On the Physics of Avalanche Formation in Confined Plasmas

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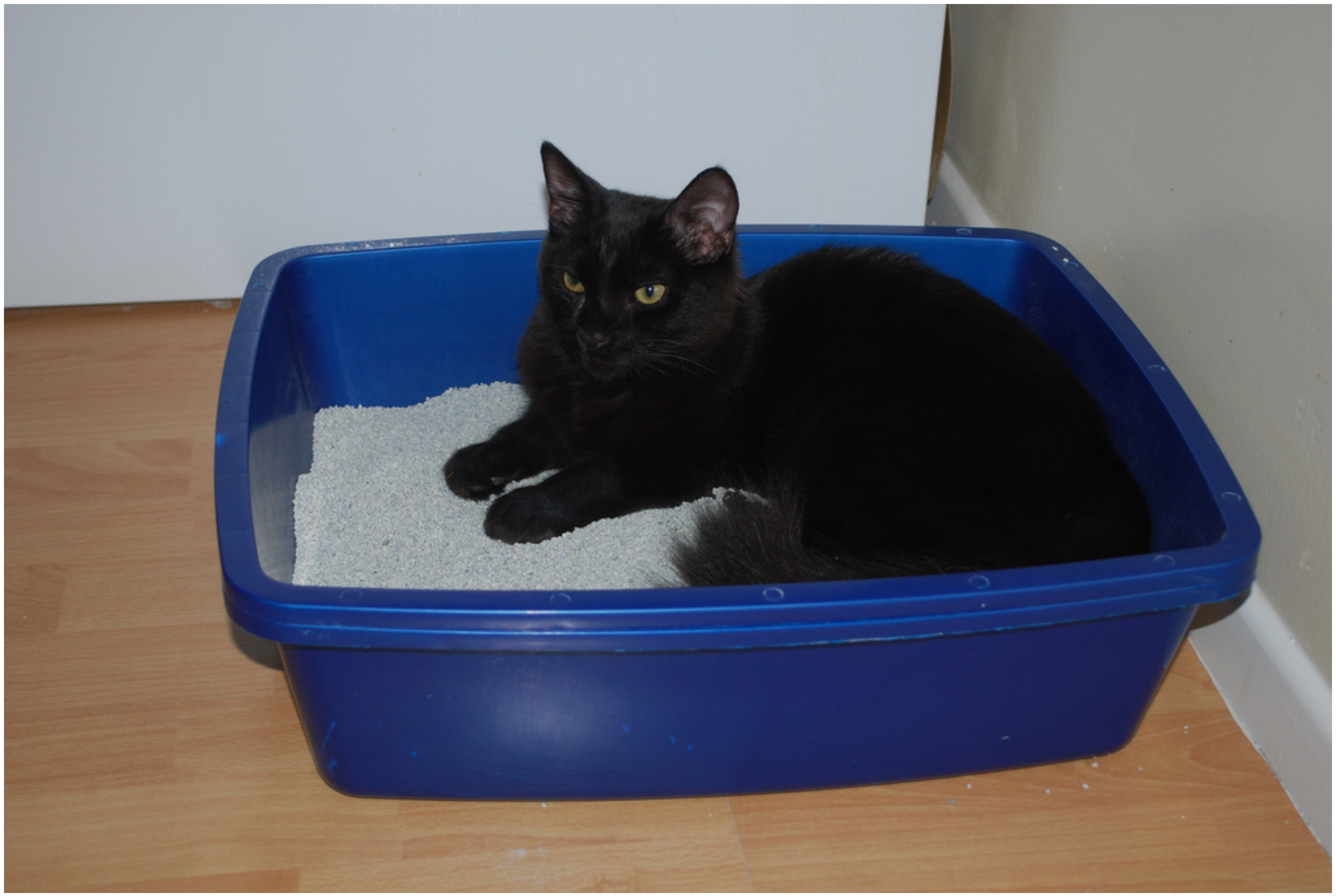
(Brexit 1776)

Aix-en-Provence

Or

What's Buried in the Pile?



Collaborators:

- Z.B. Guo, UCSD → PKU (see also, this meeting)
- A. Ashourvan, PPPL
- Y. Kosuga, Kyushu
- O.D. Gurcan, Ecole Polytechnique
- M. Malkov, UCSD
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Thesis:

- While 'avalanching', 'SOC models' widespread, little analysis of physics of the avalanching process available
- Avalanche concept is major useful output of SOC
- PV conservation + PV mixing central
  - Profile toppling inexorably coupled to zonal flow drive
  - 'avalanche' model MUST respect PV conservation
- Present day models/thinking miss this!

Outline

- Why? – scale selection problem

On one hand…

- SOC and Avalanching – basic ideas
- An MFE perspective – significance

On the other hand…

- PV mixing – zonal shearing ➔ feedback
- The crux of the Issue
- Promising directions:
  - Bi-stability
  - Phase dynamics
- Discussion

Why Avalanches?

- It's the ~~economy~~ scale selection problem, stupid!

A Simpler Problem:

→ The Sewer Pipe

- Essence of confinement problem:
  - given device, sources; what profile is achieved?
  - $\tau_E = W/P_{in}$, How optimize W, stored energy
- Related problem: Pipe flow → drag ↔ momentum flux

ΔP → pressure drop

l

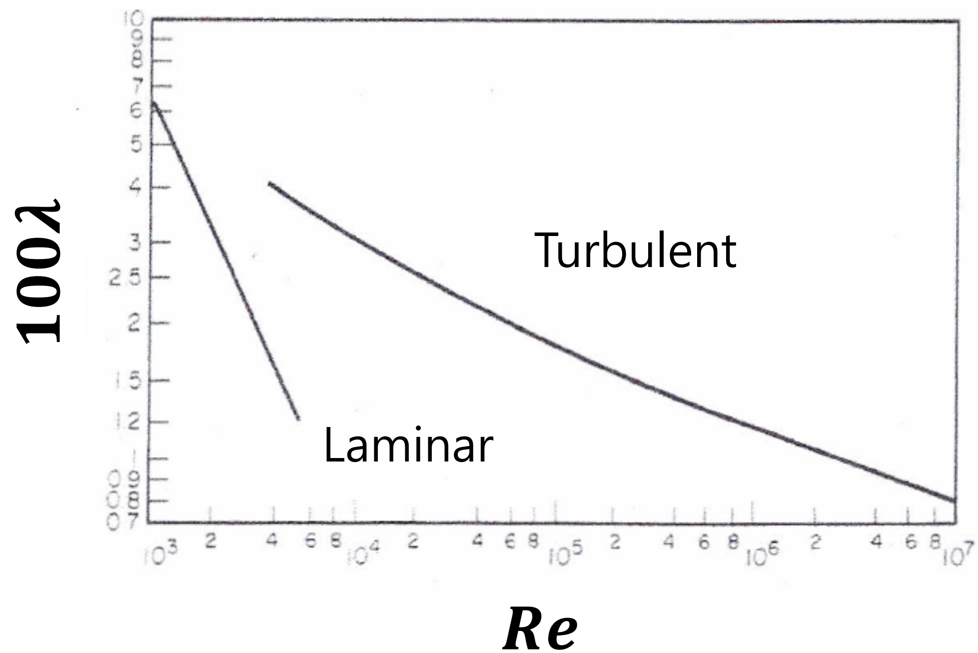
$$\Delta P \pi a^2 = \rho V_*^2 2\pi a l$$

→ friction velocity V_* ↔ u

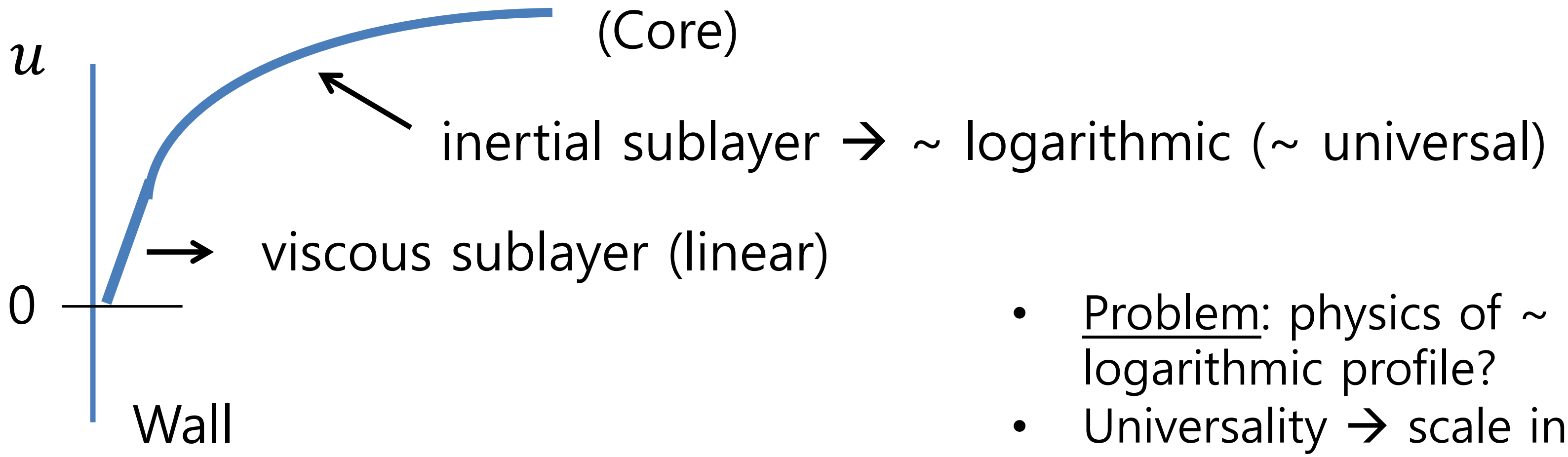
Balance: momentum transport to wall

(Reynolds stress) vs ΔP

➔ Flow velocity profile



$$\lambda = \frac{2a\Delta P/l}{1/2\rho u^2}$$



- Problem: physics of ~ universal logarithmic profile?
- Universality → scale invariance

- Prandtl Mixing Length Theory (1932)
  - Wall stress = $\rho V_*^2 = -\rho \nu_T \, \partial u / \partial x$ or: $\dfrac{\partial u}{\partial x} \sim \dfrac{V_*}{x}$ ← Spatial counterpart of K41
    - ν_T: eddy viscosity
    - x: Scale of velocity gradient?
  - Absence of characteristic scale →

$$\nu_T \sim V_* x$$

$$u \sim V_* \ln(x/x_0)$$

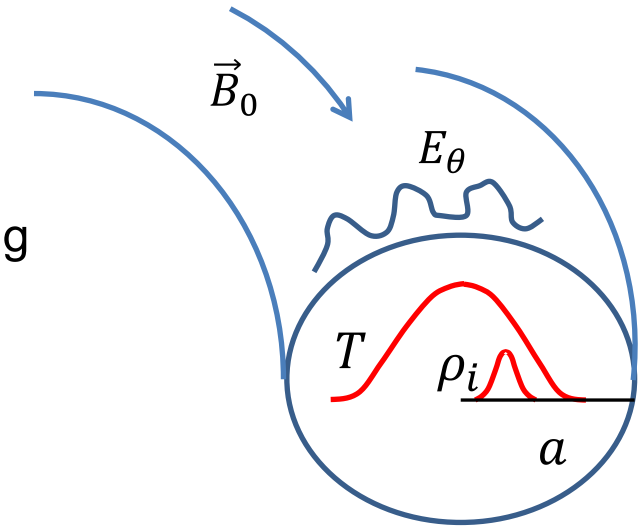
$x \equiv$ mixing length, distance from wall

Analogy with kinetic theory ...

$\nu_T = \nu \to x_0$, viscous layer → $x_0 = \nu / V_*$

En Marche! to Plasmas

Primer on Turbulence in Tokamaks I



- Strongly magnetized
  - Quasi 2D cells, Low Rossby #

* – Localized by $\vec{k} \cdot \vec{B} = 0$ (resonance) - pinning

- $\vec{V}_\perp = +\frac{c}{B}\,\vec{E} \times \hat{z}, \quad \frac{V_\perp}{l\Omega_{ci}} \sim R_0 \ll 1$
- $\nabla T_e, \nabla T_i, \nabla n$ driven
- Akin to thermal convection with: g → magnetic curvature

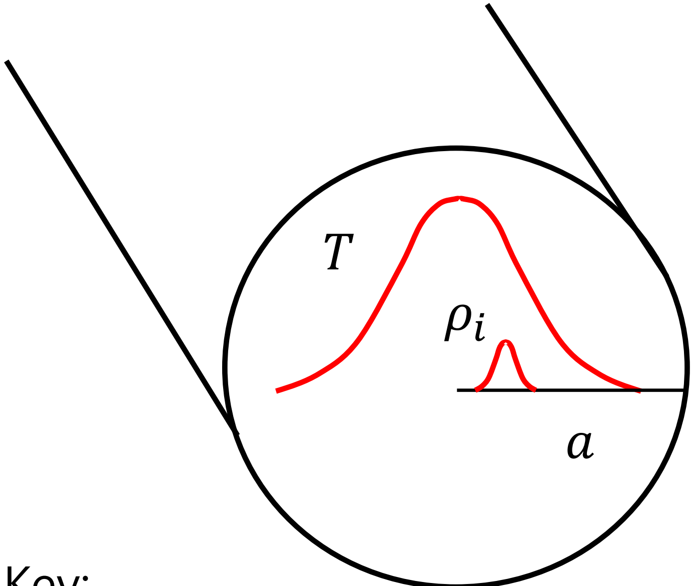
→ • $\mathrm{Re} \approx VL/\nu$ ill defined, not representative of dynamics

- Resembles wave turbulence, not high Re Navier-Stokes turbulence

→ • $K \sim \tilde{V}\tau_c/\Delta \sim 1$ → *Kubo* # ≈ 1

→ • Broad dynamic range, due electron and ion scales, i.e. a, ρ_i, ρ_e

Primer on Turbulence in Tokamaks II



Key:

2 scales:

$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

$\rho_* \equiv \rho/a$ ➔ key ratio

$\rho_* \ll 1$

- Characteristic cell scale ~ few ρ_i → "mixing length"
- Characteristic velocity $v_d \sim \rho_* c_s$
- Transport scaling: $D_{GB} \sim \rho\, V_d \sim \rho_*\, D_B$

  $D_B \sim \rho\, c_s \sim T/B$
- i.e. Bigger is better! ➔ sets profile scale via heat balance (Why ITER is huge…)
- Reality: $D \sim \rho_*^{\alpha}\, D_B\,,\ \alpha < 1$ ➔ 'Gyro-Bohm breaking'
- 2 Scales, $\rho_* \ll 1$ ➔ key contrast to pipe flow

THE Question ↔ Scale Selection

- Worst Fear (from pipe flow):
  - $l \sim a$
  - $D \sim D_B$
- Hope (mode scales)
  - $l \sim \rho_i$
  - $D \sim D_{GB} \sim \rho_* D_B$
- Reality: $D \sim \rho_*^{\alpha} D_B,\quad \alpha < 1$ → Avalanches?!

Why? What physics / competition set α?

Avalanches and SOC Ideas

- A Short OV

What is SOC? or What do we THINK it is?

(cf: Jensen)

- (Constructive)

  Slowly driven, interaction dominated threshold system

  Classic example: sandpile

- (Phenomenological)

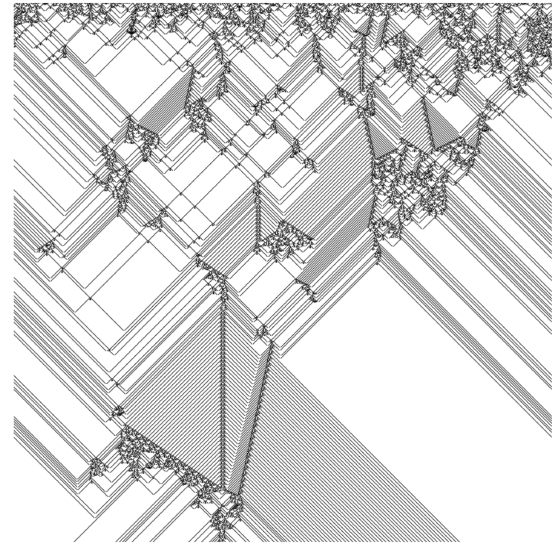
  System exhibiting power law scaling without tuning.

  Special note: 1/f noise; flicker shot noise of special interest

  See also: sandpile

N.B.: $1/f$ means $1/f^{\beta}$, $\beta \leq 1$

What is SOC?, cont'd



- Elements:

→ Interaction dominated

– Many d-o-fs { Cells / Modes }

– Dynamics dominated by d-o-f interaction i.e. couplings

➔ AVALANCHE – major useful output...

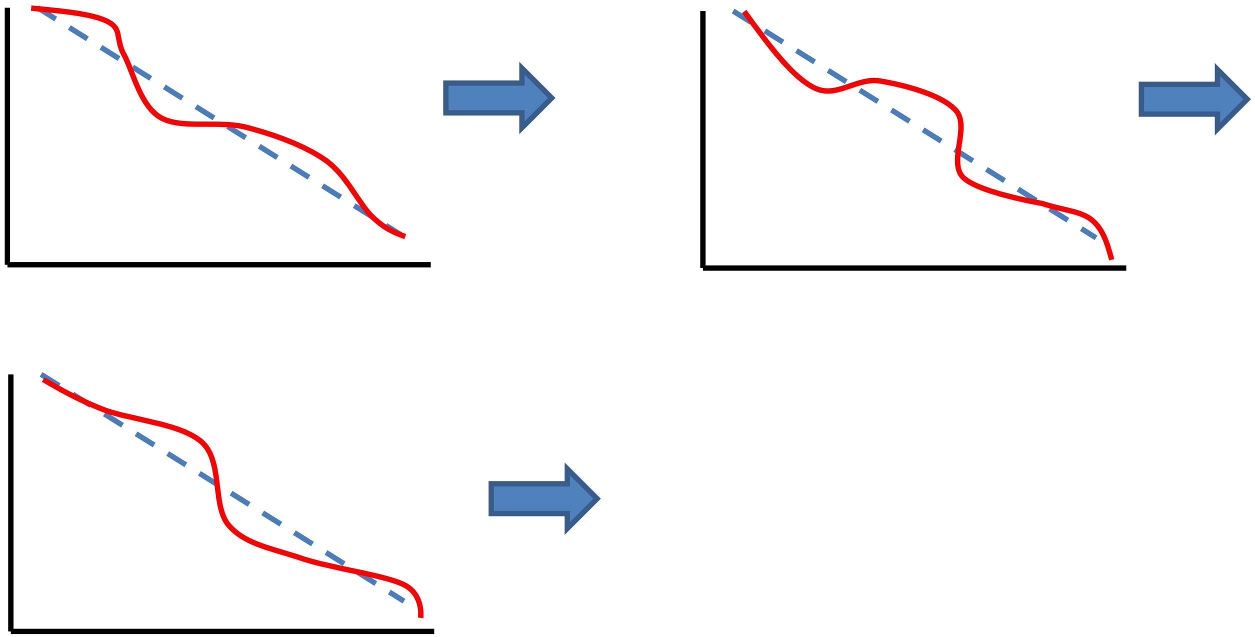
→ Threshold and slow drive

– Local criterion for excitation

– Large number of accessible meta-stable, quasi-static configuration

– 'Local rigidity' ←→ "stiffness" !?

- Multiple, metastable states

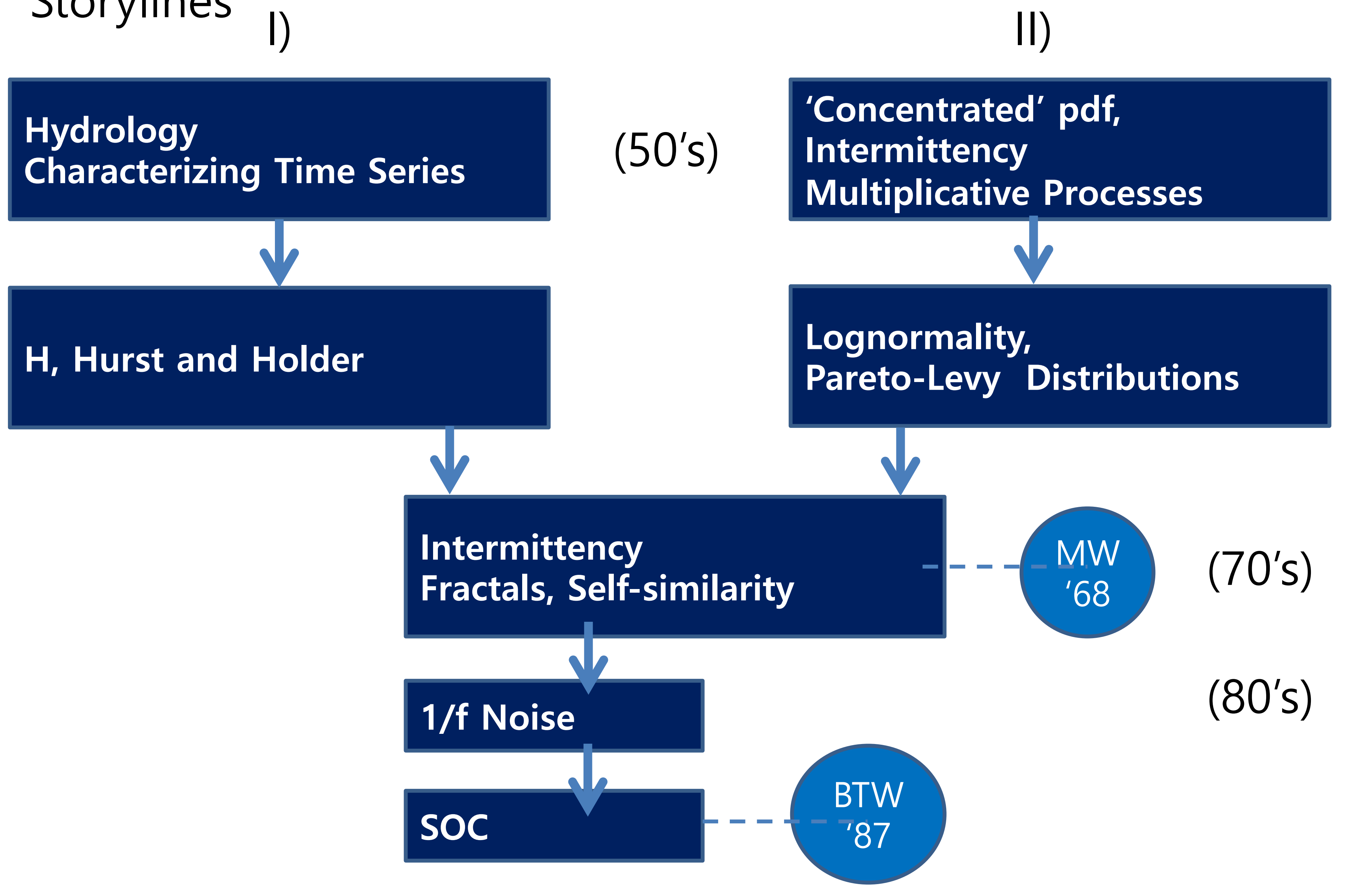


- Proximity to a 'SOC' state → local rigidity
- Unresolved: precise relation of 'SOC' state to marginal state

- Threshold and slow drive, cont'd
  - Slow drive uncovers threshold, metastability
  - Strong drive buries threshold – does not allow relaxation between metastable configurations
  - How strong is 'strong'? – set by toppling/mixing rules, box size, b.c. etc.
- Power law ←→ self-similarity
  - 'SOC' intimately related to:
    - Zipf's law: P(event) ~ 1/(size) (1949)
    - 1/f noise: S(f) ~ 1/f

A Brief Intellectual Pre-History of 'SOC'

- Storylines



- 1/f Noise?

  A few observations:

  – Zipf and 1/f related but <u>different</u>

  Zipf → $P(\Delta B) \sim 1/|\Delta B|$

  $1/f$ → $\langle(\Delta B)^2\rangle_\omega \sim 1 \,/\, \omega$

  Both embody:

    - Self-similarity
    - Large events rare, small events frequent → intermittency phenomena
    - 1/f linked to H→1

  – 1/f noise (flickers, shot…)

    - Ubiquitous, suggests 'universality'
    - Poorly understood, circa 80's

- N.B.: Not easy to get 1/f ...
- In usual approach to ω spectrum; ←→ (DIA, EDQNM, Dupree, Kadomtsev, Kraichnan, Krommes):

$$\langle \phi(t_1)\phi(t_2)\rangle = \left|\hat{\phi}\right|^2 e^{-|\tau|/\tau_c}$$

➔ $S(\omega) = \frac{1/\tau_c}{\omega^2+1/\tau_c^2} \sim \frac{1}{\omega^2}$

i.e. τ_c imposes scale, but 1/f scale free !? Key distinction of diffusion

and shot noise

- N.B.: Conserved order parameter may restore scale invariance $\nabla \cdot (\,) \rightarrow$ $1/\tau_c \rightarrow K^2 D$
- But, consider ensemble of random processes each with own τ_c (Montroll, BTW)

$$S(\omega)_{eff} = \int_{\tau_{c1}}^{\tau_{c2}} P(\tau_c) S_{\tau_c}(\omega) d\omega$$

Probability of τ_c

- And… demand $P(\tau_c)$ scale invariant, i.e.

  $P(\tau_c) = d\tau_c/\tau_c$

  $S(\omega) = \frac{\tan^{-1}(\omega\tau_c)}{\omega}\Big|_{\tau_{c1}}^{\tau_{c2}} \sim 1/\omega,$ recovers 1/f !

  → but what does it mean? …

- So, circa mid 80's, need a simple, intuitive model which:
  - Captures 'Noah', 'Joseph' effects in non-Brownian random process (H→1)
  - Display 1/f noise

SOC at last !

- Enter BTW '87:

Self-Organized Criticality: An Explanation of $1/f$ Noise (7000+ cites)

Per Bak, Chao Tang, and Kurt Wiesenfeld
Physics Department, Brookhaven National Laboratory, Upton, New York 11973
(Received 13 March 1987)

We show that dynamical systems with spatial degrees of freedom naturally evolve into a self-organized critical point. Flicker noise, or $1/f$ noise, can be identified with the dynamics of the critical state. This picture also yields insight into the origin of fractal objects.

- Key elements:
  - Motivated by ubiquity and challenge of 1/f noise (scale invariant)
  - Spatially extended excitations (avalanches)

  Comment: statistical ensemble of collective excitations/avalanches is intrinsic

  - Evolve to 'self-organized critical structures of states which are barely stable'

  Comment: SOC state ≠ linearly marginal state!

  SOC state is <u>dynamic</u>

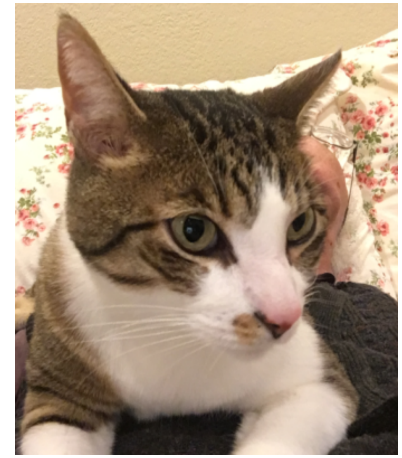
- Key elements, cont'd:
  - "The combination of **dynamical minimal stability** and spatial scaling leads to a **power law for temporal fluctuations**"
  - **"Noise propagates through the scaling clusters by means of a "domino" effect upsetting the minimally stable states"**

    Comment: space-time propagation of avalanching events
  - "The critical point in the dynamical systems studied here is an attractor reached by starting far from equilibrium

    Comment: Noise essential to probe dynamic state

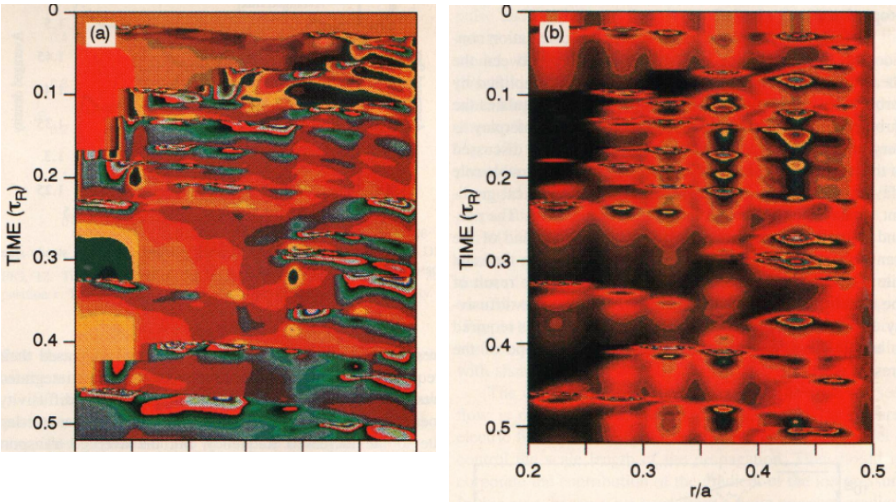
An MFE Perspective

Beyond the Box



- "Why don't you guys think outside the (sand) box and do real science?"
- Simulations! (continuum)
  - (BAC, et al '96) Flux driven resistive interchange turbulence; "weak drive"
    - Noisy source: $S_0 = S(r) + \tilde{S}$ ← noise
    - Reynolds stress driven flows, viscosity
    - Threshold: ala' Rayleigh, ∇P vs ν_c, D_c
  - Flux drive, fast gradient evolution essential, as $V_{aval} \leq V_*$

Some Findings: Avalanches happen!



– Clear difference in LHS pressure contours

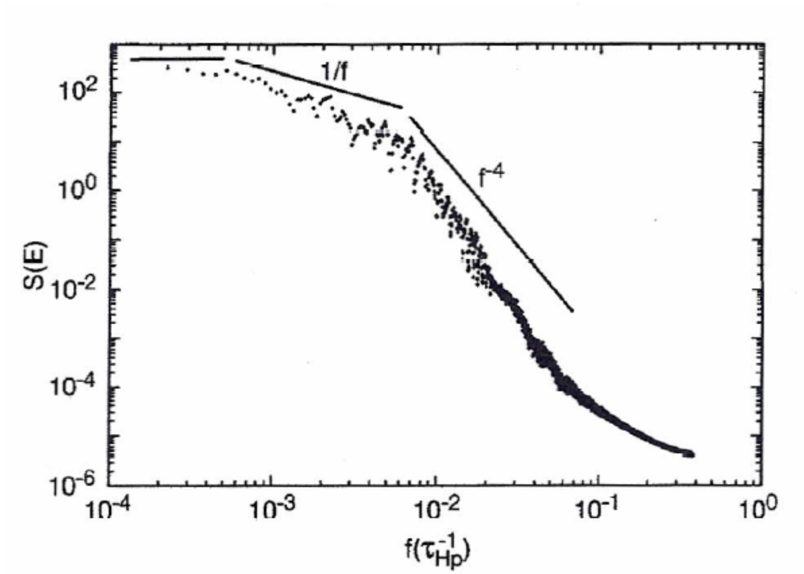
vs RHS $(e\phi/T)_{rms}$ contour

– Avalanches evident in δP

But

– Modes, resonant surfaces in $e\phi/T$

➔ illustrates collective character of avalanches

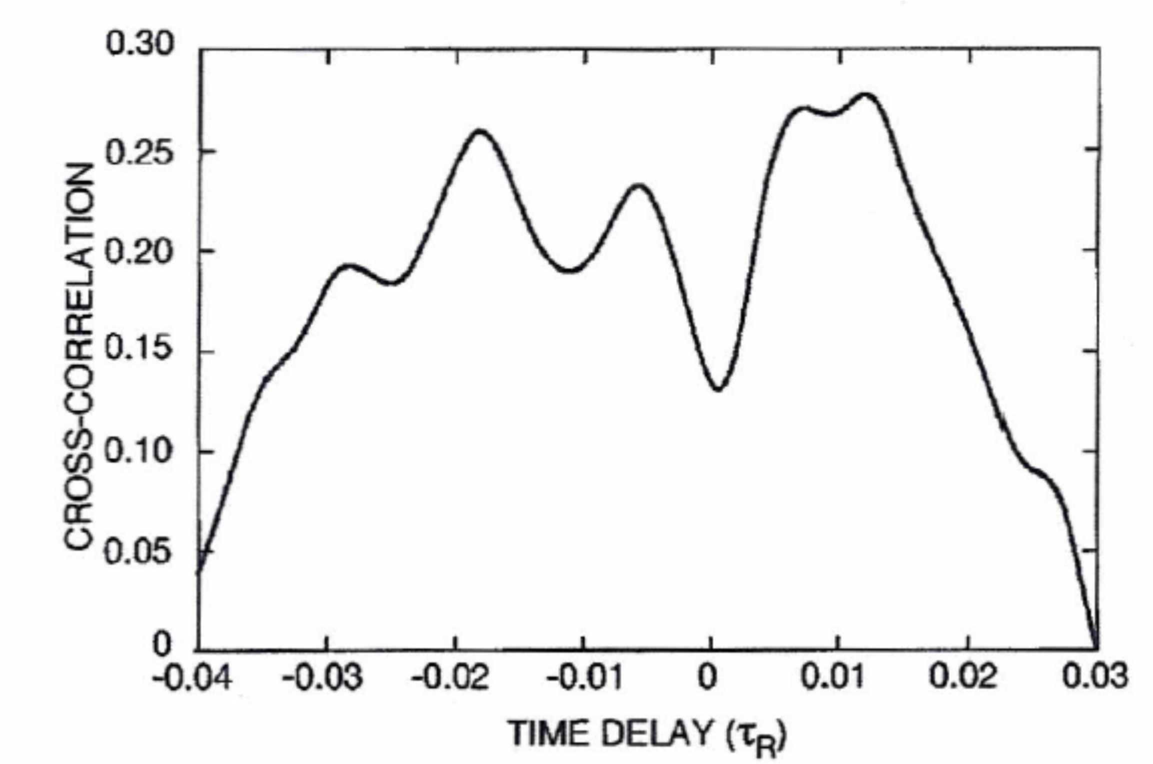


– 1/f recovered in $\langle \left(\frac{e\tilde{\phi}}{T}\right)^2 \rangle_\omega$

– Very similar to pile

– Later observed in flux

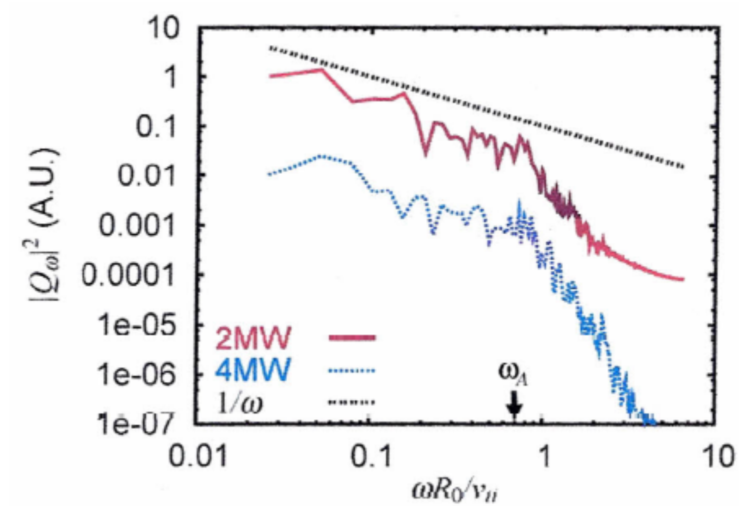
- 2 peaks in cross correlation of low frequency modulation



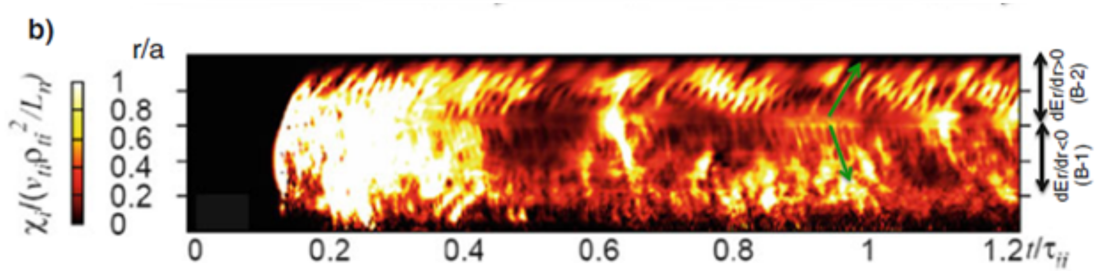
2 peaks → ingoing, outgoing avalanches

- Shear flow can truncate avalanches, ala' pile

But <u>real</u> men do gyrokinetics !



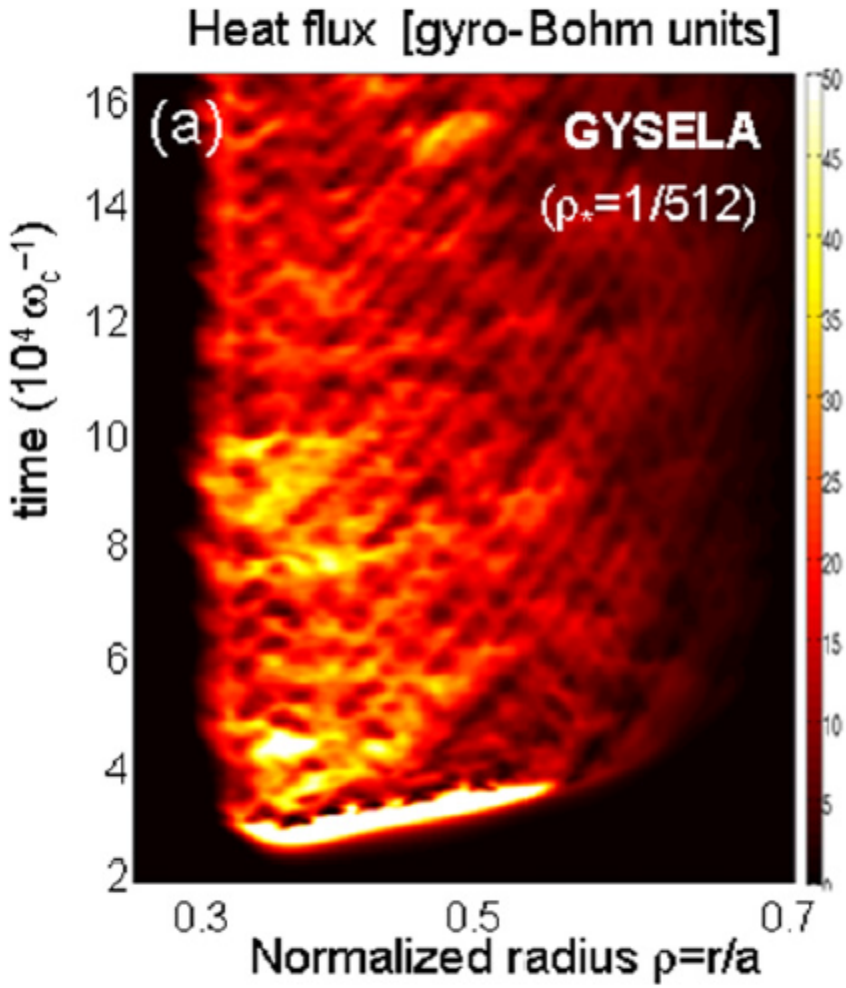
- Idomura, et al (2009)
  - Flux driven ITG, GT5D
  - Also explored intrinsic flow



- 1/f evident in $|Q_\omega|^2$
- $f^0, 1/f, 1/f^\alpha$ $(\alpha \gg 1)$ ranges, ala' pile and g-mode. Sic transit gloria GK

- GYSELA Results: Avalanches Do 'matter'

GYSELA, rhostar=1/512 [Sarazin et al., NF 51 (2011) 103023]



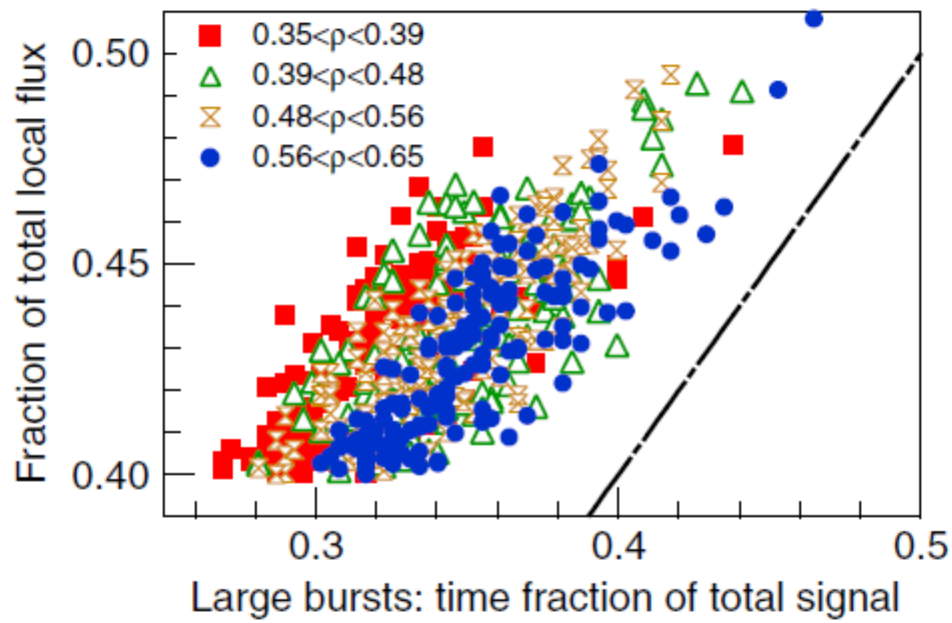


Figure 2. Fraction of the local radial turbulent heat flux carried out by a certain fraction of the largest scale bursts, as estimated from figure 1(*a*) (GYSELA data). Each point refers to one specific radial location. The colours allow one to distinguish four different radial domains. The considered time series ranges from $\omega_{c0}t = 56\,000$ to $\omega_{c0}t = 163\,000$.

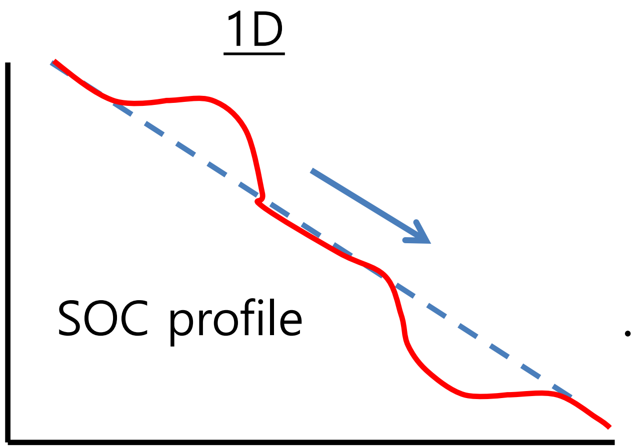
- Toward a Model Hwa, Kardar '92; P.D., T.S.H. '95; et seq.
  - **Is 'SOC' intimately connected to self-similarity, 'cascade' etc ultimately rooted in fluid turbulence – relate?**

And:

  - C in 'SOC' → criticality
  - Textbook paradigm of criticality (tunable) is ferromagnetic ala' Ginzburg, Landau → symmetry principle!?

And:

  - **Seek hydro model for MFE connections**



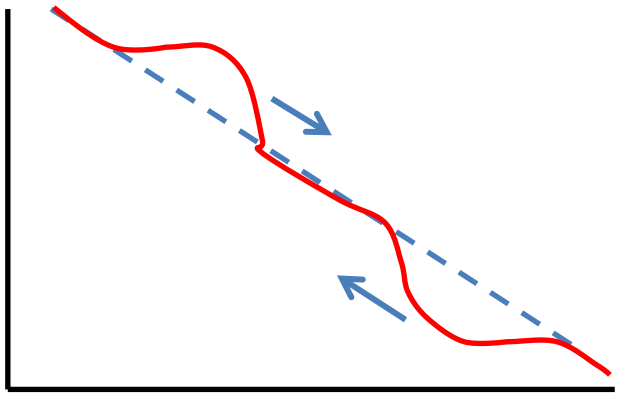
$\delta P \equiv P - P_{SOC}$ → order parameter

→ Local excess, deficit

How does it evolve?

If dynamics conservative;

- $\partial_t \delta P + \partial_x \Gamma(\delta P) - D_0 \partial_x^2 \delta P = \tilde{S}$
- Simple hydro equation
- δP conserved to $\tilde{S}$ boundary
- How constrain δP? → symmetry !
- Higher dimension, $\partial_x \to \partial_\parallel$, and $D_{\perp,0} \nabla_\perp^2$ enter



$\delta P > 0$ → bump, excess

→ Tends move down gradient, to right

$\delta P < 0$ → void, deficit

→ Tends move up gradient, to left

- Joint reflection symmetry principle

$x \rightarrow -x$

$\delta P \rightarrow -\delta P$

} ➔ $\Gamma(\delta P)$ unchanged

{ i.e. flip pile, blob → void structure → rt.

- Allows significant simplification of general form of flux:

$$\Gamma(\delta P) = \sum_{m,n,q,r,\alpha} \left\{ A_n (\delta P)^n + B_m (\partial_x \delta P)^m + D_\alpha (\partial_x^2 \delta P)^\alpha + C_{q,r} (\delta P)^q (\partial_x P)^r + \cdots \right\}$$

- So, lowest order, smoothest model:

  $\Gamma(\delta P) \approx \alpha\, \delta P^2 - D\partial_x \delta P; \quad \alpha, D$ coeffs as in G.-L.

  N.B.: Heuristic correspondence

$$\alpha \delta P^2 \leftrightarrow -\chi \left(\frac{1}{P} \nabla P|_{threshold} - \frac{1}{L_{P_{crit}}} \right) \nabla P$$

  And have:

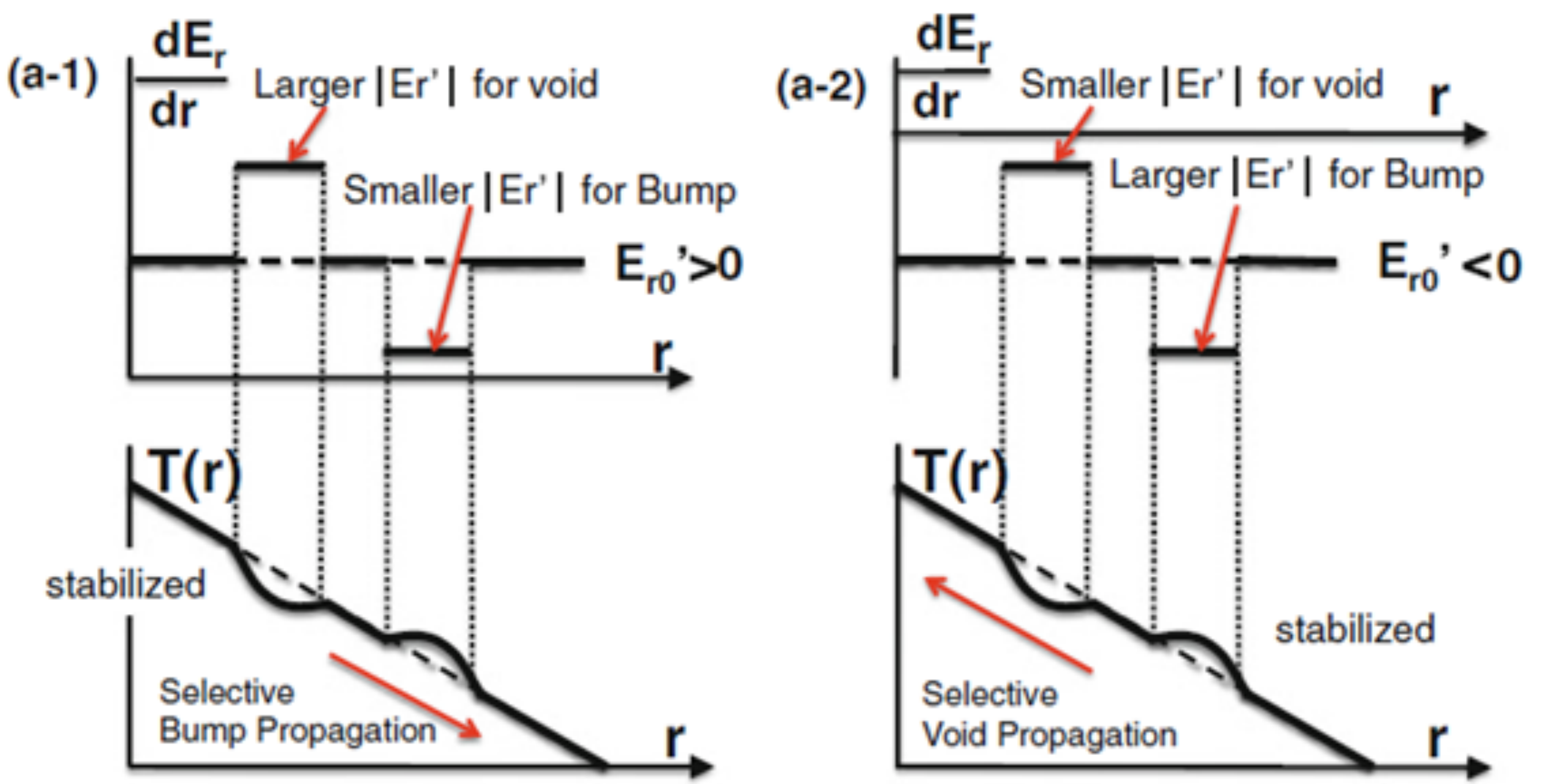
  $\partial_t \delta P + \partial_x (\alpha \delta P^2 - D \partial_x \delta P) = \tilde{s}$

  – Noisy Burgers equation
  – Solution absent noise → shock
  – Shock ←→ Avalanche
  – Manifests shock turbulence → widely studied

- More on Burgers/hydro model (mesoscale)
  - $V \sim \alpha\, \delta P$ relation → bigger perturbations, faster, over-take
  - Extendable to higher dimensions
  - Cannot predict SOC state, only describe dynamics about it. And α, D to be specified
  - $\langle \delta P \rangle$? → corrugation (!?)
  - Introducing delay time → traffic jams, flood waves, etc (c.f. Whitham; Kosuga et al '12)

- If SOC profile ≈ Marginal profile

can link E_r' to bump/void imbalance (Idomura, Kikuchi)



➔ Blobs dominate, $E_r' > 0$

Voids dominate, $E_r' < 0$

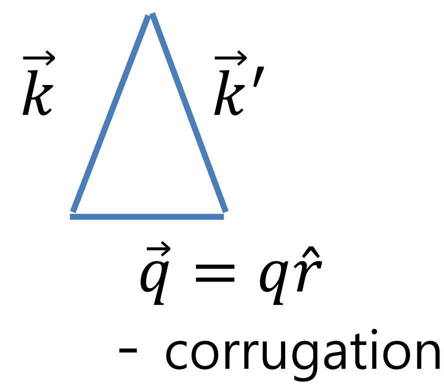
N.B. Ambient E_r'

$E_r' \leftrightarrow T''$

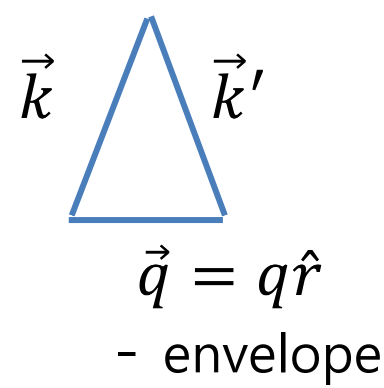
- Pivotal element of ‘SOC’ theory as connects ‘SOC’ world to turbulence world, and enables continuum analysis

➔ Points toward ensemble/gas of avalanches/pulses as natural model of avalanching transport.

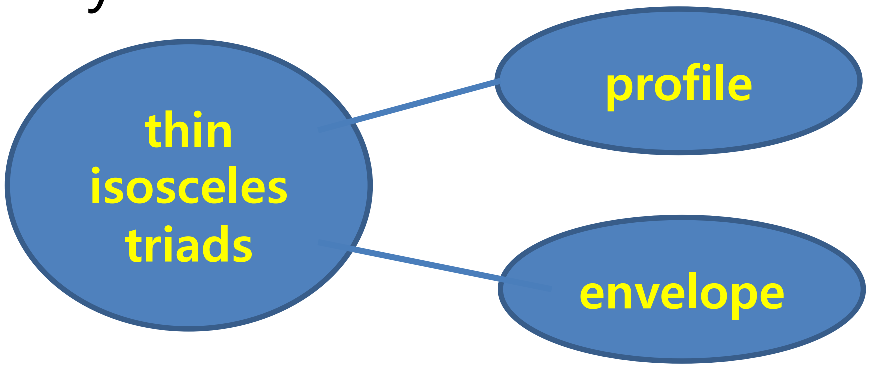
- **“Turbulence Spreading” vs “Avalanching”**
  - – Both: (non-Brownian) radial propagation of excitation
  - – Avalanching → δP overturning, ala’ Pile
    - via overturning and mixing of neighboring cells
    - Coupling via $\nabla\langle P\rangle$
    - $\partial_t \delta P \sim \partial_x\,(\alpha\delta P^2)$



  - – Turbulence spreading (t.s. by T.S.) → intensity fields, ala’ $k-\epsilon$
    - Ancient idea long used in K-ϵ models
    - via spatial scattering due nonlinear coupling
    - Couple via turbulence intensity field
    - Usually $\partial_t I \sim \partial_x(D_0 I \partial_x I)$



- **Bottom Line:**
  - Very closely linked



$\delta P \leftrightarrow I$

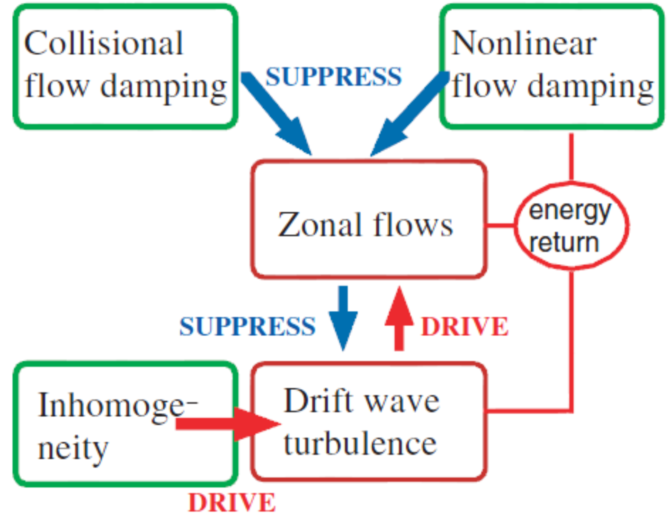
  - ~ impossible to separate
  - t.s. can persist in strongly driven, non-marginal regimes
  - Key question is penetration depth ↔ stable/damped
    - Preliminary results → weak penetration
  - See also Zhibin Guo, this meeting

But:
What of zonal flow?

"Oh there you go again…."

Feedback Loops I

- Closing the loop of shearing and Reynolds work
- Spectral 'Predator-Prey' Model



Prey → Drift waves, <*N*>

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k \langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

→ Self-regulating system → "ecology"

Feedback Loops II

- Recovering the 'dual cascade':
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high k_r { ⇒ Analogous → forward potential enstrophy cascade; PV transport
  - Predator → $|\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle$ { ⇒ growth of *n=0, m=0* Z.F. by turbulent Reynolds work; ⇒ Analogous → inverse energy cascade

- Mean Field Predator-Prey Model

  (P.D. et. al. '94, DI²H '05)

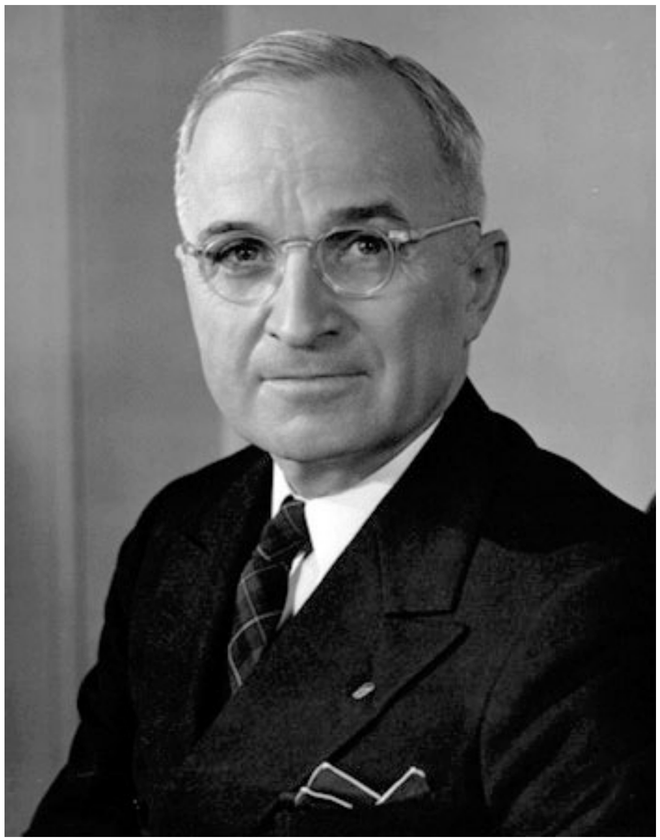
$$\frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta\omega N^2$$

$$\frac{\partial}{\partial t} V^2 = \alpha N V^2 - \gamma_d V^2 - \gamma_{NL}(V^2) V^2$$

System Status

State	No flow	Flow ($\alpha_2 = 0$)	Flow ($\alpha_2 \neq 0$)
N (drift wave turbulence level)	$\frac{\gamma}{\Delta\omega}$	$\frac{\gamma_d}{\alpha}$	$\frac{\gamma_d + \alpha_2\gamma\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
V^2 (mean square flow)	0	$\frac{\gamma}{\alpha} - \frac{\Delta\omega\gamma_d}{\alpha^2}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
Drive/excitation mechanism	Linear growth	Linear growth	Linear growth Nonlinear damping of flow
Regulation/inhibition mechanism	Self-interaction of turbulence	Random shearing, self-interaction	Random shearing, self-interaction
Branching ratio $\frac{V^2}{N}$	0	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d + \alpha_2\gamma\alpha^{-1}}$
Threshold (without noise)	$\gamma > 0$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$

This brings us to... the Crux of the matter, ...



"Give me a one handed ~~economist~~ plasma physicist"

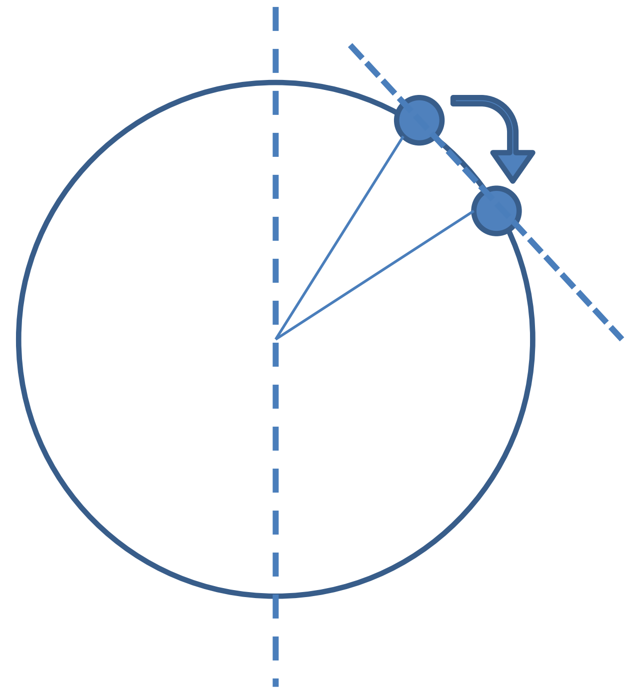
IV) Pattern Competition!

- Seemingly, two secondary structures at work:
  - Avalanche → stochastic, induces extended transport events, enhances scale
  - Zonal flow → quasi-coherent, regulates transport via shearing, self-generated, limits scale
- Both flux driven… by relaxation ∇T, etc
- Nature of co-existence or competition?

- What is PV?

$$q = ((\omega + 2\Omega)\cdot\nabla\psi)/\rho; \quad \frac{dq}{dt} = 0.$$

~developed from freezing-in law.



- Simple examples of conserved PV:
  - 2D: $\nabla^2\phi$
  - β-plane: $\nabla^2\phi + \beta y$
  - H-M: $\ln n_0 + \phi - \rho_s^2\nabla^2\phi$
  - H-W: $\ln n - \rho_s^2\nabla^2\phi$

    → $\ln n_0 + \frac{\tilde{n}}{n_0} - \rho_s^2\nabla^2\phi$

~ charge density (H-M, H-W)

PV Transport ➔ Zonal Flows

- Fundamental Idea:
  - **Potential vorticity transport** + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- **Charge Balance** → polarization charge flux → Reynolds force
  - Polarization charge ➡ $-\rho^2\nabla^2\phi = n_{i,GC}(\phi) - n_e(\phi)$
    - *polarization length scale* (ρ^2)
    - *ion GC* ($n_{i,GC}(\phi)$)
    - *electron density* ($n_e(\phi)$)
  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle \neq 0$ ⬌ 'PV transport'
    - *polarization flux* → What sets cross-phase?
  - If 1 direction of symmetry (or near symmetry):

$$-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle \quad \text{(Taylor, 1915)}$$

$-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ ➡ Reynolds force ➡ Flow

- Cannot decouple Zonal Flow and Avalanching

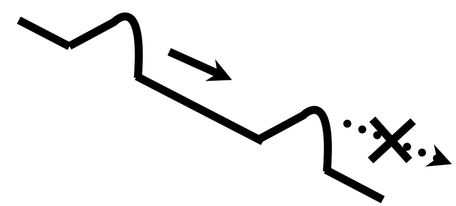
And, Avalanche Gas Unstable to Shear Formation

• How do we understand quasi-regular pattern of ExB staircase, generated from stochastic heat avalanche???

• An idea: jam of heat avalanche

(Y. Kosuga, P. H. Diamond, and Ö.D. Gürcan, PoP'12 & PRL'13)

corrugated profile ⟷ ExB staircase



→ accumulation of heat increment
→ stationary corrugated profile

→ corrugation of profile occurs by 'jam' of heat avalanche flux

* → time delay between $Q[\delta T]$ and δT is crucial element

like drivers' response time in traffic

• How do we actually model heat avalanche 'jam'??? → origin in dynamics?

An extension of the heat avalanche dynamics

- An extension: a finite time of relaxation of Q toward SOC flux state

$$\partial_t Q = -\frac{1}{\tau}\left(Q - Q_0(\delta T)\right) \qquad Q_0[\delta T] = \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

→ In principle $\tau(\delta T, Q_0)$ ⟷ large near criticality (~ critical slowing down)

i.e. enforces time delay between δT and heat flux

- Dynamics of heat avalanche:

$$\underline{\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T} - \chi_4 \partial_x^4 \delta T - \tau \partial_t^2 \delta T$$

→ Burgers
(P.D. + T.S.H. ’95)

↓ New: finite response time

→ Telegraph equation

n.b. model for heat evolution

diffusion → Burgers → Telegraph

Relaxation time: the idea

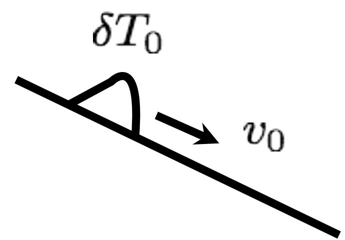
- What is ‘τ ’ physically? → Learn from traffic jam dynamics
- A useful analogy:

heat avalanche dynamics	traffic flow dynamics
temp. deviation from marginal profile	local car density
heat flux	traffic flow
mean SOC flux (ala joint relflection symmetry)	equilibrium, steady traffic flow
heat flux relaxation time	driver’s response time

- driver’s response can induce traffic jam
- jam in avalanche → profile corrugation → staircase?!?
- Key: instantaneous flux vs. mean flux

Analysis of heat avalanche dynamics via telegraph

- How do heat avalanches jam?



- Consider an initial avalanche, with amplitude δT_0, propagating at the speed $v_0 = \lambda \delta T_0$

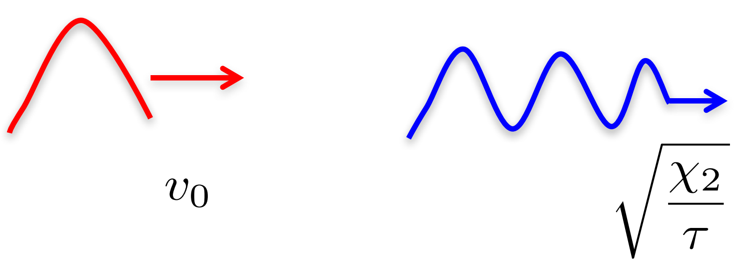
- Dynamics:

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$

pulse

'Heat flux wave': $\sqrt{\frac{\chi_2}{\tau}}$
telegraph → wavy feature

two characteristic propagation speeds



→ In short response time (usual) heat flux wave propagates faster

→ In long response time, heat flux wave becomes slower and pulse starts overtaking. What happens???

Analysis of heat avalanche jam dynamics

- In large tau limit, what happens? → Heat flux jams!!

- Recall plasma response time akin to driver's response time in traffic dynamics

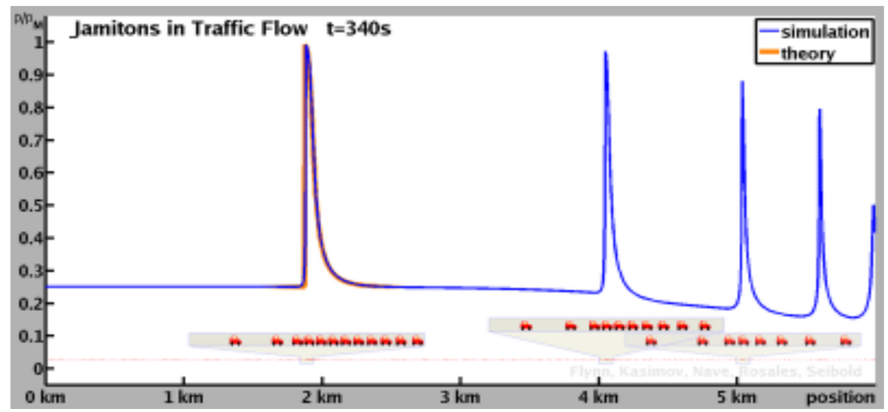
- negative heat conduction instability occurs (as in clustering instability in traffic jam dynamics)

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$

$$\rightarrow \underline{(\chi_2 - v_0^2 \tau)} \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T}$$

<0 when overtaking

→ clustering instability



n.b. akin to negative viscosity instability of ZF in DW turbulence

→ ZF as secondary mode in the gas of primary DW

→ Heat flux 'jamiton' as secondary mode in the gas of primary avalanches

Scaling of characteristic jam scale

- Saturation: Shearing strength to suppress clustering instability

Jam growth → profile corrugation → ExB staircase → $v'_{E\times B}$

→ estimate

$$\frac{\delta T}{T_i} \sim \frac{1}{v_{thi}\rho_i}\sqrt{\frac{\chi_4}{\tau}}$$

- Characteristic scale

$$\Delta^2 \sim k^{-2}(\delta T) \sim \frac{2v_{thi}}{\lambda T_i}\rho_i\sqrt{\chi_2\tau} \qquad \chi_2 \sim \chi_{neo}$$

- Geometric mean of ρ_i and $\sqrt{\chi_2\tau}$: ambient diffusion length in 1 relaxation time
- 'standard' parameters: $\Delta \sim 10\Delta_c$

Conclusion:

- PV conservation renders turbulence driven flow inseparable from avalanching.
- Avalanche gas is unstable to formation of shearing pattern.

PV Balance is “Incorruptible”…..

“… virtue, without which terror is fatal, terror without which virtue is powerless”

Promising Directions:

- Bi-stable mixing

  (A. Ashourvan, P.D. PRE Rapid Comm. ’16 & PoP ’17)

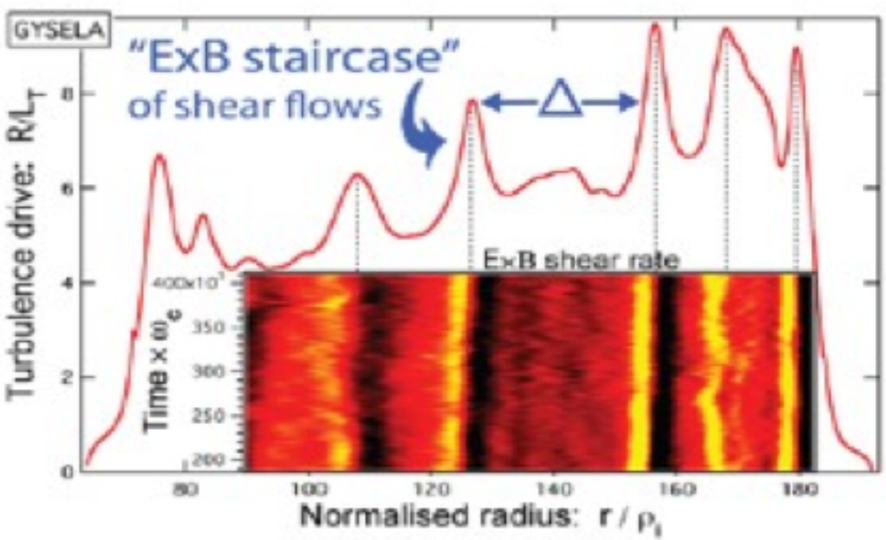
- Phase dynamics

  (Zhibin Guo, P.D. PRL ’16)

Motivation: Coherent ExB Pattern

- ExB flows often observed to self-organize in magnetized plasmas
- \`ExB staircase' is observed

(G. Dif-Pradalier, P.D. et al. Phys. Rev. E. '10)



- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases

- so-named after the analogy to PV staircases and atmospheric jets
- Step spacing → avalanche distribution outer-scale

Atmospherics, GFD.
(Dritschel and McIntyre '08)

- Interesting as:
  - Clear scale selection
  - Clear link of:

    ZF scale ←→ corrugation ←→ avalanche scale

But:

  - Systematic scans lacking
  - Somewhat difficult to capture

- Need a <u>MODEL</u> ➔ Understanding ↗ Jam ↘ <u>Bi-stability</u>

The Hasegawa-Wakatani Staircase

Profile Structure:

From Mesoscopics → Macroscopics

H-W Drift wave model – Fundamental prototype

- Hasegawa-Wakatani : simplest model incorporating instability

$$V = \frac{c}{B}\hat{z}\times\nabla\phi + V_{pol}$$

$$J_\perp = n|e|V^i_{pol} \qquad \eta J_\parallel = -\nabla_\parallel\phi + \nabla_\parallel p_e$$

$$\nabla_\perp\cdot J_\perp + \nabla_\parallel J_\parallel = 0 \quad \rightarrow \text{vorticity:} \quad \rho_s^2\frac{d}{dt}\nabla^2\phi = -D_\parallel\nabla_\parallel^2(\phi-n) + \nu\nabla^2\nabla^2\phi$$

$$\frac{dn_e}{dt} + \frac{\nabla_\parallel J_\parallel}{-n_0|e|} = 0 \quad \rightarrow \text{density:} \quad \frac{d}{dt}n = -D_\parallel\nabla_\parallel^2(\phi-n) + D_0\nabla^2 n$$

→ PV conservation in inviscid theory $\frac{d}{dt}\left(n-\nabla^2\phi\right)=0$

→ PV flux = particle flux + vorticity flux

→ zonal flow being a counterpart of particle flux

QL: $\frac{\partial}{\partial t}\langle n\rangle = -\frac{\partial}{\partial r}\langle\tilde{v}_r\tilde{n}\rangle$

→? $\frac{\partial}{\partial t}\langle\nabla^2\phi\rangle = -\frac{\partial}{\partial r}\langle\tilde{v}_r\nabla^2\tilde{\phi}\rangle = -\frac{\partial^2}{\partial r^2}\langle\tilde{v}_r\tilde{v}_\theta\rangle$

- Hasegawa-Mima ($D_\parallel k_\parallel^2/\omega >> 1 \rightarrow n\sim\phi$)

$$\frac{d}{dt}\left(\phi-\rho_s^2\nabla^2\phi\right) + v_*\partial_y\phi = 0$$

The Reduced 1D Model

Reduced system of evolution Eqs. is obtained from HW system for DW turbulence.

Variables:

$u = \partial_x V_y$ Zonal shearing field

Reduced density: $\log(N/N_0) = n(x,t) + \tilde{n}(x,y,t)$, **Vorticity:** $\rho_s^2 \nabla_\perp^2 (e\varphi/T_e) = u(x,t) + \tilde{u}(x,y,t)$

Potential Vorticity (PV): $q = n - u$, **Turbulent Potential Enstrophy (PE):** $\varepsilon = \frac{1}{2}\left\langle (\tilde{n} - \tilde{u})^2 \right\rangle$

Two components

Mean field equations:

density $\partial_t n = -\partial_x \Gamma_n + \partial_x [D_c \partial_x n]$, $\Gamma_n = \langle \tilde{v}_x \tilde{n} \rangle = -D_n \partial_x n$ → **Reflect instability**

$\text{Taylor ID}: \Pi_u = \langle \tilde{v}_x \tilde{u} \rangle = \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle$

vorticity $\partial_t u = -\partial_x \Pi_u + \partial_x [\mu_c \partial_x u]$, $\Pi_u = \langle \tilde{v}_x \tilde{u} \rangle = (\chi - D_n)\partial_x n - \chi \partial_x u$

Residual vort. flux **Turb. viscosity**

From closure

Turbulence evolution: (Potential Enstrophy)

$$\partial_t \varepsilon = \partial_x [D_\varepsilon \partial_x \varepsilon] - (\Gamma_n - \Gamma_u)[\partial_x (n - u)] - \varepsilon_c^{-1} \varepsilon^{3/2} + P$$

Turbulence spreading; Internal production; dissipation; External production $\sim \gamma\varepsilon$

Two fluxes Γ_n, Γ_u set model !

What is new in this model?

○ **In this model PE conservation is a central feature.**

○**Mixing of Potential Vorticity (PV) is the fundamental effect regulating the interaction between turbulence and mean fields. Mixing <u>inhomogeneous</u>**

○**Dimensional and physical arguments used to obtain functional forms for the turbulent diffusion coefficients. From the QL relation for HW system we obtain**

$$D_n \cong l^2 \frac{\varepsilon}{\alpha} \qquad \chi \cong c_\chi l^2 \frac{\varepsilon}{\sqrt{\alpha^2 + a_u u^2}}$$

* l **Dynamic mixing length** → **Rhines scale sets**

α **Parallel diffusion rate**

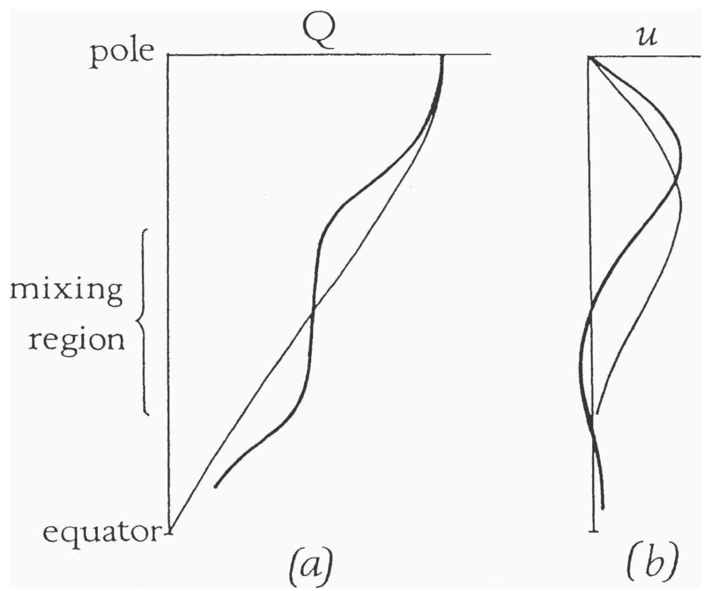
○***Inhomogeneous mixing of PV results in the sharpening of density and vorticity gradients in some regions and weakening them in other regions, leading to shear lattice and density staircase formation.***

Jet sharpening in stratosphere, resulting from inhomogeneous mixing of PV. (McIntyre 1986)

PV $Q = \nabla^2 \psi + \beta y$

Relative vorticity ($\nabla^2\psi$)

Planetary vorticity (βy)



Staircase structure

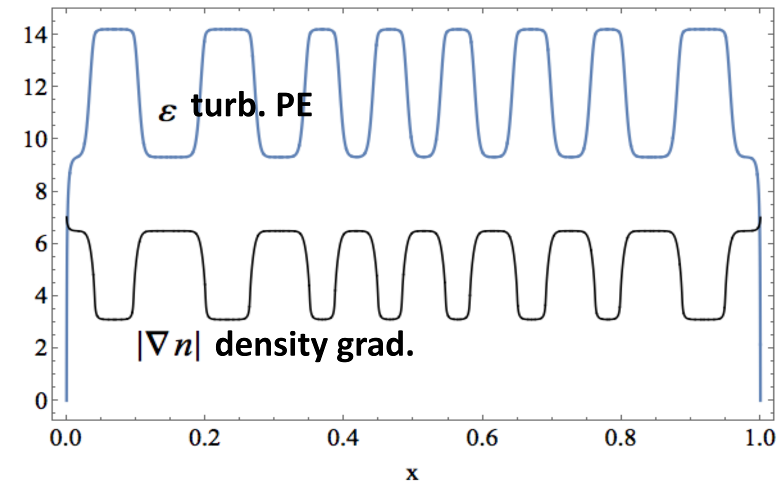
Snapshots of evolving profiles at t=1 (non-dimensional time)

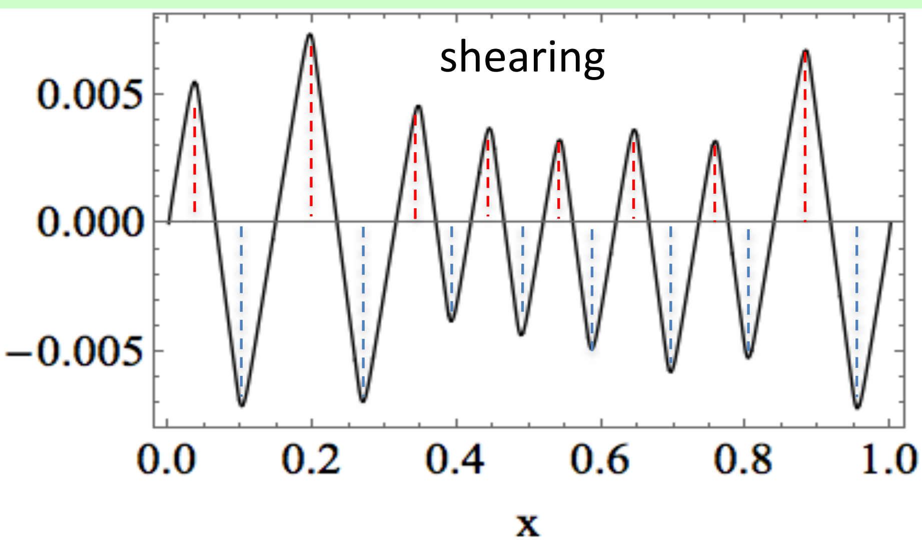
Initial conditions: $n = g_0(1-x), \quad u = 0, \quad \varepsilon = \varepsilon_0$

Boundary conditions: $n(0,t) = g_0,\ n(1,t) = 0; \quad u(0,1;t) = 0; \quad \partial_x \varepsilon(0,1;t) = 0$

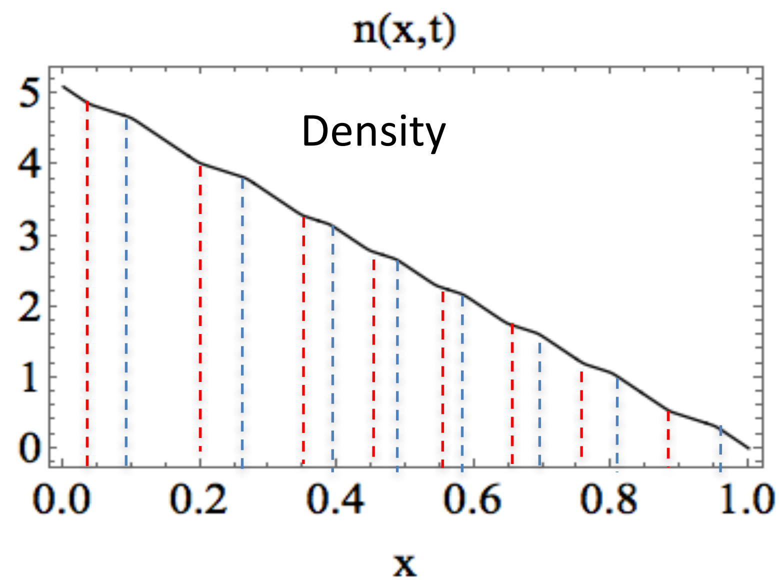
Structures:

- **Staircase in density profile:**
  - **jumps → regions of steepening**
  - **steps → regions of flattening**
- **At the jump locations, turbulent PE is suppressed.**
- **At the jump locations, vorticity gradient is positive**





Density + Vorticity lattices

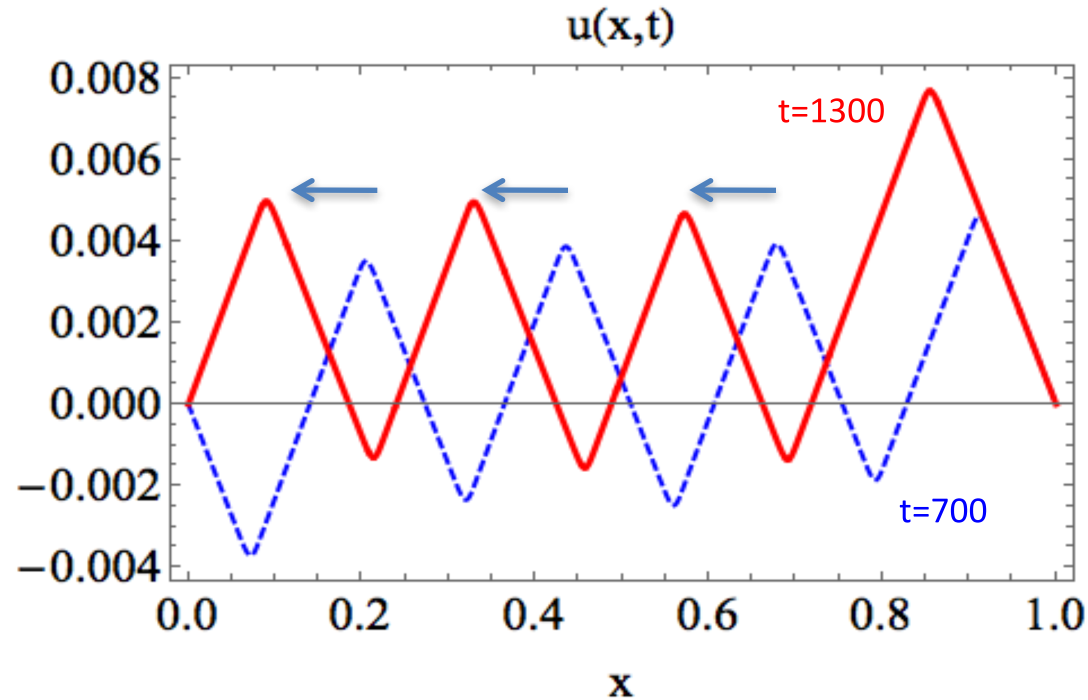


Dynamic Staircases

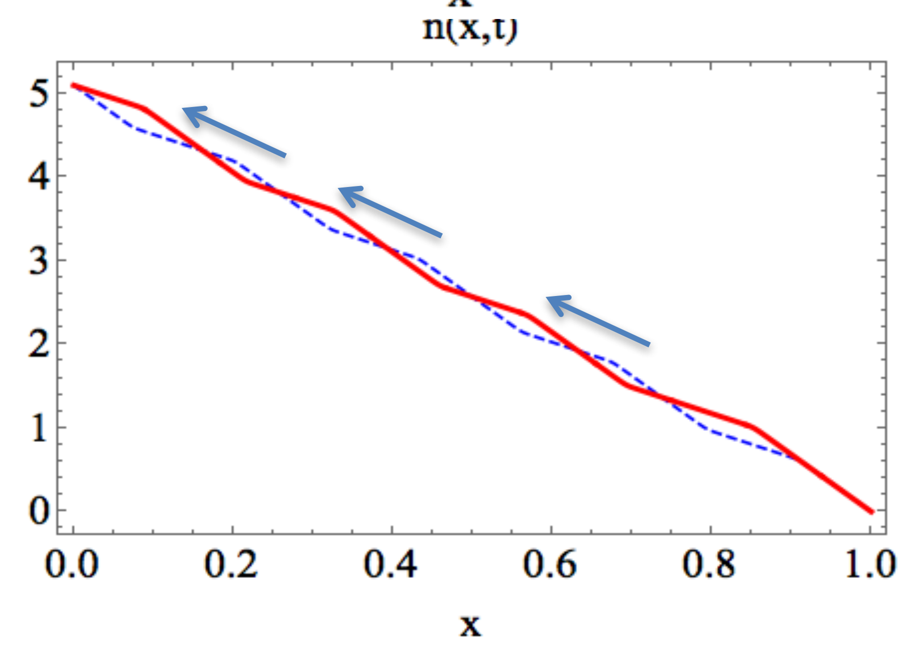
○Shear pattern detaches and delocalizes from its initial position of formation.

○Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at x=0.

○Shear lattice propagation takes place over much longer times. From t~O(10) to t~(10^4).



○Barriers in density profile move upward in an "<u>Escalator-like</u>" motion.



➔ **Macroscopic Profile Re-structuring**

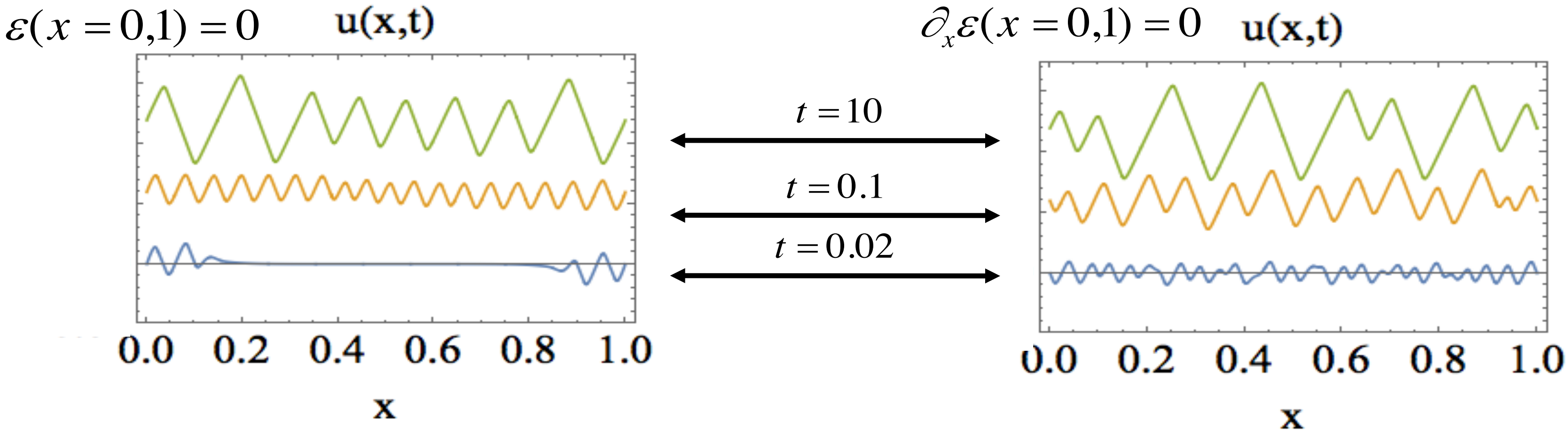
↕

'Non-locality'

Is this an inward-propagating avalanche?

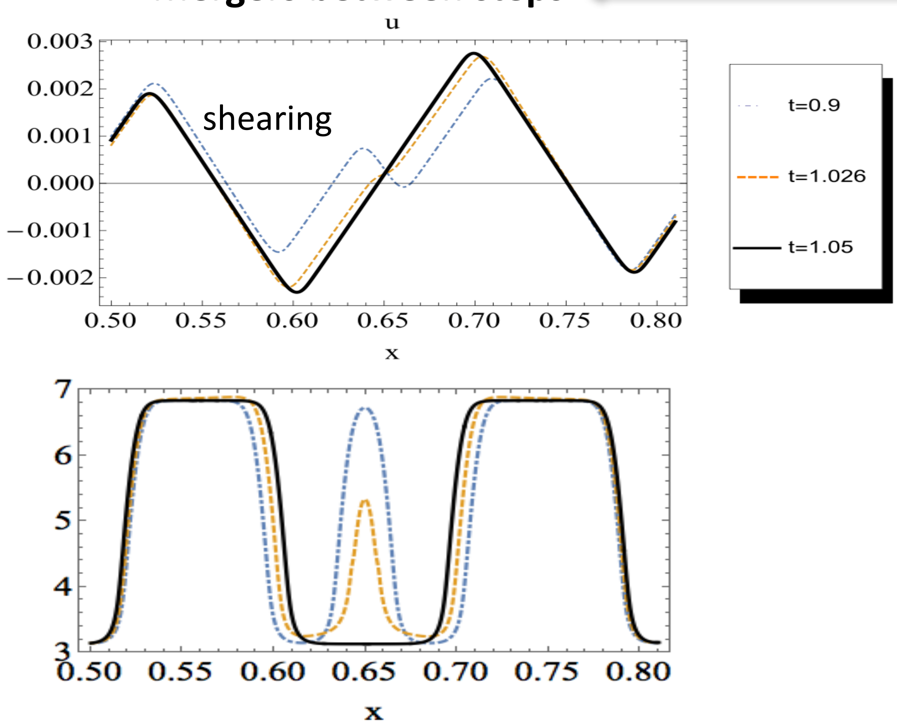
Mergers Occur

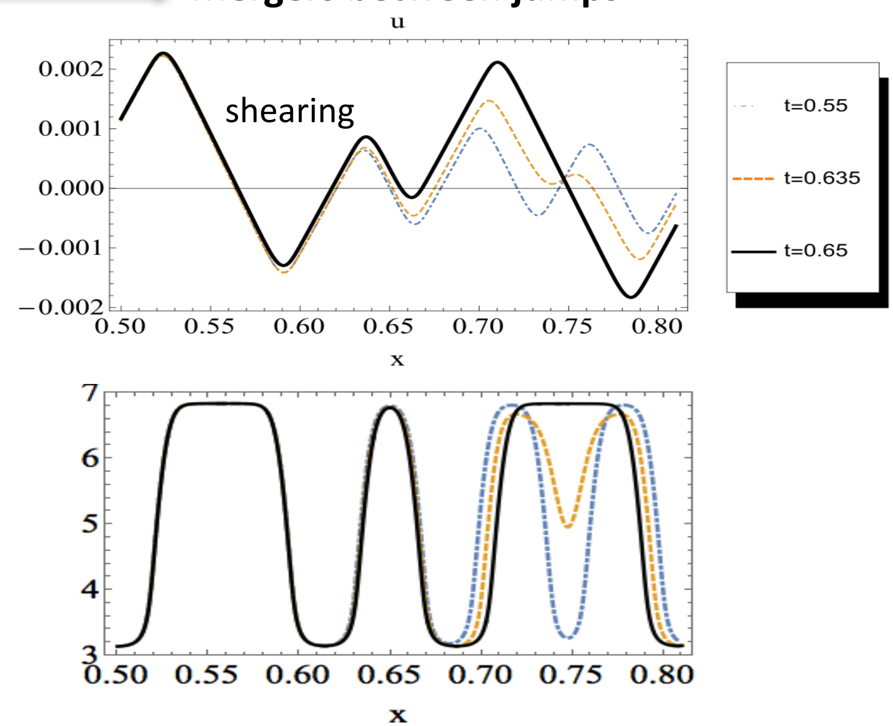
Nonlinear features develop from 'linear' instabilities



Local profile reorganization: Steps and jumps merge (continues up to times t~O(10))

Mergers between steps ⟷ **Mergers between jumps**





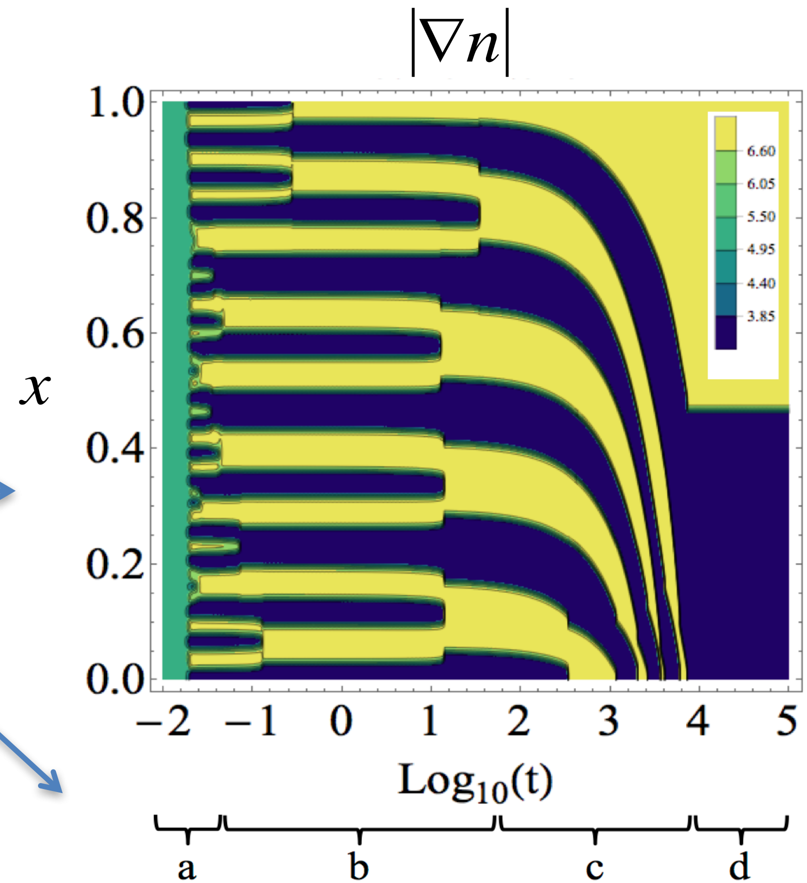
Time evolution of profiles

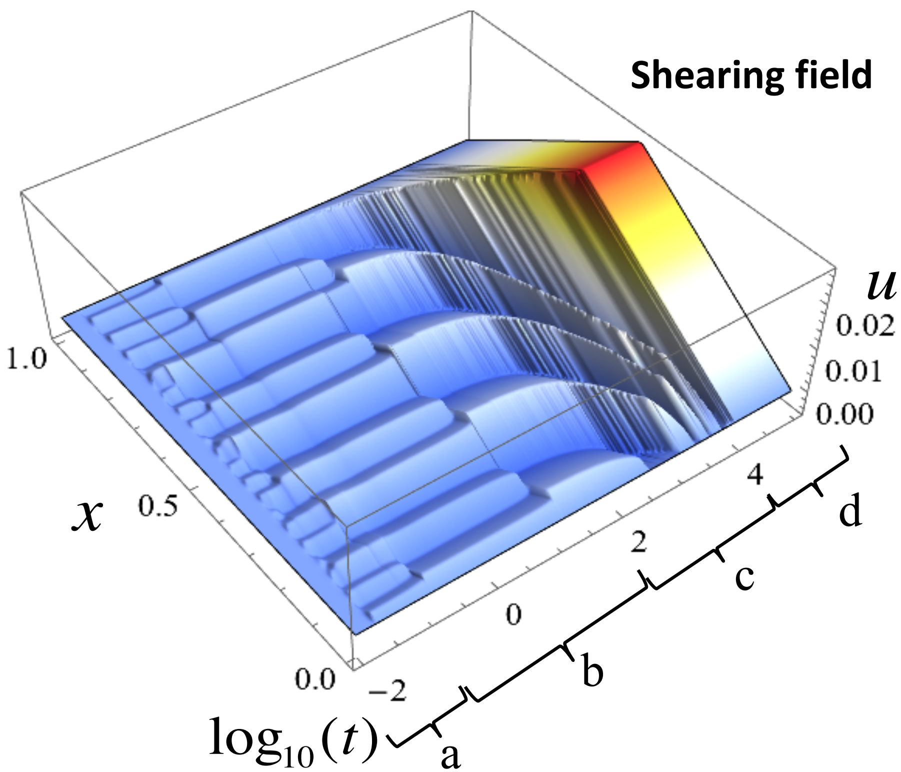
(a) Fast merger of micro-scale SC. Formation of meso-SC.

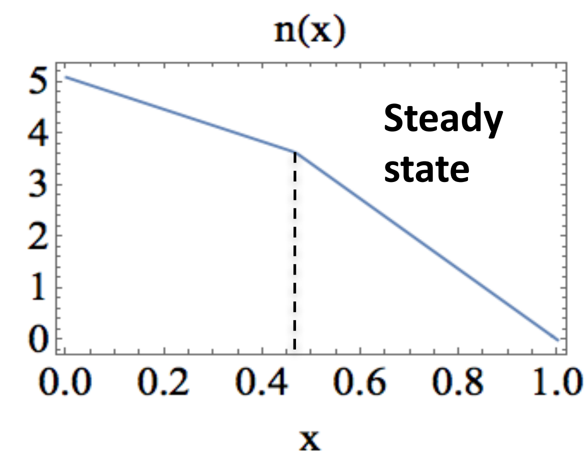
(b) Meso-SC coalesce to barriers

(c) Barriers propagate along gradient, condense at boundaries

(d) Macro-scale stationary profile







- The Point:
  - Macroscopic barrier emerges from hierarchical sequence of mergers and propagation, condensation
  - (Somewhat) familiar bi-stable transport model

But

- Barrier formation is NOT a local process!

→ Begs for flux driven, not BVP analysis!

Macroscopics: Flux driven evolution

- We add an external particle flux drive to the density Eq., use its amplitude Γ_0 as a control parameter to study:
  - What is the mean profile structure emerging from this dynamics?
  - Variation of the macroscopic steady state profiles with Γ_0. (shearing, density, turbulence, and flux).
  - Transport bifurcation of the steady state (macroscopic)
  - Particle flux-density gradient landscape

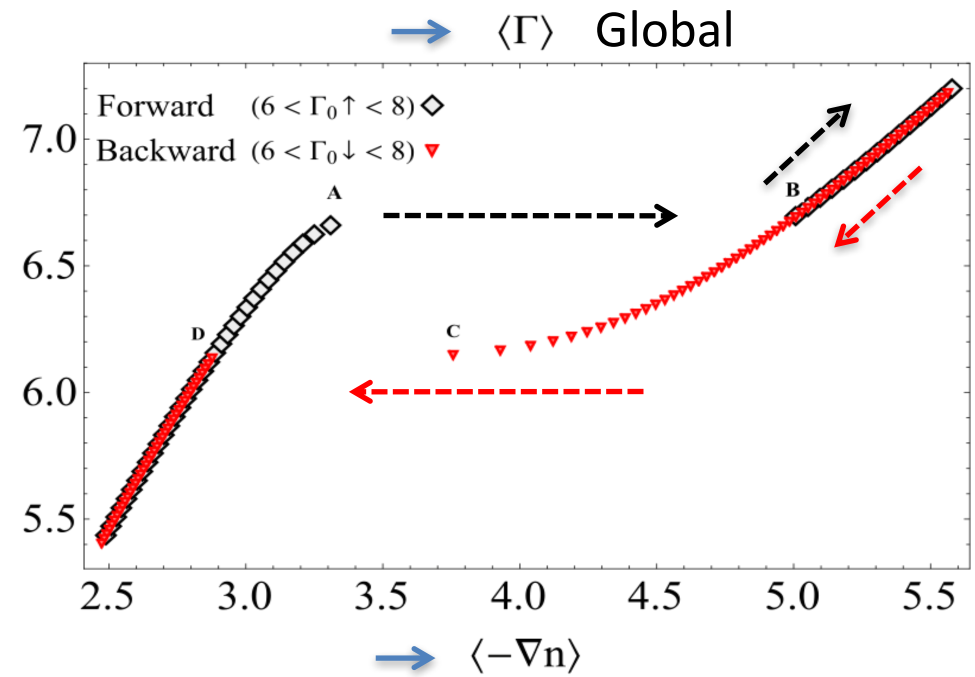
$\partial_t n = -\partial_x \Gamma - \partial_x \Gamma_{dr}(x,t)$ → Write source as $\nabla \cdot \Pi_{ex}$

External particle flux (drive) $\Gamma_{dr}(x,t) = \Gamma_0(t)\exp[-x/\Delta_{dr}]$

Internal particle flux (turb.+col.) $\Gamma = -[D_n(\varepsilon, \partial_x q) + D_{col}]\partial_x n$

Hysteresis evident in the GLOBAL flux-gradient relation

In one sim. run, from initially flat density profile, Γ_0 is adiabatically raised and lowered back down again.



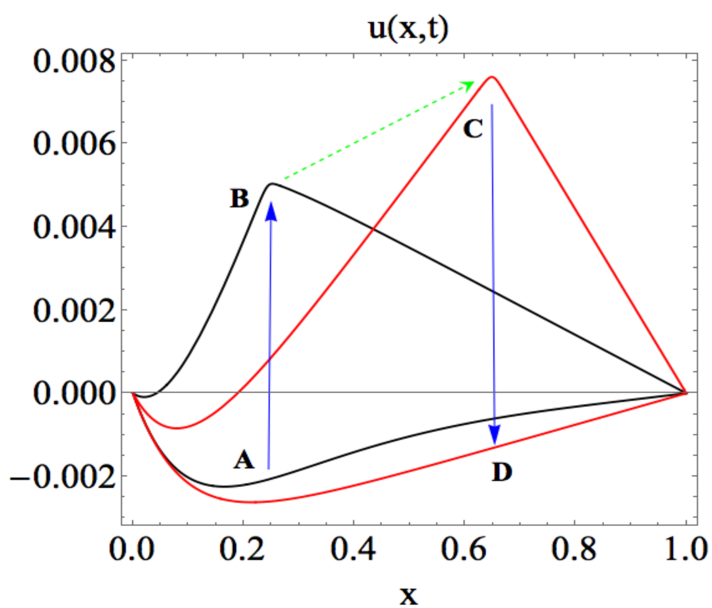
Forward Transition:
Abrupt transition from NC to EC (from A to B). During the transition the system is not in quasi-steady state.

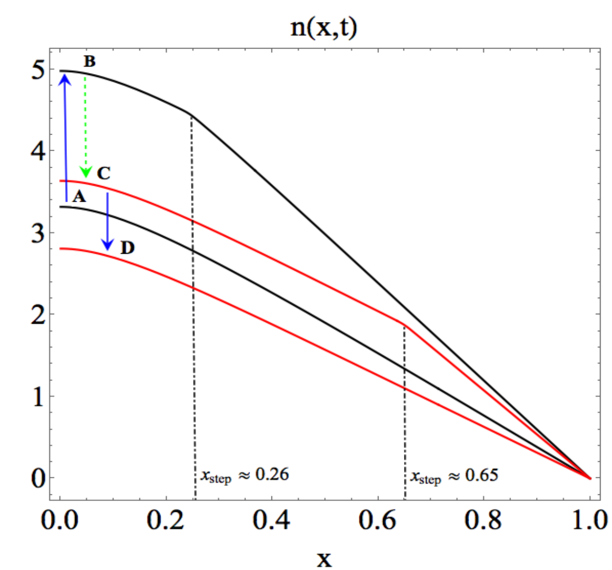
From B to C:
We have continuous control of the barrier position Barrier moves to the right with lowering the density gradient.

Backward Transition:
Abrupt transition from EC to NC (from C to D). Barrier moves rapidly to the right boundary and disappears. system is not in quasi-steady.

$$\langle \Gamma \rangle = \int_0^1 \Gamma(x) dx$$

$$\langle -\partial_x n \rangle = \int_0^1 [-\partial_x n(x,t)] dx$$



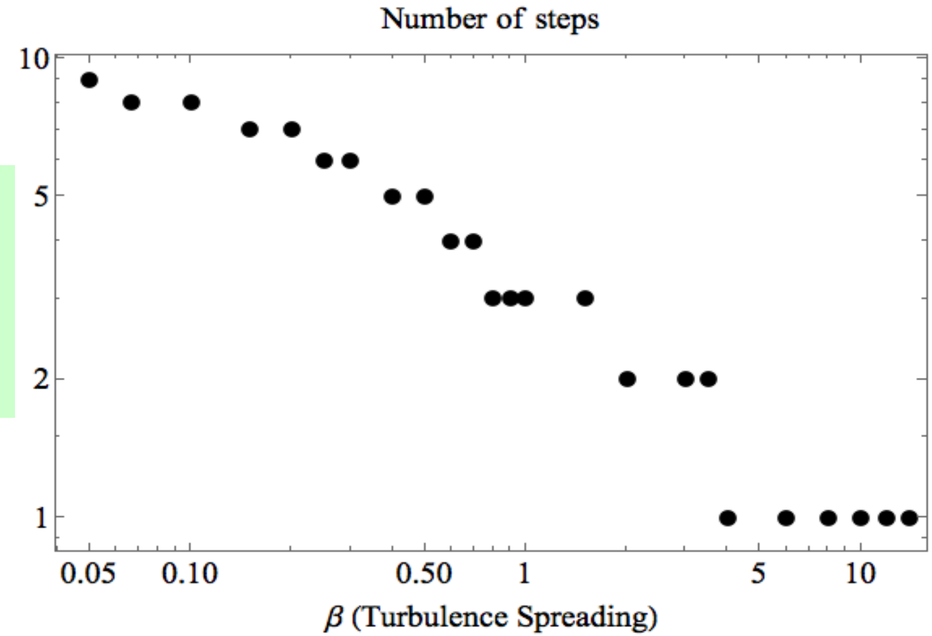


Role of Turbulence Spreading

○ **Large turbulence spreading wipes out features on smaller spatial scales in the mean field profiles, resulting in the formation of fewer density and vorticity jumps.**

$$\partial_t \varepsilon = \beta \partial_x [(l^2 \varepsilon^{1/2}) \partial_x \varepsilon] + \ldots$$

- $\beta \to 0$ excessive profile roughness



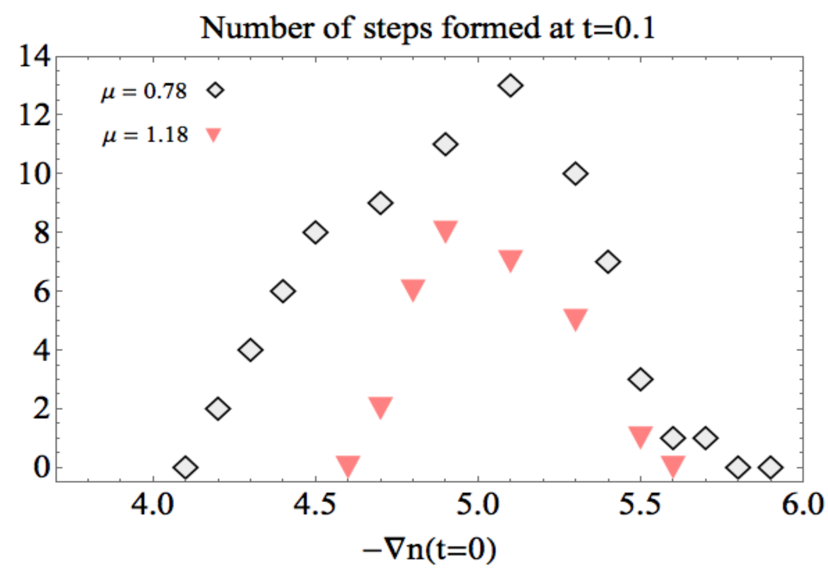
Initial condition dependence

○**Solutions are not sensitive to initial value of turbulent PE.**

○**Initial density gradient is the parameter influencing the subsequent evolution in the system.**

○**At lower viscosity more steps form.**

○**Width of density jumps grows with the initial density gradient.**



Lessons

- A) Staircases happen
  - Staircase – more generall, coherent pattern - is ‘natural upshot’ of modulation in bistable/multi-stable system
  - Bistability is a consequence of mixing scale dependence on gradients, intensity ←→ define feedback process
  - Bistability effectively locks in inhomogeneous PV mixing required for zonal flow formation
  - Mergers result from accommodation between boundary condition, drive(L), initial secondary instability
  - Staircase is natural extension of quasi-linear modulational instabilty/predator-prey model → couples to transport and b.c. ←→ simple natural phenomenon

Lessons

- B) Staircases are Dynamic
  - Mergers occur
  - Jumps/steps migrate. B.C.'s, drive all essential.
  - Condensation of mesoscale staircase jumps into macroscopic transport barriers occurs. ➔ Route to barrier transition by global profile corrugation evolution vs usual picture of local dynamics
  - Global 1st order transition, with macroscopic hysteresis occurs
  - Flux drive + B.C. effectively constrain system states.

Status of Theory

- a) Blind WTT methods can miss aspects of feedback and bistability

  b) $K - \epsilon$ genre models crude, though elucidate much

- Some type of synthesis needed

- Distribution of dynamic, nonlinear scales appear desirable

- Total PV conservation has demonstrated utility and leverage.

  Underutilized in MFE.

Phase Dynamics

→ More physics in the Reynolds stress…

- Amplitude and Phase Representation

$$\phi(\boldsymbol{x},t) = |\phi(\boldsymbol{x},t)| \exp[iS(\boldsymbol{x},t)]$$

phase field -> defines $\boldsymbol{k}$ etc.

- $S = \bar{S} + \tilde{S}$

$\tilde{S}$ -> fast, microscale

$\bar{S}$ -> slow envelope, mesoscale

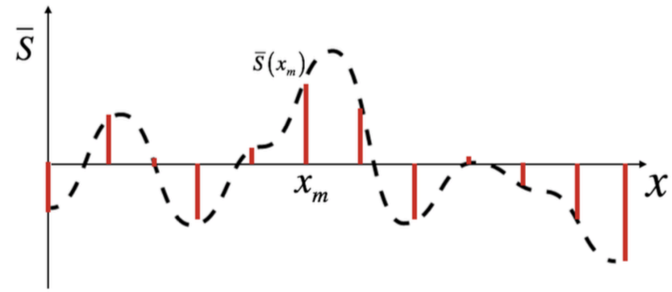


FIG. 1. Red: phase lattice; dashed black: continuous limit of the phase lattice.

- $\langle v_x v_y \rangle = 2\sum_{m'} k'_y k'_x I(m') + 2\sum_{m'} k'_y I(m') \frac{\partial}{\partial x} \bar{S}$

- $\frac{\partial}{\partial t}\langle V \rangle \cong 2k_y k_x \frac{\partial}{\partial x} I + 2k_y \frac{\partial}{\partial x} I \frac{\partial}{\partial x} \bar{S} + 2k_y I \frac{\partial^2}{\partial x^2} \bar{S} - \gamma_d \langle V \rangle$

- N.B. Phase curvature can drive flow in absence of intensity gradient.

- S evolution
- $\frac{\partial S}{\partial t} = -\omega - \boldsymbol{k} \cdot \boldsymbol{v}, \quad \omega = \omega_{\boldsymbol{k}} + 2\widehat{\omega}_{De} + k_y \langle V \rangle$

toroidal coupling

- Expand for nearest neighbor poloidal harmonics:
- $\frac{\partial}{\partial t} \bar{S} \cong -k_y \langle V \rangle - 2k_x V_D \Delta \frac{\partial}{\partial x} \bar{S} + k_x V_D \Delta^2 \left(\frac{\partial \bar{S}}{\partial x} \right)^2 + D_s \frac{\partial^2}{\partial x^2} \bar{S}$
- with: $\frac{\partial}{\partial t} I = \gamma_l I + 2k_y I \bar{S}' \langle V \rangle' + \frac{\partial}{\partial x} \left(D_T I \frac{\partial}{\partial x} I \right) - \gamma_{nl} I^2$
- and: $\frac{\partial}{\partial t} \langle V \rangle = 2k_y I \frac{\partial^2}{\partial x^2} \bar{S} + 2k_y k_x \frac{\partial}{\partial x} I - \gamma_d \langle V \rangle$
- Expanded Predator-Prey system for $\langle V \rangle, I, \bar{S}$.

- Phase steepening occurs
- $\frac{\partial}{\partial t}\bar{S}' \cong -k_y\langle V\rangle' - 2k_x V_D \Delta \frac{\partial}{\partial x}\bar{S}' +$

  $2k_x V_D \Delta^2 \bar{S}' \frac{\partial}{\partial x}\bar{S}' + D_s \frac{\partial^2}{\partial x^2}\bar{S}'$
- Balance defines ZF scale:
- $L_{ZF} \cong \frac{D_S}{2k_y V_D \Delta^2 |\delta \bar{S}'|}$

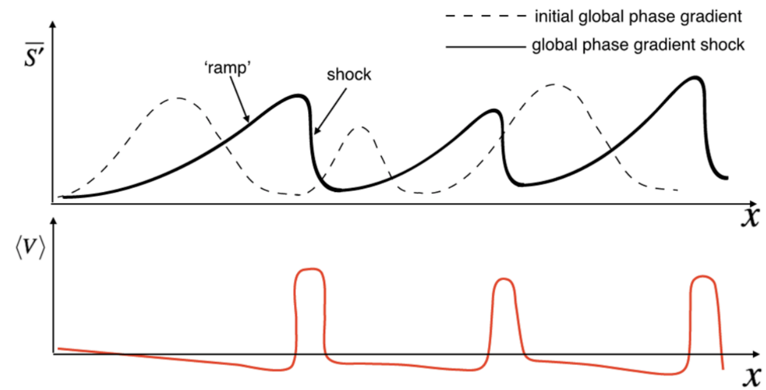


FIG. 2. Top figure: global phase gradient shock induced by magnetic toroidicity; bottom figure: staircaselike ZF bands induced by the shocks.

- Ultimately, PDF of shock layer concentrates at large scale, if phase curvature driven.

- Re-cap
  - Avalanche happen, significant contributor to transport
  - ‘Avalanche concept’ is useful (to MFE) legacy of SOC era
  - $Q, \nabla T$ temporal decay ➔ avalanche gas is ‘unstable’ to shear layer formation
  - PV balance is intrinsic to avalanche ➔ inhomogeneous mixing as regulator of avalanche field

– Envelope and K-ϵ models capture many aspect of profile dynamics and (‘weak’) non-locality while conserving PV

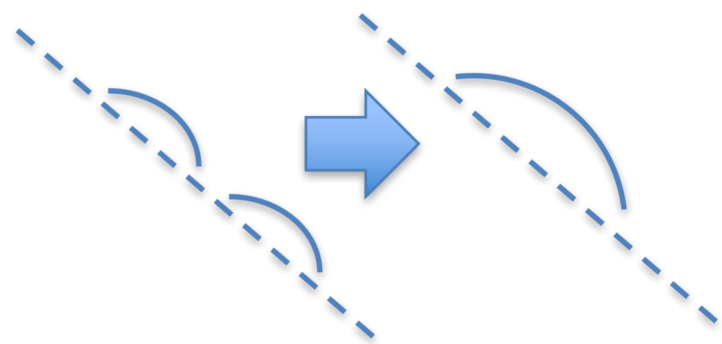
– But… represent only low order moments of pdf

– Phase dynamics is new and promising. Implications for avalanche TBD

- <u>Wish List</u>
  - (Non-trivial) kinetic equation for avalanche population + flux contribution – i.e. $f(s,r,t)$
  - Should :
    - Be multi-point (de-localized in space, time)
    - Exhibit overtaking
    - Respect PV balance
    - Capture tail of pdf

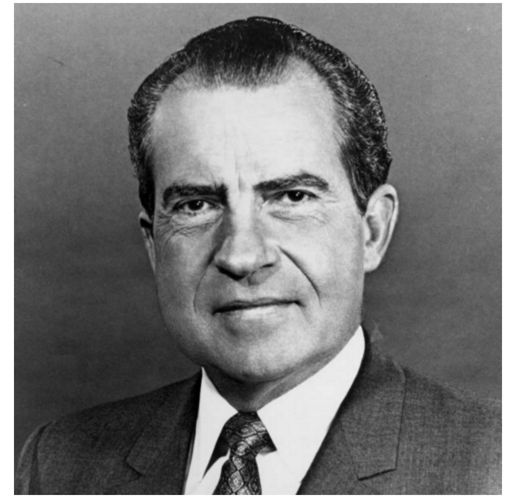
Akin:

- Schmoluchowski
- Slyozov-Lifshitz



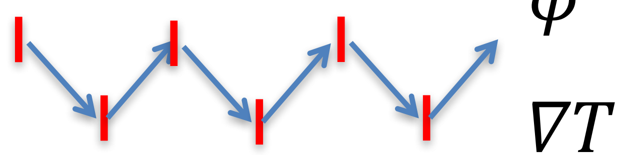
  - A step → spreading in stochastically varying ∇T profile (ZBG, PD)

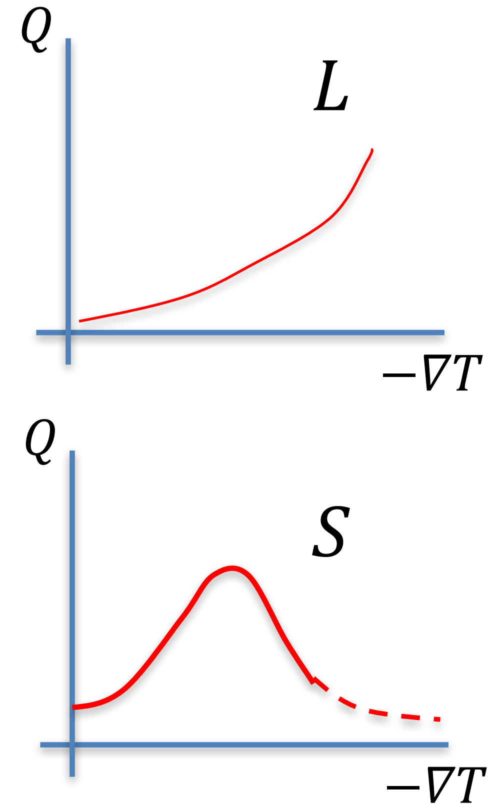
- Beyond the 'Crook' approximation
  - Key physics of heat flux relaxation time in Jam?
  - Tractable, at closure level



- Phase dynamics ←→ Avalanching?
  - $\phi \leftrightarrow \nabla T$ coupling
  - Extended phase synchronization!?

    ZF effects!?





- Multi-scale problem
  - Flux balance introduces many new twists....

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"人不知，而不愠；不亦君子乎！"

～孔子

" Not recognized by others, and yet not upset;
What a noble person! "

– Confucius
(Kongtze)

— courtesy of L. Chen, Alfven Prize Lecture, 2008

“All true genius is unrecognized.”

- Friedrich Dürrenmatt, “The Physicists”

N.B.: “The physicists” is a satiric play set in an insane asylum. It features three protagonists, one who thinks he is Newton, one who thinks he is Einstein, and one who thinks he hears the voice of the wise King Solomon.

The Evolution of Reaction to Progress in Theoretical Physics:

Stage1 : “Its wrong!”

Stage2 : “Its trivial!”

Stage3 : “I did it first !!”

- Anonymous

- Avalanche Turbulence
  - Statistical understanding of nonlinear dynamics → renormalization
  - Conserved order parameter

$$\partial_x (\alpha \delta P^2) \rightarrow \nu_T k^2 \, \delta P_k$$

$$\nu_T \approx \left(\alpha^2 S_0^2 \int_{k_{min}}^{0} dk/k^4\right)^{1/3} \rightarrow (\alpha^2 S_0^2)^{1/3} k_{min}^{-1} \sim (\alpha^2 S_0^2)(\delta l)$$

Infrared divergence due slow relaxation

  - $(\delta l)^2 \sim \nu_T \delta t$ ➔ $\delta l \sim \delta t$
    - H → 1, 'Ballistic' scaling

- Infrared trends ←→ non-diffusive scaling, recover self-similarity
- Amenable to more general analyses using scaling, RG theory

Transition to Enhanced Confinement can occur

Steady state solution for the system undergoes a transport bifurcation as the flux drive amplitude Γ_0 is raised above a threshold $.\Gamma_{th}$

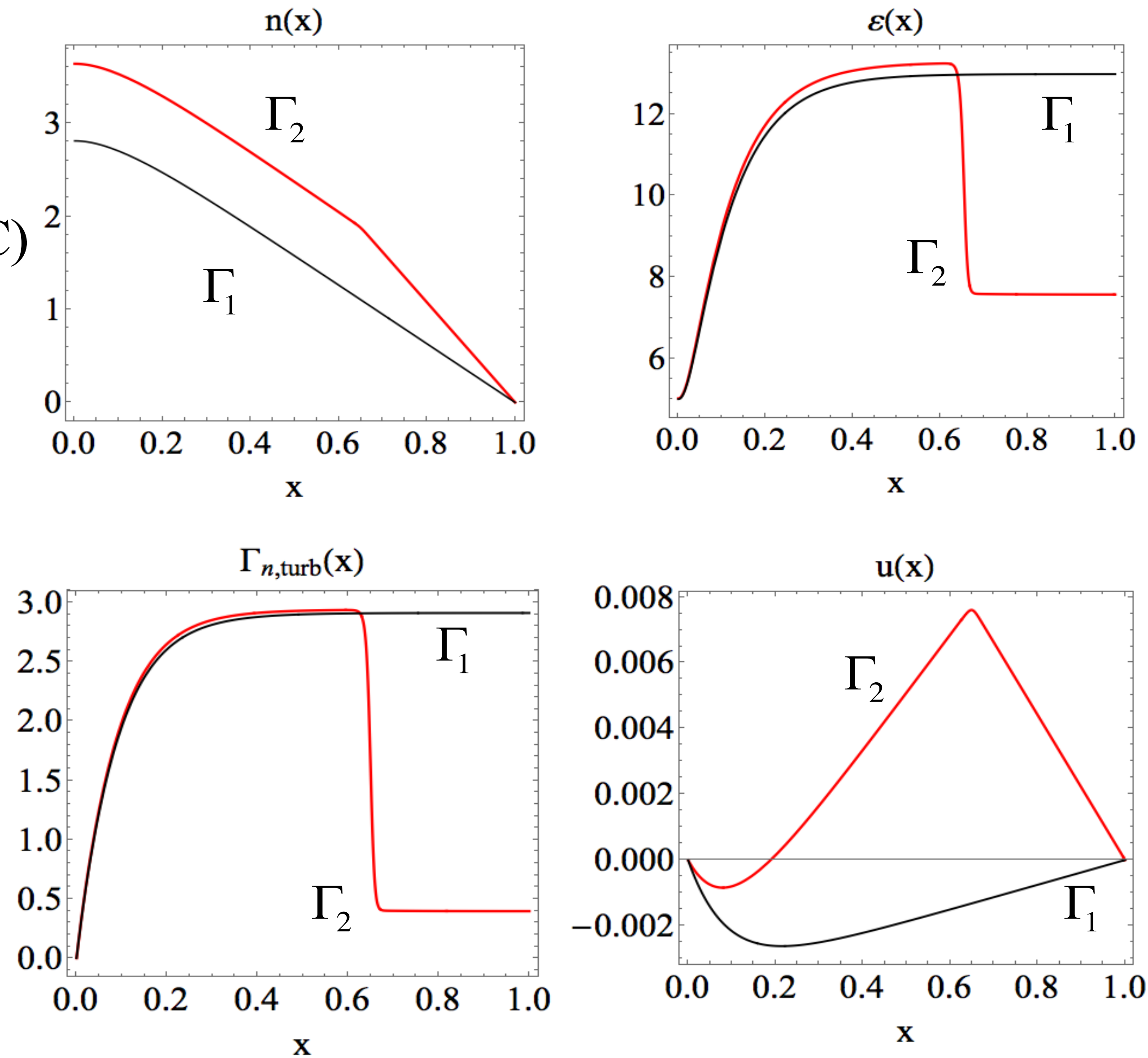
$$\Gamma_1 < \Gamma_{th} < \Gamma_2$$

$$\Gamma_0 = \Gamma_1 \rightarrow \text{Normal Conf. (NC)}$$

$$\Gamma_0 = \Gamma_2 \rightarrow \text{Enhanced Conf. (EC)}$$

With NC to EC transition we observe:

- Rise in density level
- Drop in turb. PE and turb. particle flux beyond the barrier position
- Enhancement and sign reversal of vorticity (shearing field)



- Coherent Patterns:
  - A solution to “predator-prey” problem domains via decomposition (akin spinodal)
  - Natural reduced DOF models for profile evolution
  - Realization of ‘non-local’ dynamics in transport

➔ Global bifurcation via internal re-arrangement