

# **SOL Broadening by Edge Turbulence: Experiment and Theory**

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# Collaborators

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- Computation: Nami Li, X.-Q. Xu; (LLNL)
- Experiment: Ting Long, Ting Wu (SWIP), Filipp Khabanov, Rongjie Hong, G. Mckee, Zheng Yan, G. Yu, G. Tynan (DIII-D → Frontiers Exp.), Xi Chen (GA)

# Outline

- Brief Primer on the Edge and SOL
- SOL Width Problem and the Physics of the Plasma Boundary Layer
- Some Data: Turbulence Production Ratio and its Implications
- Some Theory: Calculating the Scale of the Spreading-Driven SOL
- Some Computation: A Closer Look at Turbulence Spreading
- Open Issues and Future Plans

# Primer (Brief)

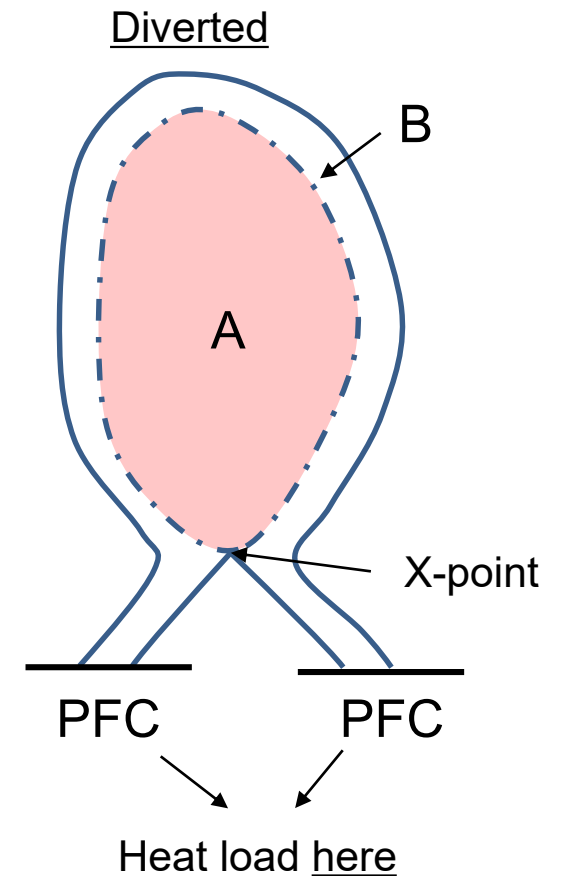
- All confinement devices have an edge and SOL (scrape-off layer)

## Fueling at Edge

- Define:
  - Confined plasma boundary
  - Connection to plasma facing components
  - SOL as confined plasma ‘boundary layer’

NB: Magnetic field lines are perp to plane, with slight tilt

A – confined plasma  
B – SOL  
Dashed – separatrix



# Primer, cont'd

- SOL:  $\nabla \cdot \vec{\Gamma} = \nabla \cdot \vec{Q} = 0$  (open lines)

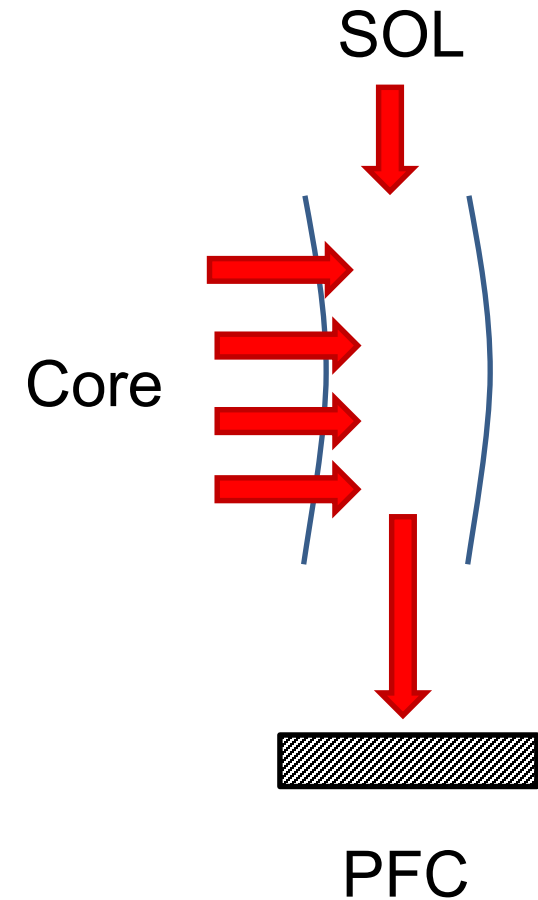
$$\Gamma_{\perp} \approx -D \partial_r n \quad (?) \quad \nabla_{\perp} \sim \partial_r \sim 1/\lambda_{\perp}$$

$$\Gamma_{\parallel} \approx \alpha c_s n \quad \nabla_{\parallel} \sim 1/L_c \sim 1/Rq$$

$$\rightarrow D \partial_r^2 n \sim \alpha n / L_c \quad \tau_{\parallel} \approx Rq / c_s$$

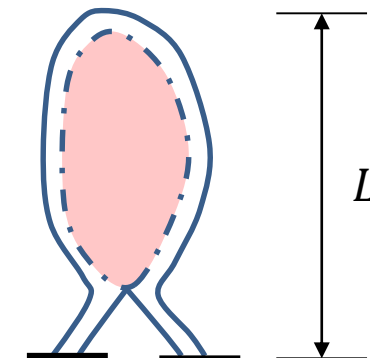
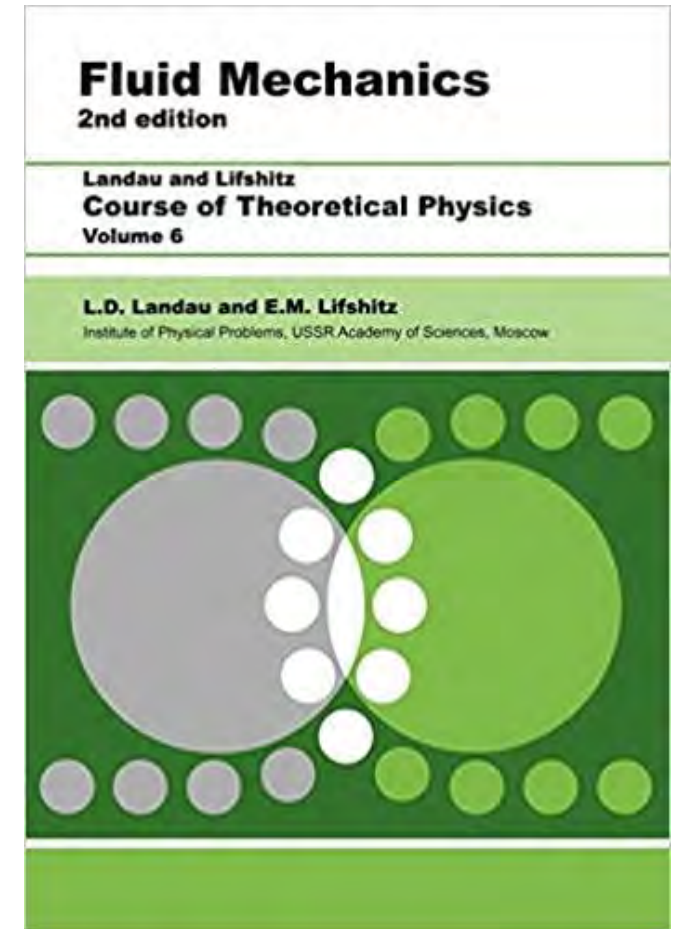
$$\lambda_{\perp} \sim (D\tau_{\parallel})^{1/2} \sim \text{crude SOL width}$$

$$\leftrightarrow 1/\tau_{\parallel} \sim \chi_{\parallel} / L_c^2 \quad \text{conduction, high density}$$



# Background

- Conventional Wisdom of SOL:  
(cf: Stangeby...)
  - Turbulent Boundary Layer, ala' Blasius, with  $D$  due turbulence
  - $\delta \sim (D\tau)^{1/2}, \tau \approx L_c/V_{th}$
  - $D \leftrightarrow$  local production by SOL instability process  
→ familiar approach,  $D$  ala' QL, ...
- Features:
  - Open magnetic lines → dwell time  $\tau$  limited by transit, conduction, ala' Blasius
  - Intermittency → “Blobs” etc. Observed. **Physics?**



# Background, cont'd

- But... Heuristic Drift (HD) Model (Goldston +)

- $V \sim V_{\text{curv}}$  ,  $\tau \sim L_c/V_{\text{thi}}$  ,  $\lambda \sim \epsilon \rho_{\theta i}$  → SOL width

- Pathetically small

- Pessimistic  $B_\theta$  scaling, yet high  $I_p$  for confinement

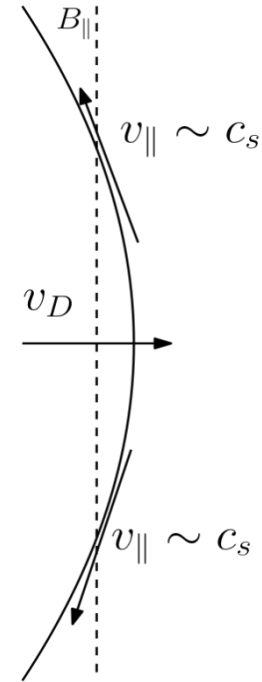
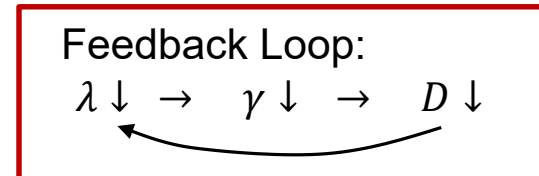
- Fits lots of data.... (Brunner '18, Silvagni '20)

- Why does neoclassical work? → ExB shear suppresses SOL modes i.e.

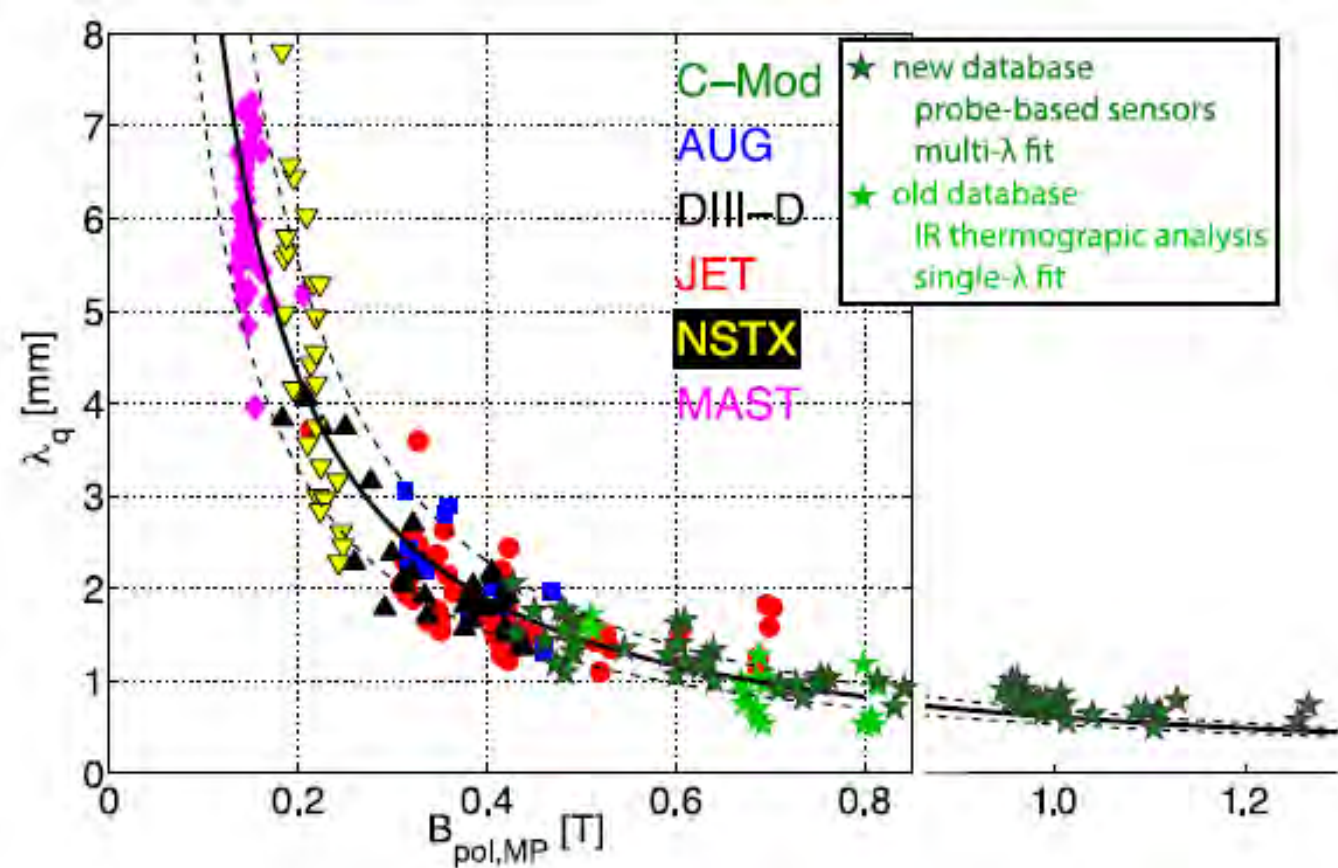
$$\gamma_{\text{interchange}} \sim \frac{c_s}{(R_c \lambda)^{\frac{1}{2}}} - \frac{3T_{\text{edge}}}{|e|\lambda^2}$$

shearing  $\leftrightarrow$  strong  $\lambda^{-2}$  scaling

from:  $\frac{c_s}{(R_c \lambda)^{\frac{1}{2}}} - \langle V_E \rangle'$



# Background: HD Works in H-mode



HD is Bad News...



# Background, cont'd

- THE Existential Problem... (Kikuchi, Sonoma TTF):

Desire  $\left\{ \begin{array}{l} \text{Confinement} \rightarrow \text{H-mode} \leftrightarrow \text{ExB shear} \\ \text{Power Handling} \rightarrow \text{broader heat load, etc} \end{array} \right. \rightarrow \text{Both to be good !}$

How reconcile? – Pay for power mgmt with confinement ?!

- Spurred:
  - Exploration of turbulent boundary states with improved confinement: Grassy ELM, WPQHM, I-mode, Neg. D ... N.B. What of ITB + L-mode edge?
  - SOL width now key part of the story
  - Simulations, Visualizations (XGC, BOUT...) ~ “Go” to ITER and all be well
- But... What’s the Physics ?? How is the SOL broadened?

# **SOL Boundary Layer:**

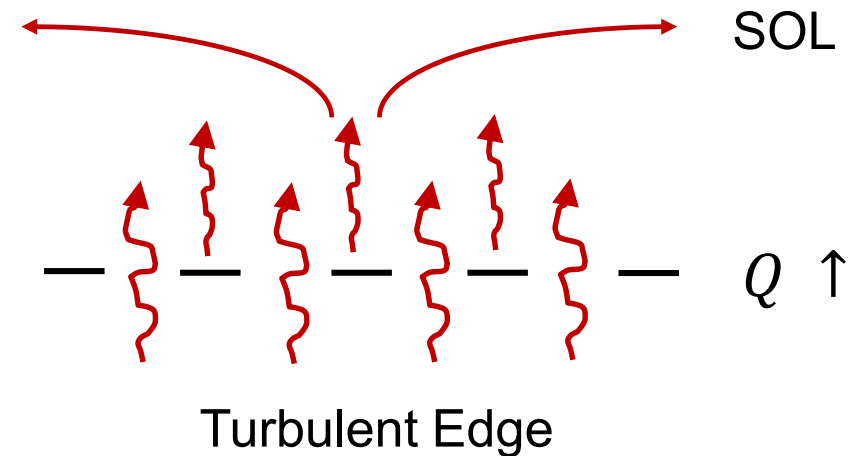
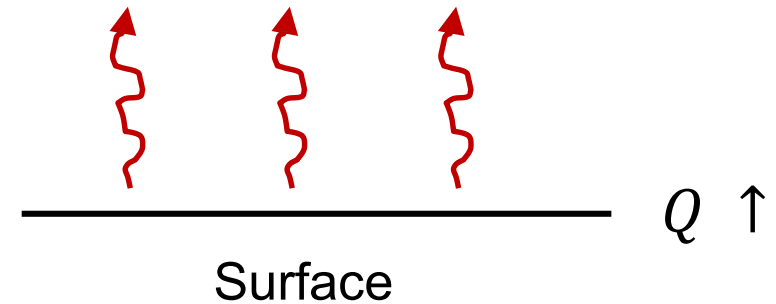
**Turbulence Production Rate and  
the Role of Spreading**

# SOL BL Problem

- Classic flux-driven BL problem
  - Heat flux at surface drives
  - Production =  $gQ$   $\tilde{V}_E \sim (gQz)^{1/3}$  etc
  - Plumes

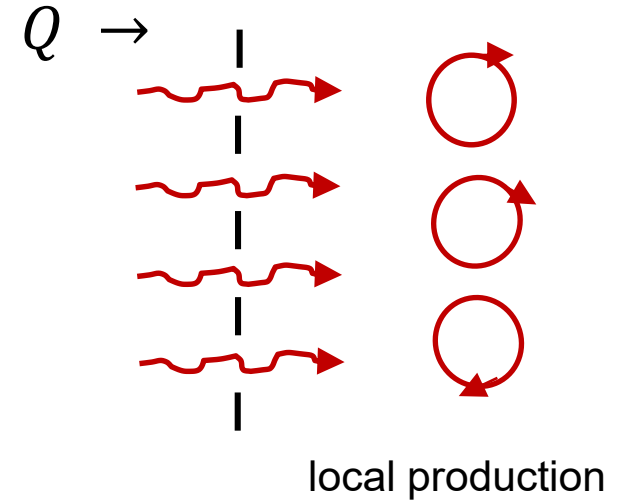
Adapt to SOL ?

- SOL
  - Open field lines
  - Turbulent energy flux and heat flux, etc drive
  - Turbulence spreading (Garbet, P.D., Hahm, ...)
  - Includes 'blobs' – c.f. Manz, 2015



# SOL BL Problem

- SOL Excitation
  - Local production (SOL instabilities)
  - Turbulence energy influx from pedestal
- Key Questions:
  - Local drive vs spreading ratio  $\rightarrow Ra$
  - Is the SOL usually dominated by turbulence spreading?
  - How far can entrainment penetrate a stable SOL  $\rightarrow$  SOL broadening?
  - Effects ExB shear, role structures ?



# Physics Issues – Part I

- Measure and Characterize Turbulence Energy Flux at LCFS
- Determine Relative Contributions of :

- Influx/Spreading thru LCFS
- SOL Production



$R_a \rightarrow$  Production Ratio

- Trends in  $\lambda_q$  and  $R_a$  vs : ExB shear, 'Blob' Fraction...

- Question: To what extent is SOL turbulence usually spreading driven?





→ Phenomenology... (see Ting Wu +, NF 2023)


# Experiments and Data Set

- HL-2A limited OH plasmas – classic “boring plasmas”
- Reciprocating probe array  $\leftrightarrow$  Outboard mid-plane
- $q_{\parallel} = \gamma J_{sat} T_e$  ,  $\gamma \equiv$  sheath transmission coefficient

N.B.:  
 $\lambda_q \rightarrow$  SOL width

- Database: ‘Garden Variety OH’  $\sim$  150 kA, 1.4T

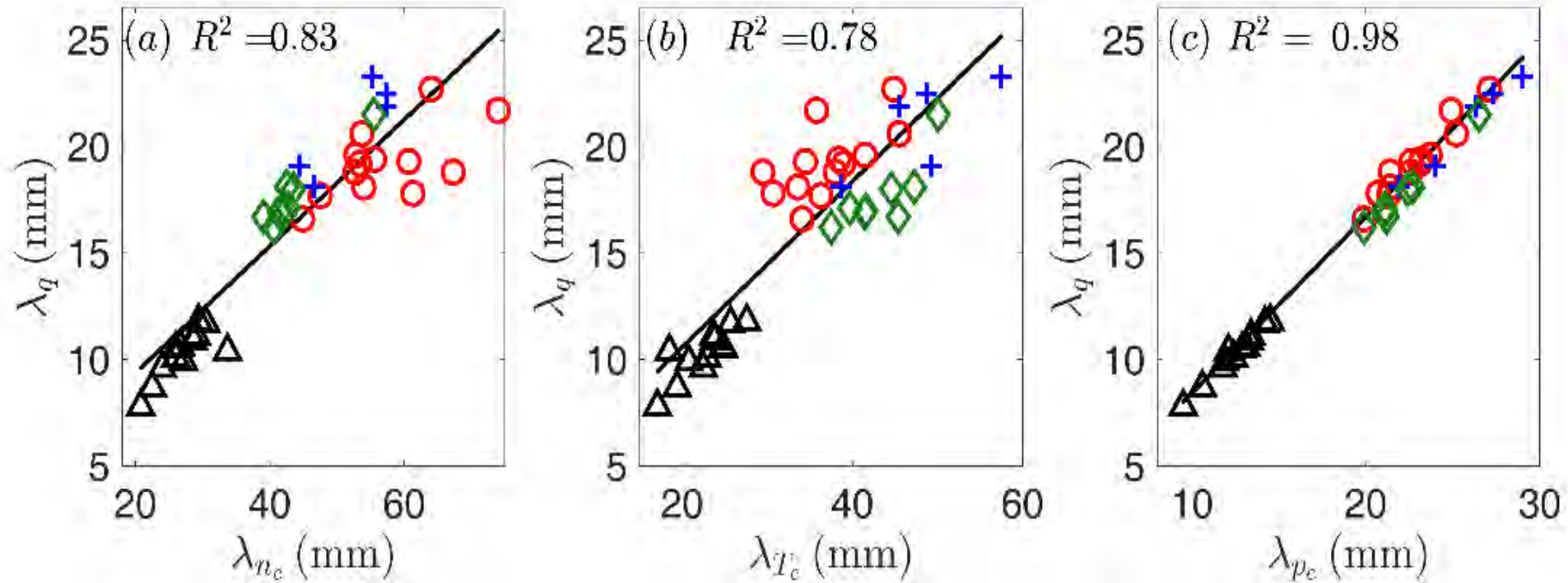
- 4 parameter subgroups  red circle  blue cross  green diamond  black triangle

- Similar, with  $\lambda_q \gg \lambda_{HD}$ , except: black triangles 

–  $\lambda_q > \lambda_{HD}$  , not  $\gg$

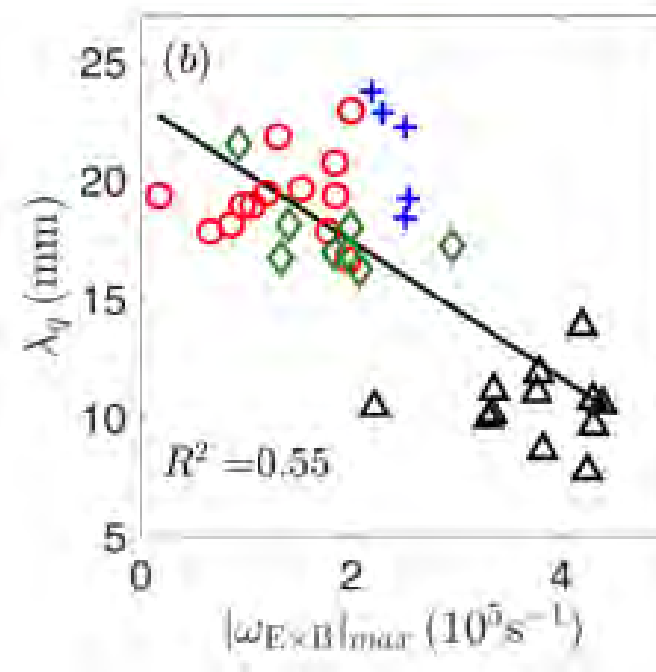
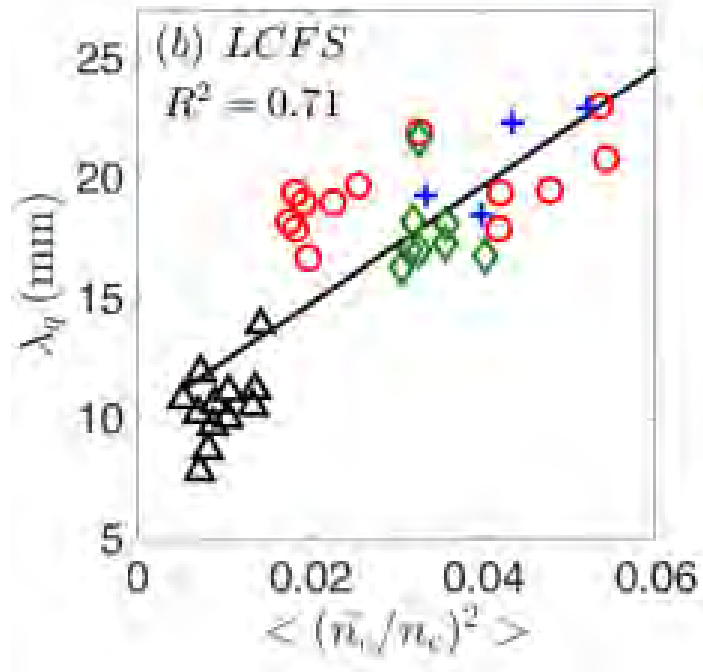
– Significant GAM activity  $\rightarrow$  stronger ExB shear

$$\lambda_{n_e} \sim \lambda_{T_e} \sim \lambda_{p_e}$$



**All SOL profiles scales comparable**

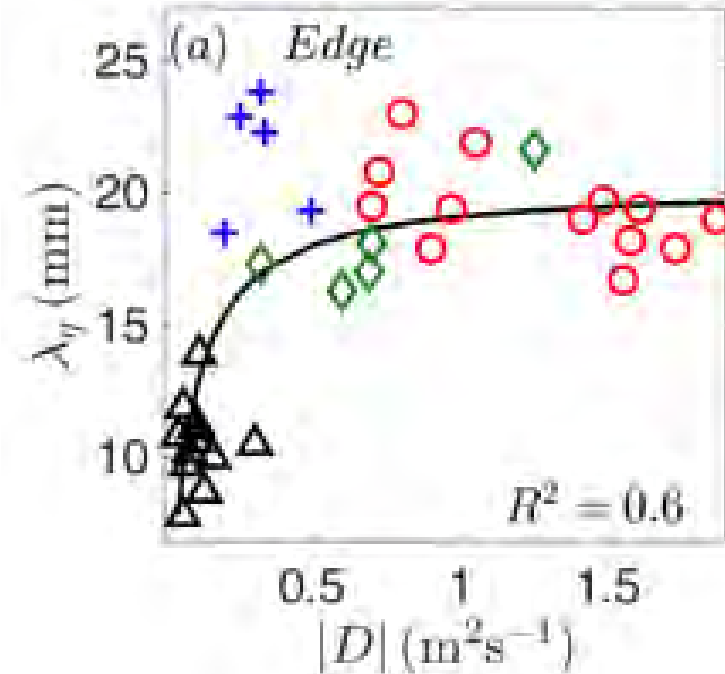
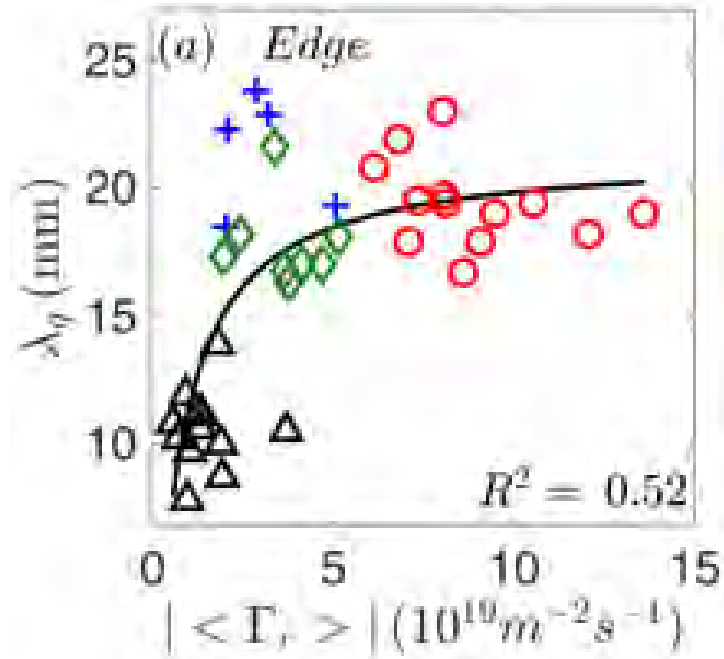
## $\lambda_q$ Trends 1 – Fluctuation Levels and Shearing



- $\lambda_q$  increases for increasing fluctuation intensity at lcfs
- $\lambda_q$  decreases for increasing ExB shear at lcfs
- Max  $\omega_{E \times B}$  at shear layer  $\sim$  lcfs

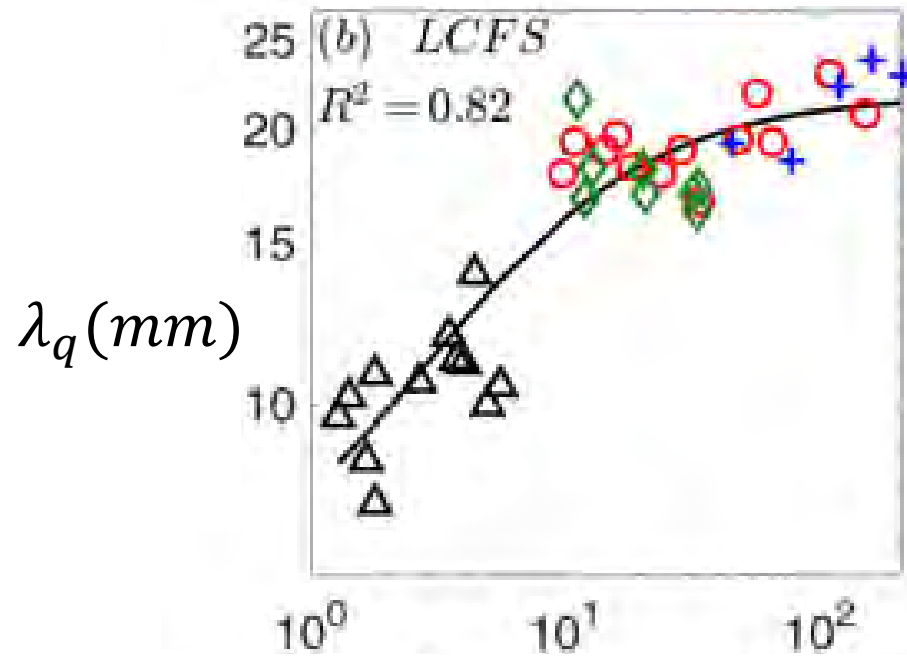


## $\lambda_q$ Trends 2 – Particle Flux and Diffusion



- $\lambda_q$  increases for increasing edge  $\Gamma_n$
- $\lambda_q$  increases for increasing edge  $D$
- ? Saturation – might expect  $\lambda \sim (D\tau)^{1/2}$  scaling ...

## $\lambda_q$ Trends 3 – Spreading !



$c_s^2 \langle \tilde{V}_r (\tilde{n}/n_0)^2 \rangle (10^8 \text{m}^3 \text{s}^{-3}) \rightarrow$  at lcfs

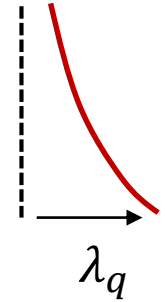
- $\Gamma_\varepsilon = c_s^2 \langle \tilde{V}_r (\tilde{n}/n_0)^2 \rangle \rightarrow$  flux of turbulence internal energy thru lcfs
- Direct measurement of local spreading flux
- Consistent with expected trend of expanded SOL width due to increasing spreading across lcfs

# SOL Fluctuation Energy – Production Ratio

1 Fluid •  $\rho \left( \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} \right) = -\nabla P + \frac{1}{c} \vec{J} \times \vec{B} + \rho g \hat{r}$

$$\nabla \cdot \vec{V} = 0, \quad \tilde{P} + \frac{\vec{B}_0 \cdot \tilde{\vec{B}}}{4\pi} \approx 0$$

SOL interchange



$$\begin{aligned} \bullet \quad \partial_t (KE)_{SOL} &= - \int_0^\lambda dr \nabla \cdot \Gamma_E + \int_0^\lambda dr \left[ \frac{c_s^2}{R} \left\langle \frac{\tilde{V}_r \tilde{n}}{n_0} \right\rangle - \langle \tilde{V}_r \tilde{V}_\perp \rangle \frac{\partial}{\partial r} \langle V_\perp \rangle \right] \\ &= -\Gamma_E|_{\lambda_q} + \Gamma_E|_{\text{cfS}} + [\text{SOL Integrated local production}] \end{aligned}$$

Fluctuation Energy Influx to SOL

•  $\Gamma_E = \langle \tilde{V}_r \tilde{V}^2 \rangle \approx c_s^2 \langle \tilde{V}_r (\tilde{n}/n_0)^2 \rangle \rightarrow$  amenable to measurement

Take: KE flux  $\sim$  Int. Energy Flux ( $\checkmark$  for drift-interchange)

this gives ...

# Aside: On Calculating the Spreading...

- Why perturbed pressure balance?
  - Else,  $\langle \vec{V} \cdot \nabla P \rangle$  and  $\langle \rho \nabla \cdot \vec{V} \rangle$  enter energy balance. Acoustic energy propagation irrelevant on  $\tau \gg \tau_{MS}$
  - Can eliminate via vorticity eqn,  $\vec{V} = \vec{E} \times \vec{B}$  etc.
- Interchange drive:  $\kappa P \rightarrow \kappa \langle \tilde{V}_r \tilde{P} \rangle \approx g c_s^2 \langle \tilde{V}_r \tilde{n} \rangle$   
as cannot measure  $\tilde{P}$  fluctuations

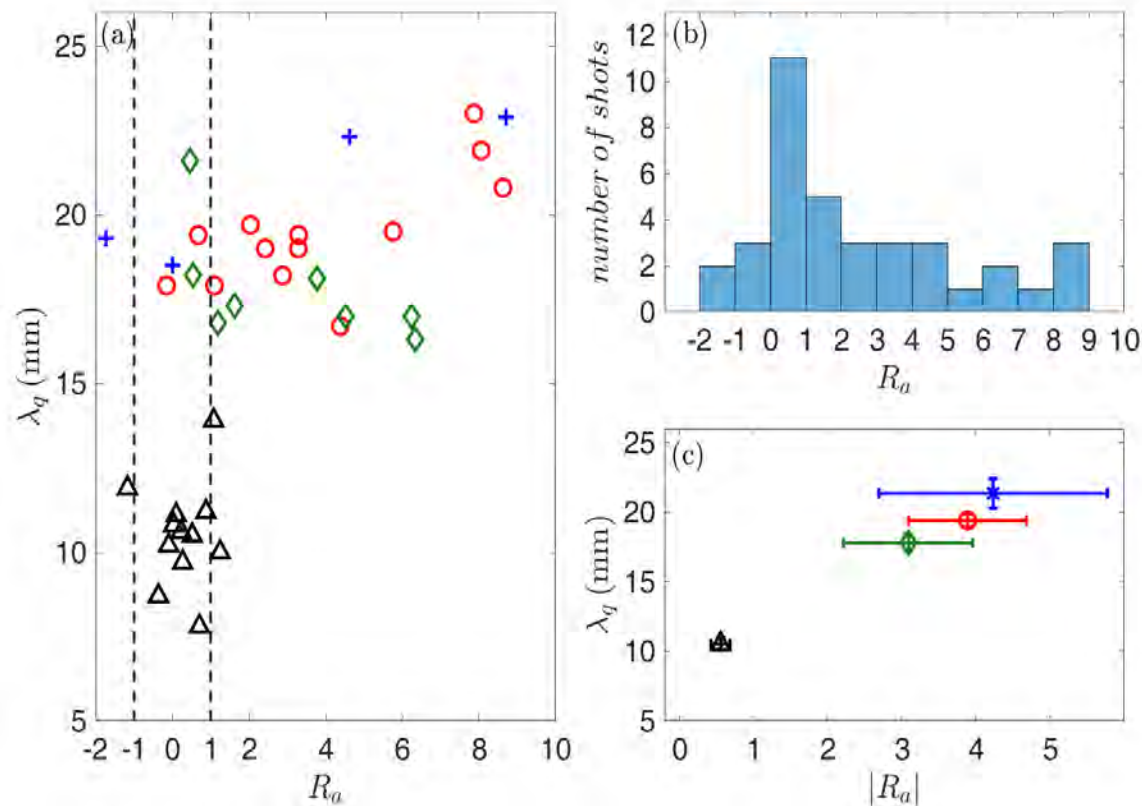
# Production Ratio, Cont'd

How important is spreading ?

$$R_a = c_s^2 \langle \tilde{V}_r (\tilde{n}/n_0)^2 \rangle \Big|_{\text{lcfs}} / \int_0^\lambda dr \frac{c_s^2}{R} \langle \tilde{V}_r \tilde{n}/n_0 \rangle$$

- Ratio of fluctuation energy influx from edge i.e. spreading drive - to net production in SOL
- $R_a < 1 \rightarrow$  SOL locally driven
- $R_a \gg 1 \rightarrow$  SOL is spreading driven
- Quantitative measurement by Langmuir probes
- N.B. very simple; likely lower bound, as local production smaller

# Production Ratio - Measurements



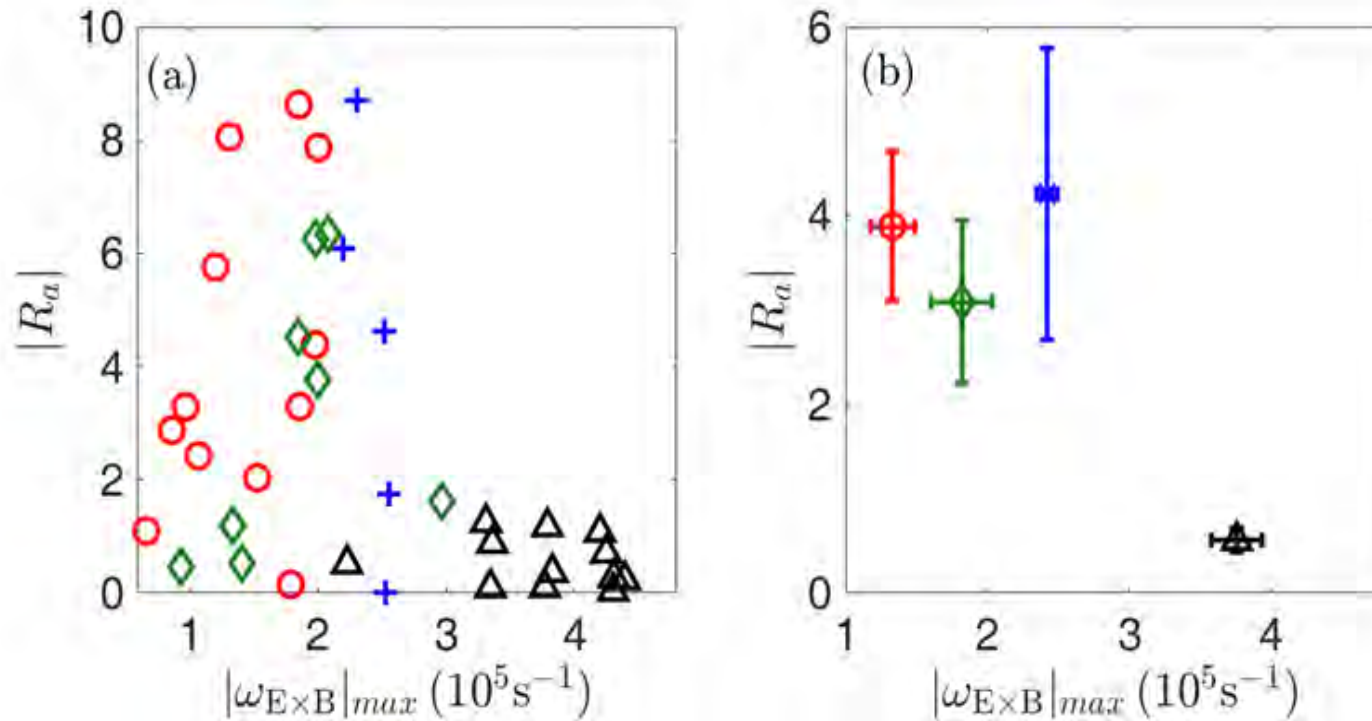
$$R_a = \frac{\text{Fluctuation Energy Influx}}{\text{SOL Local Production}}$$

- Observe:
  - $\lambda_q$  increases with  $R_a$
  - Most cases  $R_a > 1$
  - Broad distribution  $R_a$  values
  - Low  $R_a$  values  $\leftrightarrow$  strong ExB shearN.B. Non-trivial, as shear enters production, also via cross phase

- Also:
  - Some  $R_a < 0$  cases  $\rightarrow$  inward spreading  $\leftrightarrow$  local measurement trend outward
  - Some very large  $R_a$  values

What is happening?

# Production Ratio vs ExB Shear 1



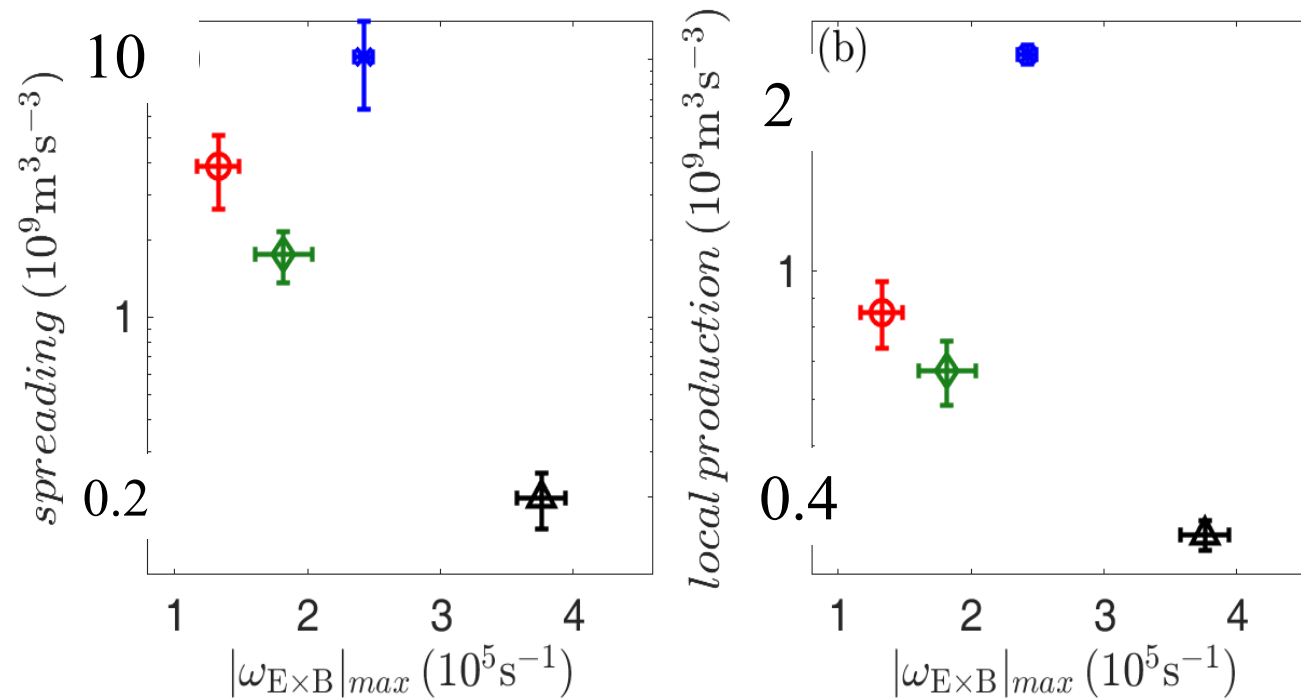
- Low values of  $|R_a|$  at high  $V_E'$
- But why?

$$R_a = c_s^2 \langle \tilde{V}_r (\tilde{n}/n_0)^2 \rangle |_{\text{lcs}} / \int_0^\lambda dr \frac{c_s^2}{R} \langle \tilde{V}_r \tilde{n}/n_0 \rangle$$

→ Expect shear inhibits both spreading and transport flux?

↔ ExB shear enters phase relation in both

# Production Ratio vs ExB Shear, cont'd

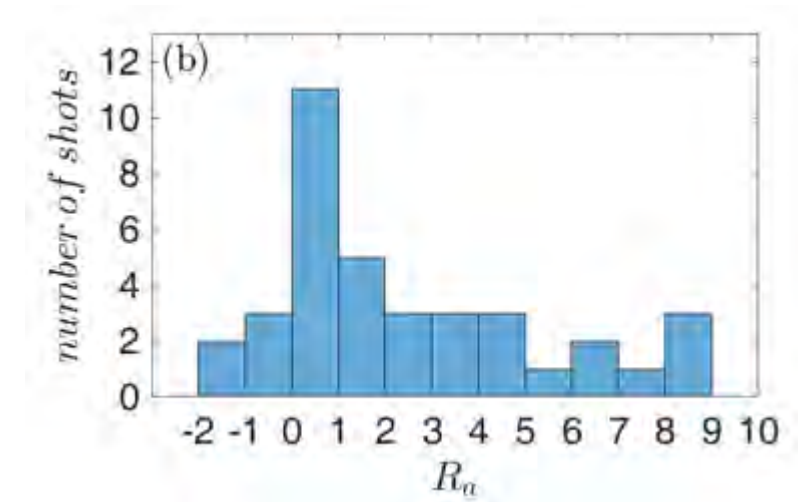


- Both spreading and local production drop due high  $V'_E$
- But spreading x (1/10) vs Production x (1/2)
- ➔ Spreading flux significantly more sensitive to  $V'_E$  than transport flux
- ↔ Triplet vs quadratic ➔ Phases?



# Large $R_a \rightarrow$ 'Blobs' ?!

- What of the large  $R_a$  values?
- Suspect – Structure Emission i.e. “blobs” !?
- Test:



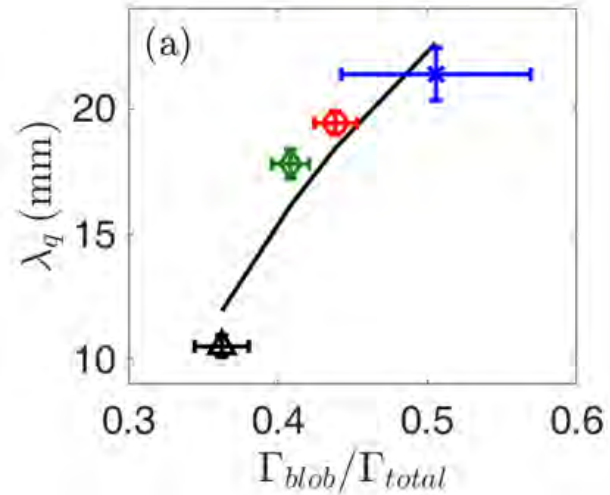
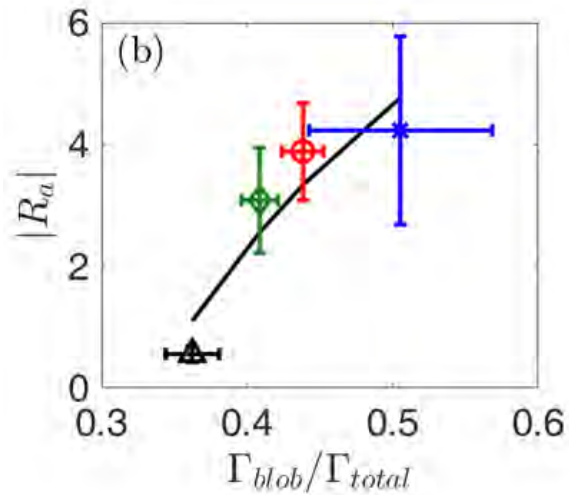
- Conditional averaging (i.e. threshold  $\tilde{n} > 2\tilde{n}_{rms} \rightarrow$  “blob”)
- Threshold arbitrary  $\rightarrow$  setting based upon previous studies
- Compute  $R_a, \Gamma$  etc. with conditionally averaged quantities

Physics of the “2” ?

Especially:  $\Gamma_{blob} / \Gamma_{total}$

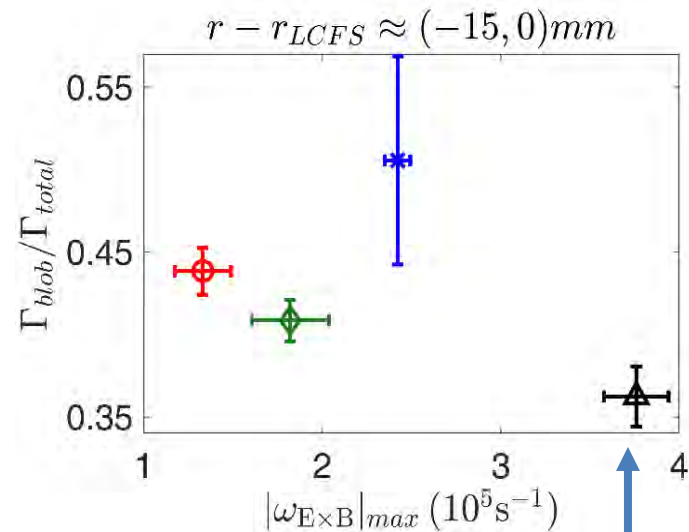
 Flux carried by “blobs”

# Large $R_a \rightarrow \lambda_q$ increases with 'blob' fraction



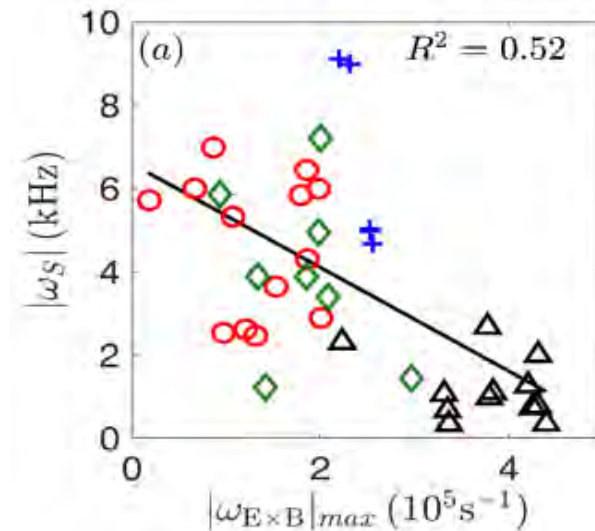
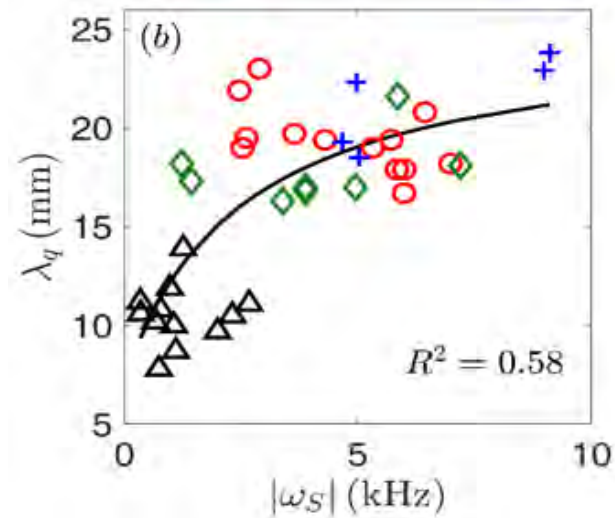
- Large  $R_a$  cases  $\leftrightarrow$  larger 'blob fraction' of flux  
 $\leftrightarrow$  spreading encompasses 'blobs' (c.f. Manz +)  $\rightarrow \langle \tilde{V}_r \tilde{n}^2 \rangle$
- $\lambda_q$  increases with  $\Gamma_b/\Gamma_{Tot}$

- High ExB shear cases  $\rightarrow$  low 'blob' fraction  
 (Consistent with Bodeo+, '03)



# Time Scales

- Spreading rates:  $\omega_s \approx -\partial_r \langle \tilde{V}_r \tilde{n} \tilde{n} \rangle / \langle \tilde{n}^2 \rangle$   
characteristic rate of spreading (Manz +)
- Shearing rate  $V'_E$



- $\lambda_q$  broadens for large  $\omega_s$
- Stronger shear reduces spreading rate

# Partial Summary

- Significant, mostly outward, spreading measured at Icfs
- Identified and calculated production ratio

$$R_a = (\text{spreading influx}) / (\text{local production})$$

- Most cases:  $R_a > 1 \rightarrow$  spreading dominant player in SOL energetics
- ExB shear reduces  $R_a \leftrightarrow$  spreading more sensitive to  $V_E'$  than transport and production – phases ?
- High  $R_a$  spreading  $\leftrightarrow$  ‘blob’ dominated dynamics  $\rightarrow$  how calculate?

YES  $\rightarrow$  SOL turbulence usually spreading driven!

“The conventional wisdom is little more than convention” - JKG

N.B. No use of closure of spreading flux

# **Calculating the Width of the Spreading-Driven SOL**

# Physics Issues – Part II

[C.f. Chu, P.D., Guo, NF 2022

P.D.+ IAEA '23]

- How calculate SOL width for turbulent pedestal but a locally stable SOL?

- spreading penetration depth

- must recover HD in WTT limit

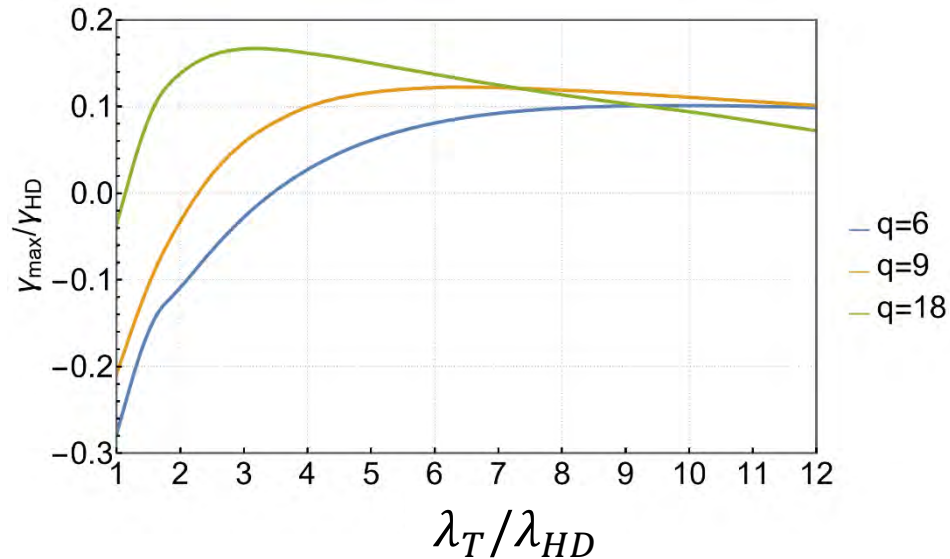
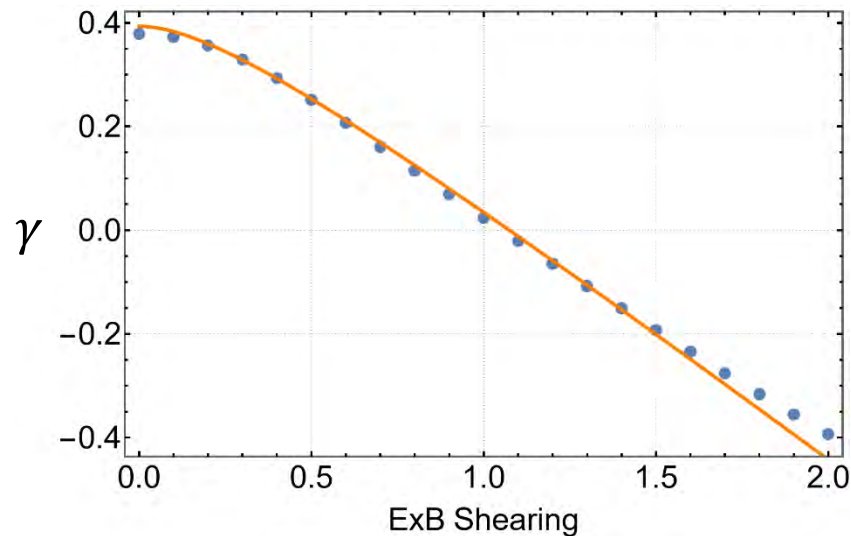
➔ • Scaling and cross-over of  $\lambda_q$  relative HD model

➔ • What is effect/impact of barrier on spreading mechanism?

- Can SOL broadening and good confinement be reconciled ?

# Model 1 – Stable SOL – Linear Theory

- Standard drift-interchange with sheath boundary conditions + ExB shear (after Myra + Krash.)



Maximal Linear Growth Rate of Interchange Mode in the SOL v.s. normalized layer width  $\lambda_D/\lambda_{HD}$  at different SOL safety factor  $q$  (with  $\beta = 0.001$ )

Linear Growth Rate of a specific mode (fixed  $k_y$ ) v.s.  $E \times B$  shear at  $q = 5, \beta = 0.001, k_y \cdot \lambda_{HD} = 1.58$ .

- Relevant H-mode ExB shear strongly stabilizing  $\gamma_{HD} = c_s/(\lambda_{HD}R)^{1/2}$
- Need  $\lambda/\lambda_{HD}$  well above unity for SOL instability.  $V'_E \approx \frac{3T_e}{|e|\lambda^2} \rightarrow$  layer width sets shear

# Model 2 – Two Multiple Adjacent Regions

- “Box Model” – after Z.B. Guo, P.D.

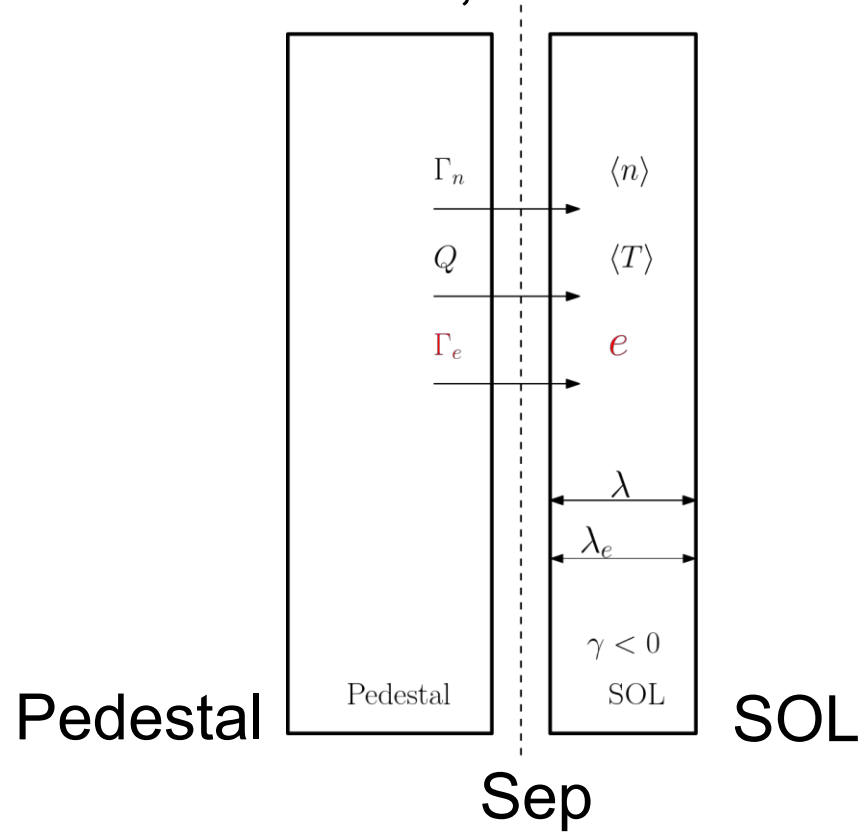


Illustration of Two Box Model: SOL driven by particle flux, heat flux and intensity flux ( $\Gamma_e$ ) from the pedestal. The horizontal axis is the radial direction, and vertical axis is the poloidal direction.

- Key Point:
  - Spreading flux from pedestal can enter stable SOL
  - Depth of penetration → extent of SOL broadening
  - Problem in one of entrainment/penetration



# Width of Stable SOL

- Fluid particle:  $\frac{dr}{dt} = V_{Dr} + \tilde{V}$ 
  - $V_{Dr}$ : drift
  - $\tilde{V}$ : fluctuating velocity

{ Dwell time  $\tau_{\parallel}$   
constrains excursion

- Dwell time:  $\tau_{\parallel}$

- $\delta^2 = \langle (\int (V_D + \tilde{V}) dt) (\int (V_D + \tilde{V}) dt) \rangle$

$\langle (\text{step})^2 \rangle = V_D^2 \tau_{\parallel}^2 + \langle \tilde{V}^2 \rangle \tau_c \tau_{\parallel}$

$= \lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2$

correlation time  
modest turbulence  $\leftrightarrow \tau_c \geq \tau_{\parallel}$

turbulence energy density

{ See also  
Fokker-Planck analysis  
i.e. drift + diffusion

- So  $\lambda = [\lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2]^{1/2} \rightarrow$  SOL width [Effects add in quadrature]

- How compute  $\varepsilon$ ?  $\rightarrow$  turbulence energy in SOL. Need relate to pedestal

- N.B. Can write:  $\lambda = [\lambda_{HD}^2 + \lambda_e^2]^{1/2}$   $\lambda_e$  is turbulent width

# Calculating the SOL Turbulence Energy 1

- Need compute  $\Gamma_e$  effect on SOL levels
- $K - \epsilon$  type model, mean field approach (c.f. Gurcan, P.D. '05 et seq)
  - Can treat various NL processes via  $\sigma, \kappa$
  - Exploit conservative form model

- $\partial_t \epsilon = \gamma \epsilon - \sigma \epsilon^{1+\kappa} - \partial_x \Gamma_e \rightarrow$  Spreading, turbulence energy flux

\*  $\left\{ \begin{array}{l} \text{Growth } \gamma < 0 \\ \text{here contains shear + sheath} \end{array} \right.$   $\rightarrow$  NL transfer  $\gamma_{NL} \sim \sigma \epsilon^\kappa$

- $\rightarrow$  • N.B.: No Fickian model of  $\Gamma_e$  employed, yet
- Readily extended to 2D, improved production model, etc.

# Calculating the SOL Turbulence Energy 2

- Integrate  $\varepsilon$  equation  $\int_0^\lambda$  ; “constant e” approximation
- Take quantities = layer average
- $\Gamma_{e,0} + \lambda_e \gamma \varepsilon = \lambda_e \sigma \varepsilon^{1+\kappa}$

Separatrix fluctuation energy flux

Single parameter characterizing spreading

So for  $\gamma < 0$ ,

$$\Gamma_{e,0} = \lambda_e |\gamma| \varepsilon + \sigma \lambda_e \varepsilon^{1+\kappa}$$

$\lambda_e$  = layer width for  $\varepsilon$

$\Gamma_{e,0}$  vs linear + nonlinear damping

- Ultimately leads to recursive calculation of  $\Gamma_e$

# Calculating the SOL Turbulence Energy 3

[Mean Field Theory]

- Full system:

$$\Gamma_{e,0} = \lambda_e |\gamma| \varepsilon + \sigma \lambda_e \varepsilon^{1+\kappa}$$

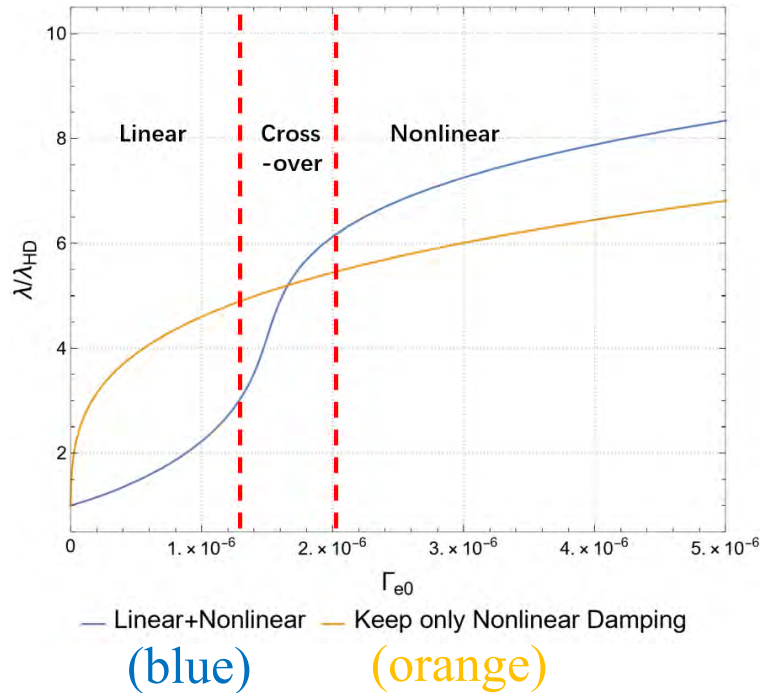
$$\lambda_e = [\lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2]^{1/2}$$

Simple model of  
turbulent SOL  
broadening

- $\Gamma_{0,e}$  is single control parameter characterizing spreading
- $\tilde{\Gamma}_{0,e}$  ? Expect  $\tilde{\Gamma}_e \sim \Gamma_0$

# SOL width Broadening vs $\Gamma_{e,0}$

- SOL width broadens due spreading



$\lambda/\lambda_{HD}$  plotted against the intensity flux  $\Gamma_{e0}$  from the pedestal at  $q = 4, \beta = 0.001, \kappa = 0.5, \sigma = 0.6$

Variation indicates need for detailed scaling analysis

- Clear decomposition into
  - Weak broadening regime  $\rightarrow$  shear dominated
  - Cross-over regime
  - Strong broadening regime
- $\rightarrow$  NL damping vs spreading } relevant

- Cross-over for:  $\langle \tilde{V}^2 \rangle \sim V_D^2 \rightarrow$  cross-over  $\Gamma_{0,e}$
- Cross-over for  $\tilde{V} \sim O(\epsilon)V_*$

# SOL Width: Some Analysis

Have  $\Gamma_{e,0} = |\gamma| e \lambda_e + \lambda_e \sigma e^{1+\kappa}$

a) Damping dominated

$$\Gamma_e \approx |\gamma| \lambda_e e \quad \lambda_q^2 = \lambda_e^2 + \lambda_{HD}^2$$

$$\lambda_q = \left[ \lambda_{HD}^2 + \left( \frac{\Gamma_e \tau_{\parallel}^2}{|\gamma|} \right)^{2/3} \right]^{1/2}$$

- Spreading enters only via  $\Gamma_e$  at sep.
- Shearing via  $|\gamma|$
- $\tau$  scalings  $\rightarrow \tau_{\parallel}$  vs  $\tau_{\parallel}^{2/3} \rightarrow$  current scaling of  $\lambda_e$  weaker

# SOL Width: Some Analysis, Cont'd

b) NL dominated

$$\Gamma_e \approx \lambda_e \sigma e^{1+\kappa} \quad \lambda_q^2 = \lambda_e^2 + \lambda_{HD}^2$$

$$\lambda_q = \left[ \lambda_{HD}^2 + \left( \frac{\Gamma_e}{\sigma} \right)^{2/(3+4\kappa)} \tau_{\parallel}^{[4(1+\kappa)/(3+2\kappa)]} \right]^{1/2}$$

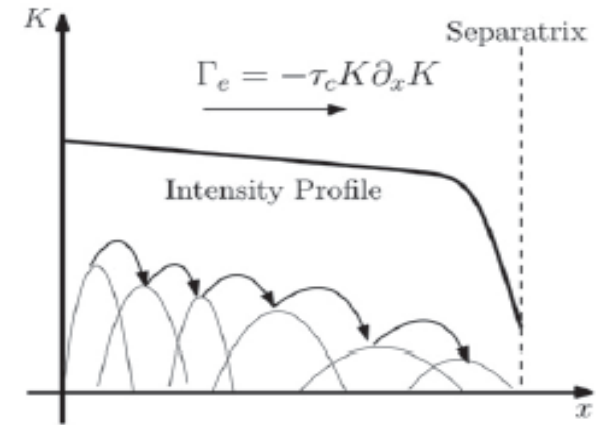
– weaker  $\Gamma_e$  scaling,  $\lambda_q \sim (\Gamma_e/\sigma)^{1/5}$  ; STT

–  $\tau_{\parallel}^{3/4}$  vs  $\tau_{\parallel}$   $\rightarrow$  weaker current scaling

# Computing the Turbulence Energy Flux 1

- Need consider pedestal to actually compute  $\Gamma_{e,0}$
- Two elements

Does another trade-off loom? -- Pedestal Turbulence: Drift wave? Ballooning?  
 -- Effect of transport barrier  $\leftrightarrow$  ExB shear layer  $\rightarrow$  barrier permeability!?



- Key Point: shearing limits correlation in turbulent energy flux

$$\text{i.e. } \Gamma_{e,0} \approx -\tau_c I \partial_x I \approx \tau_c I^2 / w_{\text{ped}} \quad (\text{Hahm, PD +})$$

ped turbulence  
intensity

correlation time  $\rightarrow$  strongly sensitive to shearing

N.B. Caveat Emptor re: intensity flux closure !



# Computing the Turbulence Energy Flux 2

- Familiar analysis for  $D \rightarrow$  Kubo

$$D = \int_0^\infty d\tau \langle V(0)V(\tau) \rangle = \int_0^\infty d\tau \sum_k |\tilde{V}_k|^2 \exp[-k_y^2 \omega_s^2 D \tau^3 - k^2 D \tau]$$

- Strong shear (relevant)

$$\tau_c = \tau_t^{1/2} \omega_s^{-1/2}$$

$$\tau_t \sim 1 / k \tilde{V}, \quad \omega_s \sim V_E'$$

Here, via RFB  $\rightarrow \omega_s = \partial_r \frac{\nabla P_i}{n|e|} \sim \frac{\rho^2}{w_{ped}^2} \Omega_{ci}$

- $\tau_c + w_{ped} +$  turbulence intensity in pedestal gives  $\Gamma_{e,0} \approx \tau_c I^2 / w_{ped}$
- Need  $\Gamma_{e,0} \geq \Gamma_{e,\min} \approx |\gamma| \lambda_{HD}^3 \tau_{\parallel}^{-2}$

# Computing the Turbulence Energy Flux 3

- Pedestal → Drift wave Turbulence
- Necessary turbulence level:

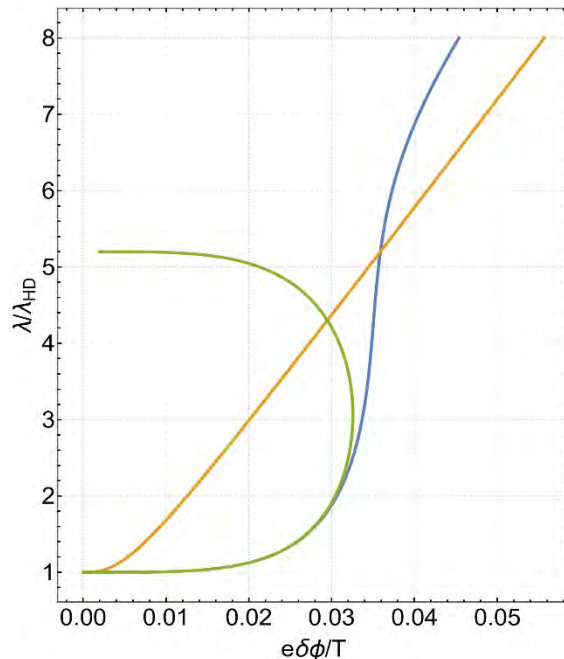
- Weak Shear  $\frac{\delta V}{c_s} \sim \left(\frac{\rho}{R}\right)^{1/2} q^{-1/4}$

- Strong Shear  $\frac{\delta V}{c_s} \sim \left(\frac{\rho}{R}\right)^{1/2} q^{-1/4} \left(\frac{w_{ped}}{\rho}\right)^{-1/8}$

blue – all damping

orange – nonlinear only

green – linear only



→  $\lambda/\lambda_{HD}$  vs  $|e|\hat{\phi}/T_e$  in pedestal

→  $\rho/R$  is key parameter

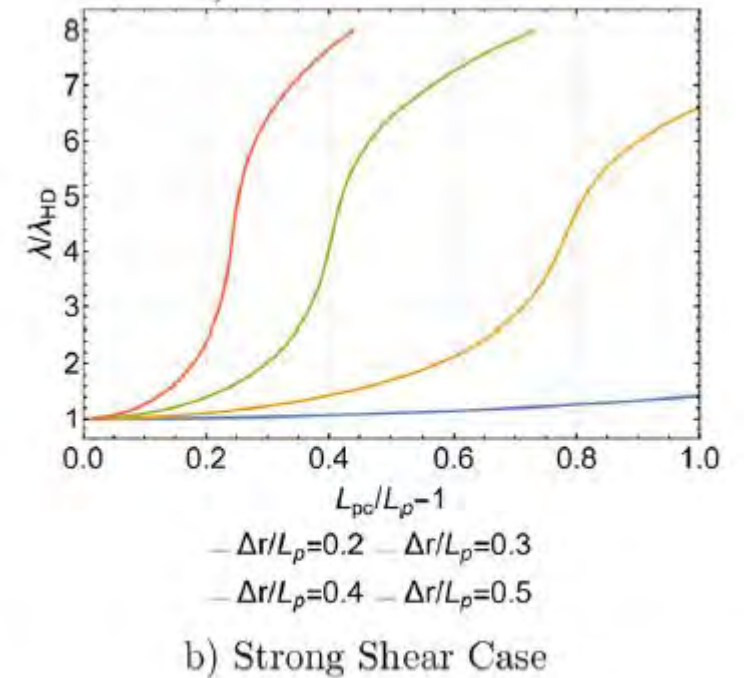
→ Broadens layer at acceptable fluctuation level

# Computing the Turbulence Energy Flux 4

- Pedestal  $\rightarrow$  Ballooning modes  $\rightarrow$  Grassy ELMs
- Necessary relate turbulence to  $L_{P,crit} / L_P - 1$
- Strong shear:

$$\frac{L_{Pc}}{L_P} - 1 \sim \left(\frac{q\rho}{R}\right)^{\frac{10}{7}} \left(\frac{R}{w_{ped}}\right)^{\frac{16}{7}} \left(\frac{w_{ped}}{\Delta_r}\right)^{\frac{16}{7}} \beta$$

- Supercriticality scales with  $\frac{\rho}{R}$ ,  $\beta_t$



**Figure 10.** Typical cases for ballooning. The normalized pedestal width  $\lambda/\lambda_{HD}$  is plotted against supercriticality  $L_{pc}/L_p - 1$  at different mode width  $\Delta/L_p$ .

# Computing the Turbulence Energy Flux 5 → Bottom Line

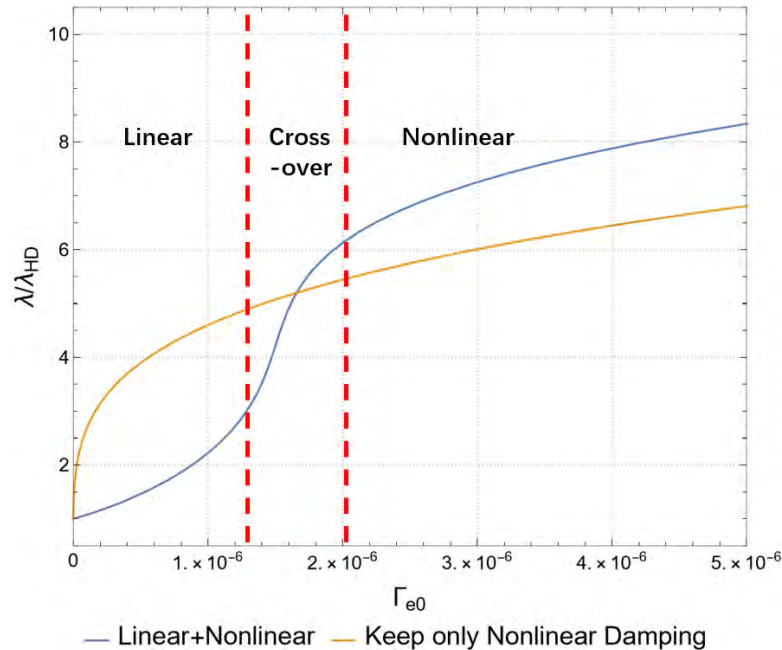
- SOL broadening to  $\lambda > \lambda_{HD}$  achievable at tolerable pedestal fluctuation levels
- DW levels scale  $\sim \left(\frac{\rho}{R}\right)^{1/2}$
- Ballooning supercritical scale  $\sim \left(\frac{\rho}{R}\right)^{10/7} \beta$
- ‘Grassy ELM’ state promising
- Sensitivity analysis → Cross over  $\varepsilon$  determined primarily by linear damping (shear). Conclusion  $\sim$  insensitive to NL saturation

# Partial Summary

- Turbulent scattering broadens stable SOL

$$\lambda = (\lambda_{HD}^2 + \varepsilon \tau_{\parallel}^2)^{1/2}$$

- Separatrix turbulence energy flux specifies SOL turbulence drive



$$\Gamma_{0,e} = \lambda_e |\gamma| \varepsilon + \lambda \sigma \varepsilon^{1+\kappa}$$

Broadening increases with  $\Gamma_{0,e}$   
cross-over for  $\langle \tilde{V}^2 \rangle \sim V_D^2$

Non-trivial dependence

- $\Gamma_{0,e}$  must overcome shear layer barrier

Yes – can broaden SOL to  $\lambda/\lambda_{MHD} > 1$  at tolerable fluctuation levels

Further analysis needed

# Some Simulation Results

(cf. Nami Li, X.-Q. Xu, P.D.; N.F.(Lett) '23)

→ BOUT++ → pedestal + SOL

→ 6 field model (“Braginskii for 21<sup>st</sup> century”)

→ Focus on weak peeling mode turbulence in pedestal

→ MHD turbulence state → small/grassy ELM, also WPQHM

# 3D Counterpart of Brunner ( $\lambda_q$ vs $B_\theta$ )

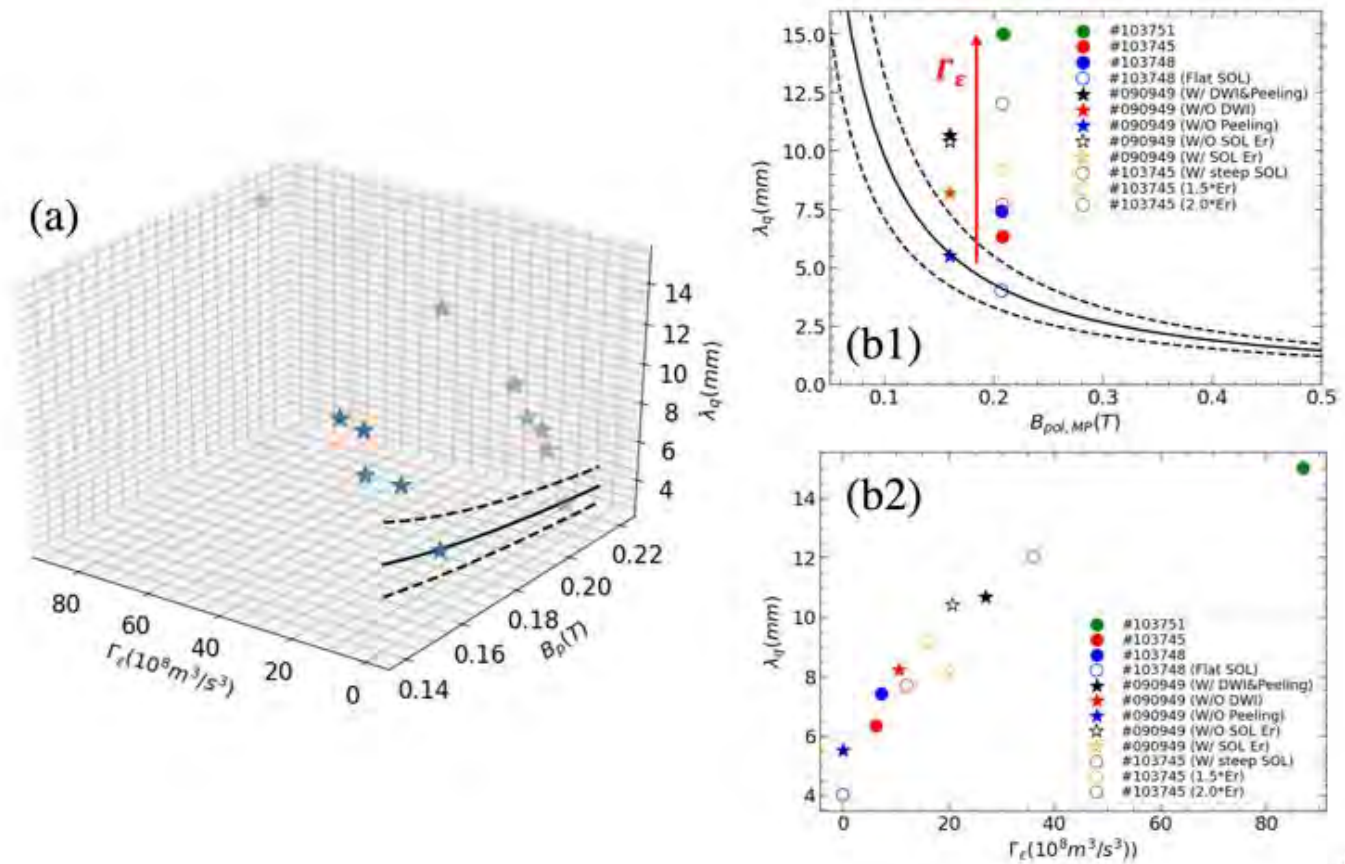


Fig. 3. (a) 3D plot of heat flux width  $\lambda_q$  vs poloidal magnetic field  $B_p$  and fluctuation energy density flux  $\Gamma_\epsilon$ ; (b) 2D plot of heat flux width  $\lambda_q$  vs poloidal magnetic field  $B_p$  (b1) and fluctuation energy density flux  $\Gamma_\epsilon$  (b2).

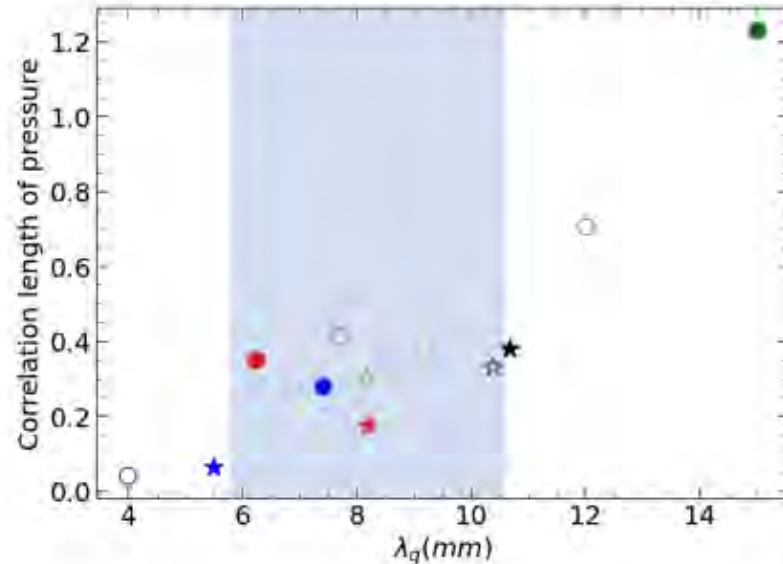
## 3D Brunner Plot – Comments

- $\lambda_q$  rises with  $\Gamma_e$
- Low  $\Gamma_e$  ,  $\lambda_q$  tracks hyperbola
- Large  $\Gamma_e$  ,  $\lambda_q$  rises above Brunner/Goldston hyperbola
- $\lambda_q$  grows with  $\Gamma_e$



# Spreading as Mixing Process ?

- Conjecture that  $\lambda_q$  should increase with pedestal mixing length  $\rightarrow \Gamma_e$



- Note division into
  - drift dominated
  - cross-over (blue)
  - turbulent

Fig 4. Radial correlation length of pressure near the separatrix vs. heat flux width  $\lambda_q$ .

# Relate Spreading to Pedestal Conditions

N.B.

- $\Gamma_e$  rises with pedestal  $\nabla P_0 \leftrightarrow$   
increased drive
- Collisionality dependence  $\Gamma_e$ :
  - high  $\rightarrow$  no bootstrap current  $\rightarrow$   
ballooning  $\rightarrow$  smaller  $l_{mix}$
  - low  $\rightarrow$  strong bootstrap  $\rightarrow$  peeling  
 $\rightarrow$  larger  $l_{mix}$

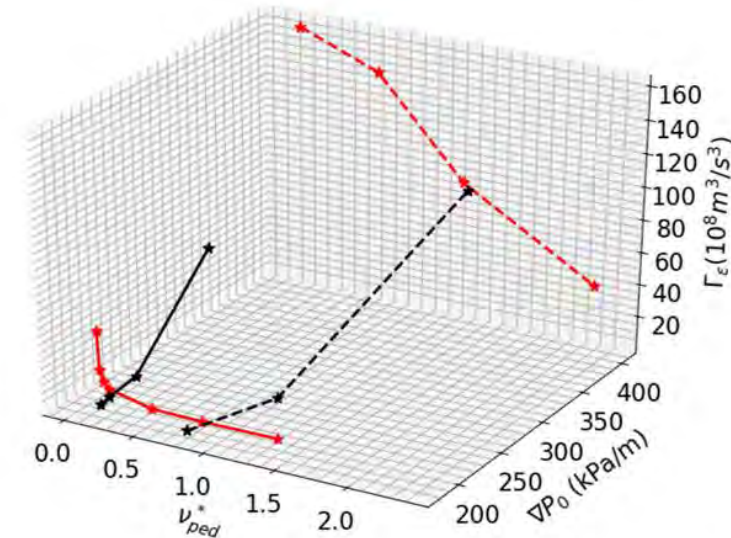


Fig. 7. 3D plot of fluctuation energy density flux  $\Gamma_e$  vs pedestal peak pressure gradient  $\nabla P_0$  and  $v_{ped}^*$ ; black curves are  $\nabla P_0$  scan with low collisionality  $v_{ped}^* = 0.108$  (solid curve) and high collisionality  $v_{ped}^* = 1$  (dashed curve); red curves are  $v_{ped}^*$  scan with small  $\nabla P_0 \sim 200 \text{ kPa/m}$  (solid curve) and large  $\nabla P_0 \sim 400 \text{ kPa/m}$  (dashed curve).

# Fundamental Physics of $\Gamma_e$

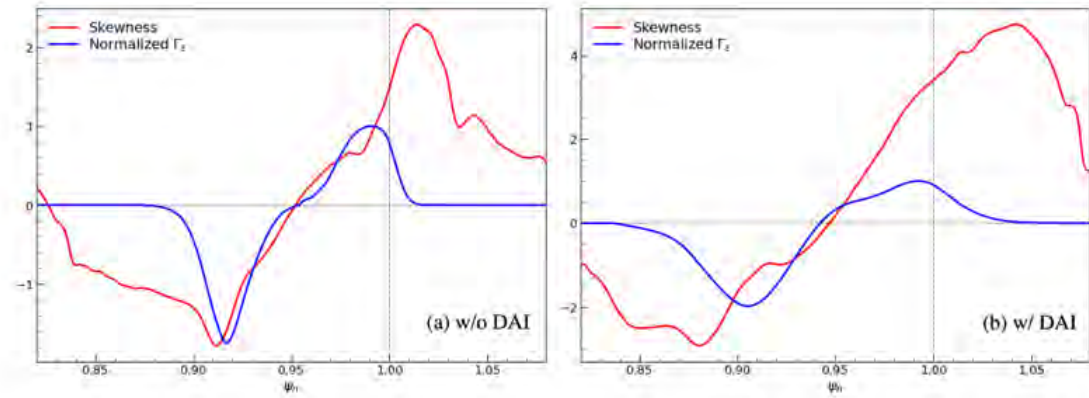


Fig. 6 Radial profiles of normalized fluctuation energy density flux  $\Gamma_e$  (blue) and skewness (red) for without (a) and with (b) drift-Alfvén instability. Here fluctuation energy density flux is normalized to the max value for each case.

- $\Gamma_e$  spreading tracks  $\tilde{P}$  skewness
  - Outward for  $s > 0 \rightarrow$  “blobs”
  - Inward for  $s < 0 \rightarrow$  “voids”
- Zero-crossings  $\Gamma_{e,s}$  in excellent agreement

# Fundamental Physics of $\Gamma_e$ , cont'd

- Spreading appears likely linked to “coherent structures”
- Likely intermittent (skewness, kurtosis related)
- Related study (Z. Li);  $Ku \sim 0.4$ , so  $\rightarrow$  if Fokker-Planck analysis

$$\frac{\partial e}{\partial t} = -\frac{\partial}{\partial x} (Ve) + \frac{\partial^2}{\partial x^2} (De) \quad \text{Convective !?}$$

Relate  $V$  to pedestal gradient relaxation event (GRE) ?!

# Broader Messages

- Turbulence spreading is important – even dominant – process in setting SOL width.  $\Gamma_{0,e}$  is critical element.  $\lambda = \lambda(\Gamma_{0,e}, \text{parameters})$
- Production Ratio  $R_a$  merits study and characterization
- ➔ • Spreading is important saturation mechanism for pedestal turbulence
- Simulation should stress calculation and characterization of turbulence energy flux over visualizations and front propagation studies.
- Critical questions include local vs FS avg, channels and barrier interaction, Turbulence ‘Avalanches’
- ➔ • Turbulent pedestal states attractive for head load management

# Open Issues

- Quantify  $\lambda = \lambda \left( \left. \frac{|e|\hat{\phi}}{T} \right|_{ped} \right)$  dependence



- Structure of Flux-Gradient relation for turbulence energy?
- Phase relation physics for intensity flux? – crucial to ExB shear effects
- Kinetics  $\rightarrow \langle \tilde{V}_r \delta f \delta f \rangle$ , Local vs Flux-Surface Average, EM
- SOL Diffusive?  $\rightarrow$  Intermittency('Blob'), Dwell Time ?
- SOL  $\rightarrow$  Pedestal Spreading ?  $\leftrightarrow$  HDL (Goldston) ?  
i.e. Tail wags Dog ? Both wagging ?  $\rightarrow$  Basic simulation, experiment ?  
Counter-propagating pulses ?

# Concluding Philosophy

- MFE relevant questions within reach in near future. Great attention to  $\lambda_q$  problem (c.f. Samuel Johnson)
- Unreasonable for tokamak experiments to probe  $\sim$  critical dynamics so as to elucidate basic questions. Simulations???
- Well diagnosed, basic experiment with some relevant features are sorely needed – akin to ‘Tube’ studies of flows, ala’ CSDX
- How?

**Thanks for Attention !**



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