Elastic Turbulence in Flatland: Interfaces Encode Memory, and so Determine Transport

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See Also: C. Chen and P.D.; NO4.00003

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Outline

- Elastic Fluids:

  Elasticity ⇔ Memory ⇔ Transport

- Active Scalar Transport in 2D MHD:

  Conventional Wisdom

- New Development: Blobs and Barriers
- Revisiting Quenching
- Inhomogeneous Mixing and Staircases
- Open Questions

Elastic Fluids

- Interal DOF exerting restoring force on fluid → "springiness"
- Examples:
  - MHD → $\vec{B},\ \vec{J}\times\vec{B}$

    Polymer hydro → Elastic element oldroyd-B

  - Spinodal Decomposition (CHNS) → droplet surface tension
- Elasticity → <u>Memory</u> → Impact on mixing?!

Active Scalar Transport in 2D MHD: Background and Conventional Wisdom

Physics: Active Scalar Transport

- Magnetic diffusion, ψ transport are cases of active scalar transport
- (Focus: 2D MHD) (Cattaneo, Vainshtein '92, Gruzinov, P. D. '94, '95)

scalar mixing – the usual

$$\partial_t A + \nabla\phi \times \hat{z} \cdot \nabla A = \eta \nabla^2 A$$

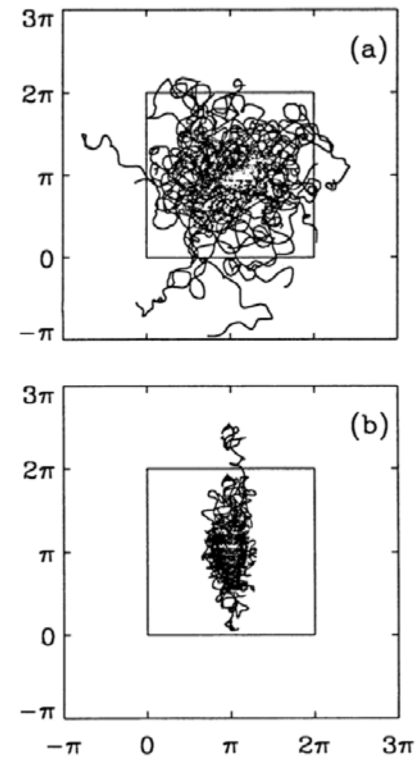
$$\partial_t \nabla^2\phi + \nabla\phi \times \hat{z} \cdot \nabla\nabla^2\phi = \nabla A \times \hat{z} \cdot \nabla\nabla^2 A + \nu\nabla^2\nabla^2\phi + \tilde{f}$$

turbulent resistivity

back-reaction

- Seek $\langle v_x A \rangle = -D_T \frac{\partial\langle A\rangle}{\partial x} - \eta \frac{\partial\langle A\rangle}{\partial x}$
- Point: $D_T \neq \sum_{\vec{k}} |v_{\vec{k}}|^2 \, \tau_{\vec{k}}^{K}$, often substantially less
- Why: <u>Memory</u>! $\leftrightarrow$ Freezing-in
- Cross Phase

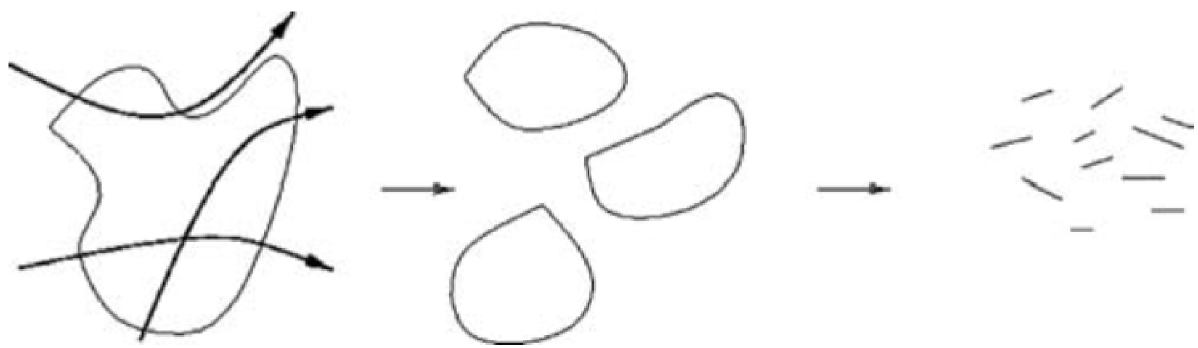
Conventional Wisdom



- [Cattaneo and Vainshtein 1991]: turbulent transport is suppressed even for a <u>weak</u> large scale magnetic field is present.
- Starting point: $\partial_t \langle A^2 \rangle = -2\eta \langle B^2 \rangle$
- Assumptions:
  - Energy equipartition: $\frac{1}{\mu_0 \rho} \langle B^2 \rangle \sim \langle v^2 \rangle$
  - Average B can be estimated by: $|\langle \mathbf{B} \rangle| \sim \sqrt{\langle A^2 \rangle} / L_0$
- Define Mach number as: $M^2 = \langle v_A \rangle^2 / \langle \tilde{v}^2 \rangle = \langle v^2 \rangle / v_A^2 = \langle v^2 \rangle / \frac{1}{\mu_0 \rho} \langle B^2 \rangle$
- Result for suppression stage: $\eta_T \sim \eta M^2$
- Fit together with kinematic stage result: $\boxed{\eta_T \sim \frac{ul}{1 + \mathrm{Rm}/M^2}}$
- Lack physics interpretation of η_T !

Origin of Memory?

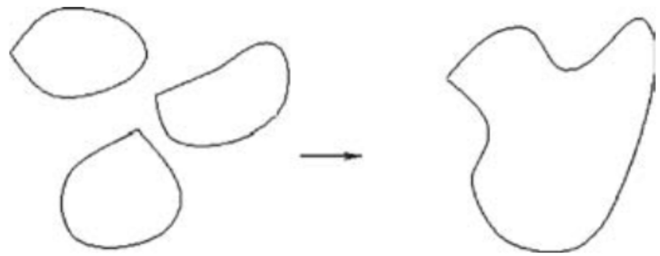
- (a) flux advection vs flux coalescence
  - intrinsic to 2D MHD (and CHNS)
  - rooted in <u>inverse cascade</u> of $\langle A^2 \rangle$ - dual cascades
- (b) tendency of (even weak) <u>*mean*</u> magnetic field to "Alfvenize" turbulence [cf: vortex disruption feedback threshold!]
- Re (a): Basic physics of 2D MHD



Forward transfer: fluid eddies chop up scalar A.

Memory Cont'd

- v.s.



Inverse transfer: current filaments and A-blobs attract and coagulate.

- Obvious analogy: straining vs coalescence; CHNS
- Upshot: closure calculation yields:

$$\Gamma_A = -\sum_{\vec{k}'}[\tau_c^{\phi}\langle v^2\rangle_{\vec{k}'} - \tau_c^{A}\langle B^2\rangle_{\vec{k}'}]\frac{\partial\langle A\rangle}{\partial x} + \cdots$$

flux of potential

competition

scalar advection vs. coalescence ("negative resistivity")

(+) (-)

N.B.:
Coalescence
→ Negative diffusion
→ Bifurcation

Conventional Wisdom, Cont'd

- Then calculate $\langle B^2 \rangle$ in terms of $\langle v^2 \rangle$

$$\partial_t A + \mathbf{v} \cdot \nabla A = -v_x \frac{\partial \langle A \rangle}{\partial x} + \eta \nabla^2 A$$

- Multiplying by A and sum over modes:

$$\frac{1}{2}[\cancel{\partial_t \langle A^2 \rangle} + \cancel{\langle \nabla \cdot \langle \mathbf{v} A^2 \rangle \rangle}] = -\Gamma_A \frac{\partial \langle A \rangle}{\partial x} - \eta \langle B^2 \rangle \qquad \frac{\partial \langle A \rangle}{\partial x} \to B_0$$

Dropped stationary case

Dropped periodic boundary → introduce nonlocality?!

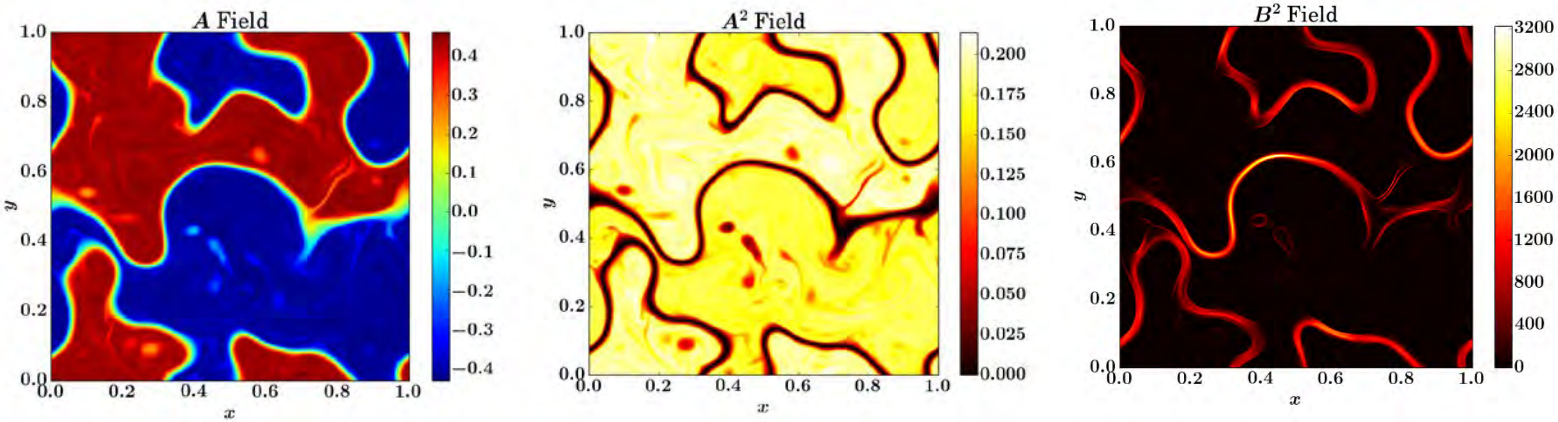
- Therefore: $\langle B^2 \rangle = -\frac{\Gamma_A}{\eta} \frac{\partial \langle A \rangle}{\partial x} = \frac{\eta_T}{\eta} B_0^2$
- Define Mach number as: $M^2 \equiv \langle v^2 \rangle / v_{A0}^2 = \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} B_0^2)$
- Result: $\boxed{\eta_T = \frac{\sum_{\mathbf{k}} \tau_c \langle v^2 \rangle_{\mathbf{k}}}{1 + \mathrm{Rm}/M^2} = \frac{ul}{1 + \mathrm{Rm}/M^2}}$
- This theory is not able to describe $B_0 \to 0$ case!

New Wrinkles

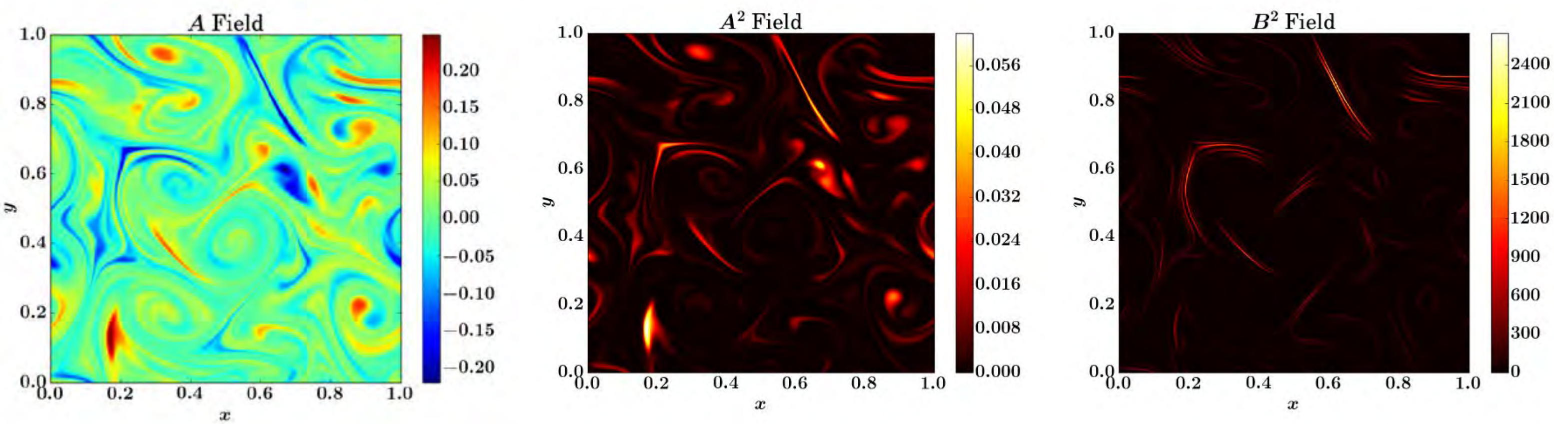
New Observations

- With no imposed B_0, in suppression stage:

Field Concentrated!



- v.s. same run, in kinematic stage (trivial):



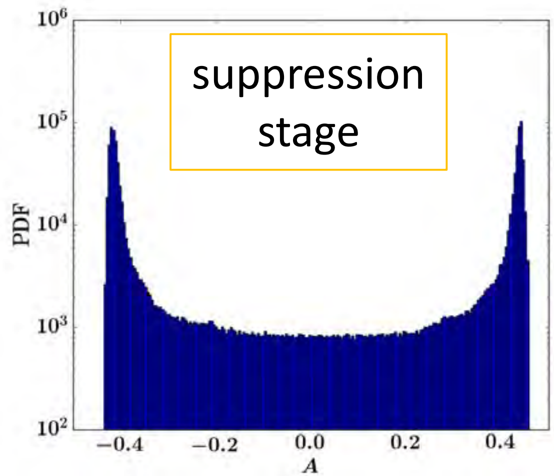
New Observations Cont'd

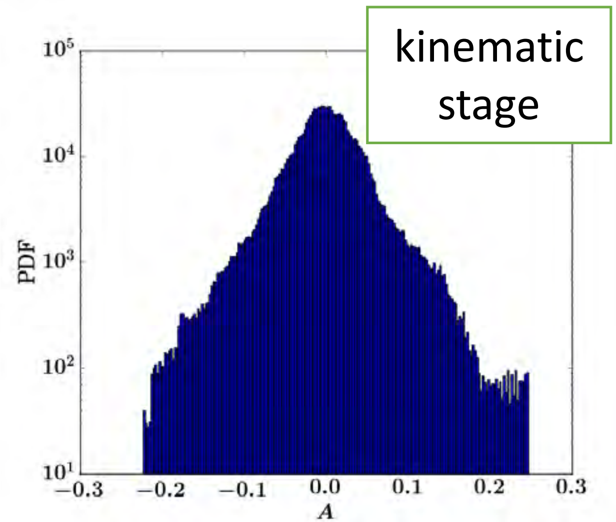
- Nontrivial structure formed in real space during the suppression stage.
- A field is evidently composed of "<u>blobs</u>".
- The low A^2 regions are 1-dimensional.
- The high B^2 regions are strongly correlated with low A^2 regions, and also are 1-dimensional.
- We call these 1-dimensional high B^2 regions ``<u>barriers</u>'', there, mixing is reduced, relative to η_K.

➔ Story one of 'blobs and barriers'

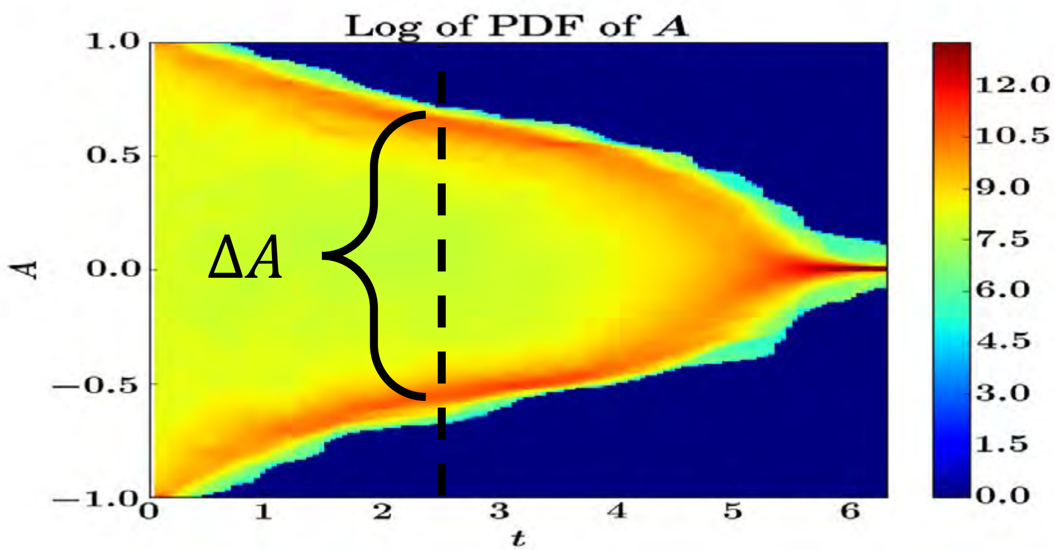
Evolution of PDF of A

- Probability Density Function (PDF) in two stage:





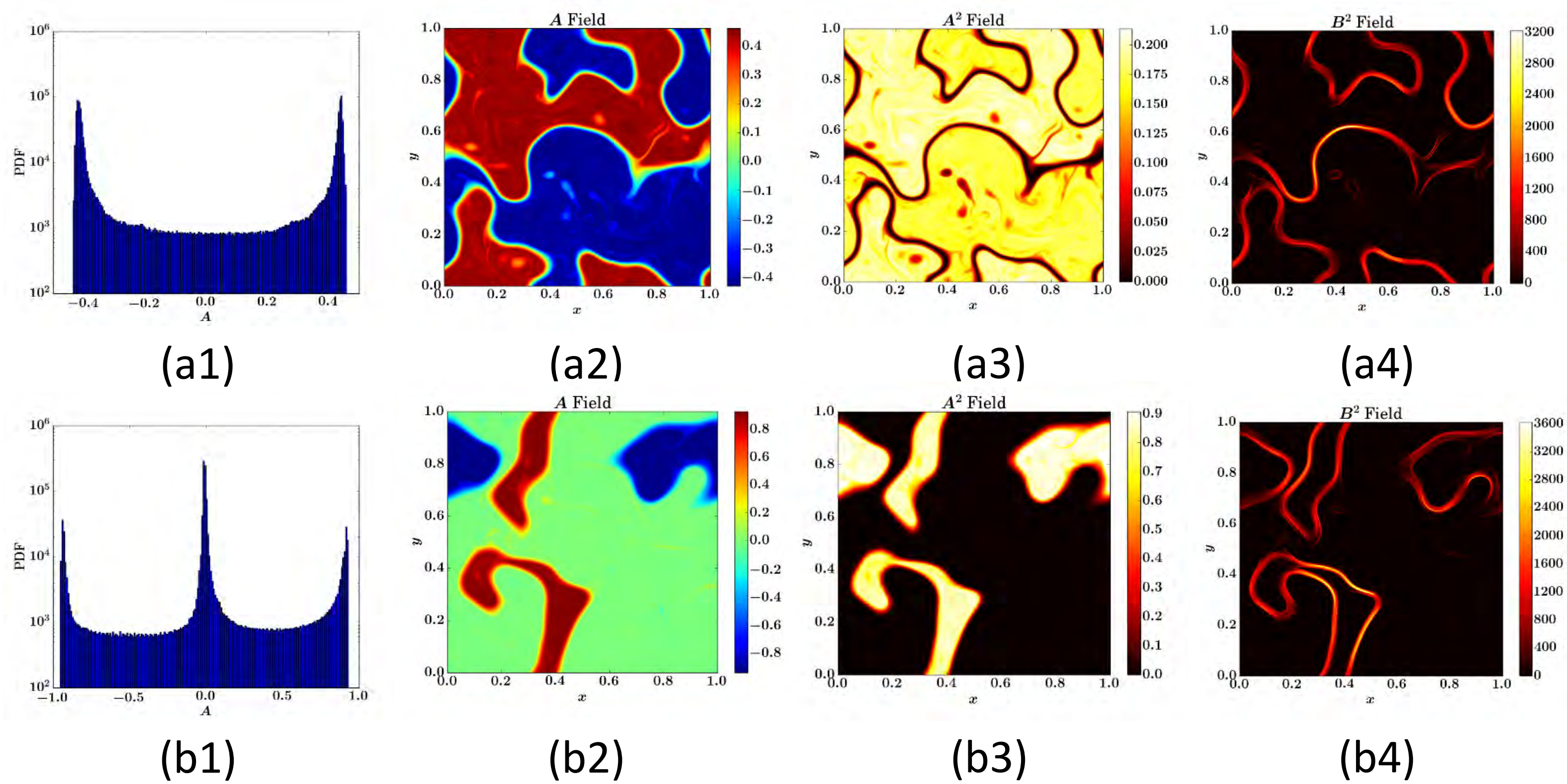
- Time evolution: horizontal “Y”.



- The PDF changes from double peak to single peak as the system evolves from the suppression stage to the kinematic stage.

Unimodal Initial Condition

- One may question whether the bimodal PDF feature is purely due to the initial condition. The answer is <u>No</u>.
- Two non-zero peaks in PDF of A still arise, even if the initial condition is unimodal.



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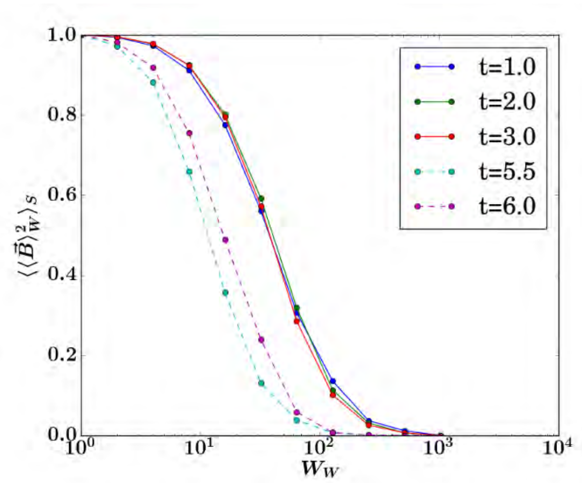
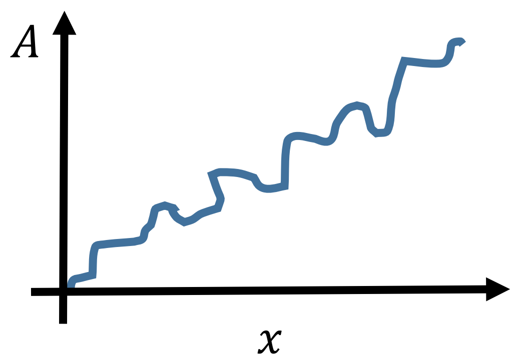
The problem of the mean field $\langle \boldsymbol{B} \rangle$ → What does mean mean?

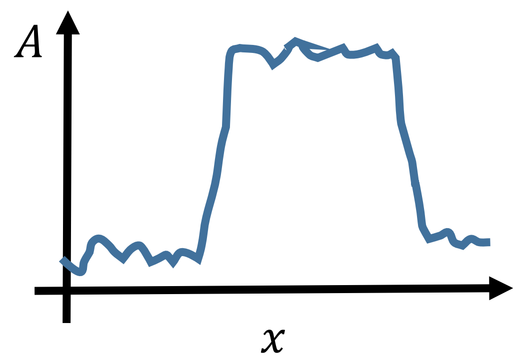
- $\langle B \rangle$ depends on the averaging window.
- With no imposed external field, B is highly intermittent, therefore $\langle B \rangle$ is not well defined.





$|\langle B \rangle| \sim \sqrt{\langle A^2 \rangle}/L_0$ ✓

v.s.



$\langle B \rangle$ not well defined

Reality

Revisiting Quenching

New Understanding

- Summary of important length scales: $l < L_{stir} < L_{env} < L_0$
  - System size L_0
  - Envelope size L_{env} → emergent (blob)
  - Stirring length scale L_{stir}
  - Turbulence length scale l, here we use Taylor microscale λ
  - Barrier width W → emergent
- Quench is not uniform. Transport coefficients differ in different regions.
- In the regions where magnetic fields are strong, Rm/M^2 is dominant. They are regions of ***barriers***.
- In other regions, i.e. Inside blobs, Rm/M'^2 is what remains. $M'^2 \equiv \langle V^2 \rangle / \left(\frac{1}{\rho} \langle A^2 \rangle / L_{env}^2 \right)$

New Understanding, cont'd

- From $\partial_t \langle A^2 \rangle = -\langle \mathbf{v} A \rangle \cdot \nabla \langle A \rangle - \nabla \cdot \langle \mathbf{v} A^2 \rangle - \eta \langle B^2 \rangle$
- Retain 2nd term on RHS. Average taken over an envelope/blob scale.
- Define diffusion (closure):

$$\langle \mathbf{v} A \rangle = -\eta_{T1} \nabla \langle A \rangle$$
$$\langle \mathbf{v} A^2 \rangle = -\eta_{T2} \nabla \langle A^2 \rangle$$

- Plugging in: $\partial_t \langle A^2 \rangle = \eta_{T1} (\nabla \langle A \rangle)^2 + \nabla \eta_{T2} \cdot \nabla \langle A^2 \rangle - \eta \langle B^2 \rangle$
- For simplicity: $\langle B^2 \rangle \sim \frac{\eta_T}{\eta} (\langle B \rangle^2 + \langle A^2 \rangle / L_{env}^2)$
- where L_{env} is the envelope size. Scale of $\nabla^2 \langle A^2 \rangle$.
- Define new strength parameter: $M'^2 \equiv \langle v^2 \rangle / (\frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2)$
- Result: $\boxed{\eta_T = \frac{ul}{1 + \mathrm{Rm}/M^2 + \mathrm{Rm}/M'^2} = \frac{ul}{1 + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle \mathbf{B} \rangle^2 / \langle v^2 \rangle + \mathrm{Rm} \frac{1}{\mu_0 \rho} \langle A^2 \rangle / L_{env}^2 \langle v^2 \rangle}}$

$$\eta_T = V\, l \,/ \left[1 + \frac{R_m}{M^2} + \frac{R_m}{M'^2}\right]$$

- Barriers:

Strong field

$$\eta_T \approx V\, l \,/ \left[1 + R_m \frac{\langle B \rangle^2}{\rho \langle \tilde{V}^2 \rangle}\right]$$

- Blobs:

Weak effective field

$$\eta_T \approx V\, l \,/ \left[1 + R_m \frac{\langle A^2 \rangle}{\rho L_{env}^2 \langle \tilde{V}^2 \rangle}\right]$$

- Quench stronger in barriers, highly non-uniform

Formation of Barriers

- How do the barriers form?

flux coalescence

$$\eta_T = \sum_{\mathbf{k}} \tau_c [\langle v^2 \rangle_{\mathbf{k}} - \frac{1}{\mu_0 \rho} \langle B^2 \rangle_{\mathbf{k}}]$$

- From above, strong B regions can support negative incremental $\eta_T \;\; \delta\Gamma_A / \delta(-\nabla A) < 0$, suggesting clustering

- $\langle \eta_T \rangle \; > \; 0$

- Positive feedback: a twist on a familiar theme

$$\begin{tikzcd}
\text{B is strong in a specific region} \arrow[r] & \text{diffusion of A is negative} \arrow[d] \\
\text{B in that region increases} \arrow[u] & \nabla\text{A increases} \arrow[l]
\end{tikzcd}$$

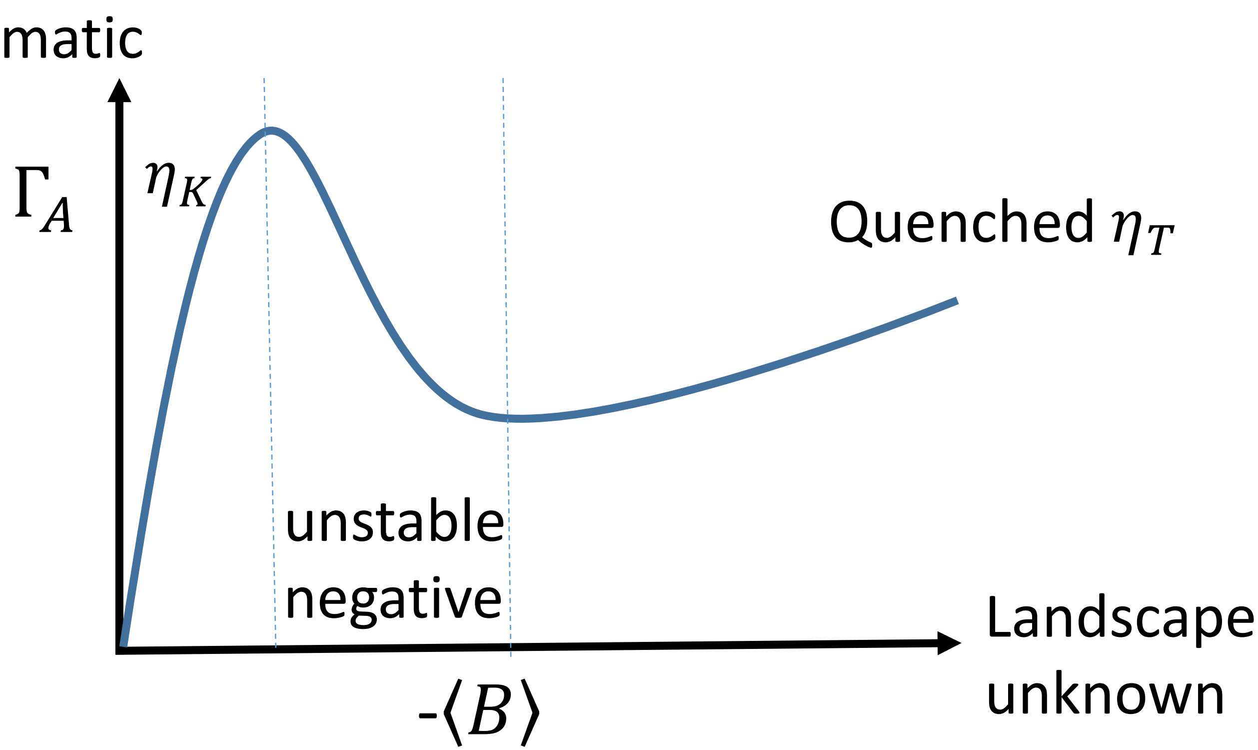
Formation of Barriers, Cont'd

- Negative resistivity leads to barrier formation.
- The S-curve reflects due to the dependence of Γ_A on B.
- When slope is negative → negative (incremental) resistivity.

Barriers ←→

Bistability of Γ_A vs ∇A

→ a familiar theme



Describing the Barriers

- How to measure the barrier width W.
- Starting point: $W \sim \Delta A / B_b$
- Use $\sqrt{\langle A^2 \rangle}$ to calculate ΔA

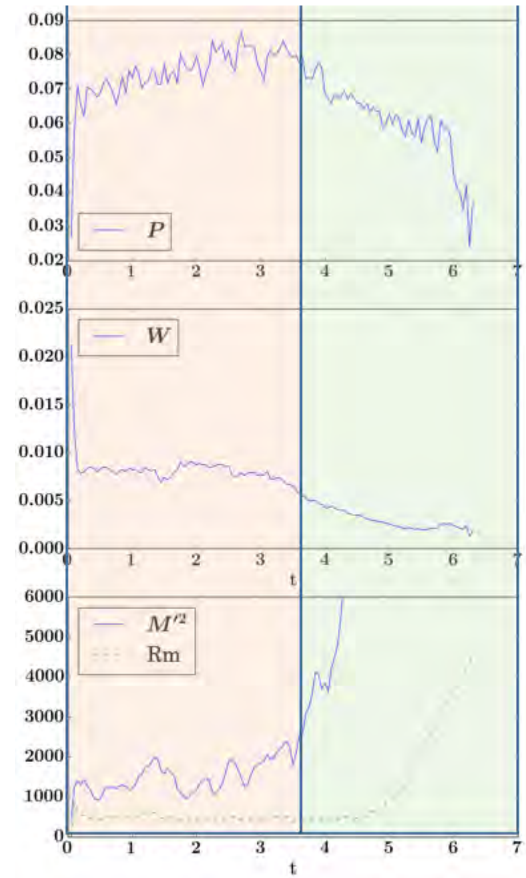
arbitrary threshold

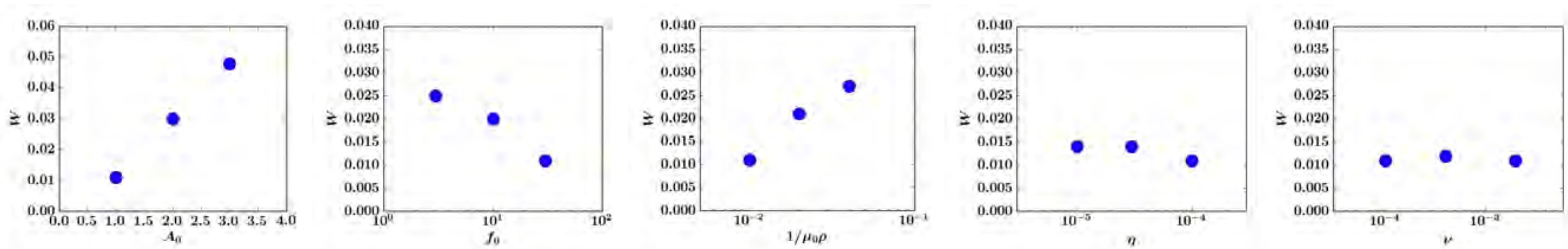
- Define the barrier regions as: $B(x,y) > \sqrt{\langle B^2 \rangle} * 2$
- Define barrier packing fracti $P \equiv \frac{\#\text{ of grid points for barrier regions}}{\#\text{ of total grid points}}$
- Use use the magnetic fields in the barrier regions to calculate the magnetic energy: $\boxed{\sum_{\text{barriers}} B_b^2 \sim \sum_{\text{system}} B^2}$
- Thus $\langle B_b^2 \rangle \sim \langle B^2 \rangle / P$
- So barrier width can be estimated by: $\boxed{W^2 \equiv \langle A^2 \rangle / (\langle B^2 \rangle / P)}$

N.B. All magnetic energy in the barriers

Describing the Barriers

- Time evolution of P and W:
  - P, W collapse in decay
  - M' rises
- Sensitivity of W:
  - A_0 or $1/\mu_0\rho$ greater → W greater;
  - f_0 greater, W smaller; (ala' Hinze)
  - W not sensitive to η or ν.



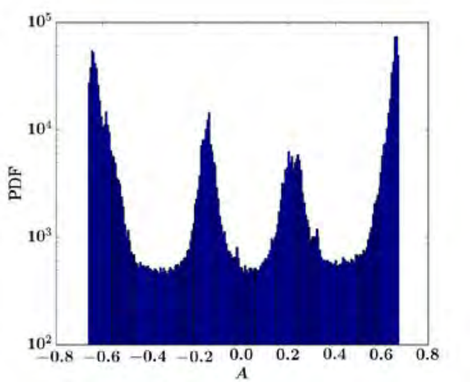


(a) (b) (c) (d) (e)

Active Scalar Staircases

Staircase (inhomogeneous Mixing, Bistability)

- Staircases emerge spontaneously! – <u>Barrier lattices</u>
- Initial condition is the usual cos function (bimodal)
- The only major sensitive parameter (from runs above) is the forcing scale k=32 (for all runs above k=5).
- Resembles the PV staircase



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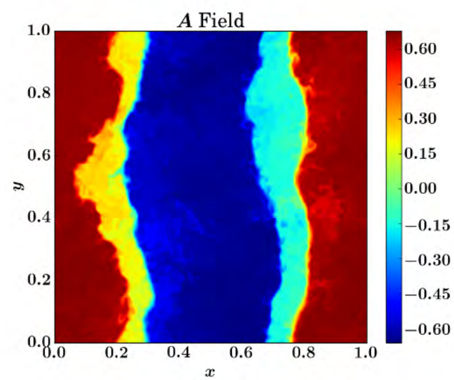

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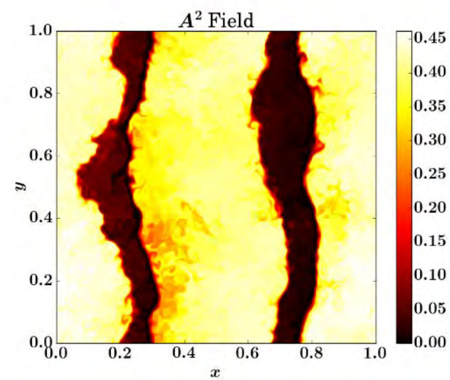

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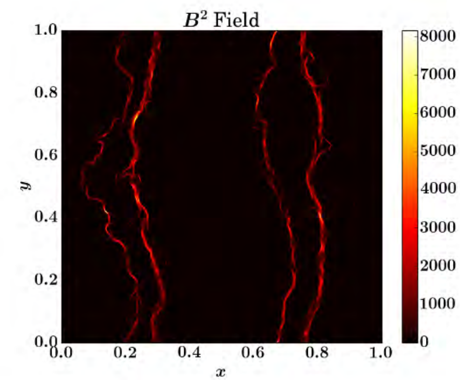


(4)

Conclusions / Summary

- Magnetic fields suppress turbulent diffusion in 2D MHD by: formation of intermittent ***transport barriers***.
- Magnetic structures:
  - Barriers – thin, 1D strong field regions
  - Blobs – 2D, weak field regions
- Quench not uniform:

$$\eta_T = \frac{ul}{1 + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle \mathbf{B}\rangle^2/\langle v^2\rangle + \mathrm{Rm}\frac{1}{\mu_0\rho}\langle A^2\rangle/L_{env}^2\langle v^2\rangle}$$

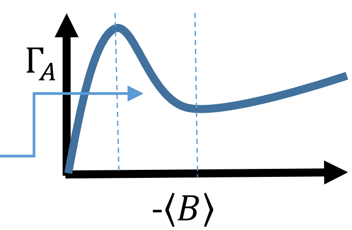
barriers, strong B

blobs, weak B, $\nabla^2\langle A^2\rangle$ remains

- Barriers form due to negative resistivity:

$$\eta_T = \sum_{\mathbf{k}} \tau_c[\langle v^2\rangle_{\mathbf{k}} - \frac{1}{\mu_0\rho}\langle B^2\rangle_{\mathbf{k}}]$$

flux coalescence



- Formation of "magnetic staircases" observed for some stirring scale

Future Works

- Extension of the transport study in MHD:
  - Numerical tests of the new η_T expression ?
  - What determines the barrier width and packing fraction ?
  - Why does layering appear when the forcing scale is small ?
  - What determines the step width, in the case of layering
- Other similar systems can also be studied in this spirit. e.g. Oldroyd-B model for polymer solutions. (drag reduction)
- Reduced Model of Magnetic Staircase

See Also:

C. Chen and P.D.: "PV Mixing in a Tangled Magnetic Field", NO4.00003

Reading

Fan, P.D., Chacon:

- PRE Rap Comm 99, 041201 (2019)
- PoP 25, 055702 (2018)
- PRE Rap Comm 96, 041101 (2017)
- Phys Rev Fluids 1, 054403 (2016)

Thank you!

Back-Up

Constitutive Relations → Deborah Number

- J. C. Maxwell:

$$(\text{stress}) + \overset{\text{relaxation}}{\tau_R}\,\frac{d(\text{stress})}{dt} = \overset{\text{viscosity}}{\eta}\,\frac{d}{dt}(\text{strain})$$

- If $\tau_R/T = D \ll 1$, stress = $\eta \dfrac{d}{dt}$(strain)

$T \equiv$ dynamic time scale

$\Pi = -\,\eta \nabla \vec{v}$ <u>viscous</u>

- If $\tau_R/T = D \gg 1$, stress $\cong \dfrac{\eta}{\tau_R}$(strain)

~ E (strain) <u>elastic</u>

- Limit of "freezing-in": D≫1 is criterion.

- D ~ Deborah Number ~ $|\nabla V|/\omega_Z$ ~ τ_{relax}/τ_{dyn}
- Limit for elasticity: $D \gg 1$ → limit for elasticity
- Why “Deborah”? →

Hebrew Prophetess Deborah:

“The moutains flowed before the Lord.” (Judges)

∴

- Revisit Heraclitus (1500 years later):

“All things flow” – if you can wait long enough

Simulation Setup

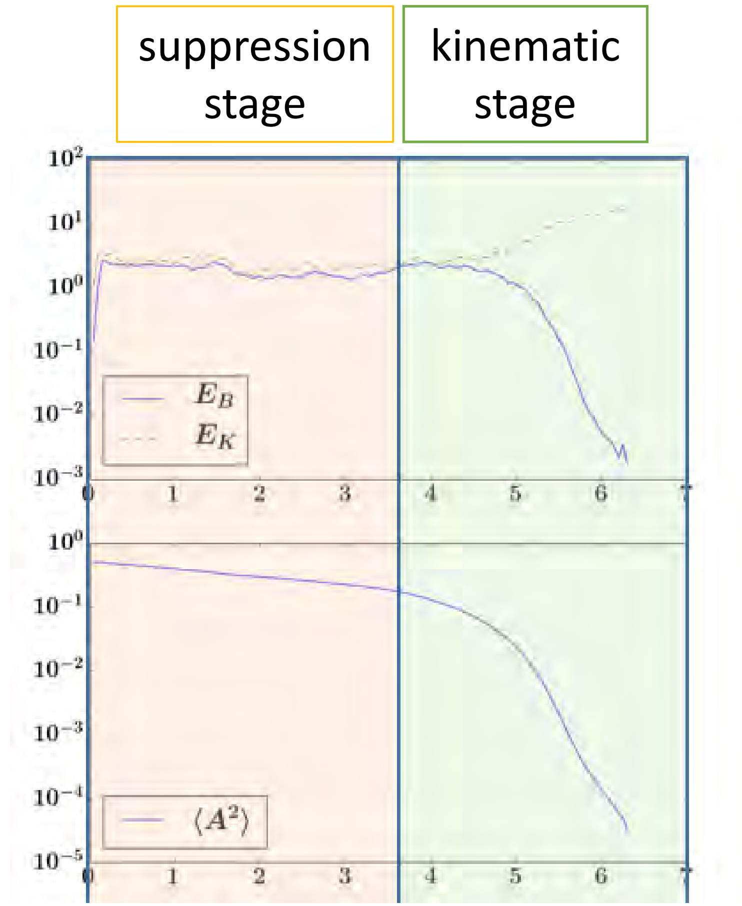
- PIXIE2D: a DNS code solving 2D MHD equations in real space:

$$\partial_t A + \mathbf{v} \cdot \nabla A = \eta \nabla^2 A$$
$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \mathbf{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega + f$$

- 1024^2 resolution.
- External forcing f is isotropic homogeneous.
- Periodic boundary conditions (both).
- Initial conditions:
  - (1) bimodal: $A_I(x,y) = A_0 \cos 2\pi x$
  - (2) unimodal: $A_I(x,y) = A_0 * \begin{cases} -(x-0.25)^3 & 0 <= x < 1/2 \\ (x-0.75)^3 & 1/2 <= x < 1 \end{cases}$

Two Stage Evolution:

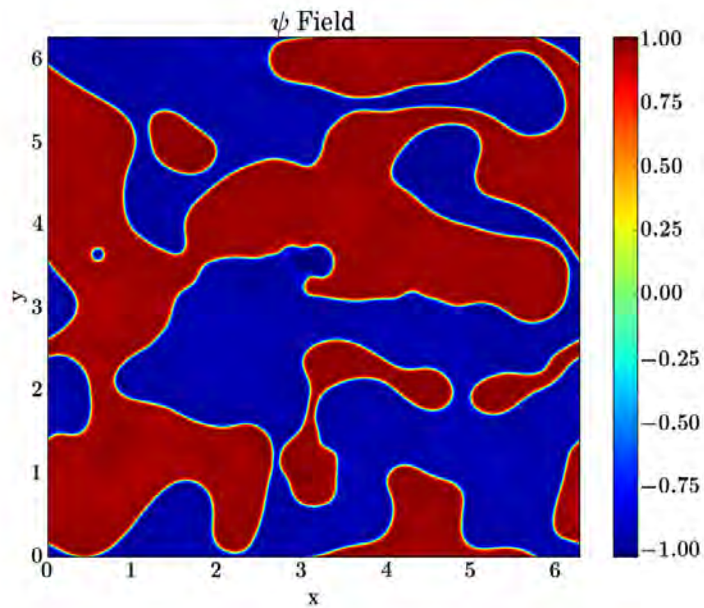
- 1. The <u>suppression</u> <u>stage</u>: the (large scale) magnetic field is sufficiently strong so that the diffusion is suppressed.
- 2. The <u>kinematic decay stage</u>: the magnetic field is dissipated so the diffusion rate returns to the kinematic rate.
- Suppression is due to the memory induced by the magnetic field.



2D CHNS and 2D MHD

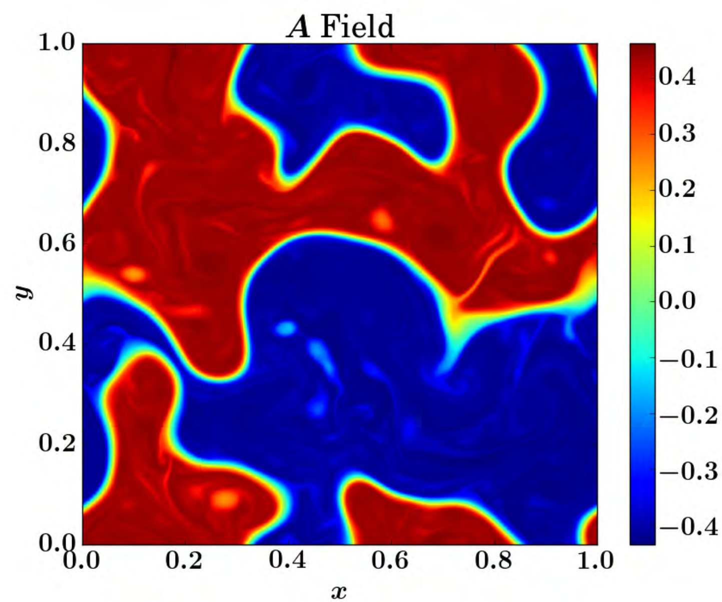
- The A field in 2D MHD in suppression stage is strikingly similar to the ψ field in 2D CHNS (Cahn-Hilliard Navier-Stokes) system:

ψ field in 2D CHNS



v.s.

A field in 2D MHD



2D CHNS and 2D MHD

	2D MHD	2D CHNS
Magnetic Potential	A	ψ
Magnetic Field	$\mathbf{B}$	$\mathbf{B}_\psi$
Current	j	j_ψ
Diffusivity	η	D
Interaction strength	$\frac{1}{\mu_0}$	ξ^2

- 2D CHNS Equations:

$$\partial_t \psi + \vec{v} \cdot \nabla \psi = D\nabla^2(-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{\xi^2}{\rho} \vec{B}_\psi \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \omega$$

$-\psi$: Negative diffusion term

ψ^3: Self nonlinear term

$-\xi^2\nabla^2\psi$: Hyper-diffusion term

With $\vec{v}=\hat{z} \times \nabla\phi$, $\omega = \nabla^2\phi$, $\vec{B}_\psi = \hat{z} \times \nabla\psi$, $j_\psi = \xi^2\nabla^2\psi$. $\psi \in [-1,1]$.

- 2D MHD Equations:

$$\partial_t A + \vec{v} \cdot \nabla A = \eta \nabla^2 A$$

$$\partial_t \omega + \vec{v} \cdot \nabla \omega = \frac{1}{\mu_0 \rho} \vec{B} \cdot \nabla \nabla^2 A + \nu \nabla^2 \omega$$

A: Simple diffusion term

With $\vec{v}=\hat{z} \times \nabla\phi$, $\omega = \nabla^2\phi$, $\vec{B} = \hat{z} \times \nabla A$, $j = \frac{1}{\mu_0}\nabla^2 A$

See [Fan et.al. 2016] for more about CHNS.

General Conclusions (MHD and CHNS)

- Dual (or multiple) cascades can interact with each other, and can modify one another.
- We show how a length scale, e.g. the Hinze scale in 2D CHNS, emerges from the balance of kinetic energy and elastic energy in blobby turbulence. → blob scale in MHD?!
- Negative incremental diffusion (flux/blob coalescence) can lead to novel real space structure in a simple system.
- Negative incremental resistivity can exist in a simple system such as 2D MHD. This results in the formation of nontrivial real space structure.