Mode Competition, Saturation Mechanisms and Spatial Patterns in Multi-Scale Turbulence - a Selected Overview

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Outline

- Preliminaries
- Conceptual Elements
  - Multi-scale problems
  - Mode competition
  - Spatial Patterns
- ‘Multi-scale’ and ‘Multi-step’: towards an improved reduced model
- Some preliminary observations
- Key open questions

Preliminaries

- Why?
  - "Shortfall problem" has resisted efforts for long time [n.b.: problem is controversial]
  - Multi-scale approach $\begin{cases} \text{Drift - } \rho_i \\ \text{ETG - } \rho_e \end{cases}$ suggested as a road forward (c.f. Holland, et. al. 2014-2017), via GYRO simulations
  - Physics of interactions not elucidated ➔ prediction is highly problematic

And

  - Physics is interesting!

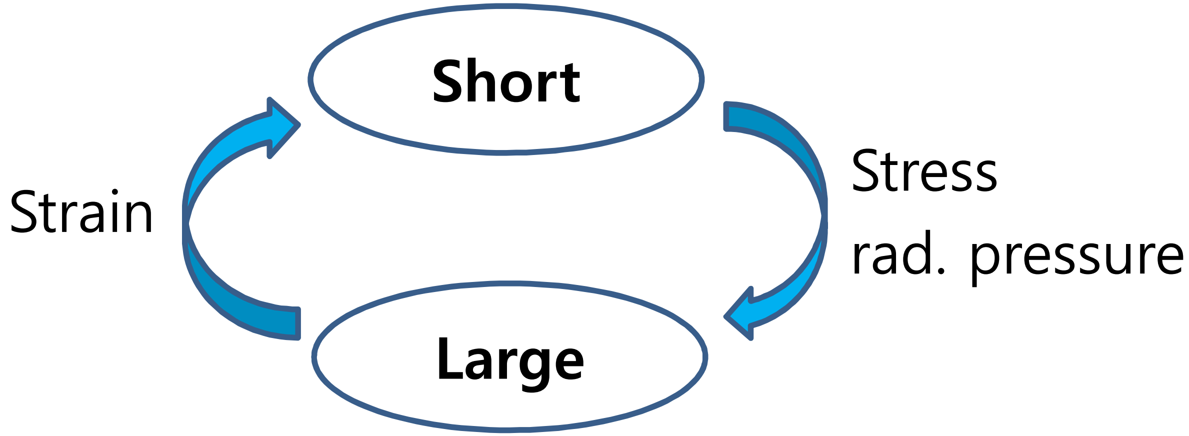
Conceptual Elements

Multi-Scale Problems

- Interactions of disparate scale populations

→ <u>Classics</u>: Langmuir turbulence

Drift wave – Zonal Flow



Langmuir collapse is pathway to singularity formation

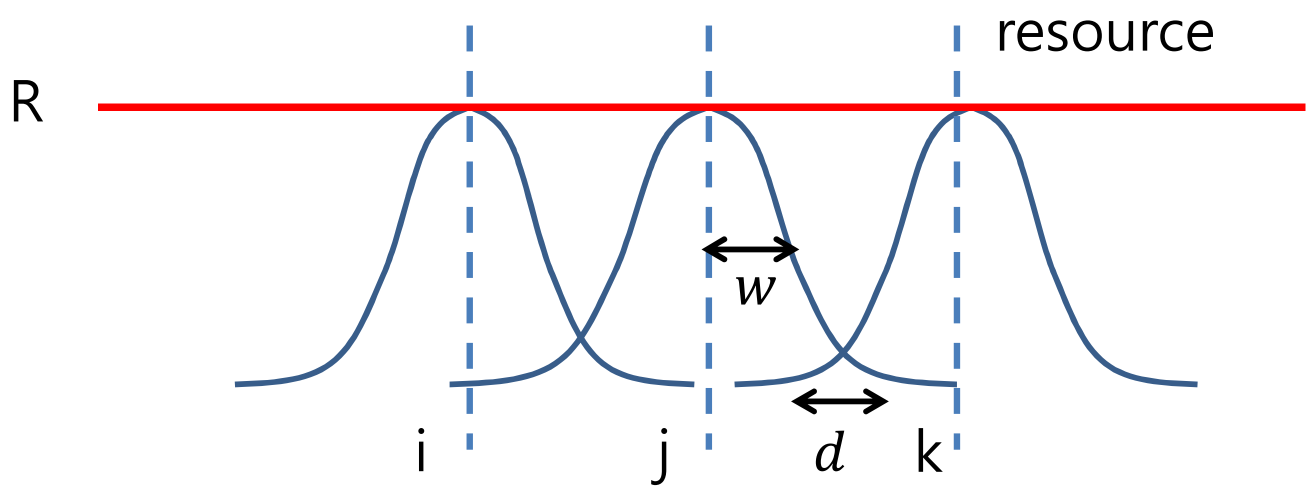
→ Familiar, though well understood in simplest cases, only

→ Interaction with free energy complicates dynamics

Mode Competition → One Complication

→ Mode competition is some set of $\left\{\begin{array}{l}\text{modes}\\ \text{population}\end{array}\right.$ competing for available drive/free energy

→ Analogy from ecology → niche overlap (c.f. R. May)



Species Utilization: $\left\{\begin{array}{l} d \rightarrow \text{spacing}\\ \text{w} \rightarrow \text{wdith}\end{array}\right.$

➔ niches

→ Question is stability of system, distribution of populations

- Populations N_i

Competition matrix → Overlap of population

$$\frac{d}{dt}N_i = N_i\left[k - \sum_{j=1}^{M}\alpha_{ij}\, N_j\right]$$

- Extends familiar predator-prey idea, where one species accesses free energy, one is symbiotic
- Structure, eigenvalues of competition matrix ➔ system state
- Noise strongly affects overlap w

Spatial Patterns

(with A. Ashourvan)

Some Questions

I)

Re: Drift-ZF Turbulence

- Impact of ZF well established
- Effectively linear modulation theory developed

Cf:

- Gurcan, P.D. ’14
- DI^2H, ‘05

But:

- What sets scale of ZF field? → V_E'
- How does modulational instability evolve nonlinearly, saturate
- N.B.: Predator-Prey feedback channel
- Saturation ←→ scale connection?

Staircase structure

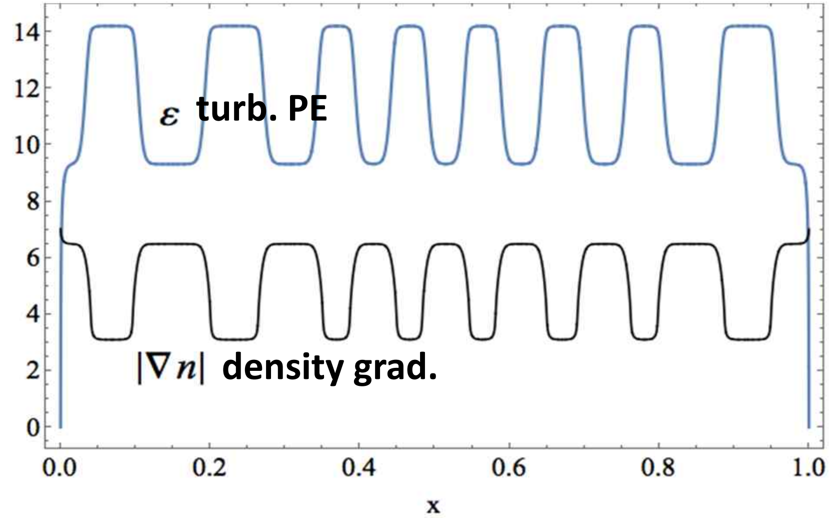
Snapshots of evolving profiles at t=1 (non-dimensional time)

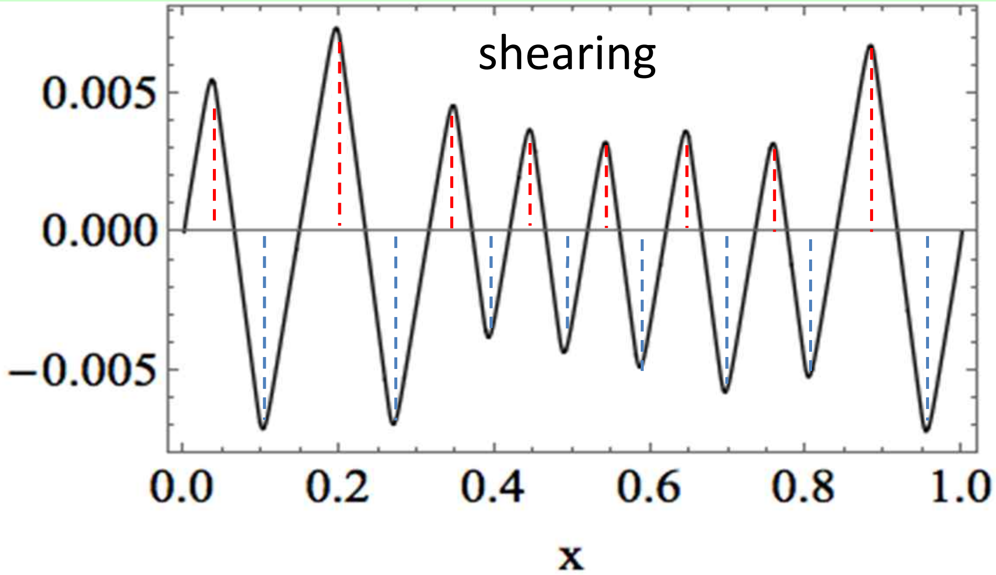
Initial conditions: $n = g_0(1-x), \quad u = 0, \quad \varepsilon = \varepsilon_0$

Boundary conditions: $n(0,t) = g_0,\ n(1,t) = 0; \quad u(0,1;t) = 0; \quad \partial_x \varepsilon(0,1;t) = 0$

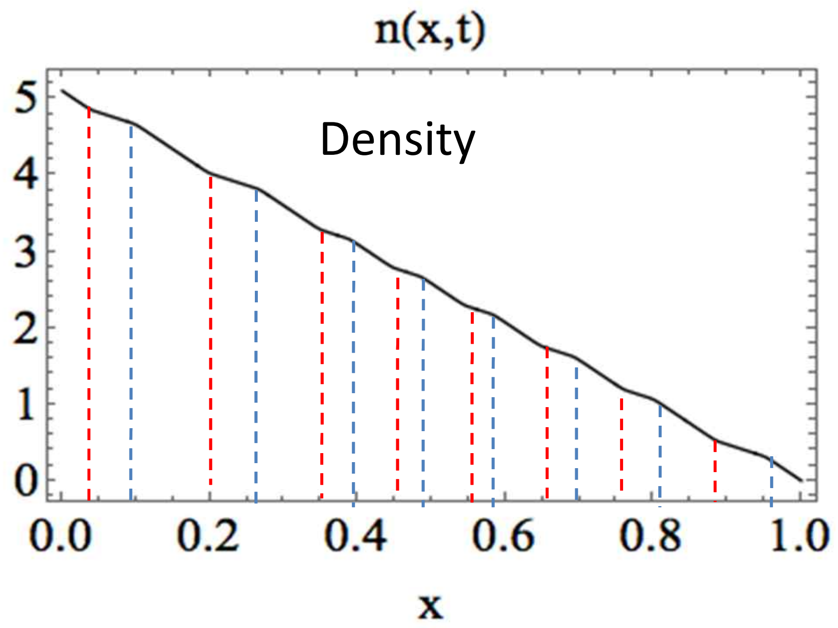
Structures:

- ○**Staircase in density profile:**
  - **jumps → regions of steepening**
  - **steps → regions of flattening**
- ○**At the jump locations, turbulent PE is suppressed.**
- ○**At the jump locations, vorticity gradient is positive**

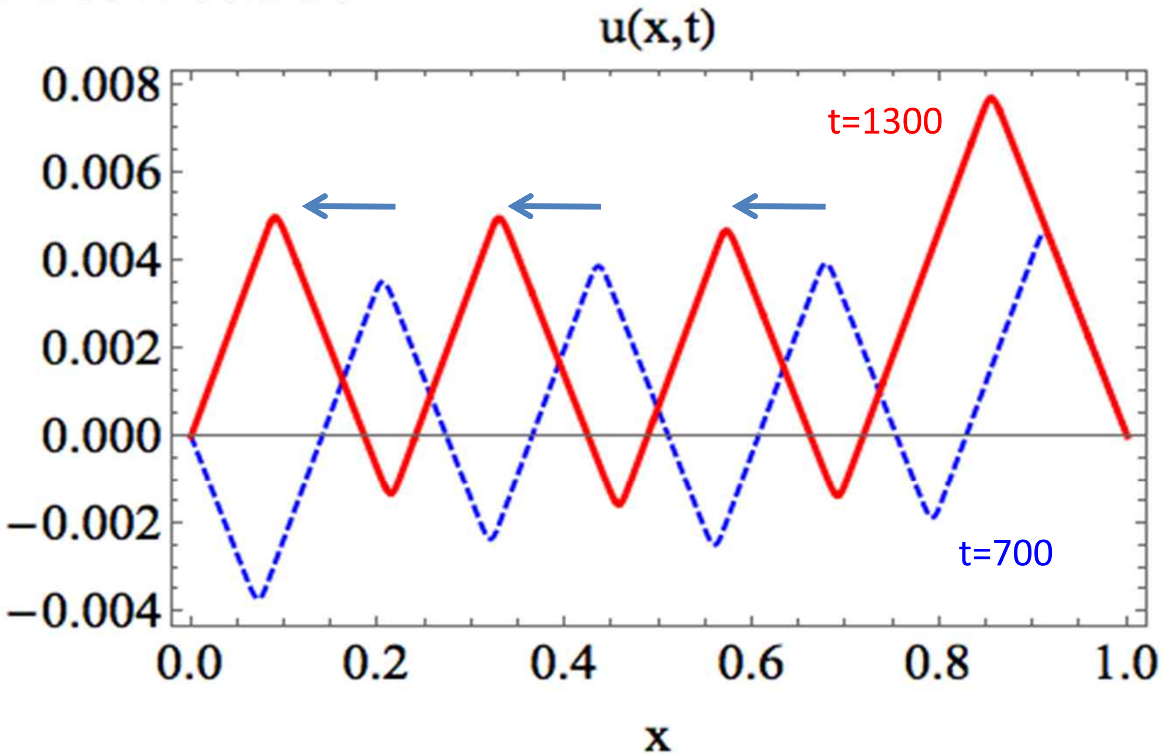




Density
+
Vorticity
lattices



Dynamic Staircases

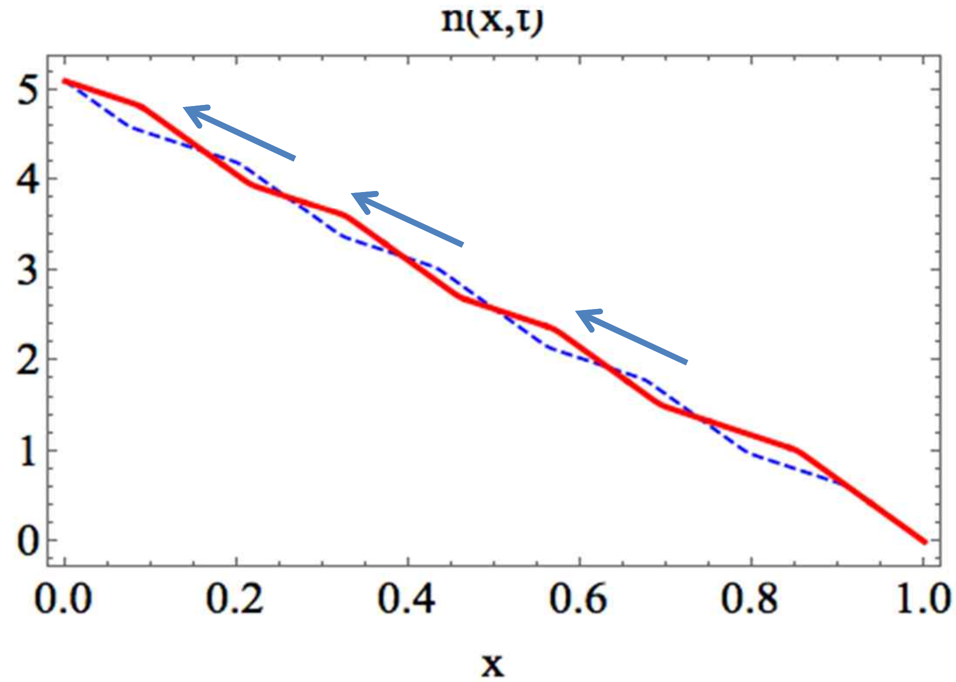


○Shear pattern detaches and delocalizes from its initial position of formation.

○Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at x=0.

○Shear lattice propagation takes place over much longer times. From t~O(10) to t~(10^4).

○Barriers in density profile move upward in an "Escalator-like" motion.



➔ **Macroscopic Profile Re-structuring**

'Non-locality'

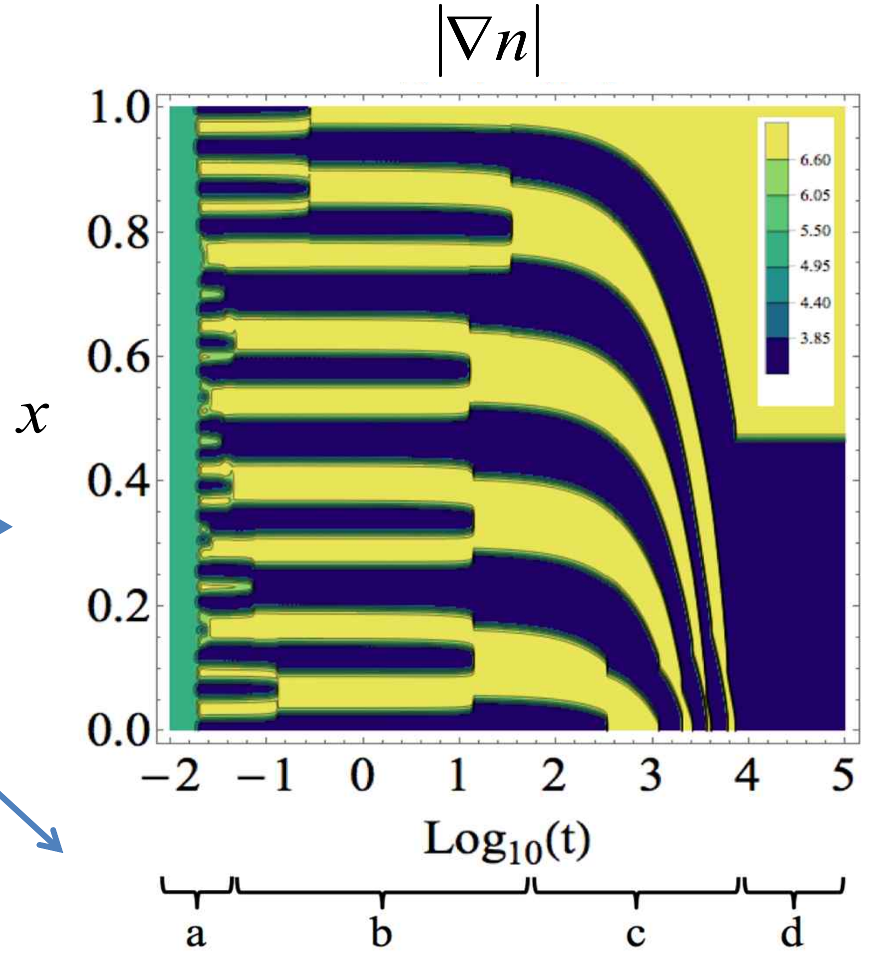
Time evolution of profiles

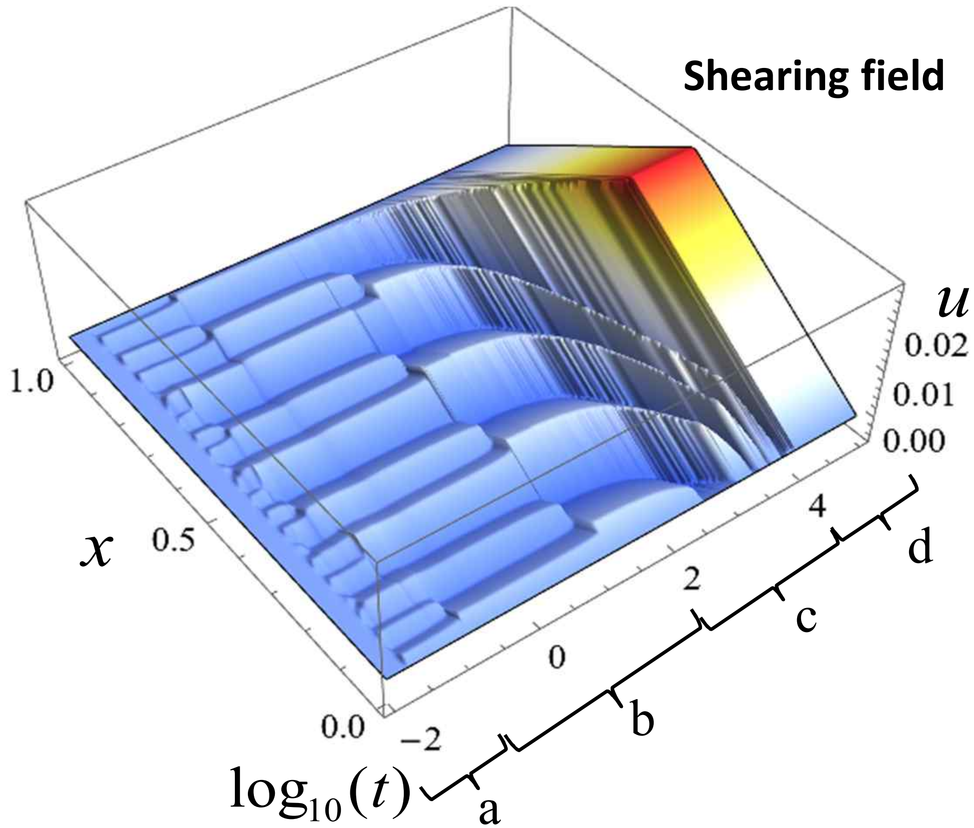
(a) Fast merger of micro-scale SC. Formation of meso-SC.

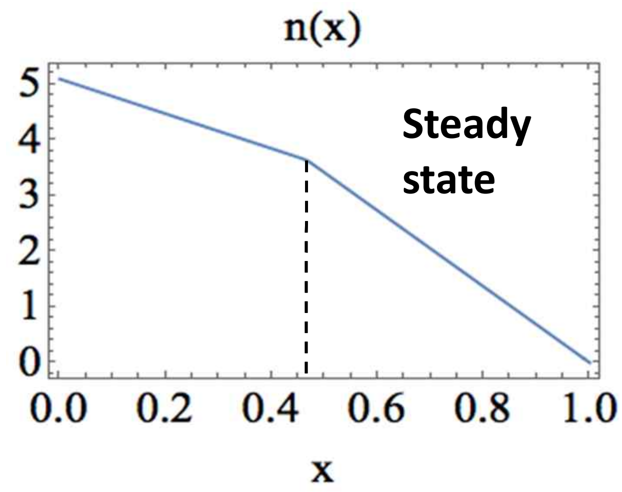
(b) Meso-SC coalesce to barriers

(c) Barriers propagate along gradient, condense at boundaries

(d) Macro-scale stationary profile





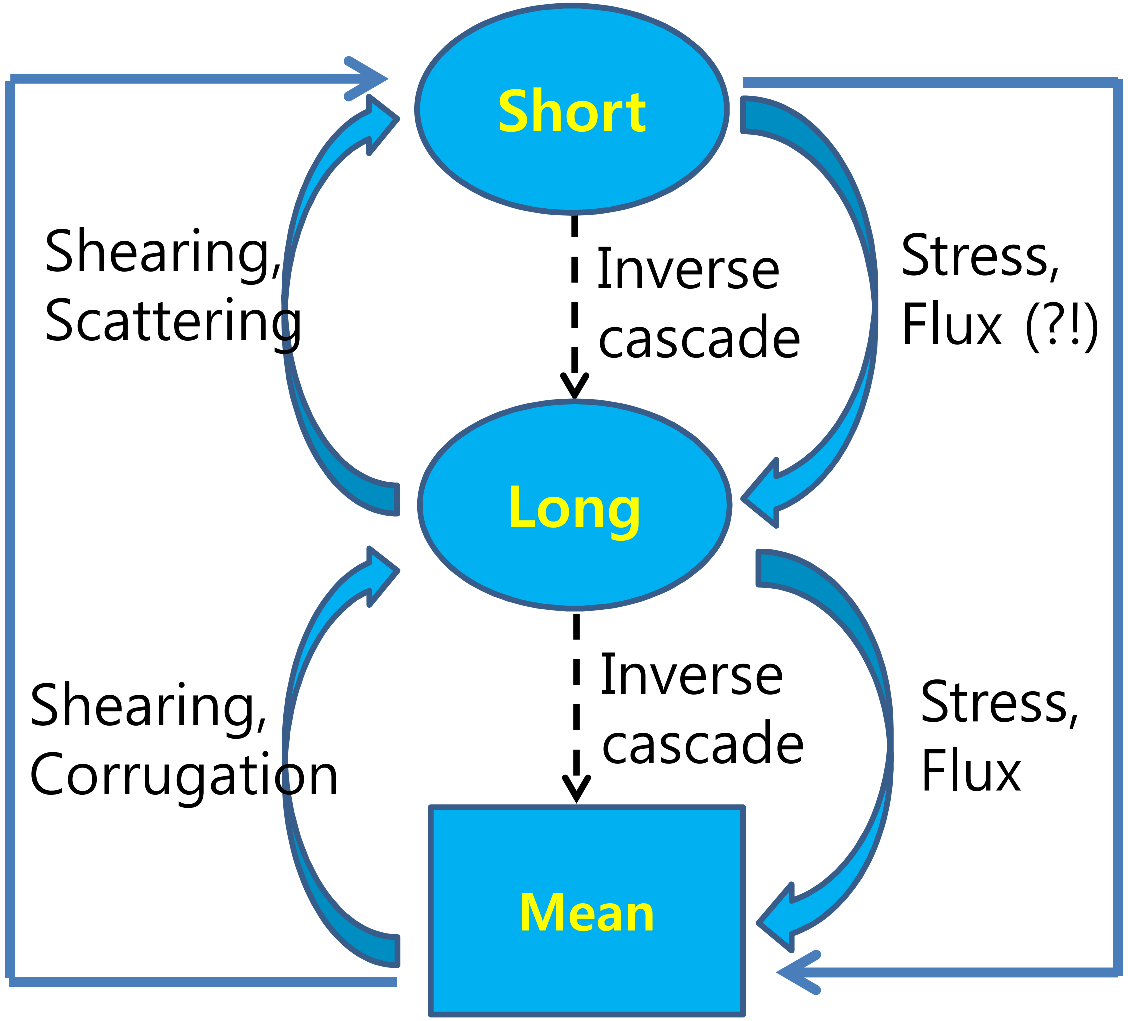


Lessons

- Staircases happen – Important spatial pattern
  - Staircase is 'natural upshot' of modulation in bistable/multi-stable system
  - Bistability is a consequence of mixing scale dependence on gradients, intensity ←→ define feedback process
  - Bistability effectively locks in inhomogeneous PV mixing required for zonal flow formation
  - Mergers result from accommodation between boundary condition, drive(L), initial secondary instability
  - Staircase is natural extension of quasi-linear modulational instabilty/predator-prey model → couples to transport and b.c. ←→ simple natural phenomenon

Towards a Reduced Model

Re-visiting Feedback Loops (Minimal)



Flux
stresses

- **Sources and Sinks**
  - $Q_e \Rightarrow \nabla T_e$, etc
  - Collisional damping of Z.F.
  - Boundary conditions, outflow

Observations re: Modelling

- H+D '04 key elements:

  ① Straining of ETG scales by D.W.'s

  $\sim \nabla_k \cdot D_k^L \cdot \nabla_k \langle N_S \rangle$ → random shearing

  ② ETG stresses negligible, in general

  ③ Noted $\nabla \langle T \rangle$ modulations by drift wave, but did not develop implication

➔ Conclusion that long wavelength activity could regulate short wavelengths, little effect short → long

➔ Expect robust large scale activity will suppress short

Other Elements

- Mode competition for free energy:

Heat source $\rightarrow \nabla T_e \longleftrightarrow \left\{ \begin{array}{l} Q_{e,\,DW} \\ Q_{e,\,ETG} \end{array} \right. \Rightarrow$ Control to population balance

N.B. Flux driven simulation ultimately required

→ Spatial scattering (i.e. 'turbulence spreading') accompanies

straining/shearing

i.e. $\partial_t \langle N_s \rangle = \nabla_x \cdot D^L \cdot \nabla_x \langle N_s \rangle + \cdots$

→ Weak nonlocality introduced

Other Elements, cont'd

- Local energy transfer, via inverse cascade

$$\frac{d}{dt}\varepsilon_{ETG} \sim -\left(\frac{\tilde{V}}{l}\right)_{ETG} \varepsilon_{ETG} \rightarrow \frac{d}{dt}\varepsilon_{I\,TG}$$

n.b.: Energy conservation!

~ effective noise source for ion scale turbulence

- Small scale $\nRightarrow$ fine scale envelope
  - Short wavelength envelope field smooth
  - Both scales contribute to ZF field
  - Enhance inertia due Boltzmann ions is 'linear' effect
  - Suggests ε_D, ε_E as fluctuation fields

A Simple, Tractable Model

- Aims are understanding/insight
- Primitive equations:
  - Fluid DWs, evolving ϕ, T_e
  - ETG, evolving ϕ, T_e
  - $\langle T_e \rangle$, $\langle \phi \rangle$ couple to both populations
- Reduced model:
  - Evolve $\langle T \rangle$, $\langle \phi \rangle$ via ion, electron scale vorticity flux, heat flux
  - Evolve ε_{DW}, ε_{ETG} envelope fields

➔ Ultimately 4 fields in (r, t)

Elements

- Mean T evolution

$$\partial_t \langle T \rangle = -\partial_r \left\{ \langle \tilde{V}_r \tilde{T} \rangle_D + \langle \tilde{V}_r \tilde{T} \rangle_E \right\} + \chi_0 \, \partial_r^2 \langle T \rangle + S_0$$

$$\langle \tilde{v} \tilde{T} \rangle_{D,E} \quad \rightarrow \quad \varepsilon_D, \ \varepsilon_E, \ \nabla T$$

$l_D, \ l_E \quad \rightarrow$ characteristic scales, evolving

- Key Point:
  - Population induced transports compete to carry heat flux
  - Spreading/spatial scattering → weak nonlocality
  - Relative thresholds important

- Mean $\langle V_E \rangle$ evolution

Weighting for electron inertia correction

$$\partial_t \langle V_E \rangle = -\partial_t \{\langle \tilde{V}_r \, \tilde{V}_\perp \rangle_D + w \, \langle \tilde{V}_r \, \tilde{V}_\perp \rangle_E\} - \mu_{ef} \, \langle V_E \rangle$$

<u>Taylor identity</u>: $-\partial_r \langle \tilde{V}_r \, \tilde{V}_\perp \rangle \;\rightarrow\; \langle \tilde{V}_r \, \nabla_\perp^2 \tilde{\phi} \rangle$

$$\langle \tilde{V}_r \, \nabla_\perp^2 \tilde{\phi} \rangle = -\chi_y \frac{\partial^2}{\partial r^2} \langle V_E \rangle + \Pi_{vort}^{res}$$

Off diagonal, ∇T driven
See Ashourvan, PD
for calculation

$$\begin{cases} \chi_y \rightarrow f_D \sqrt{E_D} \, l_{D,mix} \\ \Pi_{\nabla^2 \phi}^{res} \rightarrow f_D^{1/2} \, l_{D,mix} \sqrt{E_D} \dfrac{\Omega_i}{L_T} \end{cases}$$

; Similarly
ETG scales

- <u>Drift Wave Energetics:</u> $\varepsilon_D = \varepsilon_D(r,t)$
  → DW <u>envelope</u> scale

$$\partial_t \varepsilon_D + \partial_r \Gamma_{\varepsilon_D} = -Q_{e,turb} \frac{d}{dx}\langle T\rangle - \langle \tilde{V}_r \tilde{V}_\perp \rangle \frac{\partial}{\partial r}\langle V_E\rangle$$

Relaxation

Reynolds work

$$-\frac{f_D^{1/2}\varepsilon_D^{3/2}}{l_D} + \frac{f_E^{1/2}\varepsilon_E^{3/2}}{l_E} + \frac{d\varepsilon_D}{dt}\Big|_{cross-scale}$$

<u>Inverse cascade from ETG</u> → excitation

Γ_{ε_D} = spreading = $-l_D (f_D \varepsilon_D)^{\frac{1}{2}} \partial_r \varepsilon$

$Q_{e,turb} \frac{\partial}{\partial x}\langle T\rangle$ = ∇T relaxation ↔ $Q_{e,above}$

$\langle \tilde{V}_r \tilde{V}_\perp\rangle \frac{\partial}{\partial r}\langle V_E\rangle \Rightarrow \chi_y \left(\frac{\partial}{\partial r}\langle V_E\rangle\right)^2 + \langle V_E\rangle \Pi^{\mathrm{res}}_{\mathrm{D},\nabla^2\phi}$

- <u>ETG Energetics:</u> $\varepsilon_E = \varepsilon_E(r,t)$
  → ETG <u>envelope</u> scale

$$\partial_t \varepsilon_E + \partial_r \Gamma_E = -Q_{e,E,turb}\ \frac{d}{dx}\langle T\rangle - w_E\ \langle V_r \tilde{V}_\perp\rangle_E \frac{d}{dr}\langle V_E\rangle$$

Energy drain to DW

$$-f_E^{1/2}\ \varepsilon_E^{3/2}/l_E\ + \frac{d}{dt}\varepsilon_E|_{cross\ -scale\ ,E}$$

Γ_E = spreading = $-l_E(f_E\varepsilon_E)^{1/2}\,\partial_r\varepsilon$ ➔ significant

$$-Q_{e,turb}\ \frac{\partial}{\partial x}\langle T\rangle\ =\ l_{m\ ix,\,E}\ (f_E\varepsilon_E)^{1/2}\left(\frac{d}{dx}\langle T\rangle\right)^2$$

$$w_E\ \langle\tilde{V}_r\tilde{V}_\perp\rangle\frac{d}{dr}\langle V_E\rangle = w\ \chi_{y,E}\left(\frac{d}{dr}\langle V_E\rangle\right)^2 + \langle V_E\rangle\Pi^{res}_{E,\nabla^2\phi}$$

$$\chi_{y,E} = l_E\ (f_E\varepsilon)^{1/2},\quad \Pi^{res}_{E,\nabla^2\phi} = f_D^{1/2} l_E\sqrt{E}\ \Omega_E/L_T$$

Characteristic Scales:

- Need to specify l_D, l_E as basic correlation scales on DW, ETG scales
- These are modified by shearing, with shear computed self-consistently
- Envelope scales → i.e. $\varepsilon_D, \varepsilon_E$ scales evolved self-consistently with profiles: n.b. ε_E not obviously limited to electron scale

$$l_{mix,D}^2 \;=\; l_{0,D}^2 \;/\; \left[1 + k_D^2 \rho_s^2 \langle V_E \rangle' \tau_{c,D}^2\right];$$ with τ_c hybrid (BDT)

$$\rightarrow \;\; l_{0,D}^2 \;/\; \left[1 + \left| V_E' \;/\; k_{\perp,D}\, \rho_s\, k_m \sqrt{\varepsilon_D} \right|\right];$$ ETG, D → E

Cross-Scale Coupling: Nonlocal in scale

- Straining and Scattering of shorts by longs are essential
- Adiabatic theory for shorts → $\langle N_E \rangle$:

$$\partial_t \langle N \rangle = \nabla_x \cdot D_x \cdot \nabla_x \langle N \rangle + \nabla_k \cdot D_k \cdot \nabla_k \langle N \rangle$$

$D_x,\ D_k$ set by DW field

ε_D

$$D_x = \sum_{k,DW} \langle \vec{V}\vec{V} \rangle_k \tau_{c,k} = \sum_{k,DW} \langle \vec{V}\vec{V} \rangle_k \delta\left(\omega - k \cdot V_{gr,E}\right)$$

$$D_k = \sum_{k,DW} \left| \frac{\partial}{\partial x} \left(\omega_k \frac{\hat{T}_k}{T} + k_\theta \hat{V}_{E,k} \right) \right|^2 \tau_{c,k}$$

ε_D

Cross-Scale Coupling, cont'd

DW induced "spreading' of ETG's envelope!

$$\frac{d}{dt}\varepsilon_E\Big|_{cross-scale} = \frac{\partial}{\partial x}D_x[\varepsilon_D]\frac{\partial}{\partial x}\varepsilon_E$$

$$-\int V_{gr,E}\cdot D_k[\varepsilon_D]\cdot \nabla_k\langle N\rangle_E\, dk$$

For power law spectrum:

index

$$\frac{d}{dt}\varepsilon_E\Big|_{cross} = \text{spreading(above)} + \frac{V_{gr}}{\omega_{ETG}}D_{kr}[\varepsilon_D]\,\alpha\,\varepsilon_E$$

n.b.: $-V_{gr}/\,\omega < 0$ → backward wave ➔ straining <u>decrement</u>

➔ <u>Spectral structure</u> enters multi-scale problem. Resolution challenges to DNS – link cross-scale interaction spectral structure

So, the model

- 4 equations in r, t + mixing lengths (with shearing)
- Calculate $\varepsilon_D(r,t)$, $\varepsilon_E(r,t)$, $\langle T_e(r,t)\rangle$, $\langle V_E(r,t)\rangle$

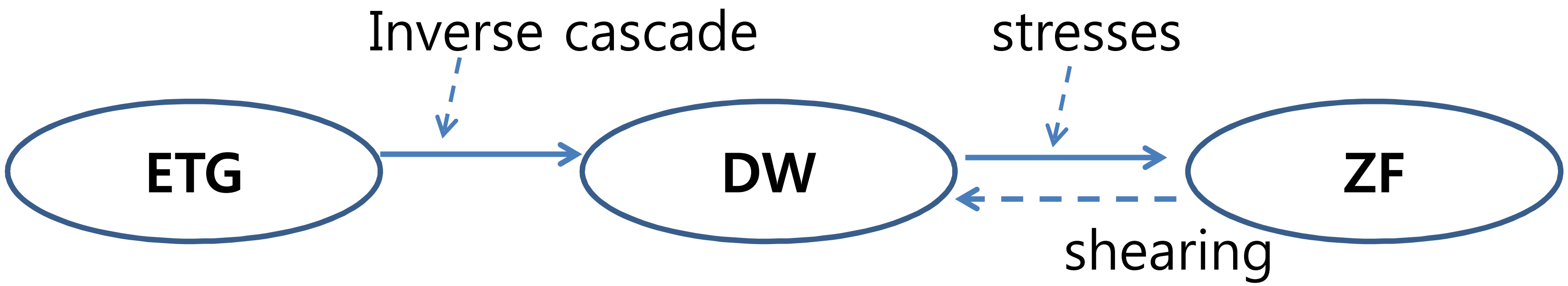
  n.b. 1 equation beyond Ashourvan, P.D. '16, '17

- Control parameters:
  - Heat source → DW, ETG compete
  - Mean flow decrements
  - $l_{0,E}$, $l_{0,\ D}$
- Un-resolved questions
  - ETG stress on DW

* – Inverse cascade to mean? → ZF $\Big\{$ Noise + Modulation

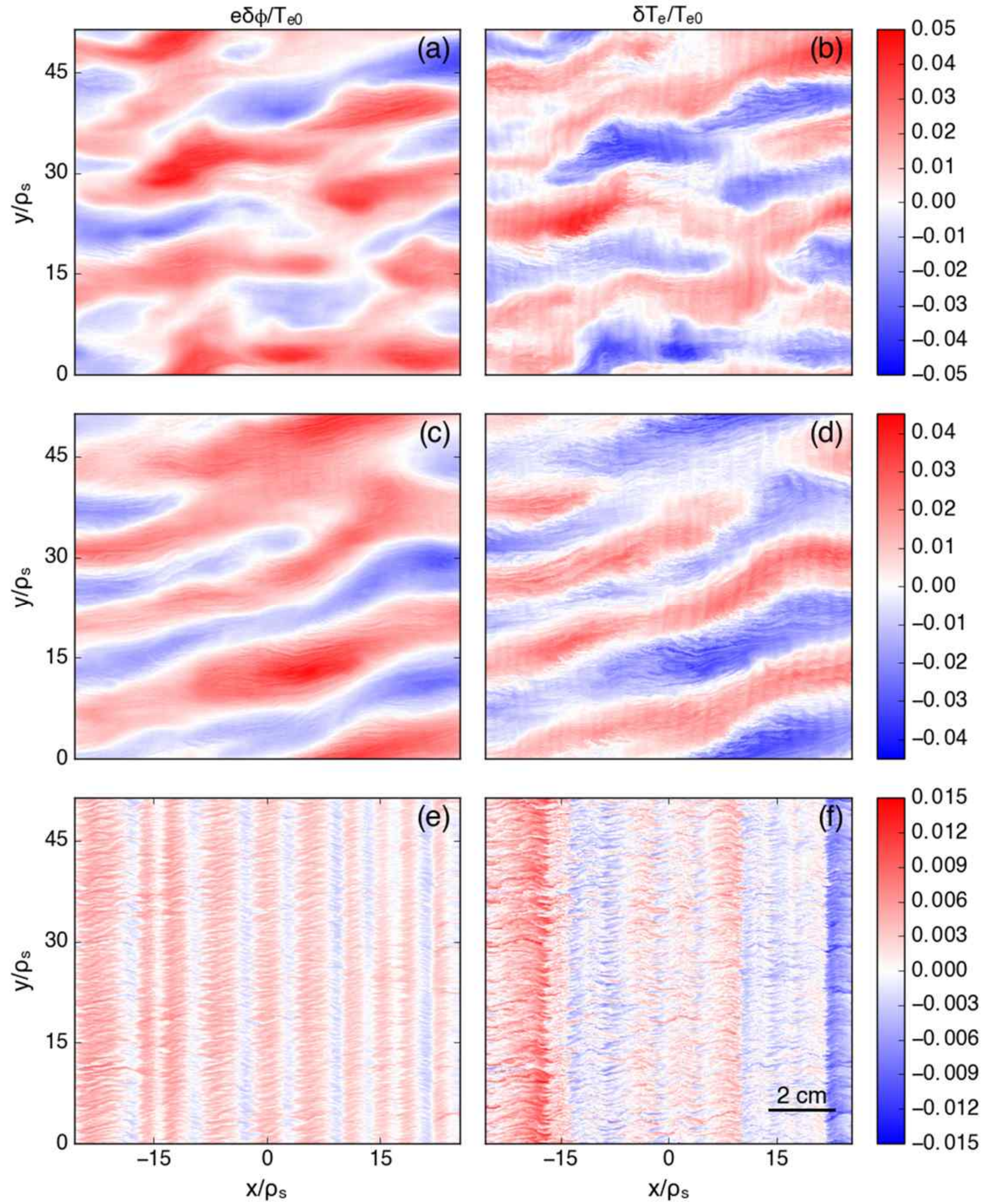
Consideration of system suggests:

- 'Dimits Shifts' like state for ETG's should exist



  - ETG can generate a state of ion scale ZF ?!
  - Regulate DW's by shearing
  - Lesson: Beware assumptions re: envelope scale

Low k
dominated



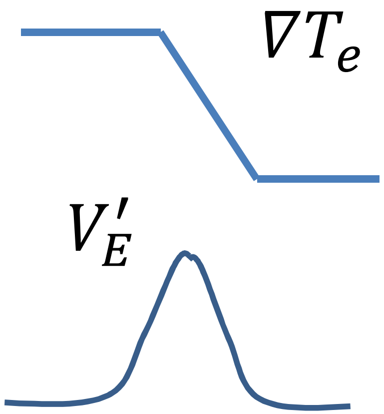
"Dimits Shift"
Regime →
ZF dominated

C. Holland, et. al.
NF 2017

- Transition from low k → 'Dimits Shift' regime as $\langle V_E \rangle'_{ext}$ increased
- How are <u>ion</u> ZF's energized in ETG regime? → inverse cascade as channel ?!

Multi-Scale, Multi-Step Staircases

- DW + ZF forms staircase structure (Dif-Pradalier, Ashourvan, P.D. '10, '16, '17 after Dritchel, McIntyre '08)
- Mechanism is ↔ Envelope scale selection
  - Inhomogeneous mixing(PV) via modulation
  - Feedback on flux

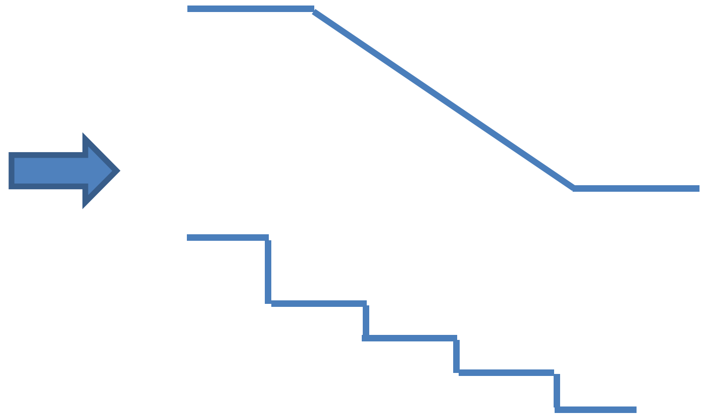


- Region of steep ∇T_e form
- DW suppressed, but ETG ~ insensitive mean $\langle V_E \rangle'$

So

- ETG staircase forming in ∇T_e jumps !?

  (survive strong DW shears?)



Or

- ETG transport limits staircase formation via feedback on ∇T_e

  (damp ion staircase as ∇T steepens?)

➔

- DW, ETG competition for free energy source is essential!
- Relative thresholds significant

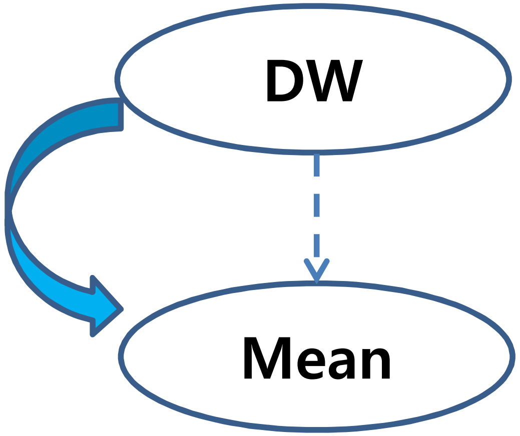
Ongoing and Plans:

- Numerical solution (1+1) of 4 equation model (easily implemented)
- Explore:
  - Threshold + flow damping scans
  - Spatial patterns – staircase (and its fate)
  - Dimits shift regimes, ETG driven ZFs structure
  - Q_e^D, Q_e^E branching ratio vs ∇T_e
  - Roles of straining, scattering in cross-scale interaction

A New Question re: Old Friend

- How understand interaction between Reynolds stresses and inverse cascade in Zonal Flow formation?

i.e.



- 'Predator-Prey' formulations have focused on Reynolds stress
- R,H retained noise, missed $\nu < 0$

- Noise alone insufficient!
- 'Diagonal' part of R.S. / vorticity flux plays essential role!

➔ coming attraction ....

Conclusion

- Multi-scale physics still yield new questions for research.
- Yet to confront non-locality, avalanching etc.
- Theory, reduced modelling necessary for understanding large scale simulations.

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