Dynamics of Turbulence Spreading and Why it's Important

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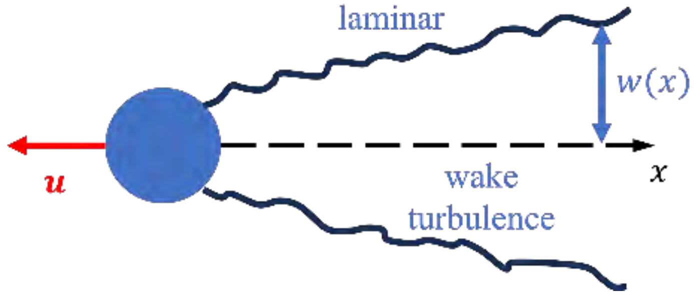
Wake-Classic Example of Turbulence Spreading

⇨



Similarity Theory

Mixing Length Theory

$W \sim (F_d/\rho U^2)^{1/3} X^{1/3}$,

$F_d \sim C_D \rho U^2 A_s$

C_D independent of viscosity at high Re

⇨ Physics: Entrainment of laminar region by expanding turbulent region.
Key is turbulent mixing. ⇨ Wake expands

⇨ Townsend '49:

— Distinction between momentum transport — eddy viscosity—and fluctuation energy transport

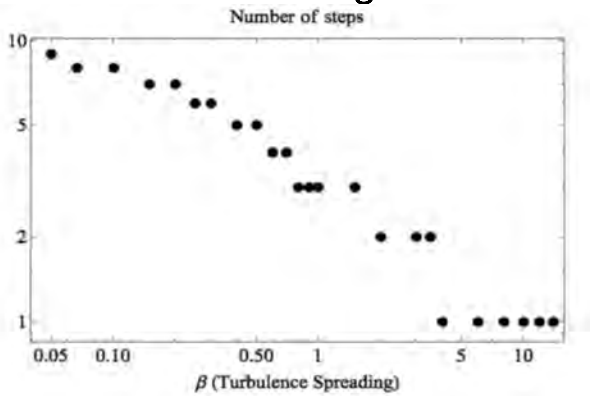
— Jet Velocity: $V = \frac{<V_{perp} * V^2>}{<V^2>}$ ⇨ spreading flux

— Failure of eddy viscosity to parametrize spreading

C.f. Ting Long, this meeting

Why Study Spreading?

⇨ Spreading strength sets staircase step size via intensity scattering. See also F. Ramirez this meeting

⇨ 

from A. Ashourvan, P.D.

⇨ Spreading potentially significant in determining

— Physical turbulence profiles
— Non-locality phenomena ⇨ K. Ida

⇨ It's observed!
— M. Kobayashi + 2022
— T. Long, T. Wu (2021, 2023)
— Estrada +

<u>Spreading in MFE Theory</u>

⇨ Numerous gyrokinetic simulations
N.B. <u>Basic</u> studies absent ...

⇨ Diagnosis primarily by: - color VG
- tracking of “Front”

⇨ Theory ⇨ Nonlinear Intensity diffusion models
⇨ Reaction-Diffusion Equations - especially Fisher + NL diffusion

Recently:

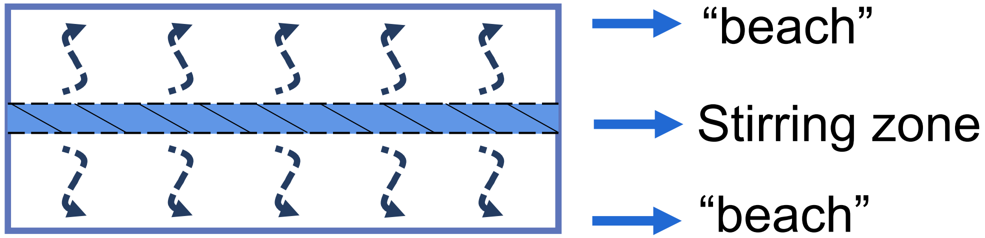
⇨ Renewed interest in context of λ_q broadening problem, cf. P. Diamond, Z. Li, Xu Chu

⇨ Simulations measure correlation of spreading $\langle \tilde{V}_r \tilde{p}\tilde{p} \rangle$ with λ_q broadening (Nami Li, P.D., Xu NF 2023)

⇨ Intermittency effects T. Wu, P. D. + 2023, A. Sladkomedova 2024,
⇕
Especially blobs, voids

Spreading Studies

⇨ 2D Box, Localized Stirring Zone



⇨ Comparison of:

System	Features
2D Fluid	Selective Decay, Vortices How to Measure Spreading?
2D MHD with weak $\underline{B_0}$ perp.	Alfvenization, Vortex Bursting, Zeldovich number
Forced Hasegawa-Mima with Zonal Flow	Waves + Eddies Multiple regimes and Mechanisms

N.B. Clear distinction between "spreading" and "avalanching"

Numerics: 2D Dedalus simulation

Box Characteristics:

- Dedalus Framework analogous to BOUT++

- Grid Size: 512×512
- Doubly Periodic boundary condition, beach regulates expansion

Forcing Characteristics:

- Superposition of Sinusoidal Forcing, vorticity
- Spectrum: Constant E(k), ensuring uniform energy distribution across wave numbers.
- Correlation Length: Approximately 1/10 of the box scale, some room for dual cascade.
- Localized through a Heaviside step function.
- Phase of forcing randomized every typical eddy turnover time

2D Fluid

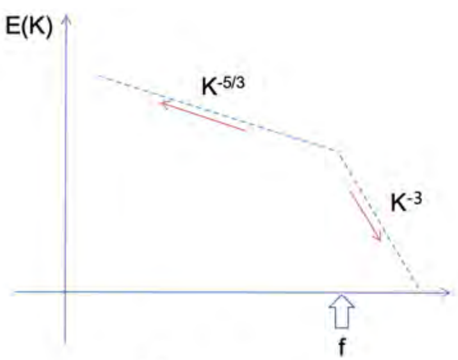
2D Fluid - the prototype

Vorticity Equation: $\frac{D\omega}{Dt} = \nu\nabla^2\omega - \alpha\omega$

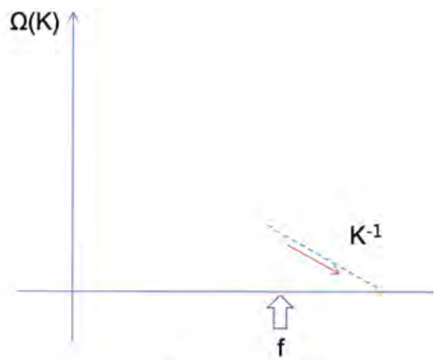
Key Physics:

- Inviscid, unforced invariants → $\begin{cases} \text{Energy } E = \int d^2x (\nabla\varphi)^2/2 \\ \text{Enstrophy } \Omega = \int d^2x (\nabla^2\varphi)^2/2 \end{cases}$

⇨ Dual Cascade Kraichnan



(3/2 law proved) Robust



("cascade" dubious)

2D Fluid, Cont'd

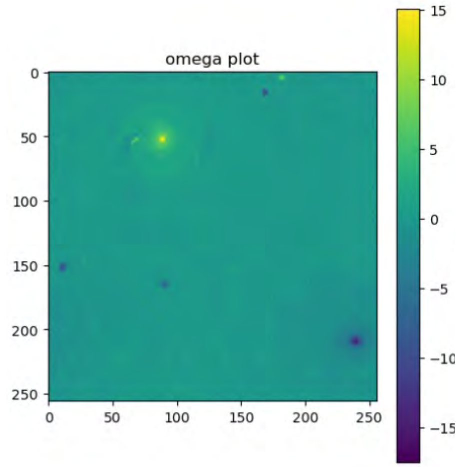
⇨ Selective Decay

Forward 'Cascade' enstrophy → Senses viscosity
Inverse 'Cascade' energy → Senses drag

For Final State of Decay:

$$\delta(\Omega + \lambda E) = 0$$ Bretherton + Haidvogel

⇨ Role Coherent Structures (Vortices)



- emergence isolated coherent vortices → survive decay
- $\frac{d}{dt}\nabla\omega = (s^2 - \omega^2)^{1/2}$

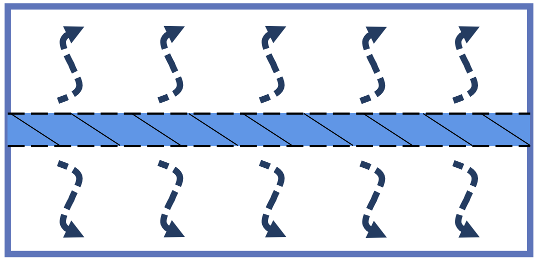
$\omega = \nabla^2\varphi$ → vorticity

$s = \partial_{xy}^2\varphi$ → shear

- Dipole vortices emerge, also

2D Fluid

⇨ Realize:



→ Forcing layer

- Most of system in state of Selective Decay !
- Need Consider / Compare :

$\langle V_y(\nabla^2\varphi)^2/2\rangle$ → Enstrophy Flux

⇩

⇨ Physical Measures of Spreading

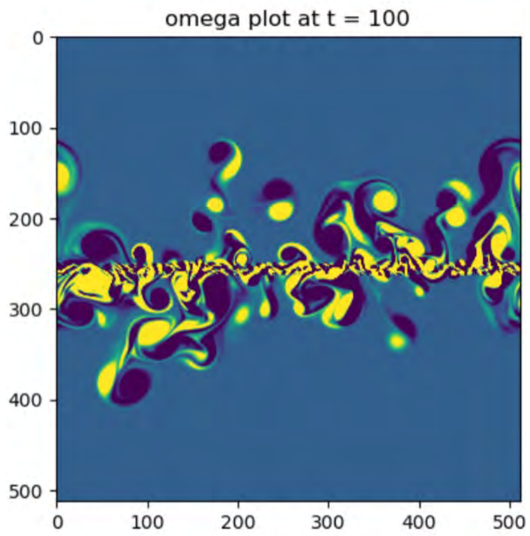
⇧

$\langle V_y(\nabla\varphi)^2/2\rangle$ → Energy Flux

as measures of “intensity spreading”. ⇨ Selective decay is radically different.

What Happens ?

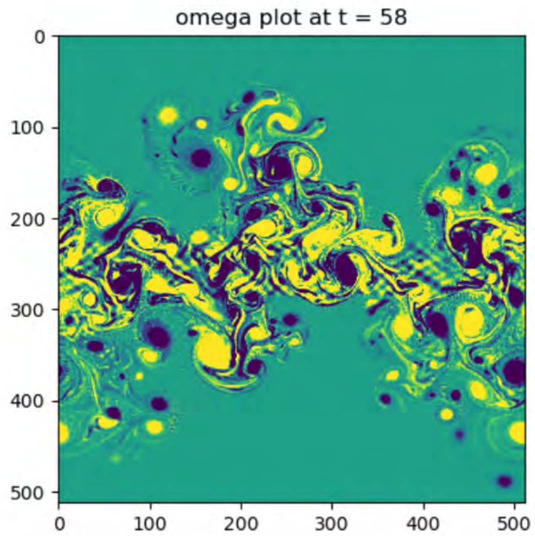
In Far Field, away from Forcing layer



Vorticity snapshot at Re~100

⇨ { Dipoles emerge
Spreading intermittent

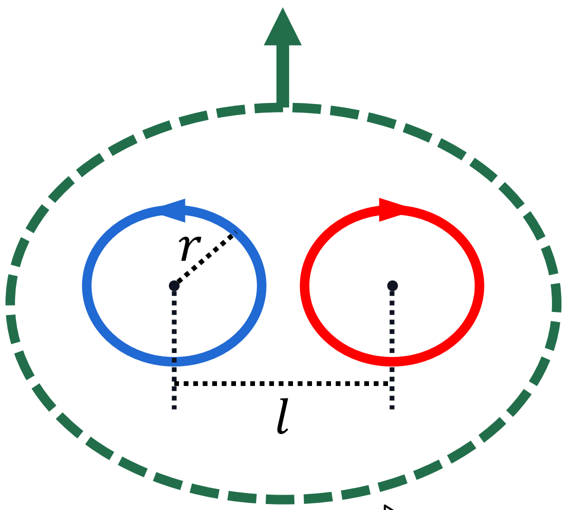
No apparent "Front "



Vorticity snapshot at Re~2000

- Dipoles, filaments, cluster
- Fractalized front

⇒ N.B. Dipole Vortex



— Uniform speed due to mutual induction

— $C = \frac{\Gamma}{l} = \frac{vr}{l}$

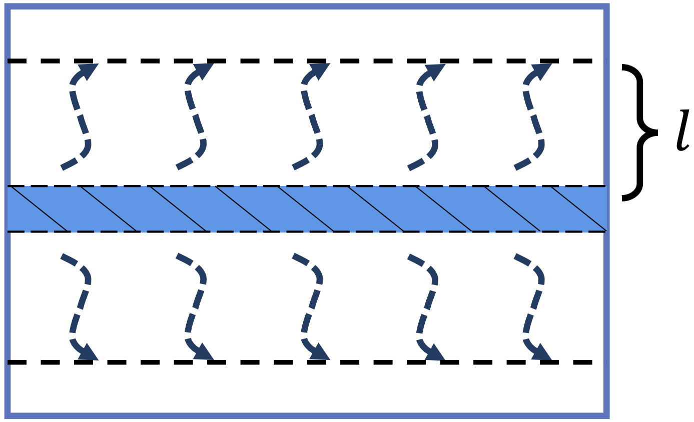
⇨ Dipole Vortices propagate at constant speed, "free flyers"

⇨ Physical origin of "ballistic spreading" ? !

i.e. ensemble dipoles expands linearly in time

On Keeping Score

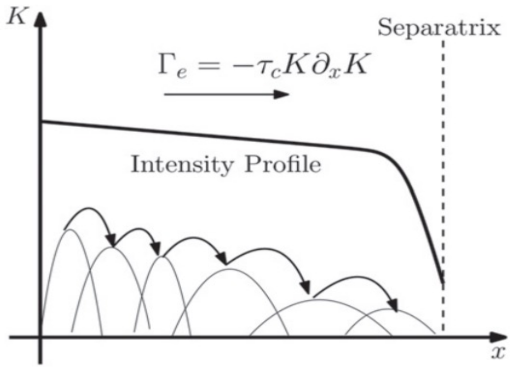
⇨ Loosely, interested in scaling of expansion of turbulent region with time



$l \sim t^{\alpha}$
α ?

⇨ Many approaches to l…

MFE favorite :



Track footprint of $|\varphi|^2$
Plot vs time,
1D projection

Keeping Score, cont'd

⇨ Approaches

Table 1: Table describing various velocity and transport parameters.

Parameter	Symbol	Equation	Description
RMS Velocity	V_{rms}	$V_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2}$	Root-mean-square velocity of turbulence, also known as turbulence intensity. This can either be measured near the forcing zone and averaged horizontally for a characteristic velocity as a basis of comparison, or measured globally to obtain global energy.
Quantity-Weighted RMS Distance	X_{W-rms}	$X_{W-rms} = \sqrt{\frac{\int \lvert\delta(x)\rvert^2 \lvert Q(x)\rvert\, dx}{\int \lvert Q(x)\rvert\, dx}}$	Quantity-weighted root-mean-square position represents the location of the quantity of interest, typically energy or enstrophy. One value is generated for each time. The quantity Q is usually energy or enstrophy.
Quantity-Weighted RMS Spreading Velocity	V_{W-rms}	V_{W-rms} is the slope of X_{W-rms} plotted against time	Quantity-Weighted RMS Spreading Velocity represents the bulk motion. This is more comprehensive than the front velocity.

N.B. :

— Quantity weighting can differ; depending on quantity

— RMS velocity sensitive to how computed

Keeping Score, cont'd

⇨ Approaches, cont'd

— Front velocity is MFE favorite sensitive to 1D projection, definition

— Transport Flux $\langle V_y E \rangle$, $\langle V_y \Omega \rangle$, most physical, clearest connection to dynamics of 2D Fluid

but: Sensitive to viscosity and selective decay

— Jet velocity very sensitive to viscosity, field chosen

Front Velocity	V_{front}	V_{front} is the slope obtained from tracking the outermost turbulent patch	This is usually comparable to V_{W-rms}, although front doesn't exist for low Reynolds number.
Transport Flux Density of certain quantity	Φ_Q	$\Phi_Q = < QV_\perp >$	The amount of certain quantity passing through a unit length per unit time; flux is the integral of flux density through the horizontal surface, which bounds half of the region and can be related to the rate of change of the quantity in that region.
Transport "jet" Velocity	V_Q	$V_Q = \frac{<QV_\perp>}{<Q>}$	Also known as normalized flux density. Average is usually taken horizontally. This velocity is separately obtained for each time.

Keeping Score, cont'd

Observation :

— Lower Re → Significant speed, 'front' fluctuations due to variability in dipole population

— Transport velocities quite sensitive to viscosity and selective decay

i.e. $\langle V_y \Omega \rangle$ drops

jet velocity $\langle V_y \Omega \rangle / \langle \Omega \rangle$ rises

especially for higher viscosity,
Due selective decay

— Formation of dipoles follows decay of enstrophy

— Dipoles ultimately determine spreading

Results

Re ~ 5000

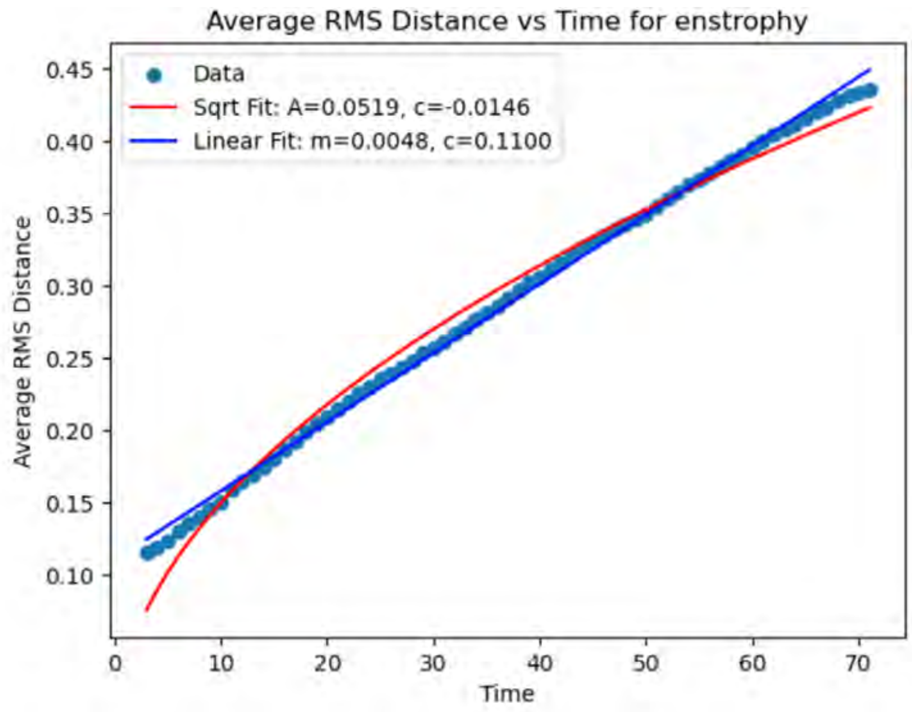
Ω–weighted
rms distance

— Constant spreading speed for enstrophy, i.e., $l \sim ct$

$\underline{\alpha = 1}$

— $c/V_{rms} \sim 0.1$

— Consistent with picture of dipole vortices carrying spreading flux



<u>Results</u>, cont'd

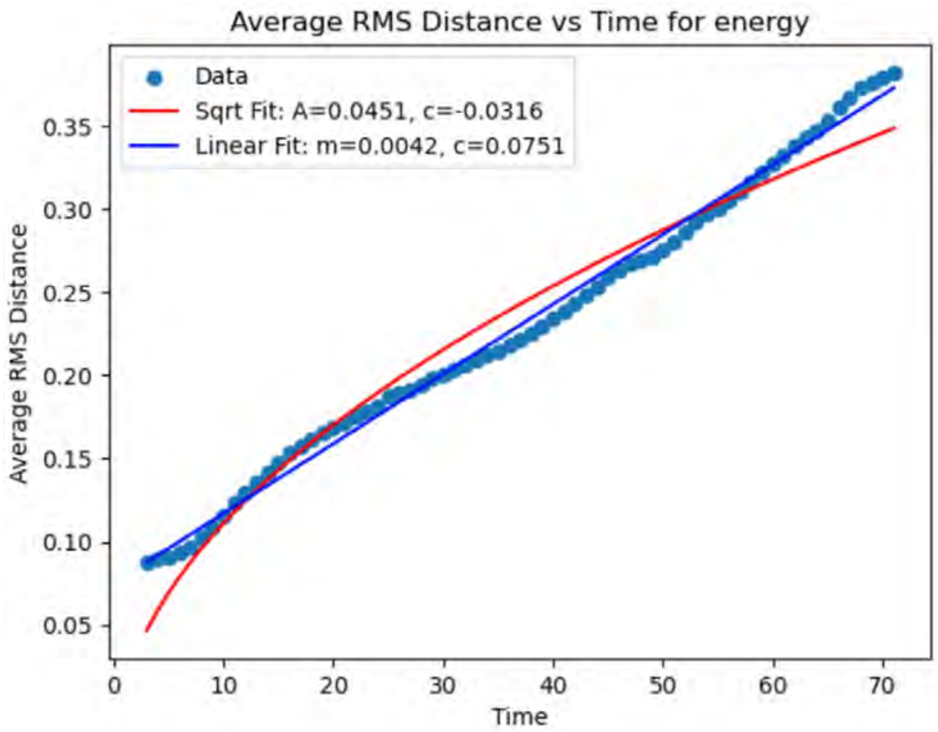
Re ~ 5000

E–weighted
rms distance

— Constant spreading speed for energy, i.e., $\alpha \simeq 1$

— $c/V_{rms} \sim 0.1$

— Lager dipoles ↔️ more energy → increases fluctuations relative to enstrophy case



Summary - 2D Fluid

— Coherent structures - Dipole vortices -
mediate spreading of turbulent region → free flyers

— Mixed region expands as $w \sim t$, consistent with dipoles.

— No discernable "Front", spreading is strongly intermittent. (space+time)

— Spreading PDF is non-trivial.

— Turbulence spreading non-diffusive.

2D MHD + Weak $\underline{B_0}$

2D MHD

- The equations: $$\frac{d}{dt}(\nabla^2\varphi) = \nu\nabla^2\nabla^2\varphi + \nabla A\times\hat{\boldsymbol{z}}\cdot\nabla\nabla^2 A + \tilde{f}$$
$$\frac{d}{dt}A = \eta\nabla^2 A$$
$$\frac{d}{dt} = \partial_t + \nabla\varphi\times\hat{\boldsymbol{z}}\cdot\nabla$$

- Inviscid Invariants: $E = \langle V^2 + B^2\rangle,\ H = \langle A^2\rangle,\ H_c = \left\langle\vec{V}\cdot\vec{B}\right\rangle \Rightarrow 0$, hereafter

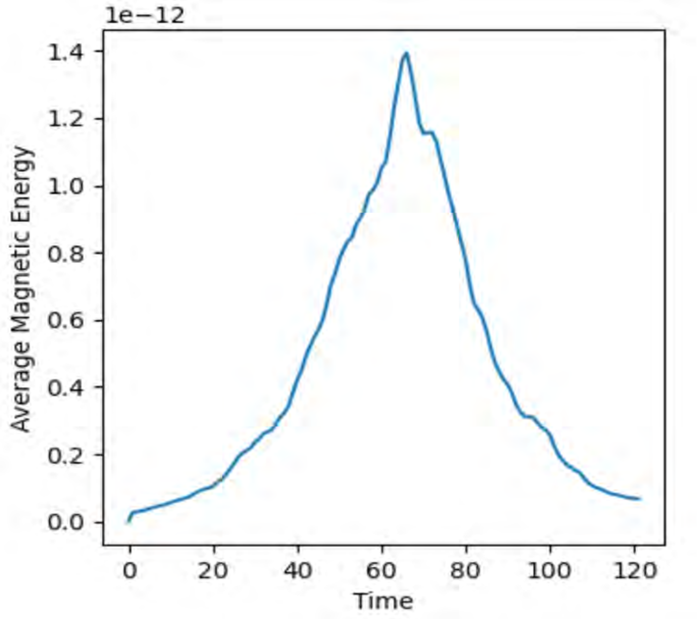
  Conservation of H is Key !

- Consider weak mean magnetic field: $B = B_0(y)\hat{x}$
  $B_0(y) \sim B_0\sin(y) \Rightarrow$ initial imposed pattern

- As before, localized forcing region, effectively unmagnetized

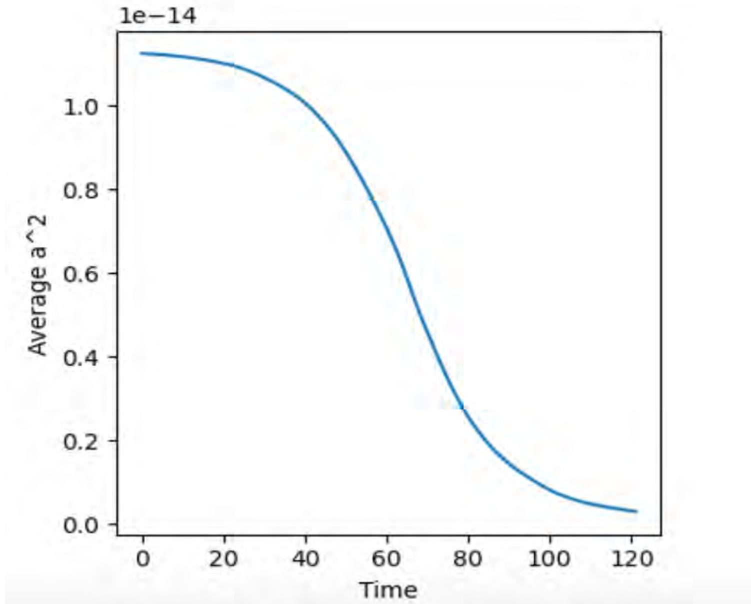
⇒ 2D MHD

- Zeldovich Theorem: No dynamo in 2D



⇨ Field ultimately decays

- Consequence of decay $\langle A^2 \rangle$



$$\frac{d}{dt}\langle A^2 \rangle = -\eta \langle B^2 \rangle$$

$$\int_0^t \langle B^2 \rangle dt \leq \frac{\langle A(0)^2 \rangle}{\eta}, \therefore \langle B^2 \rangle \text{ decays}$$

Key Physics of 2D MHD

N. B. “Z” ⇨ Zeldovich

- Lorentz force suppresses inverse kinetic energy cascade.
  Inverse cascade $\langle A^2 \rangle$ develops

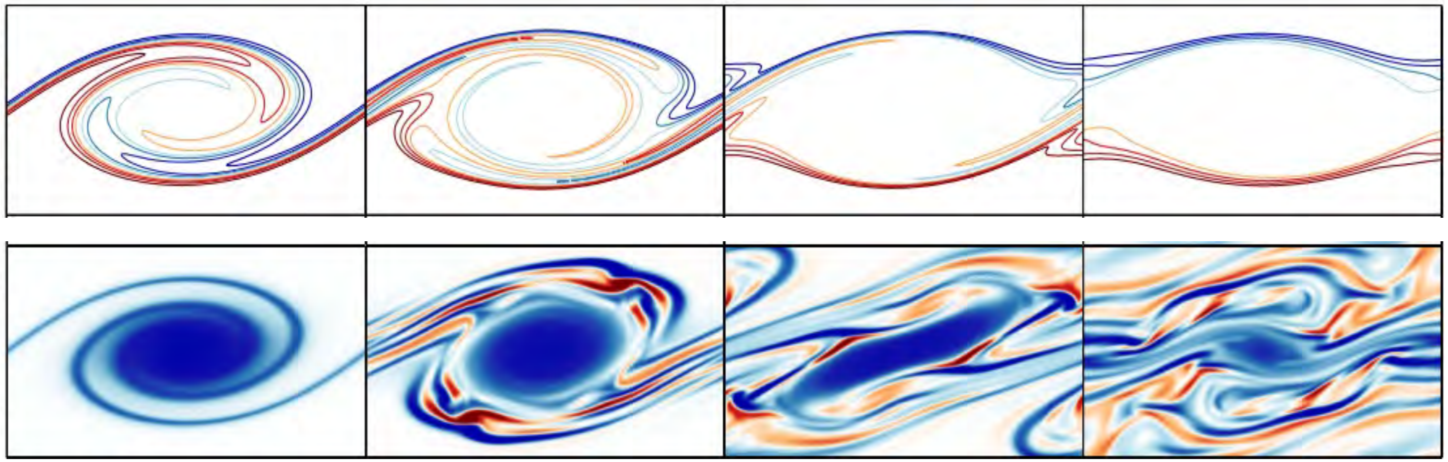
- Single Eddy: Expulsion (Weiss’66) vs. Vortex Disruption (Mak et. al 2017)

Key Parameter: $Z = Rm \frac{V_{A0}^2}{V_E^2}$

$Z \sim 1$ bounds the two regimes

Expulsion:

Vortex bursting:



from Mak et. al 2017

Key Physics of 2D MHD, cont'd

- Turbulent Diffusion: (Cattaneo + Vainshtein '92 ;
  Gruzinov + P.D. '94)

  Closure + $\langle A^2 \rangle$ conservation $\Rightarrow$ Quenched Diffusion of B - field

  From: $D_t \sim \eta_{anom} \sim \langle \tilde{V}^2 \rangle \tau_c$

  To: $D_t \sim \eta_{anom} \sim \langle \tilde{V}^2 \rangle \tau_c / \left[1 + R_m V_{A0}^2 / \langle \tilde{V}^2 \rangle \right] \sim D_{Kin} / (1 + Z)$

- Once again,

$$\boxed{\text{Key Parameter: } Z = R_m \frac{V_{A0}^2}{<\tilde{V}^2>}}$$

$< \tilde{V}^2 > \; vs \; V_E^2$

N.B.: - V_{A0} is initial weak mean magnetic field

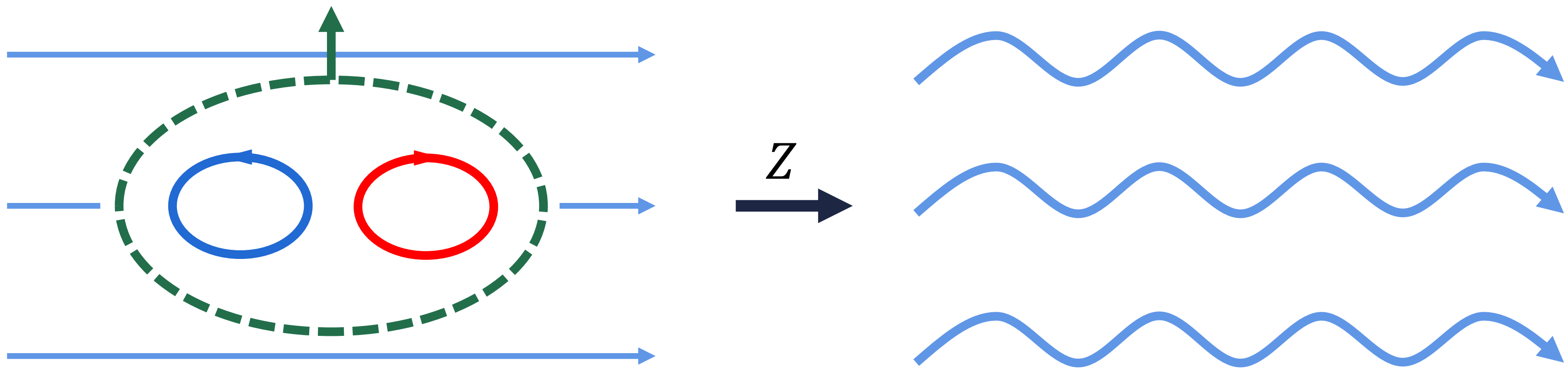
- R_m large...

<u>Crux of the Issue</u>!?

⇨ Hydrodynamics: Dipole vortex ‘Carries’ turbulence energy ⇨ spreading

⇨ But... weak $\underline{B_0}$ can ‘burst’ vortices ⇨

converts dipole kinetic energy to Alfven waves, propagating laterally, and dissipation.



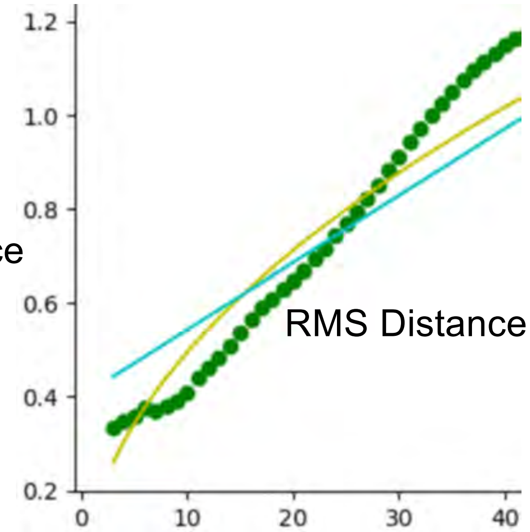
⇨ So, can a <u>weak</u> B_0 block spreading in 2D MHD ! ?

N.B. Perp Alfven waves observed

⇨ Time evolution of Spreading

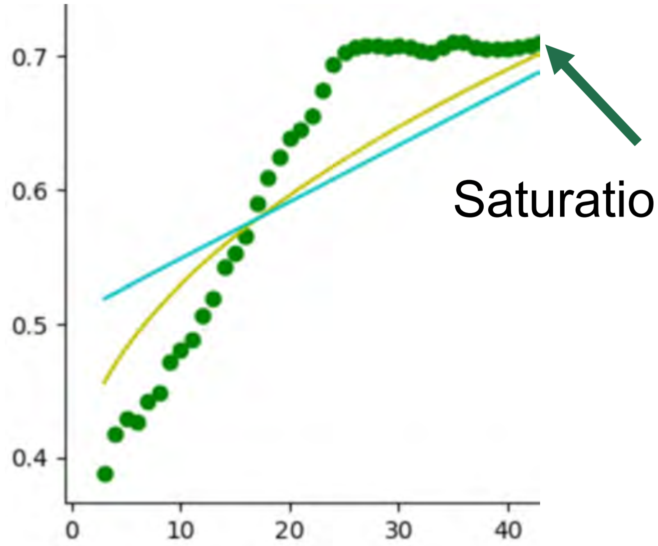
Hydro regime: $Rm = 100, Bo = 0.001, Z = 0.01$

RMS Distance



Time

MHD: $Rm = 100, Bo = 0.01, Z = 1$



Saturation at L=0.7

Time

⇨ Hydro case spreads linearly

⇨ Z=1 Case saturates.
(dipoles disrupted)

⇨ Spreading vs. Z - Turbulence

- Now consider turbulence:

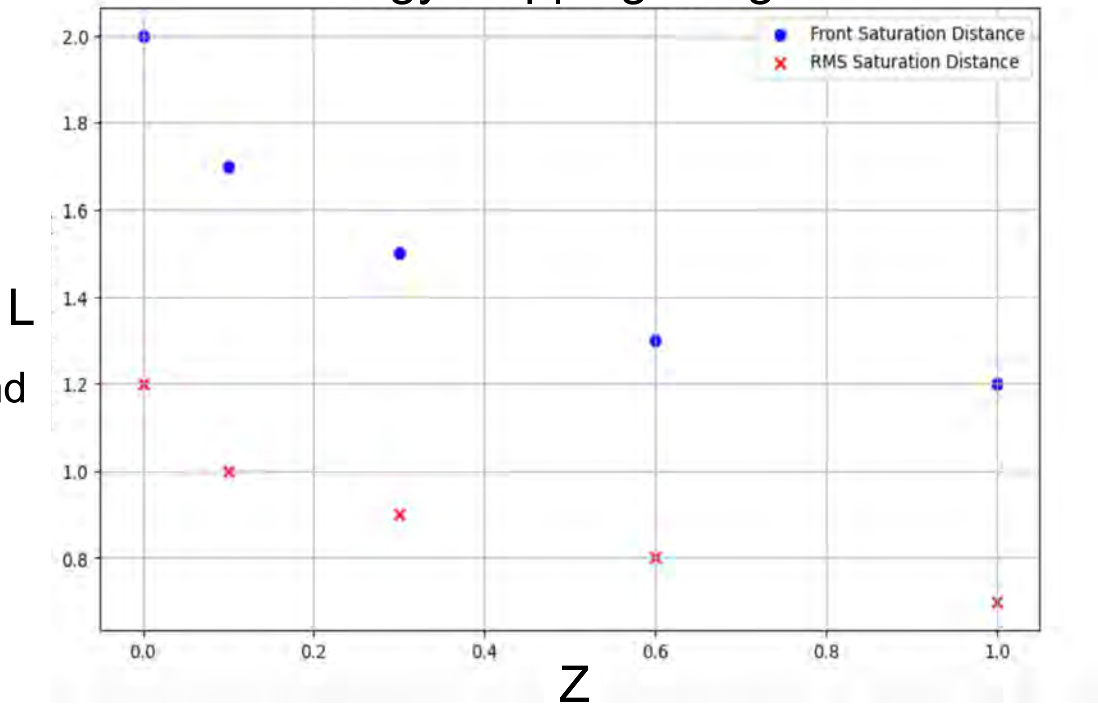
- Kinetic Energy Stopping length decreases with increasing $Z = R_m \frac{V_{A0}^2}{<V_{rms}^2>}$

  N.B. Z reflects both R_m and B_0

- Systematic difference between Front and RMS saturation evident, trends match

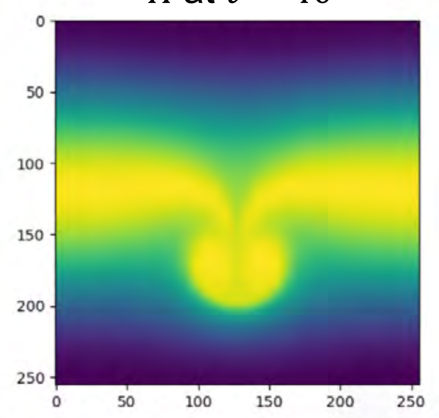
⇨ Insight from vortex studies useful

Kinetic energy stopping Length L vs. Z

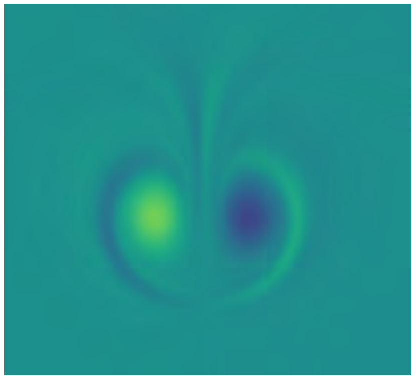


⇒ Single Dipole in weak B_0

A at $t = 40$



ω at $t = 40$



J at $t = 80$

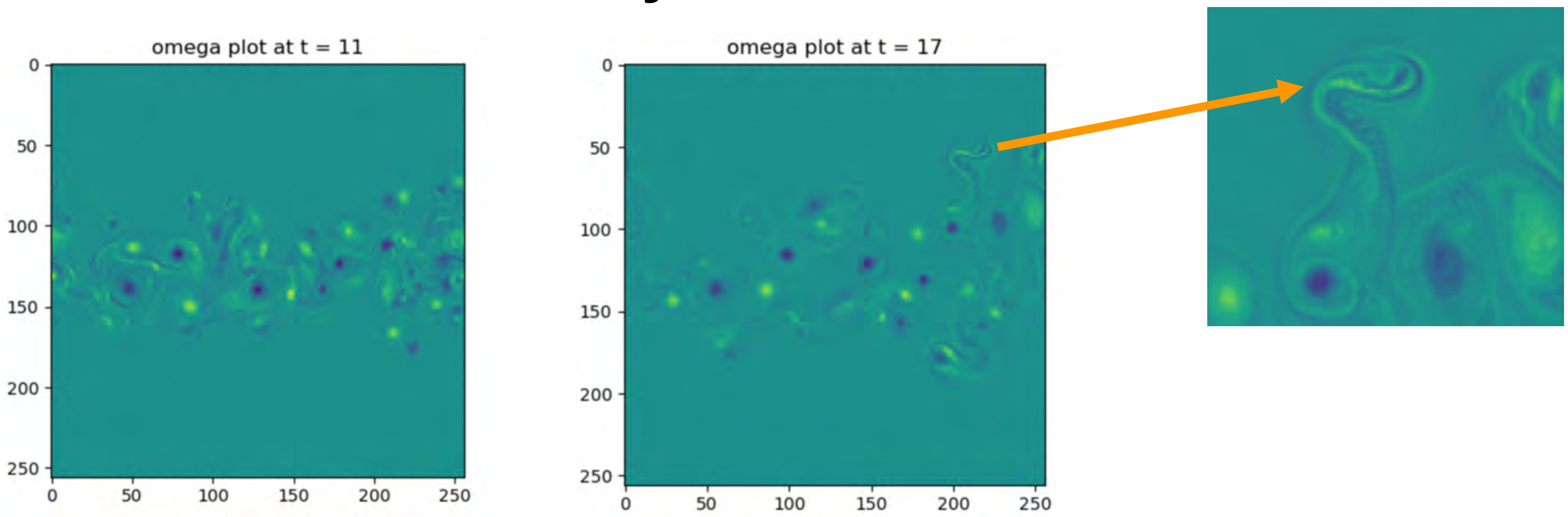
Note wrapping filament tends to cancel and push on dipole, so it distorts and ultimately bursts

Filament and vortex bursting. Concentration at small scale ⇨ fast dissipation

Connection: vortex busting ↔️ MHD cascade singularity?!

⇨ Close Look at Vorticity Field

Bursting/Filamentation



- Z=3, Rm≈50, Re≈500, B=0.01
- Dipoles evident at early times, but encounter stronger field as migrate
- Vortex bursting occurs at later times ⇨ Spreading halted.

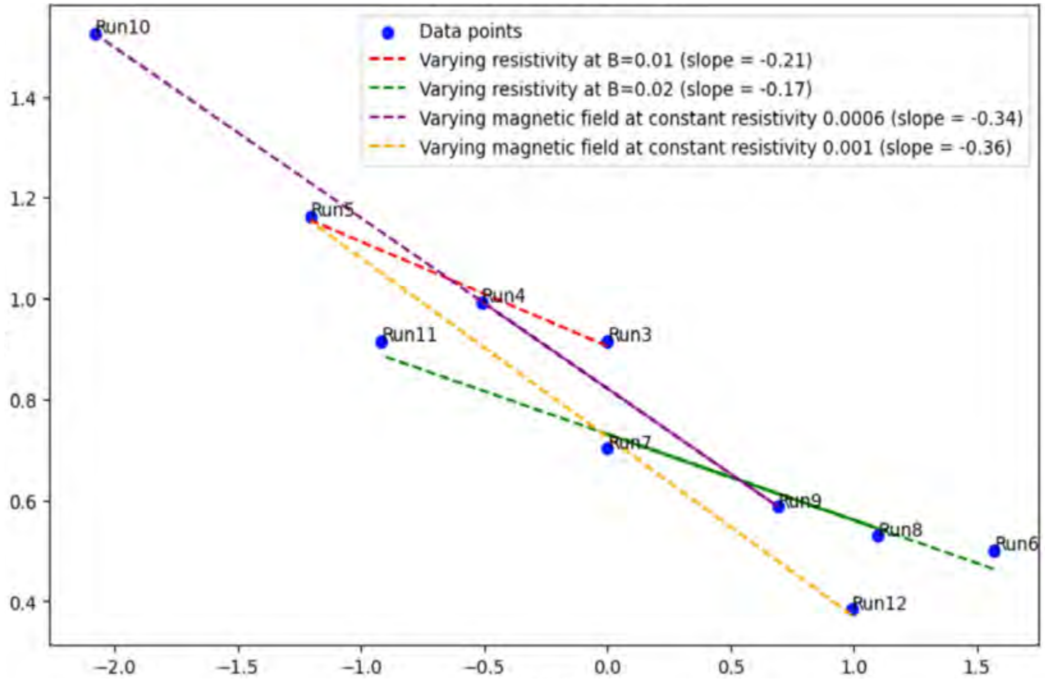
⇨ Single Dipole Penetration

- Dipole penetration decreases with increasing Z

- Evidence that varying B_0 and R_m impact penetration.

But Z is not the full story… P_m dependance?

Log-Log Plot of L against Z



$\log(L)$

$\log(Z)$

⇨ 2D MHD: Summary

- Weak B_0 enables vortex disruption

  Dipole bursting ⇨ Saturates spreading

  ⇩

- $\boxed{\text{Weak } \underline{B_0} \text{ blocks advance of kinetic energy}}$
- Process: Conversion dipole KE to Alfven waves, dissipation

- $Z = R_m \frac{V_{A0}^2}{<V_{rms}^2>}$ as critical parameter

⇨

- Reinforces notion of "free flyer dipoles" as critical to spreading

Forced Hasegawa – Mima + Zonal Flows

H-M + Zonal Flow System

- System:

$$\frac{d}{dt}\left(\tilde{\phi}-\rho_s^2\nabla_\perp^2\tilde{\phi}\right)+v_*\frac{\partial\tilde{\phi}}{\partial y}+v_{*u}\frac{\partial\tilde{\phi}}{\partial y}=\frac{\partial}{\partial r}\rho_s^2\left\langle\tilde{v}_r\nabla_\perp^2\tilde{\phi}\right\rangle+\nu\nabla^2\nabla^2(\widetilde{\phi})+\tilde{F}$$ -Waves, Eddys

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\bar{v}_z\frac{\partial}{\partial y}+\nabla\tilde{\phi}\times\hat{\mathbf{z}}\cdot\nabla$$

$$\frac{\partial}{\partial t}\nabla_x^2\bar{\phi}_z+\frac{\partial}{\partial r}\left\langle\tilde{v}_r\nabla_\perp^2\tilde{\phi}\right\rangle+\mu\nabla_x^2\bar{\phi}_z=0$$ -Zonal Flow

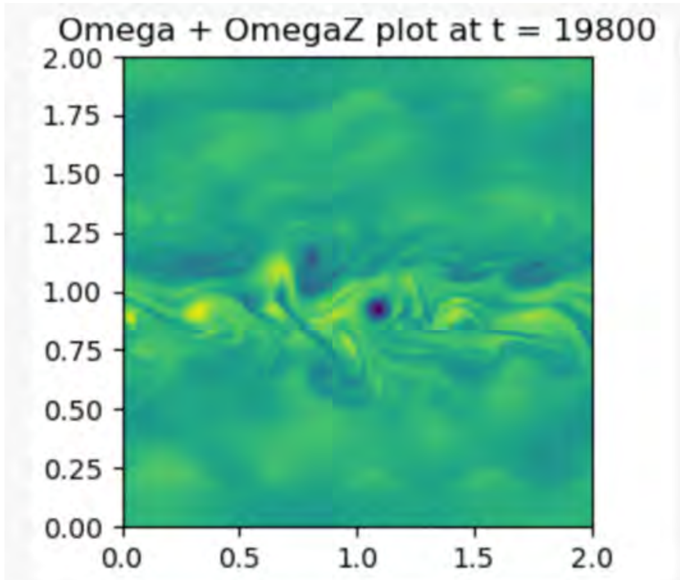
- viscosity controls small scales
- drag controls zonal flow - μ
- conserved: Energy $\longrightarrow$ $\left\langle\tilde{\phi}^2+\rho_s^2\left(\nabla\tilde{\phi}\right)^2\right\rangle+<\rho_s^2(\nabla\phi_z)^2>$

  Potential Enstrophy $\longrightarrow$ $\underbrace{\left\langle\left(\tilde{\phi}-\rho_s^2\nabla^2\tilde{\phi}\right)^2\right\rangle}_{\text{Waves}}+\underbrace{<(\rho_s^2\nabla^2\phi_z)^2>}_{\text{ZF}}$

Typical saturated snapshot(Kubo 0.2)

— Dipoles disappear

— Large coherent vortex



Total vorticity snapshot at the end. Steady state; Turbulent in the center only. Dipole isn't a steady structure in this system; instead, we get single vortex that looks like Jupiter's eye, which is not gonna move by itself

Total Vorticity: $\nabla^2(\tilde{\phi} + \phi_z)$

H-M + Zonal Flow System, cont'd

→ Now: *waves* $\omega = \omega_*/(1 + k_\perp^2 \rho_s^2)$, $\underline{v_{gr}}$

eddies $\tilde{v}$

zonal mode (symmetry)

$\left\{\begin{array}{l} \tilde{v} \text{ vs } v_* \rightarrow \\ \text{mixing length} \end{array}\right.$

i.e. ⇒ Energy Flux: $\left\{\begin{array}{l} \sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}} \rightarrow \text{and other} \\ \langle \tilde{v}_r \xi \rangle \rightarrow 3^{rd} \text{ order} \end{array}\right.$

N.B. 2 channels for "turbulence spreading"

- Waves/Wave transport
- Turbulent mixing

-Branching ratio, vs. Ku number ?

For clarity; Contrast:

⇨ **Spreading in presence of fixed, externally prescribed shear layer**

⇨ **Here:** → Forcing $\rightarrow \begin{Bmatrix} \text{Waves} \\ \text{Eddies} \end{Bmatrix}$ → Zonal flow (self-generated)

∴ $\boxed{\text{forcing } (\tilde{v}_{rms}, Re) + \text{drag} \Rightarrow \text{control parameters}}$

⇨ "weak" and "strong" Turbulence Regimes

$$v_{gr} \text{ vs } v_r \rightarrow \frac{\langle \tilde{v}_r \xi \rangle}{\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}}} \rightarrow \frac{\tilde{v}_r \tau_c f}{\Delta_c} \rightarrow Ku$$

coherency factor

$\Delta_c \sim v_{gr} \tau_c$

⇨ $Ku < 1 \rightarrow$ wave dominated spreading

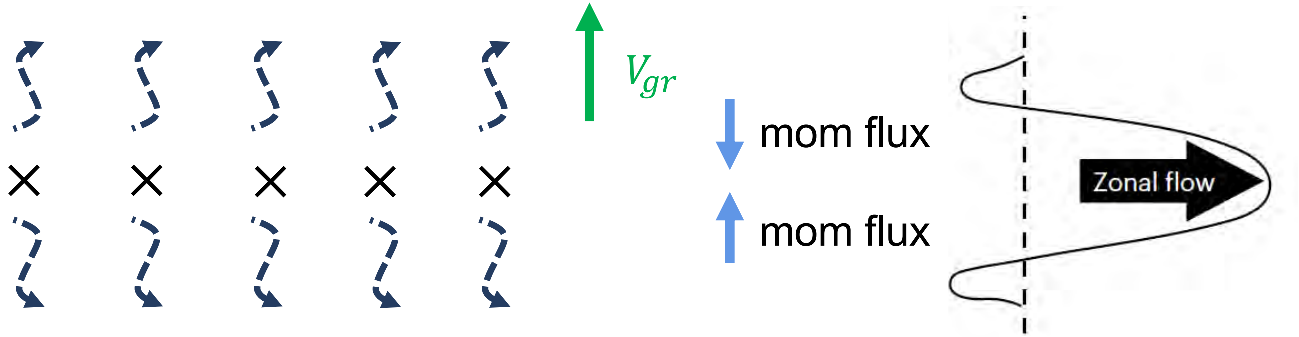
$Ku > 1 \rightarrow$ mixing dominated spreading

H-M + Zonal Flow System, cont'd

→ Enter the Zonal Flow…

- Multiple channels for NL interaction
- But with ZF ⟷ eddy, wave coupling to ZF dominant
- ZF is the mode of minimal inertia, damping, transport

$\Rightarrow$ energy coupled to ZF ($\tilde{v}_r = 0$) cannot "spread", unless recoupled to waves

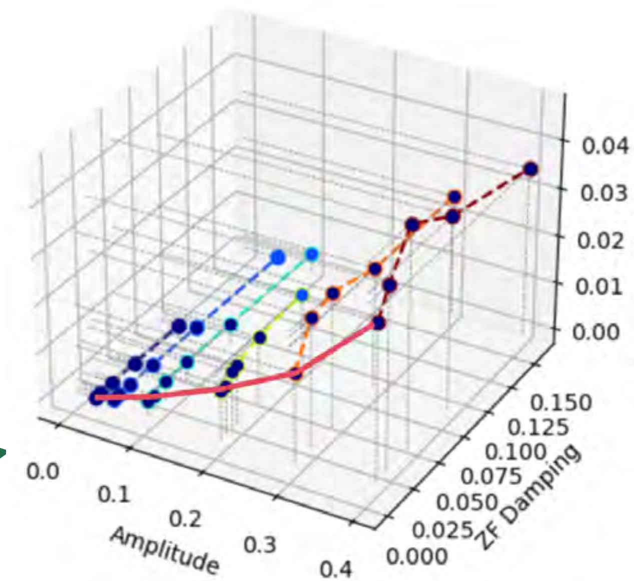


→ Degradation of ZF (back transfer) is crucial to spreading

<u>Results</u>

Potential Enstrophy Flux

Dimits Regime



averaged spatially and temporally

— Potential enstrophy flux generally <u>increases</u> as drag increases. "Dimits regime" for turbulence spreading. Spreading diminishes as power coupled to Z.F.

— Self-generated barrier to spreading.

— For A increasing, PE flux rises sharply, even for weak ZF damping. Fate of ZF?

— "KH-type" mechanism loss of Dimits regime at high Kubo # - characterization?

Results, Cont'd
Wave Energy Flux

Wave Energy Flux $< -\frac{\partial \phi}{\partial t} \nabla \phi >$ $\longleftrightarrow$ $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) E_{\boldsymbol{k}}$

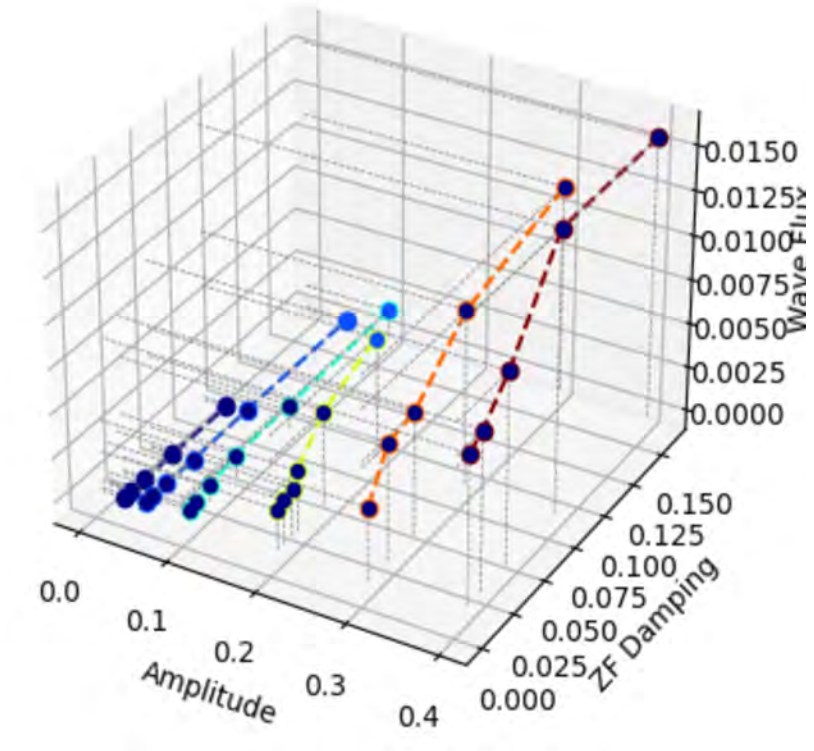
– Dimits regime at low forcing and ZF damping

–Increases with ZF damping and forcing amplitude

– Dominant K_x increases under ZF decorrelation

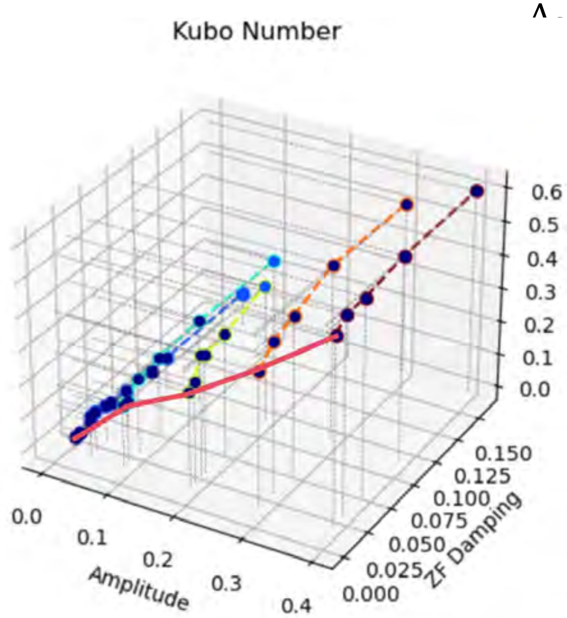
– Spectrum condensation towards low k with inverse cascade

implication for v_{gr} and $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) E_{\boldsymbol{k}}$

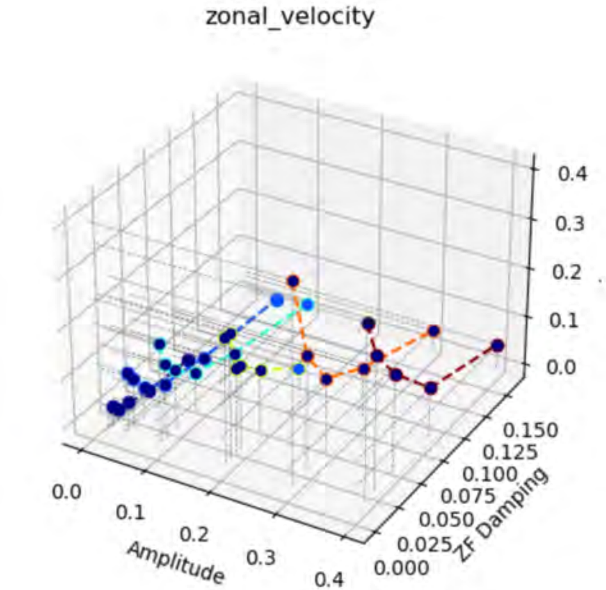


<u>Results</u>, Cont'd

$\frac{\tilde{v}_r \tau_c f}{\Delta_c}$ where $\Delta_c \sim < K_x^2 >^{-1/2}$

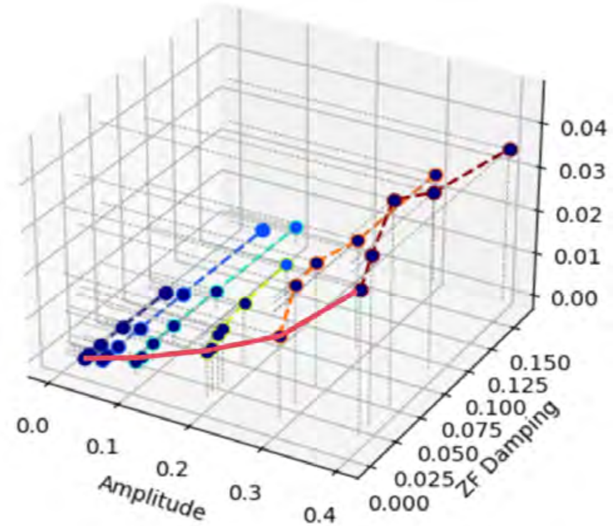


Fluctuation intensity <u>increases</u> as drag increases



Zonal velocity <u>decreases</u> with increasing drag

→Spreading and Fate of Zonal Flows



→ Spreading rises for increased forcing, even for $\mu \to 0$

→ Dimits regime destroyed. How?

$\underline{\Rightarrow}$ Seems necessary for spreading in systems with ZF

→ Animal Hunt for linear instabilities(KH, Tertiary …) seems pointless in turbulence

→ Instead, $P_{\mathrm{Re}} = -\langle \widetilde{V_x}\widetilde{V_y}\rangle \cdot \dfrac{\partial \overline{V}_y}{\partial x}$ Power transfer fluctuations → flow

$P_{Re} < 0$: Wave → ZF transfer

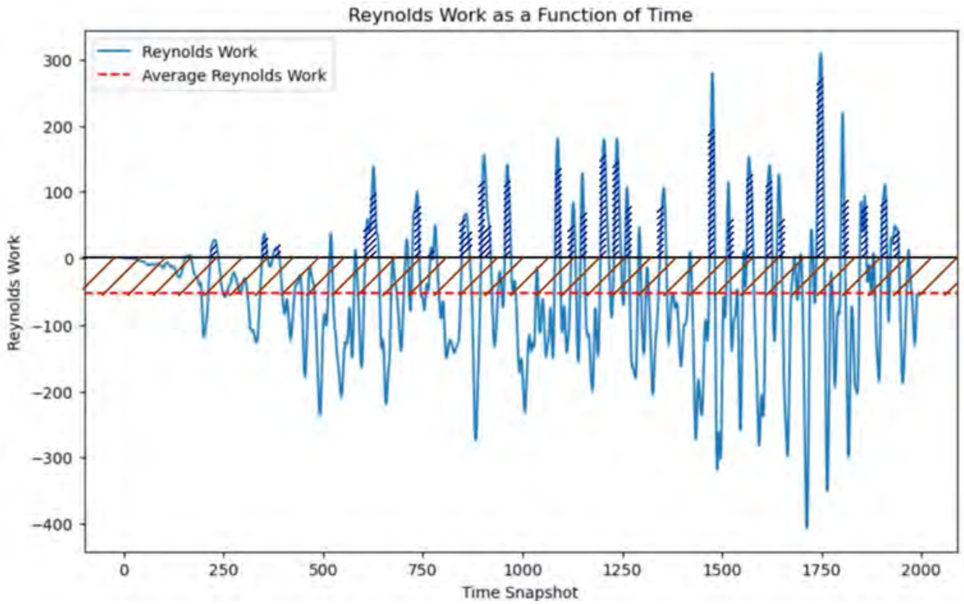
$P_{Re} > 0$: ZF → Wave transfer ⇒ ZF decay

Quantifying Wave-ZF Power transfer

$$1/2 * \frac{\partial \overline{V}_y^2}{\partial t} = \omega_Z < \widetilde{v_x}\widetilde{v_y} > -drag * \overline{V}_y$$

Reynolds power

We quantify ZF → Waves Power Transfer as the ratio of the area above the axis to work done on the zonal flow.



Reynolds power vs time

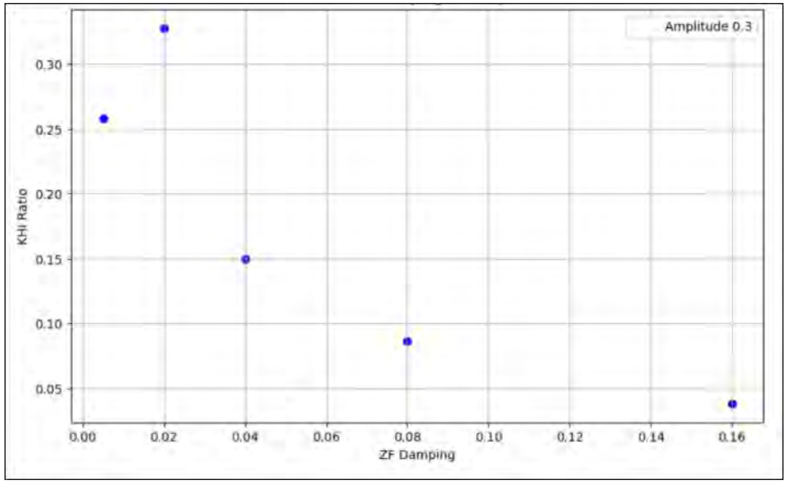
$P_{Re} < 0 \Rightarrow$ Wave → ZF transfer

$P_{Re} > 0 \Rightarrow$ ZF → Wave transfer

<u>Results</u>, Cont'd

P_{Re} *ratio vs ZF damping*

Dimits Regime



— The ratio generally decreases as a function of ZF damping

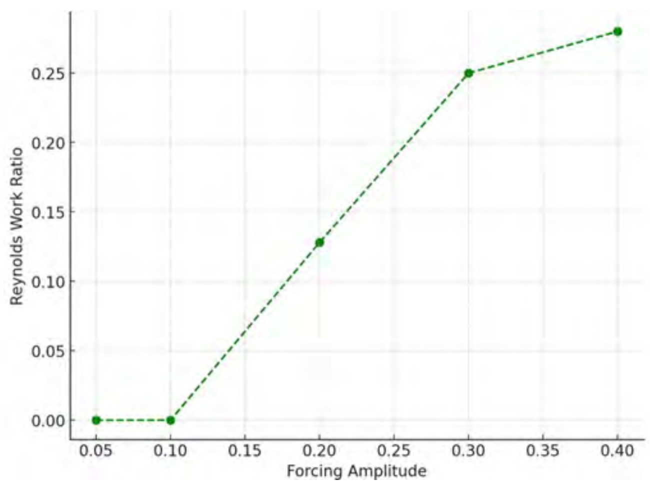
— The ratio increases for increasing Kubo number

— Possible improvement: Non-local transfer ala' closure, instead QL

$$P_{\mathrm{Re}} \approx -(V_{\mathrm{rms}}^2 \cdot \tau_{\mathrm{corr}}) \cdot \omega_{\mathrm{zonal}}^2$$

Results, Cont'd, Reynolds power vs Kubo

P_{Re} $ratio\ vs\ forcing\ amplitude$



Preliminary
→ Explore other FOMs

— The ratio goes up as a function of Kubo number

— Indicates that re-coupling of ZF energy to turbulence increases for stronger forcing

— Avoids instability morass

Summary - Drift Wave Turbulence

→ Spreading fluxes mapped in forcing, ZF damping parameter space

→ Dominant mechanism ⟷ Ku (waves vs mixing) , Both waves and mixings in play.

→ Dimits-like regime discovered

→ ZF quenching intimately linked to spreading

→ $P_{Re} > 0$ bursts track breakdown of Dimits regime and onset turbulent mixing

→General Summary

→ Coherent structures dipoles frequently mediate spreading

←→ underpin “ballistic scaling”

→ Spreading dynamics non-diffusive; Conventional wisdom misleading, or worse.

N.B. stay tuned for talks by Alsu, Ting, Filipp

→ In DWT, wave propagation and turbulent mixing both drive spreading

→ ZF quenching critical to spreading in DWT. Power coupling most useful to describe ZF quench.

→Future Plans

— High resolution studies

— Understand ZF quenching physics and calculate power recoupling-general case, GK formulation?

— Spreading in Avalanching. Relative Efficiency? Spreading and Transport? Flux-driven H-W System. Potential Enstrophy Flux!?

More general:

— Is spreading mechanism universal? Seems unlikely

— Towards a model, models… Ku~1 is an interesting challenge

— Relation/connection of DW+ZF spreading and Jet Migration (L. Cope)