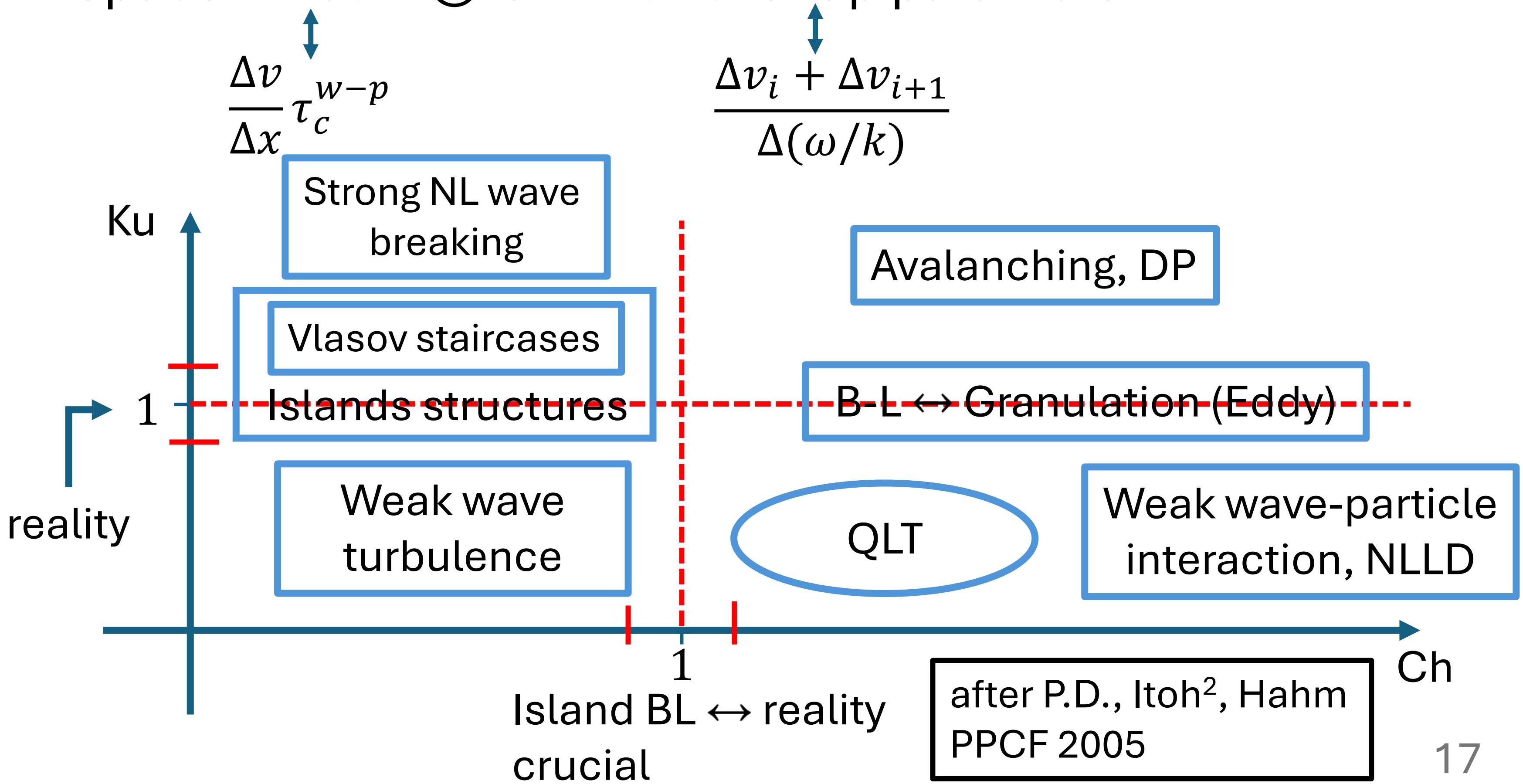
Basics cont'd: Location in the Conventional Grand Scheme (after Sagdeev + Galeev, '67; P.D., Itoh^2, 2010)



Basics, cont'd

- Mapping the Phenomenology → where does QLT apply?

Space: Kubo # ⊗ Chirikov overlap parameter



after P.D., Itoh[2], Hahm PPCF 2005

Basics, cont'd: Energetics — How do the books balance?

→ Easily shown: Resonant particles + waves conserve

$$\partial_t(RPKED) + \partial_t(WED) = 0$$

$$\partial_t(RPKED) = \int \frac{mv^2}{2}\frac{\partial}{\partial v} D_R \frac{\partial\langle f\rangle}{\partial v}, \quad \partial_t(WED) = \sum_k 2\gamma_k\omega_k \frac{\partial\epsilon}{\partial\omega}\bigg|_{\omega_k}\frac{|E_k|^2}{8\pi}$$

→ Also: $\partial_t(PKED) + \partial_t(EED) = 0$

and

→ $\partial_t(RPMD) + \partial_t(WMD) = 0$

$\sim$ Wave energy density flux,
pseudo momentum

$$\partial_t(PMD) = 0$$

Basics, cont'd: Comments on Energetics

- RPKED vs WED is natural, and most physical balance
- Energetics drives 2 component/2 fluid picture of dynamics, as resonant particles + waves or resonant particles + quasi-particles
- Leads to picture of waves as quasi-particle gas $\Rightarrow$ wave kinetic description.

  i.e., $\partial_t(RPKED) + \partial_t(N\omega) = 0$, etc.

Basics, cont'd

→ QLT is a system

$$\langle f(v,0)\rangle$$

$$\downarrow$$

$$\epsilon(k,\omega) = 0 \quad \rightarrow \quad \omega = \omega(k) = \omega_k^{Re} + i\gamma_k$$

$$\downarrow$$

$$\partial_t |E_k|^2 = 2\gamma_k |E_k|^2 \quad \rightarrow \quad \text{update fields}$$

$$\downarrow$$

$$\frac{\partial\langle f\rangle}{\partial t} = \frac{\partial}{\partial v} D(v) \frac{\partial\langle f\rangle}{\partial v} \quad \rightarrow \quad \text{update } \langle f\rangle$$

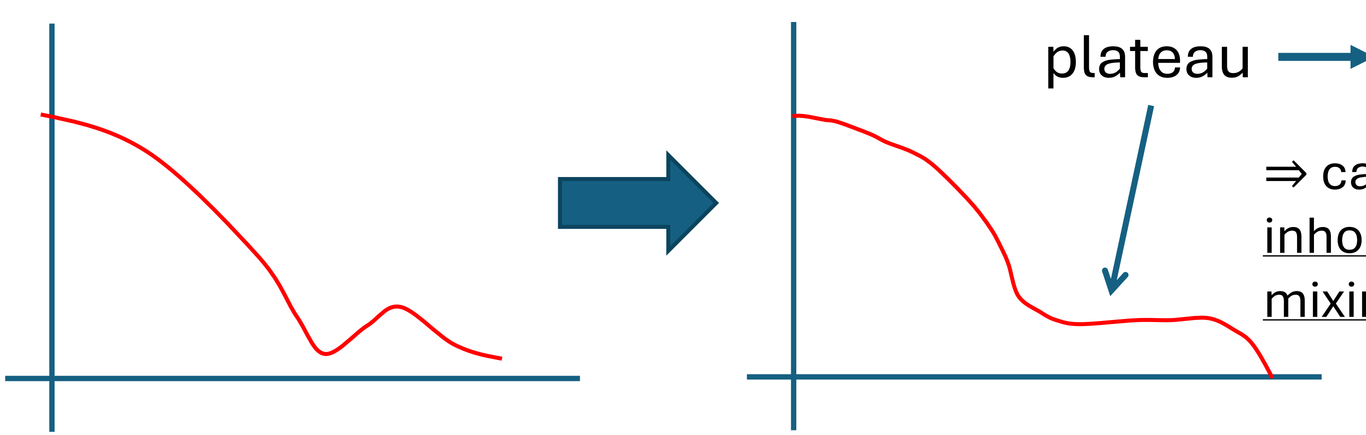
$$\downarrow$$

to $\gamma_k = 0$

Basics, cont'd:

- Outcome → Saturation?!

– B-O-T: Plateau formation



$$\frac{\partial\langle f\rangle}{\partial v} \to 0$$

⇒ case of <u>inhomogeneous mixing</u>

– Prediction for $\left|\tilde{E}_{sat}\right|^2/4\pi nT$ when plateau formed

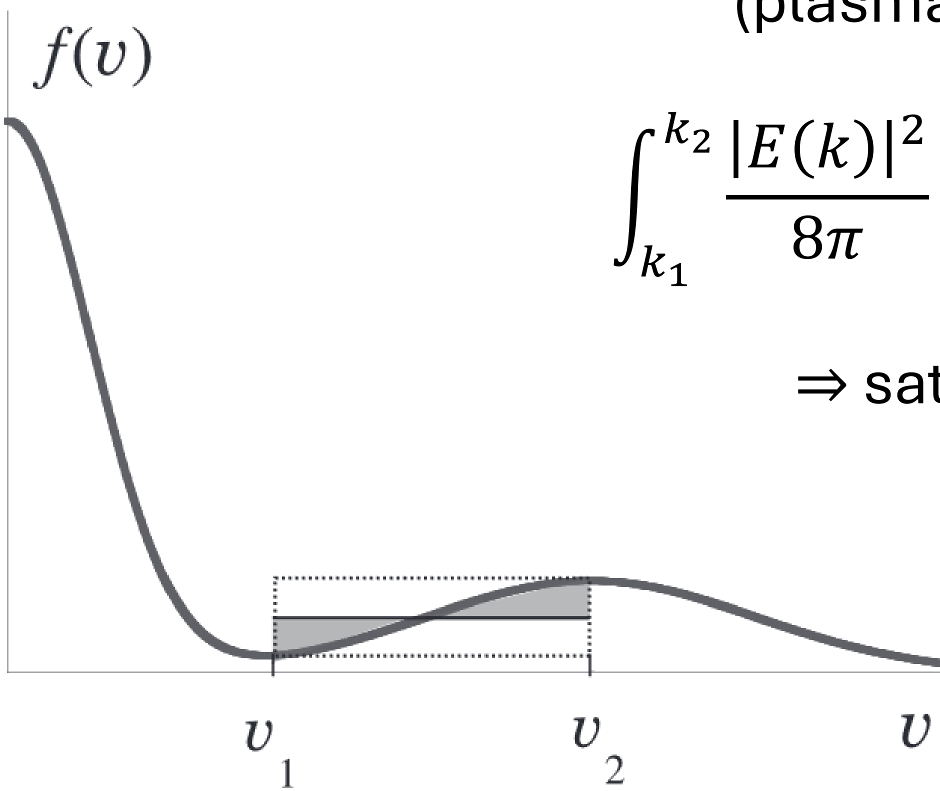
– But: Inhomogeneous mixing (local) on tail drives
  <u>global</u> re-adjustment.

  a) Non-resonant particles "heated" by finite amplitude spectrum

  b) "Heating" is one-sided, due momentum conservation.

Basics, cont'd

→ Plateau Formation: Saturation Level



(plasma waves)

$$\int_{k_1}^{k_2} \frac{|E(k)|^2}{8\pi} d^2k = -\frac{1}{2}\Delta\left(\int_{v_1}^{v_2} dv \frac{mv^2}{2}\langle f\rangle\right)$$

⇒ saturated field energy level

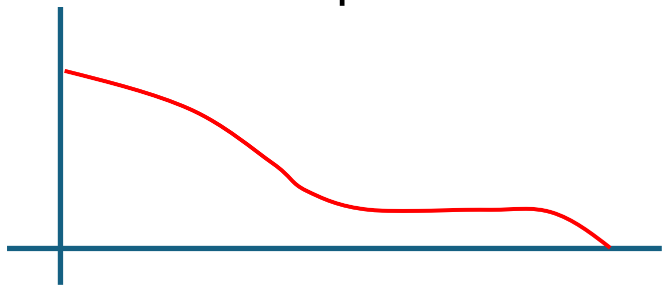
Basics, cont'd

- Why Plateau?
  - In collisionless, un-driven system, need at stationarity:

    $\int dv D_R (\partial \langle f \rangle / \partial v)^2 = 0$

  - So either:

    i) $\partial \langle f \rangle / \partial v = 0$, where $D(v) \neq 0$ on interval → plateau with finite amplitude waves



ii) Or $D_R = 0$ → fluctuations decay everywhere, $\gamma_k < 0$

Basics, cont’d

- If ii), can show from QL system:

- $\langle f(v,t)\rangle = \langle f(v,0)\rangle + \frac{\partial}{\partial v}\left(\frac{D_R(v,t)-D_R(v,0)}{\pi\omega_{pe}^2 v^2}\right)$

- If $D_R \rightarrow 0$ as t increases $\langle f(v,t)\rangle \approx \langle f(v,0)\rangle$

  ($D_R(0)$ negligible)

- But $D_R \rightarrow 0$ requires $\frac{\partial\langle f\rangle}{\partial v} < 0$, while $\frac{\partial\langle f(v,0)\rangle}{\partial v} > 0$ → contradiction!

So

- i) applies → plateau forms

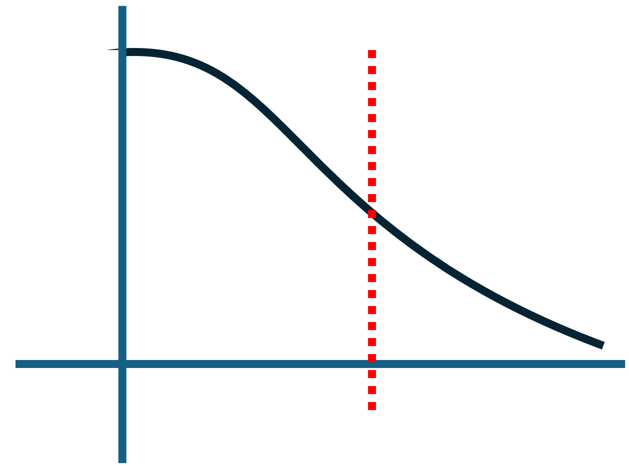
Basics, cont’d

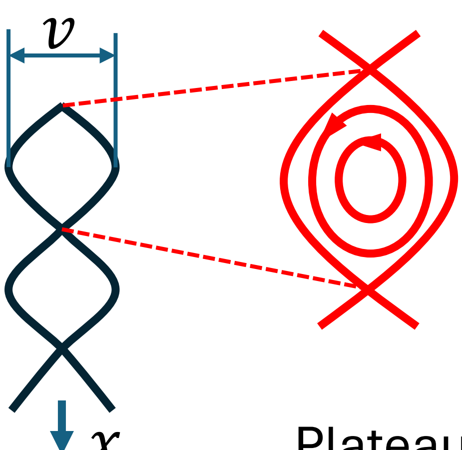
→ What of Single Large Wave?

“large” $\Rightarrow k\Delta v_T \sim \omega_b > \gamma \Rightarrow$

$\Delta v_T \sim (q\phi/M)^{1/2}$

Trapping in phase space island





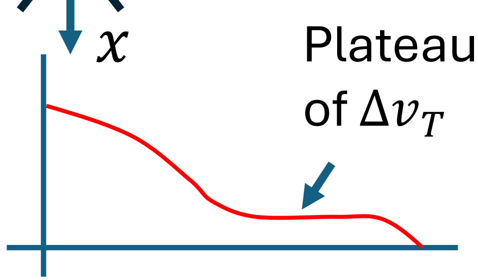
- Differential rotation in island

+

- Coarse graining $c(f)$

Mixing via straining + diffusion (in v)

So



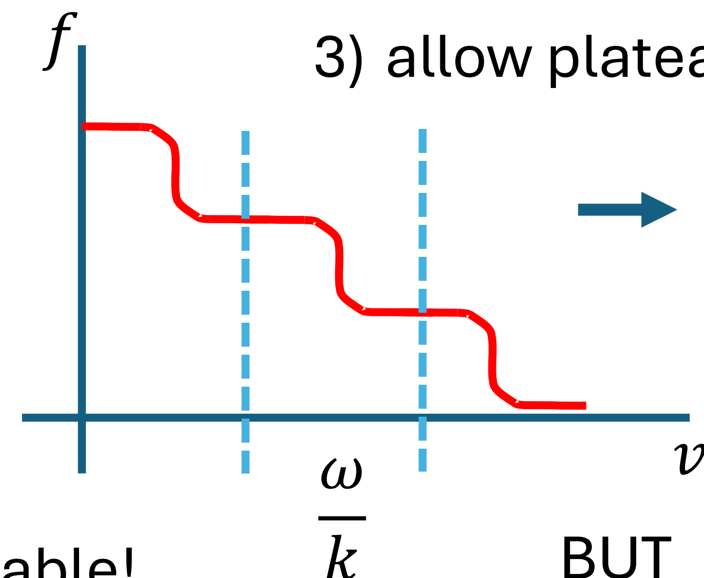
akin Homogenization ala’ AFD, GFD

cf. Shapiro, O’ Neil

Basics, cont'd

Speculation: How to form a simple staircase in v?

1) select resonant waves, of large amplitude
2) ensure $\Delta v_T < \frac{\omega}{k}|_i - \frac{\omega}{k}|_{i+1}$
   So islands not close to overlap
3) allow plateau formation



Staircase in v !?

Sequence of inhomogeneous mixing regions

Characterized by Δv_T vs. $\Delta(\omega/k)$

Testable!

BUT

No bistability, ∽ curve
(cf. M. Vergassola)

Beyond QLT:
Nonlinear Wave-Particle Interaction

N.B. In turbulence

→ G. Falkovich: “you should calculate the next order term before declaring victory”

- At stochastic acceleration level:

$$D = \int_0^\infty d\tau \frac{q^2}{m^2} \langle E(t+\tau)E(t)\rangle$$

- Retain orbit perturbation

$$\begin{aligned} E(x(t),t) &= E(x_0(t) + x_1(t) + \cdots) \\ &\approx E(x_0(t),t) + x_1(t)\frac{\partial}{\partial x}E(x_0(t),t) + \cdots \end{aligned}$$

valid for $\tau_{ac} < \tau_b$ cf: Dupree + Manheimer ‘67

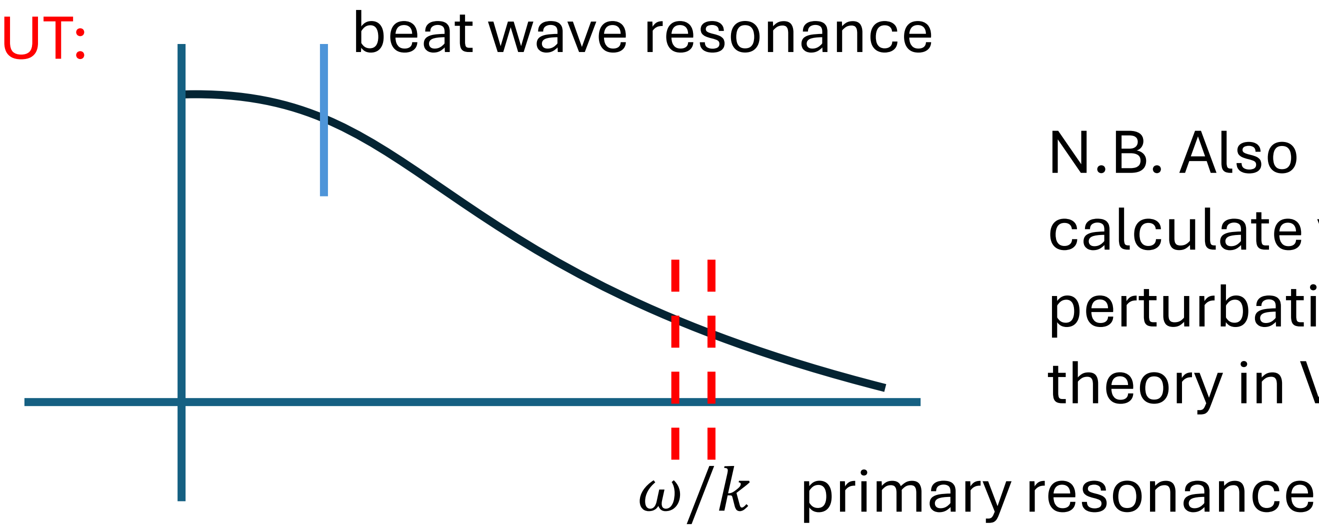
- So $D = D^{(2)} + D^{(4)}$

$$D^{(4)} = \frac{\pi q^2}{m^2}\sum_{k,k'}|E_k|^2|E_{k'}|^2\left(\frac{k-k'}{(kv-\omega)(k'v-\omega')}\right)^2 \delta\big((k-k')v - (\omega-\omega')\big)$$

$$D^{(4)} \sim \sum_{k,k'} \left|\tilde{E}_k\right|^2 \left|\tilde{E}_{k'}\right|^2 (cc)^2 \delta\big((k-k')v - (\omega - \omega')\big)$$

beat wave resonance

- Nominally $D^{(4)} \sim \mathcal{O}\left(\tilde{E}^2/4\pi n T\right) D^{(2)}$
- BUT:



N.B. Also calculate via h.o. perturbation theory in Vlasov

really $\tilde{E}^2/4\pi n T$ vs. Gaussian

- Promising channel for ions in CDIA, Drift-ITG etc.
- For flux transport, see Shane Keating, P.D., JFM

→ Resonance Broadening → Physics of Strong Wave-Particle Scattering (Dupree '66 et seq.)

Linear response:

$$\delta f_{k,\omega} = -\frac{q}{m} e^{-ikx} \int_0^{\tau} d\tau e^{i\omega\tau} u(-\tau) \left[e^{ikx} E_{k,\omega} \frac{\partial \langle f \rangle}{\partial v} \right]$$

$u(-\tau)e^{ikx} = e^{ikx_0(-\tau)} = e^{-ikv\tau}$ → integrate along unperturbed orbits

Now: $x(-\tau) = x_0(-\tau) + \delta x(-\tau)$ → integrate along scattered orbits

↓

statistically distributed, avg. over

$$\delta f_{k,\omega} = -\int_0^{\infty} d\tau e^{i(\omega - kv)\tau} \left\langle e^{ik\delta x(-\tau)} \right\rangle \frac{q}{m} E_{k,\omega} \frac{\partial \langle f \rangle}{\partial v}$$

But $\delta x = -\int_0^{\tau} d\tau' \delta v(-\tau')$ $\qquad D = D_v$

$$\left\langle e^{ik\delta x(-\tau)} \right\rangle = \exp[-k^2 D t^3/6] = \exp[-\tau^3/\tau_c^3]$$

RBT

So

$$\delta f_{k,\omega} = -\frac{q}{m}\int_0^\infty d\tau \exp\left[i(\omega - kv)\tau - \frac{\tau^3}{\tau_c^3}\right] E_{k,\omega}\frac{\partial\langle f\rangle}{\partial v}$$

$1/\tau_c \sim (k^2 D_v/6)^{1/3}$ → particle decorrelation rate (scattering time to decorrelate by k^{-1} from upo)

$1/k\tau_c \sim \Delta v$ → Broadened resonance width

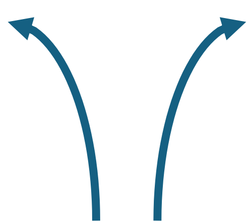
For 'eddy' of resonant, turbulent phase space fluid:

$k^{-1}, \Delta v \rightarrow$ size

$\tau_c \rightarrow$ time scale

N.B. $\Delta v \tau_c \sim k^{-1}$

Similar approach to Rhines, Young, Moffatt, Kamkar

$1/\tau_c$  → ~ Lyapunov exponent for resonant particle orbits

cf. Rechester +

Resonance Broadening Theory

- RBT is a crude propagator renormalization

$$-i(\omega - kv) \rightarrow -i(\omega - kv) \underbrace{- \frac{\partial}{\partial v} D \frac{\partial}{\partial v}}_{self-energy}$$

- A plethora of additional terms exists, but physics is not understood. (cf. Krommes, P.D., Itoh[2], ...)
- Of course, 'rigorous' approach $\Rightarrow$ Non-Markovian renormalization

$$\frac{\partial}{\partial v} D \frac{\partial}{\partial v} \rightarrow \frac{\partial}{\partial v} D_{k,\omega} \frac{\partial}{\partial v}$$

D for resonant particles is Markovian

But $D_{k,\omega} = \sum_{k',\omega'} \left| E_{k',\omega'} \right|^2 \pi \delta(\cancel{\omega} + \omega' - (\cancel{k} + k')v) \rightarrow D$

$$\omega = kv$$

Challenges to Quasilinear Theory

Challenges

*

- Mode coupling
- Resonance broadening

- Phase space eddies
- Dynamical friction

→ Stochastic view

→ Dupree, Kadomtsev...

- BGK modes
- Phase space holes, water bag models

- Phase space vorticities
- Drag, wake

→ Coherent view

→ Lynden-Bell, Berk, Roberts, Feix, Schamel

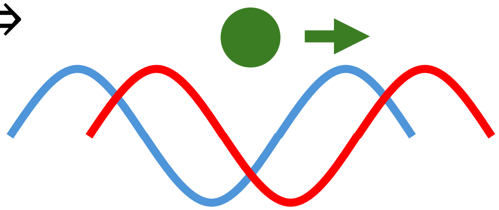
<u>Phase space granulations</u> → Enhanced Cerenkov emission

Fluctuation constituent in addition to waves → major impact on **dynamics?!**

Granulations/Eddies

- Eddies in phase space, as well as eigenmodes. Relevance of $\delta f \sim f^c$ dubious

- Eddies $\leftrightarrow$ strongly correlated particles $\Rightarrow$ enhanced Cerenkov emission $\Rightarrow$ Will granulations couple to available free energy more effectively than waves? Enhanced growth?



- To describe granulation dynamics, formulate theory for evolution $\langle \delta f(1) \delta f(2) \rangle$, with $\delta f = f^c + \tilde{f}$. Reminiscent of Pouquet + approach, as opposed to Mean Field Electrodynamics.

Plan for Discussion:

- Approaches to Physics of Granulations

- Adam, Laval, Pesme (ALP): Predicted Multiplicative Enhancement of Growth.
  $\rightarrow$ Concrete, Testable Prediction ...

- Traveling Wave Tube Experiment $\leftrightarrow$ Dedicated test of QL

  $\rightarrow$ Test ALP prediction

- Understanding the Outcome ...

- <u>Granulations</u>
  - Mode coupling mediated by resonant particles (k-space)
  - Distorts distribution, so: akin eddy, vortex (real (phase) space)
  - $\delta f = f^c + \tilde{f}$ → granulation $\Rightarrow \langle E\delta f \rangle \rightarrow -D\frac{\partial \langle f \rangle}{\partial v} + F$
  - Calculate $\langle \tilde{f} \rangle^2$ via $\langle \delta f^2 \rangle$+extraction $\langle f^{c2} \rangle$ etc.
  - Poisson equation → $\tilde{f}$ induces dynamical friction (i.e. drag)

Granulations alter relaxation

$$\partial_t \langle \delta f^2 \rangle + T_{1,2} \langle \delta f^2 \rangle = D \left(\frac{\partial \langle f \rangle}{\partial v} \right)^2 - F \left(\frac{\partial \langle f \rangle}{\partial v} \right)$$

Relative scattering, streaming

RHS→ $\langle f \rangle$ relaxation

$$\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial v} \left[D \frac{\partial \langle f \rangle}{\partial v} - F \right]$$

Theory (1):

$$\frac{\partial}{\partial t}\langle \delta f \delta f\rangle + \left(v_1 \frac{\partial}{\partial x_1} + v_2 \frac{\partial}{\partial x_2}\right)\langle \delta f \delta f\rangle$$
$$+ \frac{\partial}{\partial v_1}\langle E(1)\delta f(1)\delta f(2)\rangle + \frac{\partial}{\partial v_2}\langle E(2)\delta f(2)\delta f(1)\rangle$$
$$= -\langle E(1)\delta f(2)\rangle \frac{\partial\langle f\rangle}{\partial v}\Big|_{v_1} - \langle E\delta f(1)\rangle \frac{\partial\langle f\rangle}{\partial v}\Big|_{v_2}$$

Closure + Relative Coordinates (x_-, v_-):

$$T_{1,2} = v_- \frac{\partial}{\partial x_-} - \frac{\partial}{\partial v_-} D_{Rel} \frac{\partial}{\partial v_-}$$

$$D_{Rel} = D_{1,1} + D_{2,2} - D_{1,2} - D_{2,1}$$

e.g. Bivariate Fokker-Planck

$$\lim_{x_-, v_- \to 0} D_{Rel} = 0$$

(important!)

Theory (2)

$$\text{RHS} = -\langle E(1)\delta f(2)\rangle \frac{\partial \langle f\rangle}{\partial v_1} - \langle E(2)\delta f(1)\rangle \frac{\partial \langle f\rangle}{\partial v_2}$$

$\delta f = f^c + \tilde{f}$, Poisson Eqn.

$$\text{RHS} = D\left(\frac{\partial \langle f\rangle}{\partial v}\right)^2 - F\left(\frac{\partial \langle f\rangle}{\partial v}\right)$$

"QL" piece

granulation piece (dynamical friction)

RHS gives growth of granulations via interaction with $\partial\langle f\rangle/\partial v$

$$\text{RHS} \leftrightarrow \frac{\partial \langle f\rangle}{\partial t} = \frac{\partial}{\partial v}\left[D\frac{\partial \langle f\rangle}{\partial v} - F\right]$$ $\rightarrow$ mean relaxation feeds $\langle \delta f^2\rangle$

Structurally similar to Balescu-Lenard Theory

$\therefore$ screened granulation $\leftrightarrow$ screened particle

- <u>Implications</u> → mode coupling enters growth dynamics
  - Dynamical friction enters relaxation, and mean ←→ fluctuation coupling
  - Interspecies drag can solve stationarity problem

And:

  - Introduces new routes to relaxation, subcritical growth via collisionless momentum transfer by structures
  - Prediction of subcritical CDIA instability (Dupree '82) → partially vindicated (Lesur +, 2014)

- Adam, Laval, Pesme (ALP): A Testable Prediction

1980, et seq. re: Granulations

- Enhanced B-O-T Growth

$$\frac{\partial}{\partial t}\langle \delta f^2 \rangle + \left(v_- \frac{\partial}{\partial x_-} - D_{Rel} \frac{\partial^2}{\partial v_-^2} \right) \langle \delta f^2 \rangle = D \left(\frac{\partial \langle f \rangle}{\partial v} \right)^2$$

+ Poisson Eqn

$\# = 4 \sum J_n(n)^2/n = 1.668$

$\Rightarrow \gamma^2 \rightarrow (\#)\gamma_{lin}^2$ for B-O-T

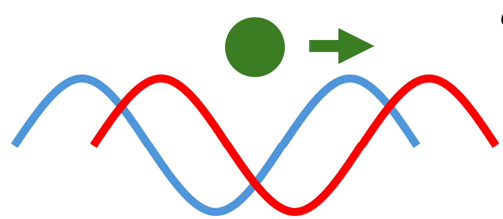
Multiplicative correction

$\# \leftrightarrow \tau_{cl}/\tau_c$

Physics: "The modification is a consequence of wave emission by strongly correlated resonant particles".

—Attracted wide attention ...

(N.B.: Big Noise ...)



"clump" emission

Where are we?

- long standing, well established QLT
- Serious theoretical questions, culminating

  in a testable prediction
- simulation results scattered

The Quasilinear Experiment

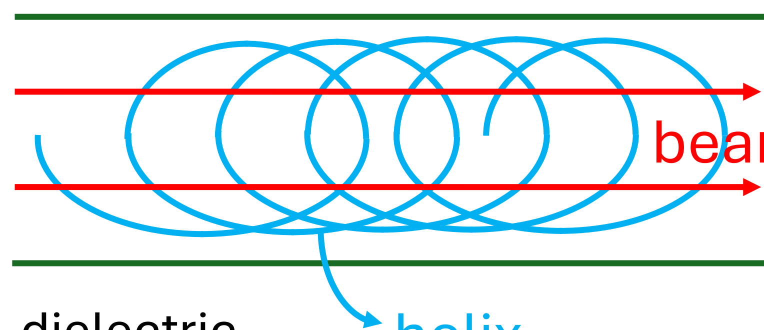
ala’

“Let the cannon decide!”

— Ultima Ratio Regis

Rejoinder: N.B. Be careful what you ask for ...

- TWT experiment (Tsunoda et al 1989, 1990)
- $\boxed{\text{‘Simulate’ B-O-T via}}$
  - Beam → resonant particles
    $\bar{v}, \overline{v^2}$
  - Slow wave helix → non-resonant, dielectric



- Could program variety of spectral perturbations, and control phase initialization — test RPA
- Can measure:
  - net growth of perturbations
  - fluctuation spectrum

key:
use of slow wave
helix avoids
problematic ion noise

- TWT Apparatus

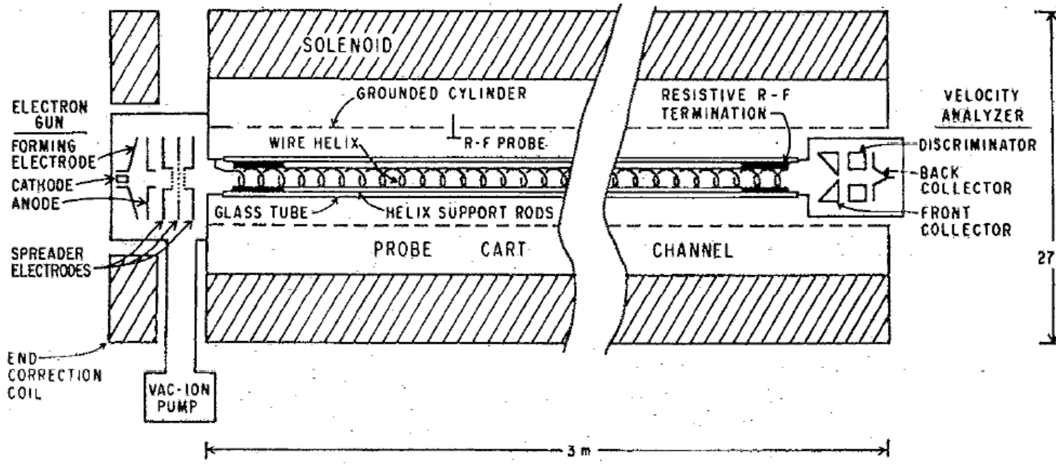
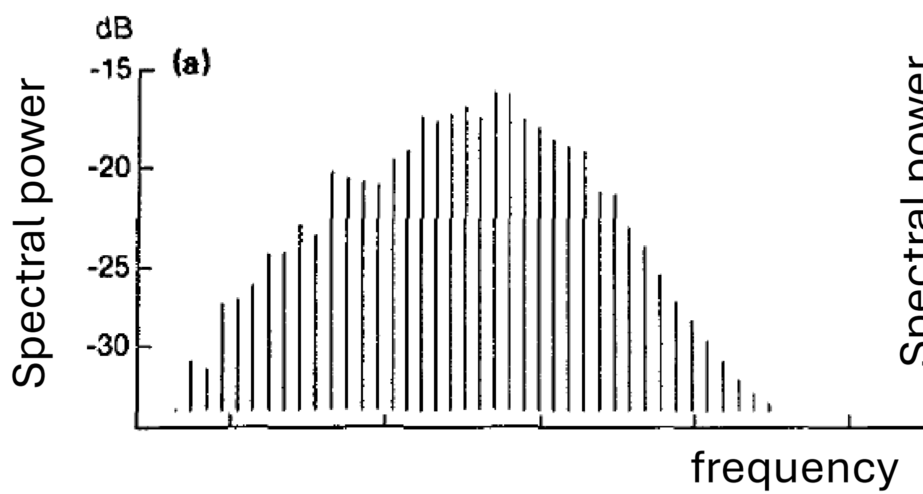
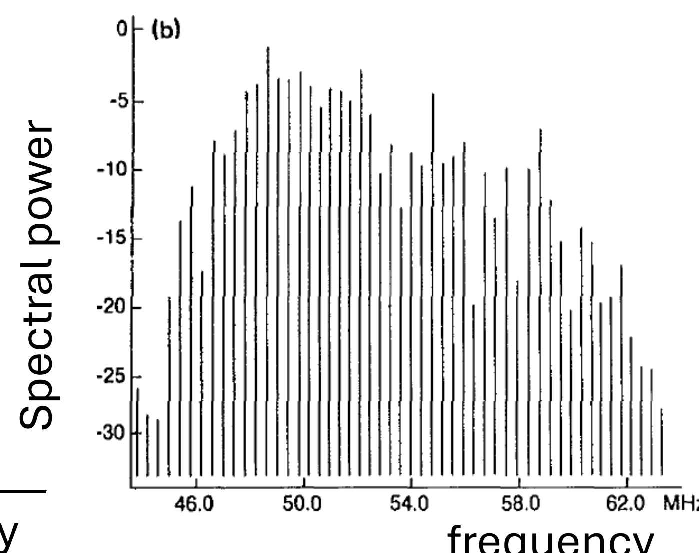


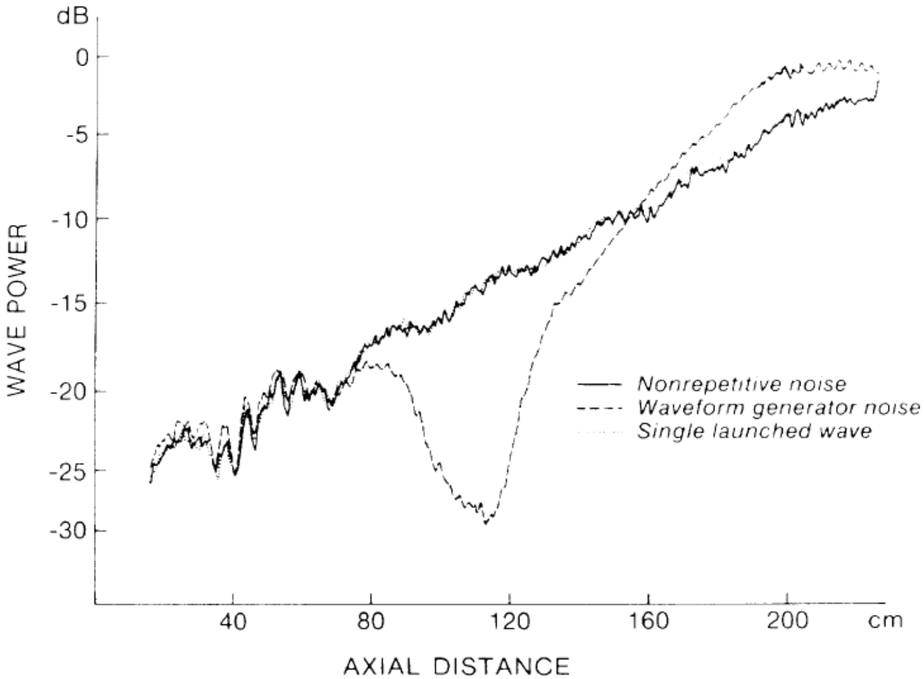
FIG. 1. Schematic diagram of the apparatus (not drawn to scale).

- Spectral evolution → evidence for mode coupling mediated by resonant particles





- The reckoning:



Dashed → one mode in smooth spectrum

Dotted → linear (single, weak mode)

Solid → non-rep noise

- “no deviation of frequency, ensemble averaged growth from Landau, to 10%”

- Message: mode coupling via resonant particles occurs, yet growth tracks linear Landau, QLT “works” for γ

- Comments
  - TWT results effectively vindicated QLT ala’ 60’s and demolished ALP.
  - Much more might have been extracted by TWT
    - Studies of nonlinear transfer
    - Effect of adjustable dissipation in slow wave structure (see later)
    - Coordinated numerical simulation effort → ideal venue for validation of Vlasov codes
  - Time to re-visit TWT or variant? —TBC

Twitter Summary:

QLT is not dead yet

The Aftermath —

what, really, was this argument about?

- What Happened? Why QLT clearly deficient yet predicts growth?

Conclusion, Tsunoda:

“To sum up, we have shown that the quasilinear theory description of our experiment is incomplete. The correct nonlinear description of our experiment has yet to be found. **An important clue may be the existence of statistical or dynamical conservation law governing mode coupling effects.**”

- Comments, cont'd

  - Thoughts on the outcome (Liang, P.D. '93)

  - Gist: momentum conservation

  Well known: Balescu-Lenard evolution of 1D stable plasma

  leaves $\partial_t \langle f \rangle = 0$

  i.e. Like particle, momentum and energy conserving collision

  leave final state = initial state

  ∴ 1D, 1 species granulations not effective for relaxation

  - Difference here: System not stationary → growing waves

- Analysis, key points:

$$\left(\partial_t + T_{1,2}\right)\langle \delta f(1)\delta f(2)\rangle = S(v)$$

$$S(v) = -2\frac{q}{m}\,\langle \tilde{E}\delta f\rangle\,\partial\langle f\rangle/\partial v$$

- For $S(v)$:

$$\frac{q}{m}\langle \delta E(1)\delta f(1)\rangle = \sum_k{}'\left(-k^2\frac{q^2}{m^2}\langle\phi_k\phi_{-k}\rangle\pi\delta(\omega_k-kv)\frac{\partial f_0}{\partial v}-ik\frac{q}{m}\langle\phi_k\tilde{f}_{-k}\rangle\right)e^{2\gamma_k t}$$
$$=\sum_k{}'\left[-k^2\frac{q^2}{m^2}\pi\delta(\omega_k-kv)\frac{\partial f_0}{\partial v}\left(\frac{4\pi n_0 q}{k^2}\right)^2\int\frac{dv_1\,dv_2}{|\epsilon(k,\omega_k+i\gamma_k)|^2}\langle\tilde{f}_k(v_1)\tilde{f}_{-k}(v_2)\rangle\right.$$
$$\left.-k\frac{q}{m}\left(\frac{4\pi n_0 q}{k^2}\right)\frac{\operatorname{Im}\epsilon(k,\omega_k+i\gamma_k)}{|\epsilon(k,\omega_k+i\gamma_k)|^2}\int dv'\langle\tilde{f}_k(v')\tilde{f}_{-k}(v)\rangle\right]e^{2\gamma_k t}.$$

$$\downarrow$$

- Further:

$$\frac{q}{m}\langle\delta E(1)\delta f(1)\rangle = -\sum_k{}'k\frac{q}{m}\frac{\gamma_k\partial\epsilon'(k,\omega_k)/\partial\omega}{|\epsilon(k,kv+i\gamma_k)|^2}\times\langle\tilde{\phi}_k\tilde{f}_{-k}(v)\rangle e^{2\gamma_k t}.$$

- N.B.: $S(v)\sim\gamma_k$ as electrons exchange momentum with waves, **only,** here

- Results:

- For $S(v)$:

$$S(v) = 2k^2 \frac{q}{m} \frac{\gamma_k/\omega_k}{\epsilon''(k,\omega_k) + \gamma_k \partial\epsilon'(k,\omega_k)/\partial\omega} \frac{\partial f_0}{\partial v} \times \langle \tilde{\phi}_k \tilde{f}_{-k}(v) \rangle e^{2\gamma_k t}$$
$$= \frac{2}{\pi} \frac{q}{m} \frac{k^4}{\omega_k \omega_p^2} \frac{\gamma_k^L \gamma_k}{\gamma_k - \gamma_k^L} \langle \tilde{\phi}_k \tilde{f}_{-k}(v) \rangle e^{2\gamma_k t},$$

- For γ_k:

$\sim \tau_{ac} < \tau_c < \gamma_k^{-1}$:

$$\gamma_k \approx \gamma_k^L \left(1 - \frac{2A(k)}{\pi} \frac{\gamma_k^L}{\omega_k}\right)^{-1} \approx \gamma^L \left(1 + O\left(\frac{\gamma^L}{\omega_k}\right)\right)$$

$\sim \tau_{ac} < \gamma_k^{-1} < \tau_c$:

$$\boxed{\gamma_k \equiv \gamma^L \left(1 + \frac{2A(k)}{\pi\beta} \frac{1}{\omega_k \tau_c}\right) \approx \gamma^L \left[1 + O\left(\frac{1}{\tau_c \omega_k}\right)\right]}$$

- Small additive correction to linear growth rate!

- Comments

  - Compare:

    - ALP: $\gamma \approx \# \, \gamma^L$

    - LD: $\gamma \approx \gamma^L \, (1 + \epsilon)$

  ALP inconsistent with TWT results

  LD within error bars

  - QLT ‘61 (seemingly) vindicated for Gentle B-O-T, single species

$*$ - LD explains how reconcile observation of mode coupling with QL growth

  But

  - Is the B-O-T representative? CDIA? Other?
    Is the “simplest problem” too simple?

Recent Progress

— A Sample

Recent Progress (Lesur, Kosuga, P.D.)

- Subcritical growth in the B-B model (Lesur, P.D. 2013; P.D., Lesur, Kosuga Aix Fest 2009)
  - What is B-B (Berk-Breizman) model?
  - B-B ('99) based on reduced model of energetic particles (i.e. alphas) resonant with Alfven wave (TAE). Point is that resonant particle distribution evolves like 1D plasma, **near resonance**
  - Reduction is somewhat controversial, still
  - Analogy: beam, helix ←→ TWT

    EP's, bulk motion in AW ←→ tokamak

  Both are beam-driven instabilities

- For EP distribution

RHS → collision operator

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{qE}{m}\frac{\partial f}{\partial v} = -\gamma_a \delta f + \frac{\gamma_f^2}{k}\frac{\partial \delta f}{\partial v} + \frac{\gamma_d^3}{k^2}\frac{\partial^2 \delta f}{\partial v^2}$$

$$E = re(Z), \qquad f = f_0 + \delta f$$

$$\frac{dZ}{dt} = -\frac{m\omega_p^2}{4\pi n q}\int f e^{-i\,\varepsilon}\, dv - \gamma_d Z$$ ← key difference

dissipation in feedback loop

- Note: collisions and 'extrinsic' γ_d

  * γ_d resembles dissipative helix response in TWT

  → momentum, energy exchange channel ?!

- Linearly $\gamma = \gamma_{kin} - \gamma_d$

- Useful to exploit analogy with QG fluid

  - So 'phasetrophy' $\psi_s = \int_{-\infty}^{\infty} dv \langle \delta f_s^2 \rangle$

  - Wave energy $W = nq^2 \langle E^2 \rangle / m\omega_p^2$

- So, for <u>single structure</u> (with single wave)
  - For ψ: $$\frac{d\Psi_s}{dt} = -2\frac{q_s}{m_s}\int_{-\infty}^{\infty} \frac{df_{0,s}}{dv} \langle E\,\delta f_s \rangle\, dv - \gamma_\Psi^{\mathrm{col}} \Psi_s$$
  - For W: $$\frac{dW}{dt} + 2\gamma_d W = -2\sum_s u_s q_s \int \langle E\,\delta f_s \rangle\, dv \qquad u_s = \omega_p/2k$$
  - Akin to Charney-Drazin theorem $$\frac{dW}{dt} + 2\gamma_d W = \sum_s \frac{m_s u_s}{d_v f_{0,s}} \left(\gamma_\Psi^{\mathrm{col}} + \frac{d}{dt} \right) \Psi_s$$

- Approximate solution :

$$\boxed{\gamma_\psi \approx \frac{16}{3\sqrt{\pi}} \frac{\Delta v}{v_R} \frac{\gamma_{L,0}}{\omega_p} \gamma_d}$$

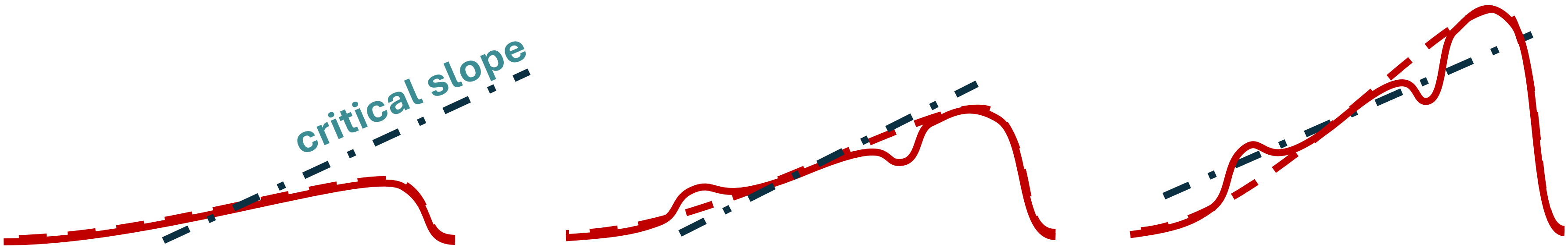
- Nonlinear, $\Delta v \sim (q\phi/m)^{1/2}$
- Exploits γ_d (dissipation)

  i.e. can have $\gamma_{L,0} - \gamma_d < 0$ but $\gamma_\psi > 0$

- $\gamma_{L,0} > 0$ ←→ free energy

Subcritical instability

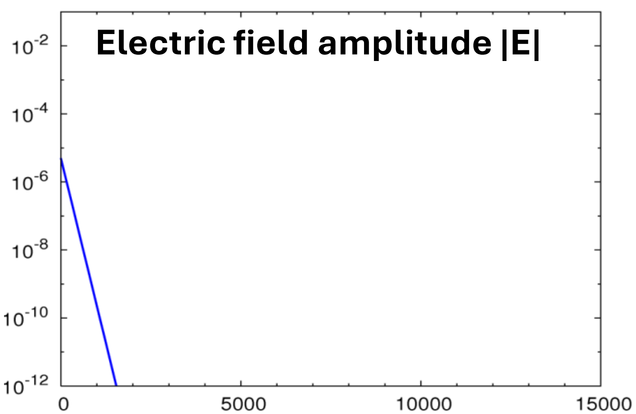
Linear growth rate $\gamma \approx \gamma_L - \gamma_d$ $\Rightarrow$ Critical slope $\gamma_L = \gamma_d$



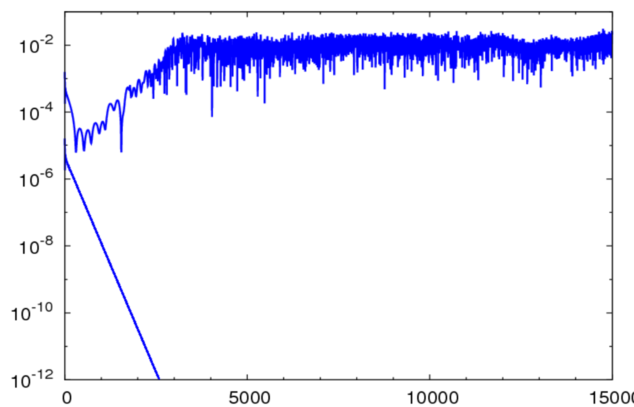
Stable, $\gamma < 0$

Nonlinearly unstable, $\gamma < 0$ (Subcritical instability)

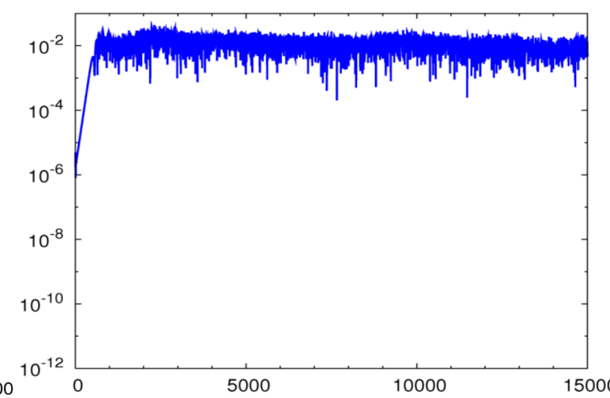
Unstable, $\gamma > 0$



Time (ωt)



Time (ωt)



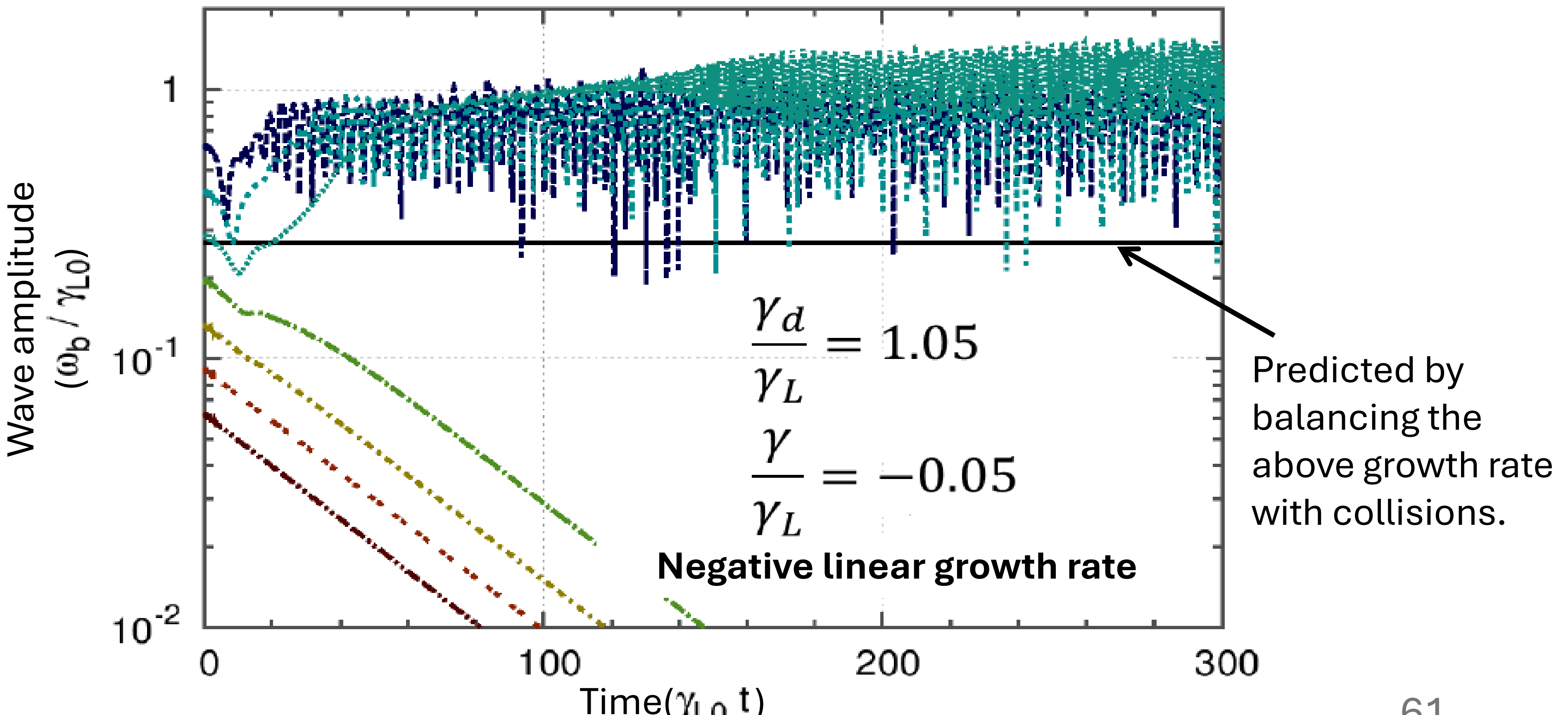
Time (ωt)

Nonlinear growth rate

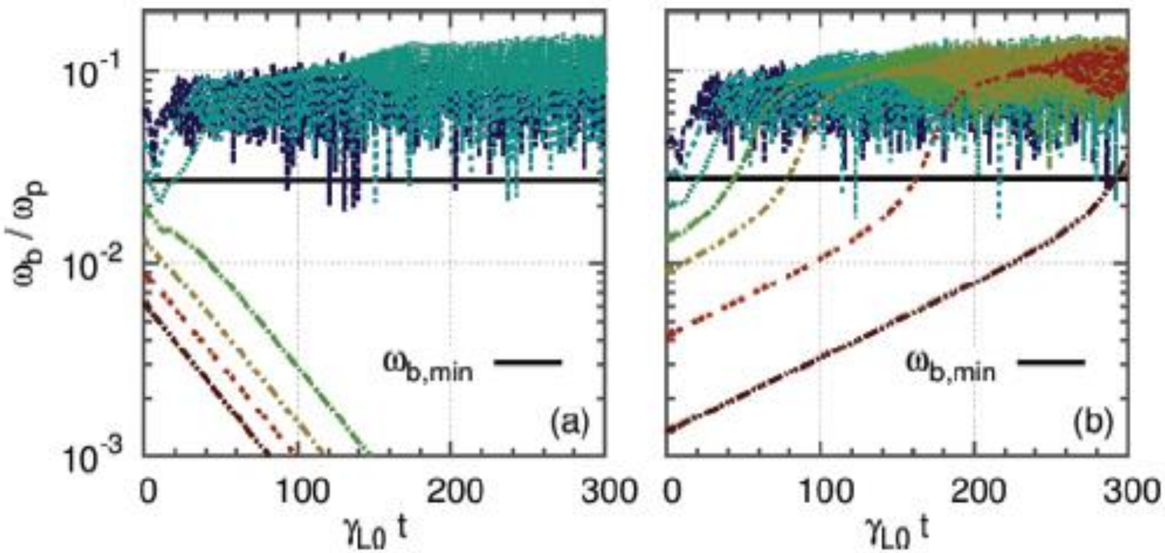
Lesur, Diamond, PRE 2013

$$\gamma_\Psi \approx \frac{16}{3\sqrt{\pi}} \frac{\Delta v}{v_R} \frac{\gamma_L}{\omega_p} \gamma_d$$

$\Rightarrow$ **Nonlinear growth does not require that $\gamma_{L,c} > \gamma_d$**



- Perhaps more convincing:



- Point is that even weak linear instability can be swamped by nonlinear growth → note for weak linear instability, saturation levels match those for nonlinear instability

- Establishes existence of robust exception to QLT61 ! Clearly related to γ_d dissipation channel. Limited to single structure.

Conclusion

Thoughts for Discussion

- Where does this story stand?
  - QLT '61 vindicated for relaxation of single species B-O-T, its paradigmatic example
  - 1D conservation constraints allow reconciliation of mode coupling with observed Landau growth. This interpretation raises (implicitly) the question of how representative the classic B-O-T is.

But

- Significant departures from QLT61 appear in (even 1D) systems with multiple energy-momentum exchange channels, usually associated with multi-species
  - B-B via γ_d
  - CDIA, though structure required.

  Signature of nonlinear growth observed in simulations.

- Role of strong wave-particle resonance and phase space structure in even simple drift-zonal systems is not understood and merits further study. Systems with drift resonances are especially tantalizing.
  - Subcritical growth?
  - Role of granulations—and phase space dynamics— in avalanching? (nucleation process.)
  - Granulation interaction with zonal flows?

Why Drift Resonance?

(P.D. + IAEA '82; Y. Kosuga, P.D., 2012 et seq.)

- $\delta f \rightarrow g$—bounce avg distribution

$$-i(\omega - \overline{\omega}_D \epsilon)\tilde{g}_k + (\tilde{v}_{E\times B} \cdot \nabla \tilde{g})_k = i\frac{|e|}{T}(\omega - \omega_{*T})\phi_k \langle f \rangle$$

- $\Delta(\omega - \overline{\omega}_D \epsilon) \sim |\Delta k_\theta| \left| \frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta} \right|$ ~ ala' 1D.

  but modes weakly dispersive ⇒

  τ_{ac} long, Ku large.

- Looks like a granulation paradise …

Related

→ Drift/Rossby Turbulence + Zonal Flow

Turbulence, for drag$\rightarrow 0$?

→ Appeal to shear flow instability (c.f. G. Esler…)

but need Model of PV mixing and transport?

$\Rightarrow$ QLT for PV.

See J.C. Li, P.D., PoP 2018

- What to Do?
  - Revitalize TWT (start over!) , in coordination with modern simulation program
    - Allow variable slow wave structure dissipation → γ_d as in B&B → test Lesur, P.D. model?
    - Study mode coupling, beat resonance (NLLD) phenomena
  - Is a (philosophically) similar CDIA experiment possible? Many testable predictions on the record. Consider multi-ion species to deal with m/M issue. Negative ion plasma to deal with mass ratio?!
  - While corresponding basic experiment dubious, Darmet model simulation program appears doable and interesting.

Closing Thoughts:

“Truth is never pure, and rarely simple.”

— Oscar Wilde

→ Plenty to be done on QLT …

→ I hope the zealots are unhappy!