Heat Flux Jamming and Corrugation Patterns in Confined Plasma

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- With: Yusuke Kosuga, Ozgur Gurcan, Pierre Guillon
- Phil Trans 2026, in press

  Special issue on: “Anti-diffusion …”

  and

  References therein

Outlook:

- Barriers, Corrugations as Jamming phenomena

Jam ←→ localized reduction in {traffic / heat(turbulent)} flux

- Jams, Jam trains emergent in traffic flow

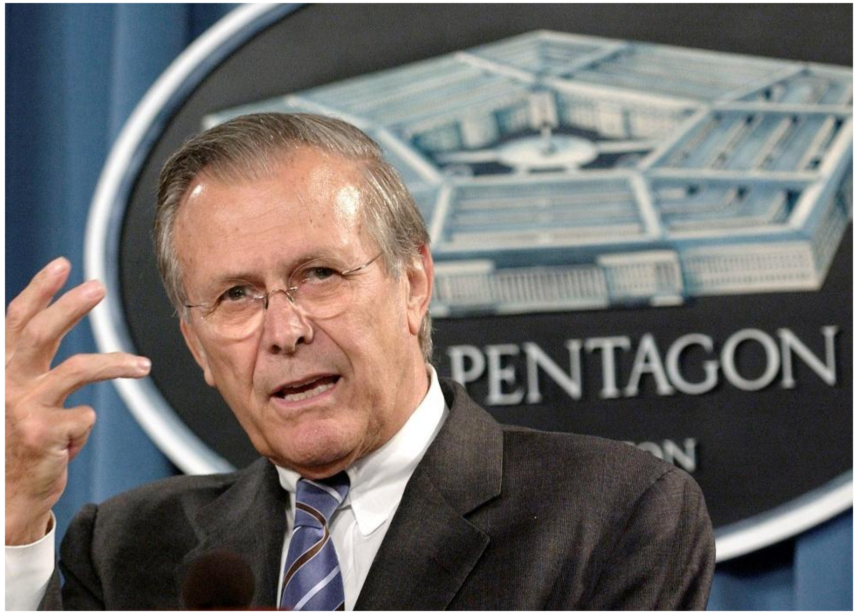
➔

- Implications for MFE?

N.B.: Informal, pedagogic

Rumsfeld Matrix (c.f. Hollis Robbins)

Known Knowns	Known Unknowns
Unknown Knowns	Unknown Unknowns (i.e. Black Swan)



Here: uncovering an unknown known…

Outline

- Traffic Jams and their simple models
- Avalanches and their models
- Flux Jams
- *The Delay Time* - central issue
- Commentary

Traffic Jams

→ 1D

→ cf. Lighthill and Whitham, et. seq.

- sequence in Proc. Roy. Soc.
- "Linear and Nonlinear Waves"

Kinematic Waves

- Conserved order parameter $\rho(x)$ - density

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\big(\rho V(\rho)\big) - D\,\frac{\partial^2 \rho}{\partial x^2} = 0$$

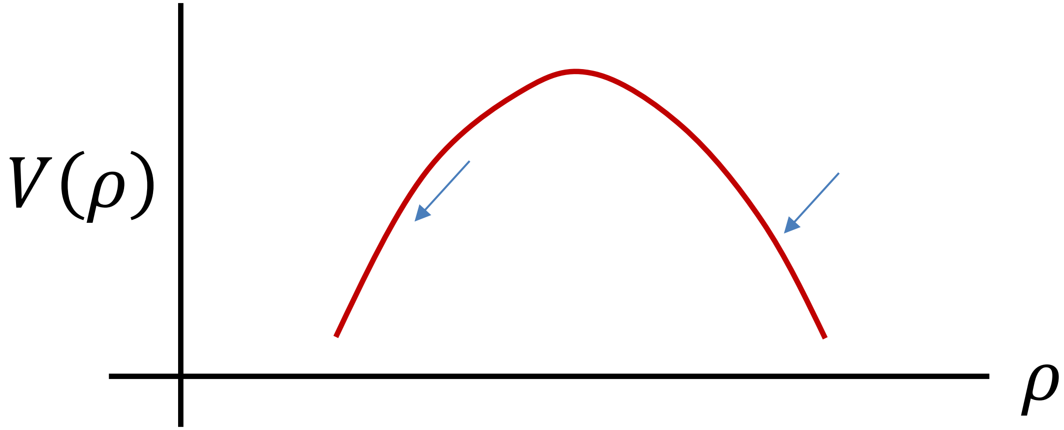
$V(\rho)$ specified – “kinematic”, contrast: gas dynamics

Can drive from CA-like automata rules

D ? → “anticipation” – driver sees ahead

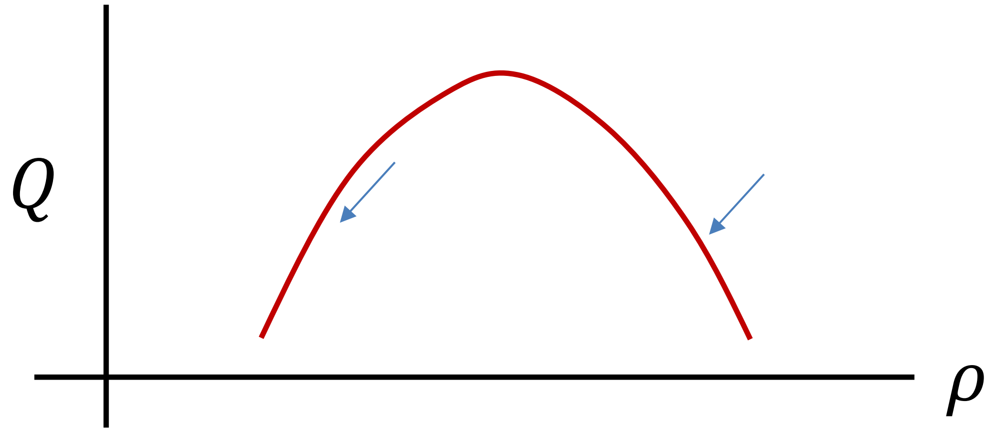
- $V(\rho)$ posited – empirics (1950’s U.K.)
  - Low density, ala’ gas dynamics
  - $V(\rho) \sim \rho$ → Burgers equation

➔ shock



Kinematic Waves, cont'd

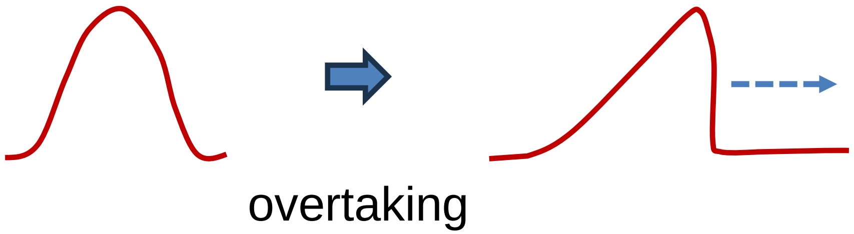
- Convenient: Flux $\rho\, V(\rho) = Q(\rho)$
  - High density
  - Bottle necks
  - i.e. increasing ρ slows down → larger ρ etc…



- Flow perturbation speed $c(\rho) = \frac{dQ(\rho)}{d\rho}$
  - $V > 0,\ \ \frac{dQ(\rho)}{d\rho} > 0 \ \rightarrow\ c(\rho) > 0$: perturbation along flow, increases with ρ → conventional shock
  - $V < 0,\ \ \frac{dQ(\rho)}{d\rho} < 0 \ \rightarrow\ c(\rho) < 0$: perturbation opposite flow, decreases with ρ → jam

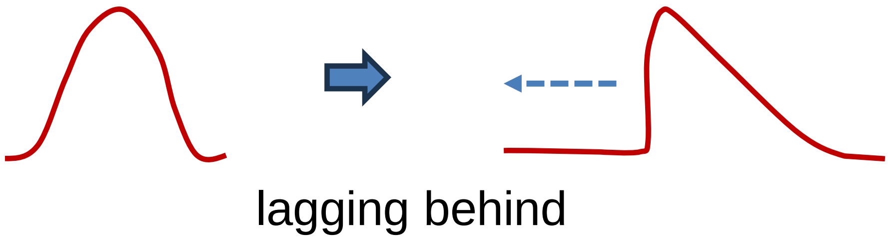
Kinematic Waves, cont'd

- $c(\rho) > 0$



Forward shock, as usual

- $c(\rho) < 0$



Backward shock

- Backward shock → entry to bottleneck, i.e. toll collection
  - High density perturbation propagates upstream, against flow

  → Jam

Kinematic Waves and MIPS

- MIPS – motility induced phase separation (Cates & Tailleur)
  - Self-propelled particles slow down in dense regions $V \sim 1 \,/\, n^{\alpha}$
  - Reduced motility → accumulation → further slow down → clustering
  - Broadly applicable to active fluids, biophysics etc. Independent of microscopics
- 1D condition: $V' / \, V < -1/\rho$

i.e. $\delta Q \,/\, \delta\rho < 0$ → MIPS = Jaming, ala' Lighthill / Whitham

Dynamics with Relaxation

- Generalized model → usual variables

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V) = 0$$

Noise + Diffusion?

$$\frac{\partial V}{\partial t} + V\frac{\partial}{\partial x}V = -\frac{1}{\tau_R}\left(V - V(\rho) + \frac{D}{\rho}\,\partial_x \rho\right)$$

Derived as moments of CA-model – car following theories

  - V relaxes to $V(\rho)$ → kinematic value
  - V → individual velocity
  - $V(\rho)$ → bulk velocity
  - τ_R → individual driver reaction flow relaxation (i.e. large for DWI…)
  - $(D/\tau_R)^{1/2}$ → speed → breaks rescaling

Dynamics with Relaxation, cont'd

- Linear stability analysis $\rho = \rho_0 + r$

$$\frac{\partial r}{\partial t} + c_0 \frac{\partial r}{\partial x} = D \frac{\partial^2 r}{\partial x^2} - \tau_R \left(\frac{\partial}{\partial t} + V_0 \frac{\partial}{\partial x} \right)^2 r$$

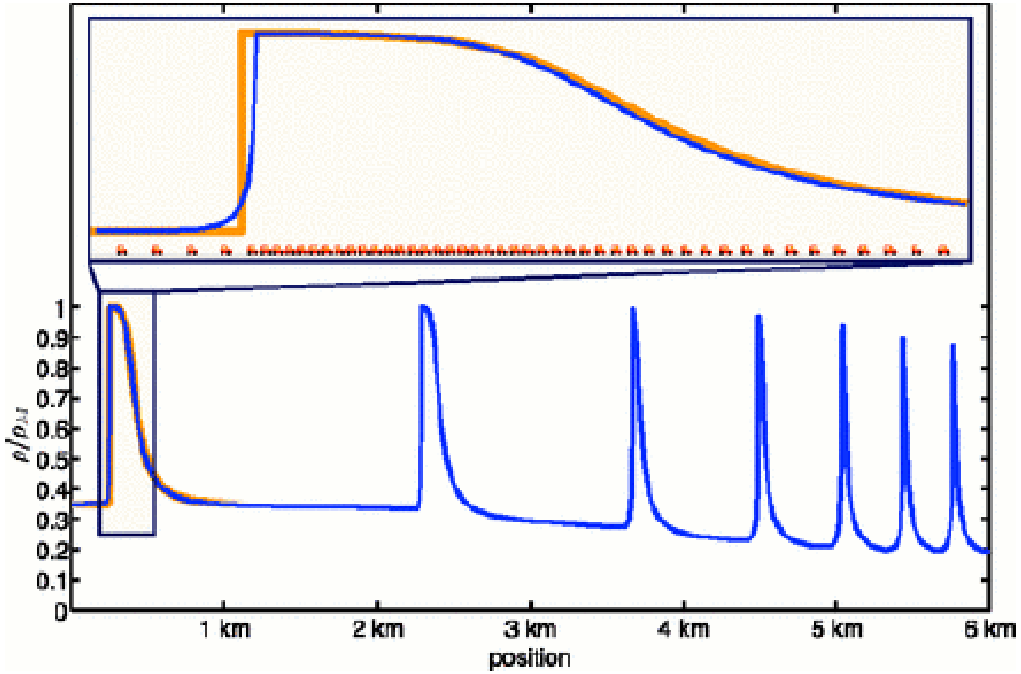
$$c_0 = \frac{d}{dr} \left(\rho V(r) \right) \Big|_{\rho_0} = V_0 + \rho_0 V_0'$$

so for $f = f(x - c_0 t)$

$$\frac{\partial r}{\partial t} + c_0 \frac{\partial r}{\partial x} = \underbrace{\left(D - \tau_R (V_0 - c_0)^2 \right)}_{\text{effective diffusivity} < 0 \text{ possible !}} \frac{\partial^2 r}{\partial x^2}$$

- For long τ_R, negative diffusivity → jam weak dependence on sign of V_0'

Dynamics with Relaxation, cont'd



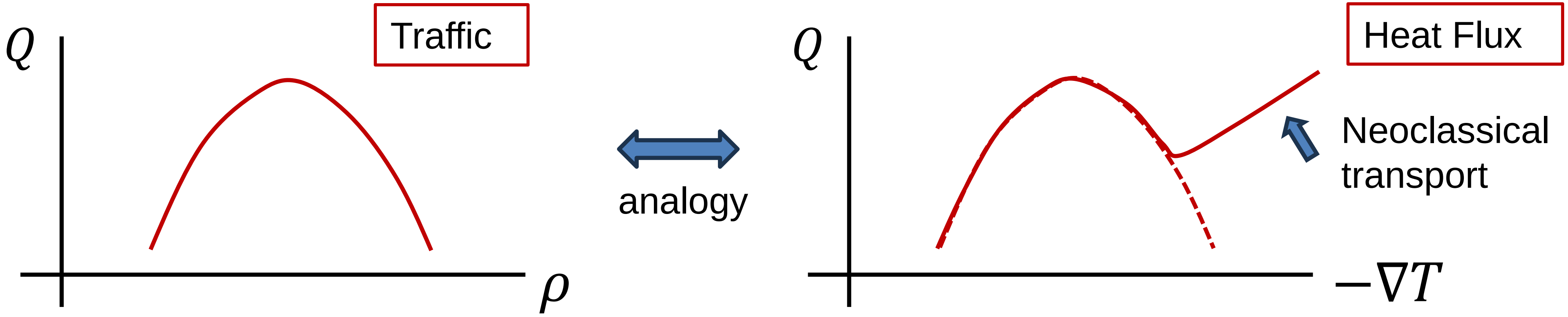
'Jamiton' train – nonlinear wave in traffic flow

➔ Density corrugation array ?!

Observations

- Kinematic wave theory requires:

$\frac{dQ}{d\rho} < 0$ for jam; independent of microscopic

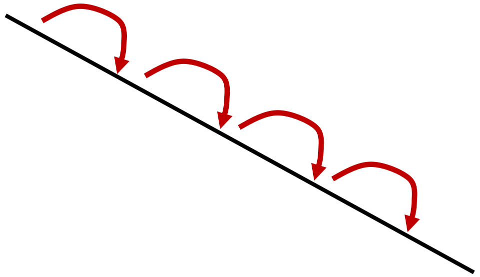


"Ideal" mechanism – flux roll-over

- Dynamic wave theory requires:
  - $[D - \tau_R(V_0 - c_0)^2] < 0$, clusters form → jams
  - Long τ_R "Dissipative" mechanism, sign V' irrelevant

Avalanches

- Intermittent, radially extended transport events
- Propagating fronts:
  - Correlated overturnings
  - Gradient relaxation
- Canonical Cartoons:



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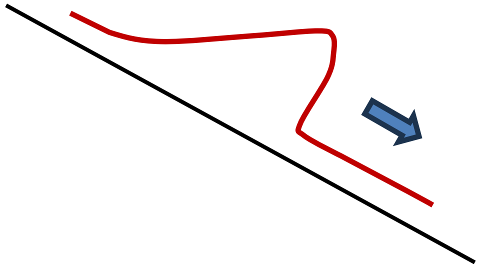


Natural to think of as bursty perturbations in heat flow

- Statistical distribution
- Encompasses turbulence spreading → c.f. Thursday talk
- Avalanching more general than SOC

Avalanches, cont'd

- As with magnetism, etc. forego micro-details for mesoscopic hydrodynamics approach, ala' TDGL

- For conserved heat

$$\delta T = T - T_{self\ organized}$$

local excess, deficit

$$\frac{\partial}{\partial t}\delta T + \partial_x[Q(\delta T)] - \chi_0\, \partial_x^2 \delta T = \tilde{s}$$

Analogous to kinematics

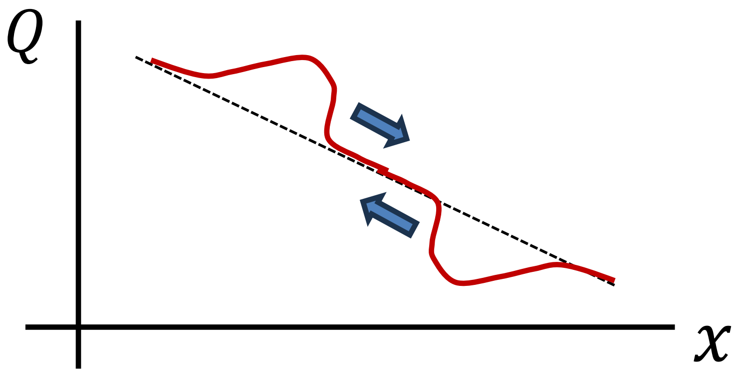
deviation-induced flux

- δT → pulse, avalanche envelope

- → $\delta T \sim \tilde{I}$ ➔ intensity perturbation

Avalanches, cont'd

- How constrain $Q[\delta T]$? (Hwa, Karder, P.D., Hahm)



$\delta T > 0$ bump → tends more down gradient, to right

$\delta T < 0$ void → tends more up gradient

∴

Can AI/ML deduce structure of $Q(\delta T)$ from numerical data?

- Joint reflection symmetry principle

$X \rightarrow -X$

$\delta T \rightarrow -\delta T$

} $Q[\delta T]$ unchanged (recall TDGL derived from parity $\eta \leftrightarrow -\eta$)

<u>So</u>

- $Q[\delta T] = \sum A_n(\delta T)^{2n} + B_m(\partial_x \delta T)^m + C_p(\partial_x^2 \delta T)^{2p} + \cdots$

Avalanches, cont'd

- Lowest order, smoothest model

$$Q(\delta T) \approx \alpha\, \delta T^2 - D\, \partial_x \delta T, \quad \alpha, D \text{ t.b.d.}$$

- Heuristic correspondence

$$\alpha\, \delta T^2 \;\leftrightarrow\; -\chi \left(\frac{1}{T} \nabla T - \frac{1}{L_{T_{crit}}} \right) \nabla T$$

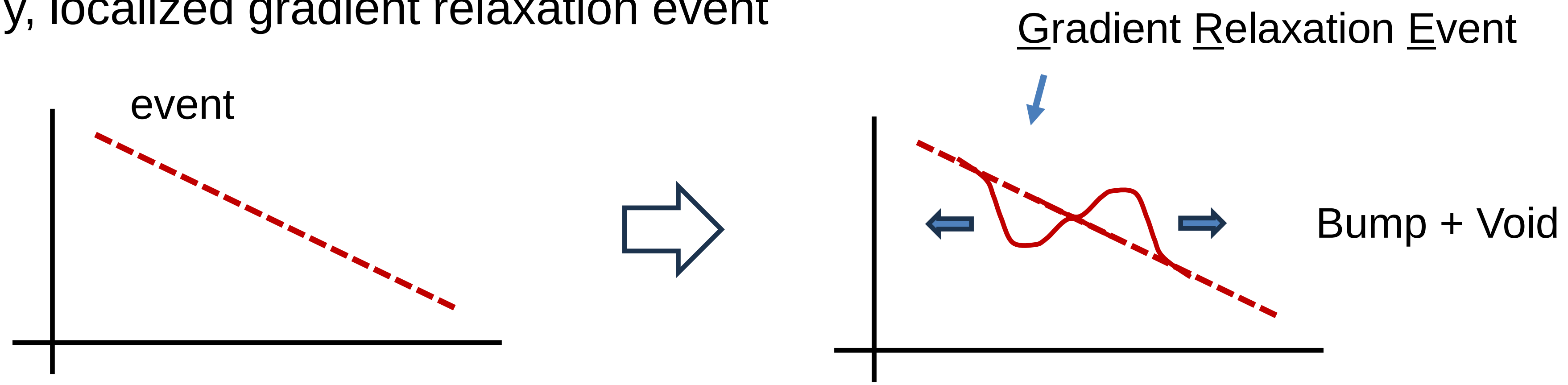
δT → threshold
δT → gradient

i.e. threshold!

- Noisey Burgers (ala' $V(\rho) \sim \rho$) N.B. If D in KW ➔ should there be noise ?!
  - Shock solution
  - Shock ←→ avalanche
  - Shock turbulence – favorite toy model
  - Velocity ~ pulse size

Bumps and Voids

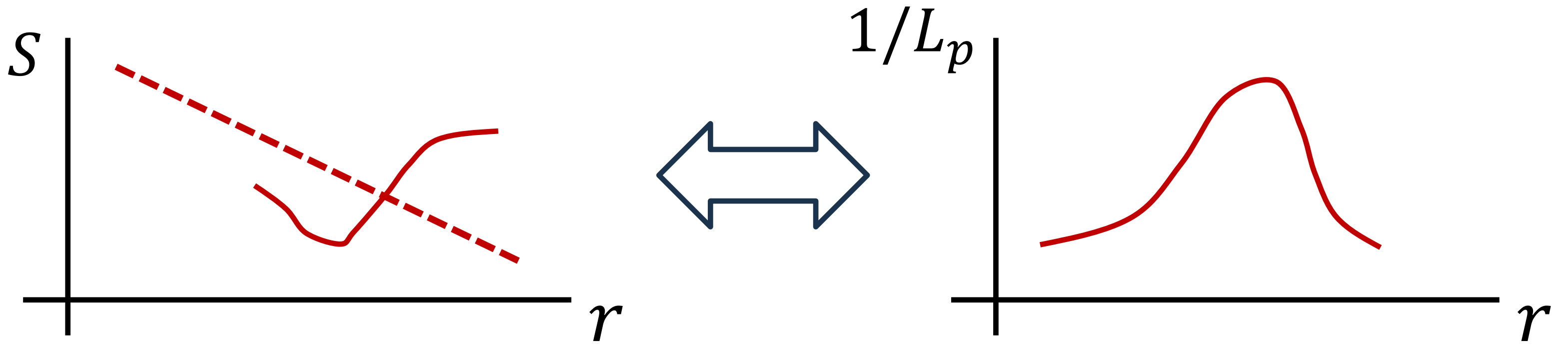
- Generically, localized gradient relaxation event



blob + void – edge
clump + hole – EP

co-existing populations

- Edge Signature



Skewness → 0 near separatrix

Jamming vs Avalanches ?

- Joint reflection symmetry constraints:

$$Q = Q(\delta T^{2n}, (\partial \delta T/\partial x)^{m}, (\partial^2 \delta T/\partial x^2)^{2p}, etc)$$

- But flux roll-over possible !
- More general form:

$$Q(\delta T) = \frac{\alpha}{2}\,(\delta T)^2 \Big/ f\left(1 + \beta(\delta T)^{2n} + \gamma\left(\frac{\partial \delta T}{\partial x}\right)^{2m} + \cdots\right)$$

(1) (2)

  - Perfectly admissible in JRS
  - (1) → n > 1 ➔ roll over for large pulse
  - (2) → $(\partial \delta T/\partial x)^{2m}, (\partial^2 \delta T/\partial x^2)^{2p}$ etc reminiscent of Hinton …

Jamming vs Avalanches, cont'd

- General JRS – consistent form admits jams
- Simplest form:

$$Q(\delta T) = \frac{\alpha}{2}(\delta T)^2/[1+\beta(\delta T)^{2n}]$$

- Jams occur:

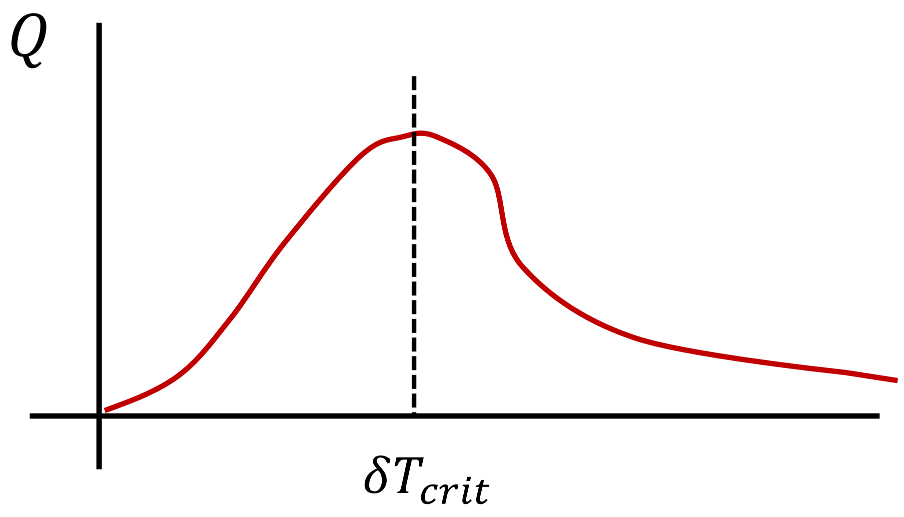
$$\frac{dQ}{d\delta T} = \frac{\alpha\delta T}{1+\beta(\delta T)^{2n}}\left[\frac{1-(n-1)\beta\delta T^{2n}}{1+\beta(\delta T)^{2n}}\right] < 0$$

$$\delta T > \delta T_{crit} = (\beta/n-1)^{1/2n} \quad \underline{n>1}$$

- β defines suppression

Jamming vs Avalanches, cont'd

- $\delta T > \delta T_{crit}$ to jam defines threshold – change branch



Jams result from co-existence of two branches $Q(\delta T)$ curve

- Pinning sites for jam formation ←→ arise via structure $\beta(x)$
- Possible route to staircase via array of local maxima in $\beta(x)$ – quasi-periodic modulations. i.e. competition of emergence vs pinning

Comments:

- No shortage of feedback loops to produce multi-branched $Q[\delta T]$

  ➔ $E \times B$ shear, diamagnetism …

- Geometry ←→ $\beta(x)$ pins jam location

  Low q's – jam

- Determine $Q(\delta T)$ structure via response to modulation? – Task for ML

- Flux roll-over counter-intuitive in TDGL context

‘Dissipative’ Heat Flux Jamming

- Here explore heat flux jams not requiring multi-branch co-existance

  ➔ mechanism for corrugation, staircases, etc.

- Approach via analogy

  Traffic: $\rho, V(\rho), V, \tau_R$

  N.B. $V(\rho)$ a characteristic / expected speed. Instantaneous speed V relaxes to $V(\rho)$ in τ_R

  Heat: $\delta T, Q(\delta T), Q, \tau_R$

  Q → instantaneous heat flux, evolves in time

  τ_R → relaxation time $Q \rightarrow Q(\delta T)$ → generalized transport prediction

Heat Flux Equations

- Standard: $Q = -\chi \nabla T$

- Systematic / Evolutionary (Perturbative):

$$\frac{\partial Q}{\partial t} + \partial_x \Gamma_Q(\varepsilon) = -k_\perp^2\, D(\varepsilon)(Q + \chi(\varepsilon)\partial_x \langle T \rangle)$$

$Q \equiv$ evolving heat flux $\quad k_\perp^2 D(\varepsilon) \equiv$ relaxation rate (turbulent scattering)

$-\chi \nabla T \equiv$ mean heat flux $\quad \varepsilon \equiv$ turbulence energy $\sim \delta T$

- Simplified form, m.f. → JRS

$$\frac{\partial Q}{\partial t} = -\frac{1}{\tau_R}\left(Q - Q_0(\delta T)\right) \qquad \text{now: } Q_0 = \frac{\alpha}{2}(\delta T)^2 - \chi_2\, \partial_x \delta T + \chi_4\, \partial_x^3 \delta T$$

τ_R ?! - t.b.d.

System

$$\frac{\partial}{\partial t}\delta T + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} = -\frac{1}{\tau_R}\left(Q - Q_0(\delta T)\right)$$

$$\frac{\partial}{\partial t}\delta T + \tau_R \frac{\partial^2}{\partial t^2}\delta T + \alpha\, \delta T \frac{\partial}{\partial x}\delta T = \chi_2\, \partial_x^3 \delta T - \chi_4\, \partial_x^4 \delta T + \tilde{S}$$

- <u>Telegraph equation structure</u> ➔ familiar from HPP analysis (phenomenological)
- Really Telegraph-Burgers…

Microphysics of τ_R - Physics Origin!?

- τ_R emerges from evolution of Q

  Q → quadratic ➔ correlation function moment

- So physics τ_R underpinned by $\langle \tilde{f}(1)\tilde{f}(2)\rangle$

  2pt correlation equation:

$$\frac{\partial}{\partial t}\left\langle \tilde{f}(1)\tilde{f}(2)\right\rangle + (\mathrm{V}_1 \cdot \nabla_1 + V_2 \cdot \nabla_2)\left\langle \tilde{f}(1)\tilde{f}(2)\right\rangle + P_1\langle f_1\rangle + P_2\langle f_2\rangle + \nabla \cdot \Gamma_{Q_1} + \nabla \cdot \Gamma_{Q_2} = 0$$

$P = \left\langle \tilde{V}_{E\times B}\tilde{f}\right\rangle \cdot \nabla$ → production

$\Gamma_Q = \left\langle \tilde{V}_{E\times B}\tilde{f}(1)\tilde{f}(2)\right\rangle$ → turbulent mixing of correlation (phasetropy cascade)

$\therefore\ \tau_R \sim \tau_C$ → mixing time. TBC (as you knew all along…)

Microphysics of τ_R

- $\tau_R \sim \tau_C$ → "critical slowing down" phenomena near marginality !

So expect

$$1/\tau_c \sim 1/\tau_0 \left(R/L_T - R/L_{T_{crit}}\right)^{\mu}$$ typically $\mu < 1$

μ → stiffness exponent ~ threshold deviance τ_0 → characteristic time (ω_0^{-1})

- τ_R diverges near criticality / marginality

∴ expect jam formation there

- But: how near is "near"? → question of the ages…

How near is "near" ?

- Recall telegraph-Burgers equation
- Linearization – abt pulse

$$\partial_t \delta\tilde{T} + c_0 \partial_x \delta\tilde{T} + \tau_R \partial_t^2 \delta\tilde{T} = \chi_2 \partial_x^2 \delta\tilde{T} - \chi_4 \partial_x^4 \delta\tilde{T}$$

$c_0 \approx \alpha\delta T$ → avalanche speed (bigger → faster)

χ_2 → background heat diffusion ~ χ_{neo}

- $\delta\tilde{T} = f(x - c_0 t, \tau)$

$$\frac{\partial f}{\partial \tau} = (\chi_2 - \tau_R c_0^2)\partial_x^2 f \ - \chi_4 \ \partial_x^4 f$$

⇩

must < 0 for jam on any scale → delay time vs χ_{neo} !

How near is "near" ?

$$\frac{c_0^2}{\chi_{neo}} > \frac{1}{\tau_0}\left(R/L_T - R/L_{T_c}\right)^{\mu}$$

$c_0 \sim \alpha\delta T$

dedicated to GDP

i.e. Avalanche speed must exceed speed of diffusion in a relaxation time

- Weak residual / neoclassical diffusion +

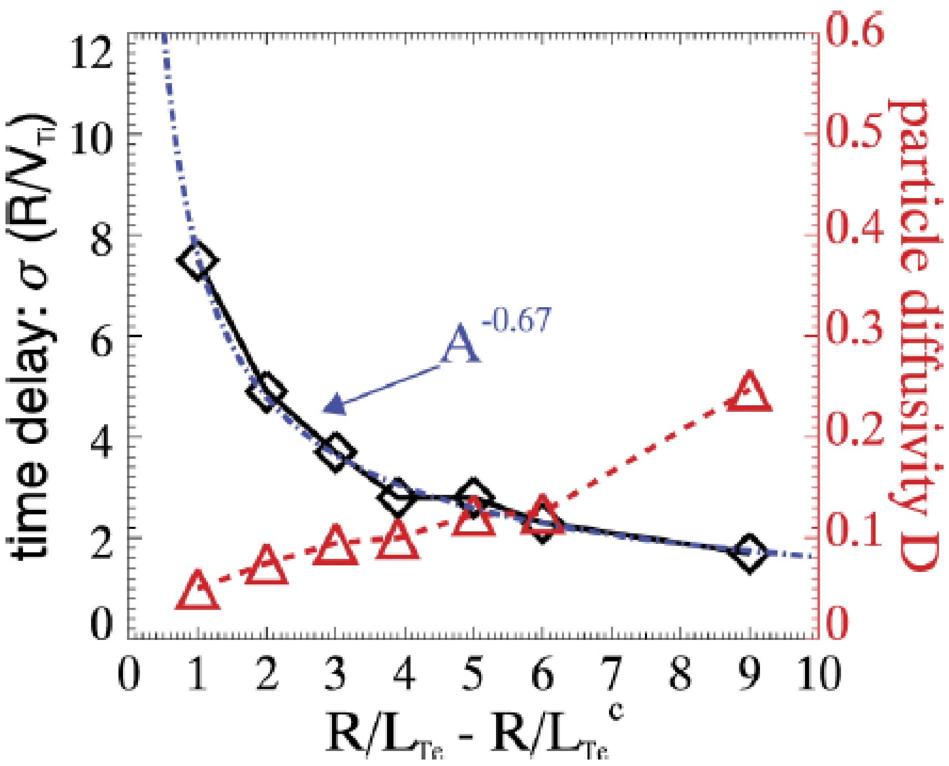
Larger avalanche speed favors larger τ_R and jamming

←→ Power and neoclassical scaling trends testable

i.e. $\alpha\delta T$ – modulation amplitude

χ_{neo} - predictable scalings, especially I_p

How near is “near” ?



GKPSP (δf) simulation by Lei Qi +

→ GYSELA comparison?

Systematic Analysis (Kosuga+)

$$\tau_R > \frac{\chi_2}{c_0^2}\left(1 + \chi_4 \frac{k^2}{\chi_2}\right)$$

$$k_{max}^2 \sim \frac{\alpha \delta T}{2(\chi_{neo}\chi_4)^{1/2}}$$

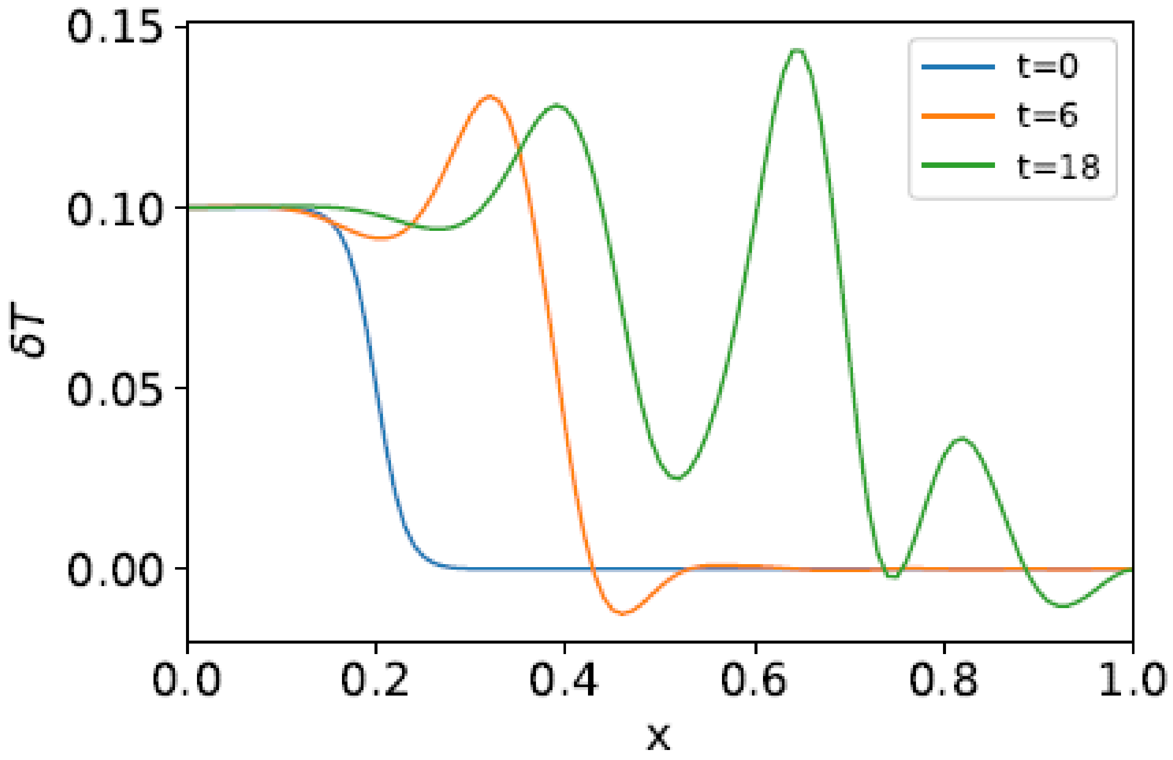
- With $E \times B$ shear

$$\frac{\delta T}{T} \sim \left(\frac{1}{v_{thi}\rho_i}\right)\left(\frac{\chi_4}{\tau_R}\right)^{1/2}$$ → corrugation amplitude

$$\Delta_{max}^2 \sim \rho_i \frac{2 v_{thi}}{\alpha T_i}\left(\frac{\chi_2 \tau_R}{\rho_i^2}\right)^{1/2}$$ → corrugation scale

$$\rho_i < \Delta_{max} < L_T$$

Jamiton Train



What have we learned?

- Both ideal and dissipative mechanisms / routes to jams, entirely consistent with JRS
- Ideal jams occur via saturation of Q with increasing $|\delta T|$
- For dissipative jams, can quantify extent of range for criticality phenomena
  → how near is “near” ?
- Expect testable scalings

What next?

- Explore synergy between ideal and dissipative processes (Kosuga+)
- Effect nucleation – pinning i.e. $\alpha = \alpha_0 \cos(kx + \varphi)$
- Characterize jam trains → scale of staircase
- * Q_k scale dependency, spectral transfer trends – τ_R trends

Thought

- Declarations of “near marginal stability” should always be accompanied by estimates of how near…

- Models should predict this.