Spreading and Entrainment in Drift Wave – Zonal Flow Turbulence: A Basic Study

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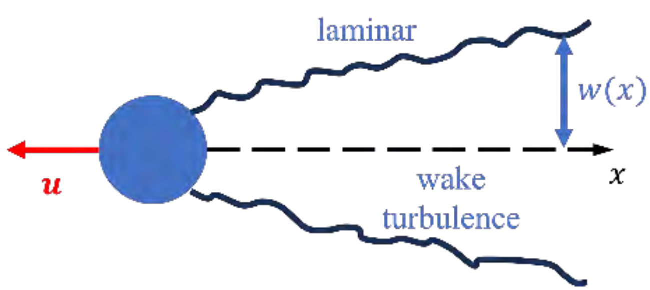
Wake-Classic Example of Turbulence Spreading

⇨ 

Similarity Theory
Mixing Length Theory

$W \sim (F_d/\rho U^2)^{1/3} X^{1/3}$,

$F_d \sim C_D \rho U^2 A_s$

C_D independent of viscosity at high Re

⇨ Physics: Entrainment of laminar region by expanding turbulent region.
Key is turbulent mixing. ⇨ Wake expands

Turbulence Spreading

⇨ Townsend '49:

— Distinction between momentum transport — eddy viscosity—and fluctuation energy transport
— Failure of eddy viscosity to parametrize spreading
— Jet Velocity: $V = \frac{<V_{perp} * V^2>}{<V^2>}$ ⇨ spreading flux FOM

Forced Hasegawa – Mima + Zonal Flows

H-M + Zonal Flow System

- System:

PV forced

$$\frac{d}{dt}\left(\tilde{\phi} - \rho_s^2 \nabla_\perp^2 \tilde{\phi}\right) + v_* \frac{\partial \tilde{\phi}}{\partial y} + v_{*u} \frac{\partial \tilde{\phi}}{\partial y} = \frac{\partial}{\partial r} \rho_s^2 \left\langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \right\rangle + \nu \nabla^2 \nabla^2 (\widetilde{\phi}) + \tilde{F}$$ -Waves, Eddys

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{v}_z \frac{\partial}{\partial y} - \nabla \tilde{\phi} \times \hat{\boldsymbol{z}} \cdot \nabla_\perp$$

$$\frac{\partial}{\partial t} \nabla_x^2 \bar{\phi}_z + \frac{\partial}{\partial r} \left\langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \right\rangle + \mu \nabla_x^2 \bar{\phi}_z = 0$$ -Zonal Flow (Axisymmetric)

N.B. $\bar{\phi}_z = \bar{\phi}_z(x)$, only.

N.B. : Electrons Boltzmann for waves, not for Zonal Flow

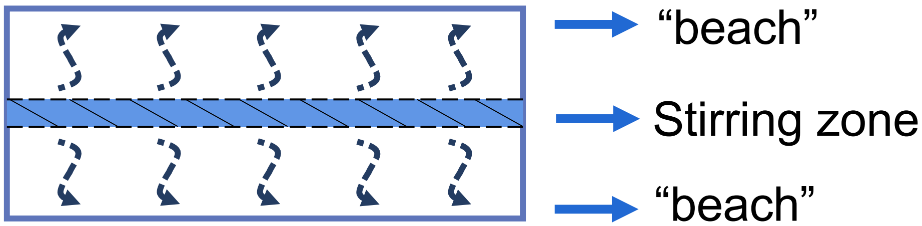
- viscosity controls small scales
- drag controls zonal flow - μ
- conserved: Energy $\longrightarrow \left\langle \tilde{\phi}^2 + \rho_s^2 \left(\nabla \tilde{\phi}\right)^2 \right\rangle + \langle \rho_s^2 (\nabla \phi_z)^2 \rangle$

  Potential Enstrophy $\longrightarrow \underbrace{\left\langle \left(\tilde{\phi} - \rho_s^2 \nabla^2 \tilde{\phi}\right)^2 \right\rangle}_{\text{Waves}} + \underbrace{\langle (\rho_s^2 \nabla^2 \phi_z)^2 \rangle}_{\text{ZF}}$

N.B. Energy, Pot Enstr. exchange between Waves and ZF possible.

Spreading Studies - Numerical Experiments

- 2D Box, Localized Stirring Zone



- Comparison of:

System	**Features**
2D Fluid	Selective Decay, Vortices How to Measure Spreading?
2D MHD with weak $\underline{B_0}$ perp.	Alfvenization, Vortex Bursting, Zeldovich number
Forced Hasegawa-Mima with Zonal Flow	Waves + Eddies + ZF Multiple regimes and Mechanisms

N.B. Clear distinction between "spreading" and "avalanching"

Numerics: 2D Dedalus simulation

- Dedalus Framework

Box Characteristics:

- Grid Size: 512×512
- beach regulates expansion

Forcing Characteristics:

- Superposition of Sinusoidal Forcing, vorticity
- Spectrum: Constant E(k), ensuring uniform energy distribution across wave numbers.
- Correlation Length: Approximately 1/10 of the box scale, some room for dual cascade.
- Localized through a Heaviside step function.
- Phase of forcing randomized every typical eddy turnover time

H-M + Zonal Flow System, cont'd - channels

→ Now: *waves* $\omega = \omega_* / (1 + k_\perp^2 \rho_s^2)$, $\underline{v_{gr}}$

eddies $\tilde{v}$

zonal mode (symmetry)

$\left\{ \begin{array}{l} \tilde{v} \text{ vs } v_* \rightarrow \\ \text{mixing length} \end{array} \right.$

i.e. ⇒ Energy Flux has two components:

$\left\{ \begin{array}{l} \sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}} \rightarrow 2^{nd} \text{ order in } e\tilde{\phi}/T \\ \langle \tilde{v}_r \xi \rangle \rightarrow 3^{rd} \text{ order in } e\tilde{\phi}/T \end{array} \right.$

N.B. 2 channels for "turbulence spreading"

- Waves/Wave transport
- Turbulent mixing

-Branching ratio, vs. Ku number ?

Channels, cont'd

$\Longrightarrow$ **Spreading in presence of fixed, externally prescribed shear layer**

$\Longrightarrow$ **Here:** $\rightarrow$ Forcing $\rightarrow \left\{ \begin{matrix} \text{Waves} \\ \text{Eddies} \end{matrix} \right\} \rightarrow$ Zonal flow (self-generated)

$\therefore$ $\boxed{\text{forcing } (\tilde{v}_{rms}, Re) + \text{drag} \Rightarrow \text{control parameters}}$

$\Longrightarrow$ "weak" and "strong" Turbulence Regimes

coherency factor

v_{gr} VS $v_r \rightarrow \dfrac{\langle \tilde{v}_r \xi \rangle}{\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}}} \rightarrow \dfrac{\tilde{v}_r \tau_c f}{\Delta_c} \rightarrow Ku$ $\Longleftrightarrow$ 2nd vs 3rd order energy flux

$\Delta_c \sim v_{gr} \tau_c$

$\Longrightarrow$ $Ku < 1 \rightarrow$ wave dominated spreading

$Ku > 1 \rightarrow$ mixing dominated spreading $\Longrightarrow$ ~ 2D fluid

Channels, cont'd

$But \rightarrow$ Enter the Zonal Flow

- Multiple channels for NL interaction
- But with ZF ⟷ eddy, wave coupling to ZF dominant
- ZF is the mode of minimal inertia, damping, transport

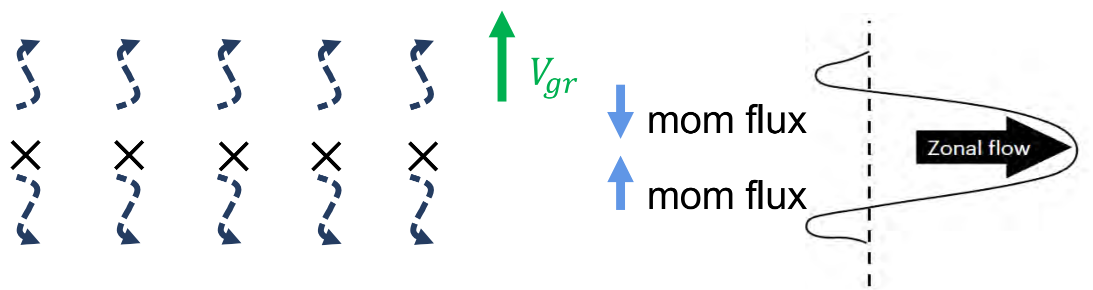
$\Rightarrow$ energy coupled to ZF ($\tilde{v}_r = 0$) cannot "spread", unless recoupled to waves

Waves:

$$\frac{\partial}{\partial t}\left(1 + k_\perp^2 \rho_s^2\right)\tilde{\phi} = \ldots\ldots$$

ZF:

$$\frac{\partial}{\partial t}\left(k_r^2 \rho_s^2\right)\bar{\phi}_z = \ldots\ldots$$



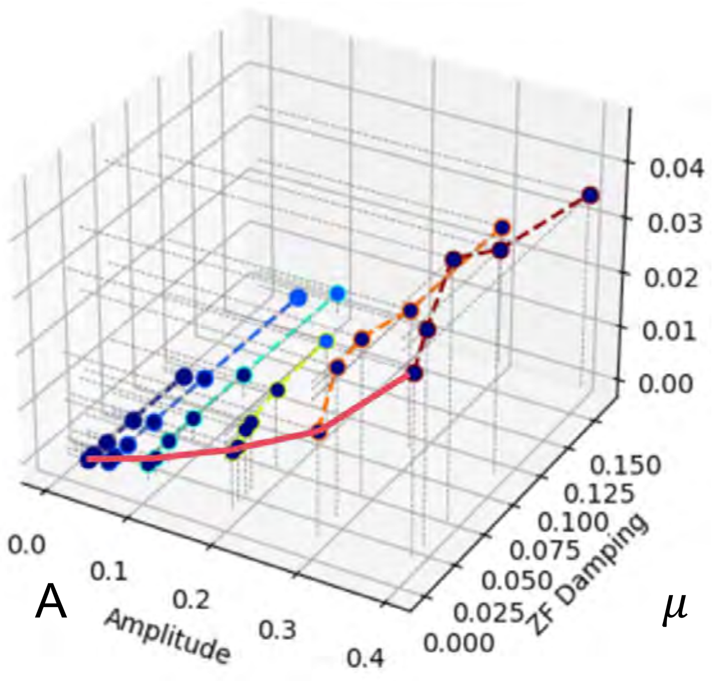
$\rightarrow$ Degradation of ZF (back transfer) is crucial to spreading

$\therefore$ μ must regulate spreading. What of $\mu \rightarrow 0$ regimes?

$\rightarrow$ Revisit collisionless NL dissipation problem

Results

FOM – Fluctuation Potential Enstrophy Flux



"Dimits" Regime

A

μ

averaged spatially and temporally

Turbulent Mixing

- Potential enstrophy flux generally <u>increases</u> as drag increases. "Dimits regime" for turbulence spreading. Spreading diminishes with power coupled to Z.F. (Fixed, spatially)
- <u>Z.F. is self-generated barrier to spreading</u>
- For A increasing, PE flux rises sharply for weak ZF damping. Fate of ZF? "KH-type" mechanism loss of Dimits regime at higher A? Characterization??

N.B. "Dimits Regime"= Condensation of energy into ZF for weaker forcing.

Results
Wave Energy Flux

Wave Energy Flux $< -\frac{\partial \phi}{\partial t} \nabla \phi >$ $\longleftrightarrow$ $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) E_{\boldsymbol{k}}$

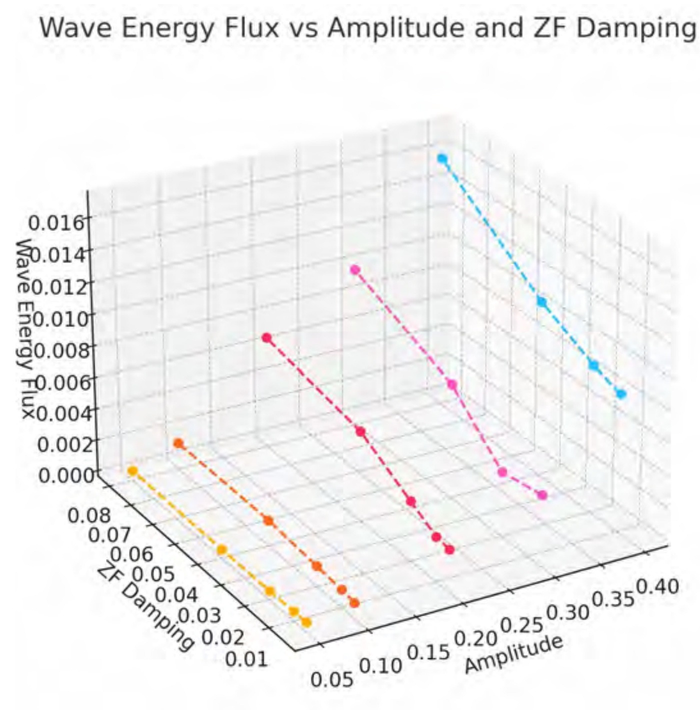
for drift waves

– Dimits regime at low forcing and ZF damping
–Increases with ZF damping and forcing amplitude

– Dominant K_x increases due ZF decorrelation
– Spectrum condensation towards low k with inverse cascade

implication for v_{gr} and $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) E_{\boldsymbol{k}}$

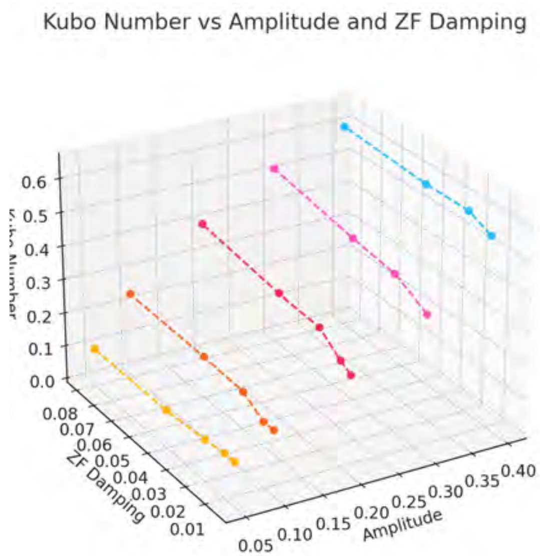
– Take note of increasing W.E. flux as $\mu \to 0$,
A increases.

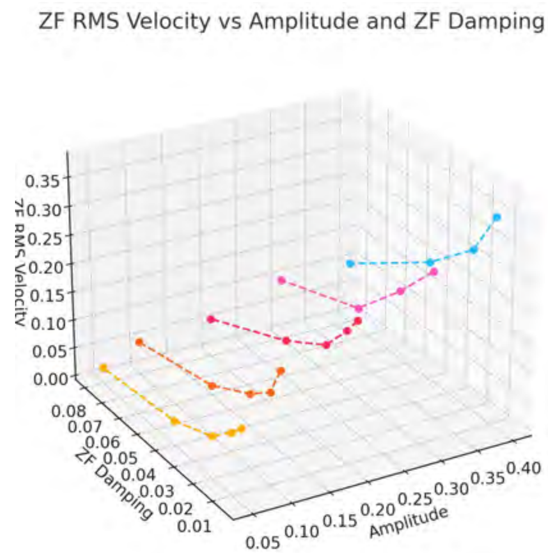


Results, Cont’d

$\frac{\tilde{v}_r \tau_c f}{\Delta_{c_c}}$ where $\Delta_c \sim < K_x^2 >^{-1/2}$

⇨

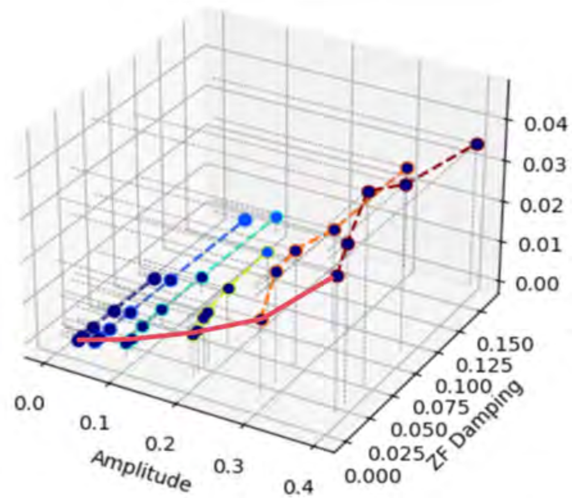




Fluctuation intensity increases as drag increases

Zonal velocity decreases with increasing drag (clear)

→Spreading and Fate of Zonal Flows



→ Spreading rises for increased forcing, even for $\mu \to 0$

→ Dimits regime destroyed. How?

$\underline{\Rightarrow}$ Seems necessary for spreading in systems with ZF

→ Animal Hunt for linear instabilities(KH, Tertiary …) seems pointless in turbulence

→ Instead, $$P_{\mathrm{Re}} = -\langle \widetilde{V_x}\widetilde{V_y} \rangle \cdot \frac{\partial \overline{V}_y}{\partial x}$$ Power transfer [fluctuations → flow]

$P_{Re} < 0$: Wave → ZF transfer

$P_{Re} > 0$: ZF → Wave transfer ⇒ ZF decay

Aside:

— Of course, evokes 'happy memories' of studies of limitation of Dimits shift in G.K.

— But identification of 'Tertiary Instability', "R-K." etc not useful alone-effective noise !?

— Seek insight to and quantification of return of energy from Z.F. to turbulence, as control parameters scanned ➔ Reynolds Power density

— Goal is nonlinear ZF decay model for improved Predator-Prey system

— N.B. Reynolds power density used widely in data analysis

Quantifying Wave-ZF Power transfer

$$1/2 * \frac{\partial \overline{V}_y^2}{\partial t} = \omega_Z < \widetilde{v_x}\widetilde{v_y} > -drag * \overline{V}_y$$

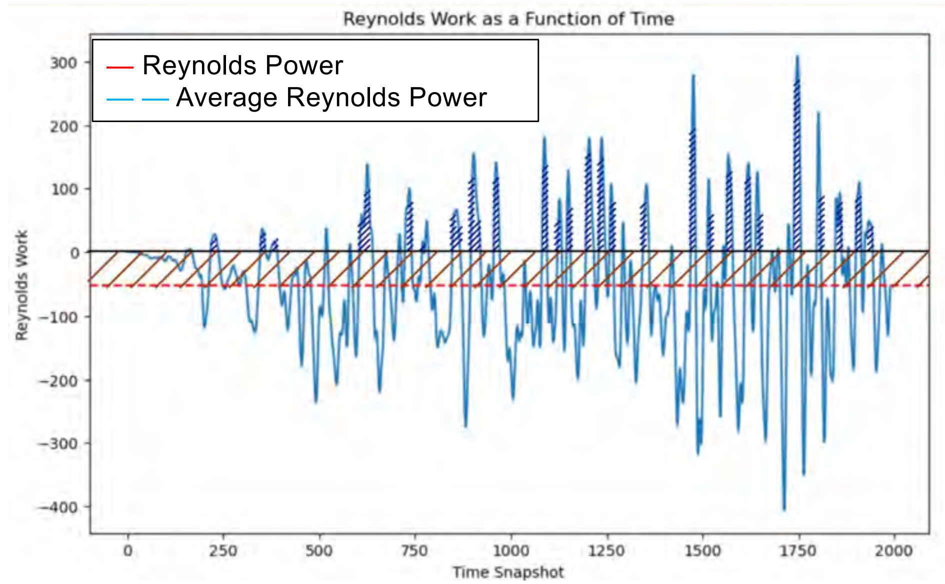
Reynolds power

We quantify ZF → Waves Power Transfer as the ratio of the area above the axis to mean work done on the zonal flow.

N.B.:

$$P_{\mathrm{Re}} = -\langle \widetilde{V_x}\widetilde{V_y} \rangle \cdot \frac{\partial \overline{V}_y}{\partial x} \rightarrow D_t(\partial V_y \ /\partial x)^2?$$

Mixing length model fails capture 2 signs



Reynolds power vs time

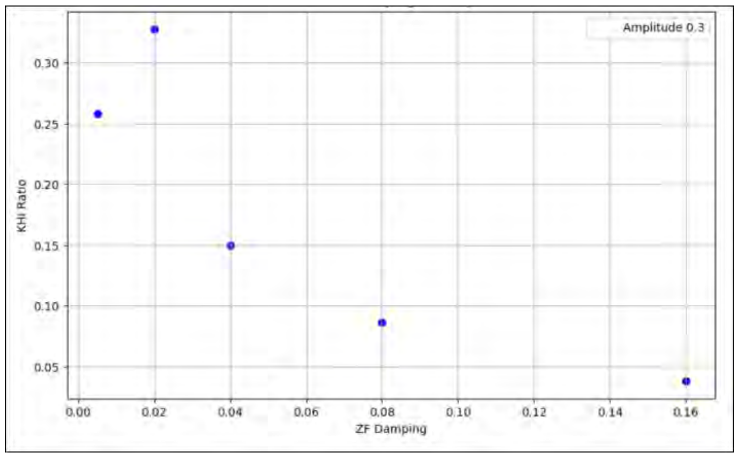
$P_{Re} < 0 \Rightarrow$ Wave → ZF transfer

$P_{Re} > 0 \Rightarrow$ ZF → Wave transfer

Return Fraction $= \frac{Area\ above\ axis}{Area\ of\ mean}$

Results, Cont'd

P_{Re} *ratio vs ZF damping*



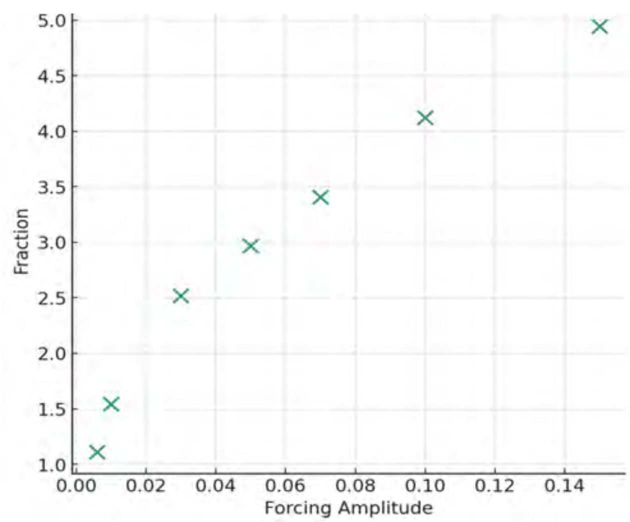
Dimits Regime

— The ratio generally decreases as a function of ZF damping

⟺ Damped Zonal Flow More Stable.

<u>Results</u>, Cont'd, P_{Re} Ratio vs Forcing Strength

$P_{Re}\ ratio\ vs\ forcing\ amplitude$

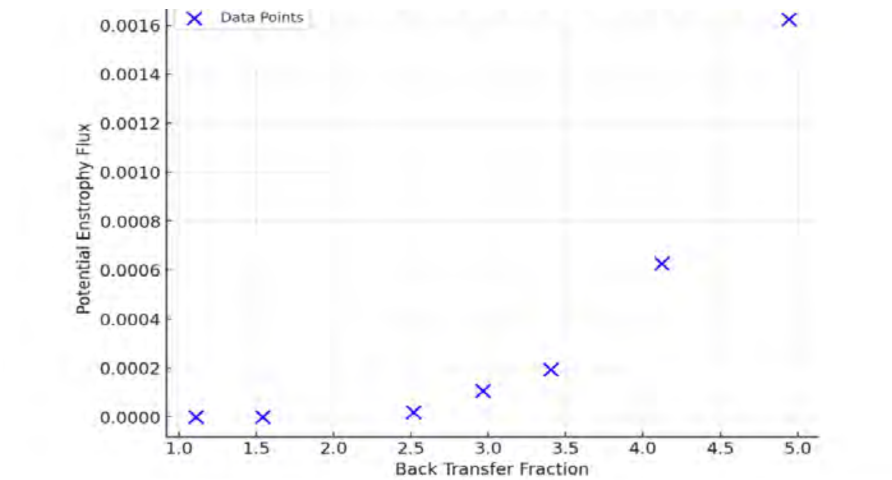


Preliminary
→ Explore other FOMs

- Indicates that re-coupling of ZF energy to turbulence increases for stronger forcing
- This approach avoids instability morass → amenable to parametrization
- Significant nonlinear recoupling energy to waves

Results, Cont'd

– Potential Enstrophy Flux vs. Energy Return Fraction

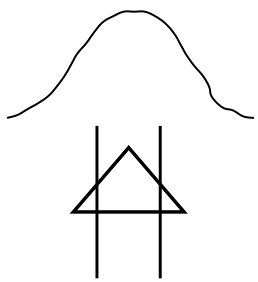


– Potential Enstrophy Flux rises rapidly with fraction of energy return from zonal flow

– Turbulence spreading closely related to zonal flow relaxation

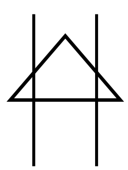
Related Problem: Jet Migration(Laura Cope)

<u>i.e.</u> - Here:



$\Rightarrow$ turbulence patch propagates,
drags ZF/Jet along

- There:



$\Rightarrow$ Jet migrates
<u>but</u> Migration enabled by dynamics of fluctuation field

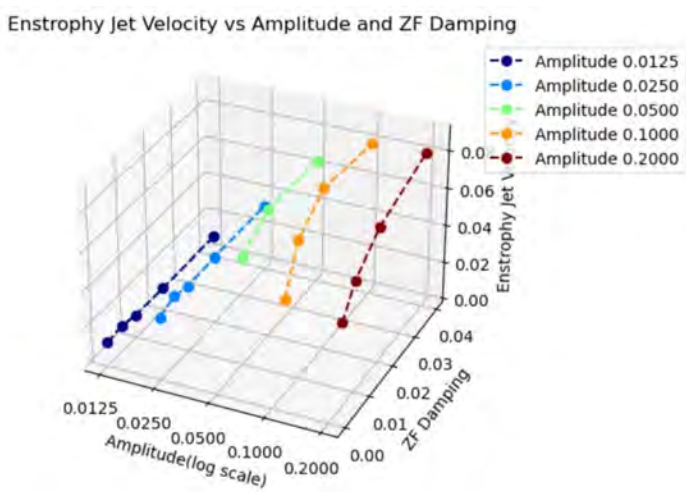
$\Rightarrow$ Zonon

So Jet Velocity !?

→ As waves/eddys drag along zonal flow, Jet velocity(ala' Townsend) is related to Jet Migration.

so

→ Enstrophy Jet Velocity?!



- Now familiar trends
- Seems semi-quantitatively consistent with Cope results.

Summary - Drift Wave Turbulence

→ Spreading fluxes mapped in forcing, ZF damping parameter space

→ Dominant mechanism ⟷ Ku (waves vs mixing) , Both waves and mixings in play.

→ Dimits-like regime discovered. Fixed ZF pattern.

→ ZF quenching intimately linked to spreading

→ $P_{Re} > 0$ bursts track breakdown of Dimits regime and onset turbulent mixing
Spreading increases.

→General Summary

→ In DWT, wave propagation and turbulent mixing both drive spreading

→ ZF quenching critical to spreading in DWT. Power coupling most useful to describe ZF quench.

→ Closely related to jet migration.

→Future Plans

— High resolution studies
— Understand ZF quenching physics and calculate power recoupling-general case, GK formulation?
— What is physics of P_{Re}>0 bursts? - shedding?

— Spreading in Avalanching. Relative Efficiency? Spreading and Transport? Flux-driven H-W System. Potential Enstrophy Flux!?

More general:
— Is spreading mechanism universal? Seems unlikely
— Towards a model, models… Ku~1 is an interesting challenge
— Relation/connection of DW+ZF spreading and Jet Migration (L. Cope)
— Is Directed Percolation of any use in this?
Ideas, Approaches-yes?! Details-??