THC/P3-30

New Results in Negative Viscosity Models for Fusion Plasma Dynamics

-- Zonal Scale Selection, Staircases, Dynamical Symmetry Breaking

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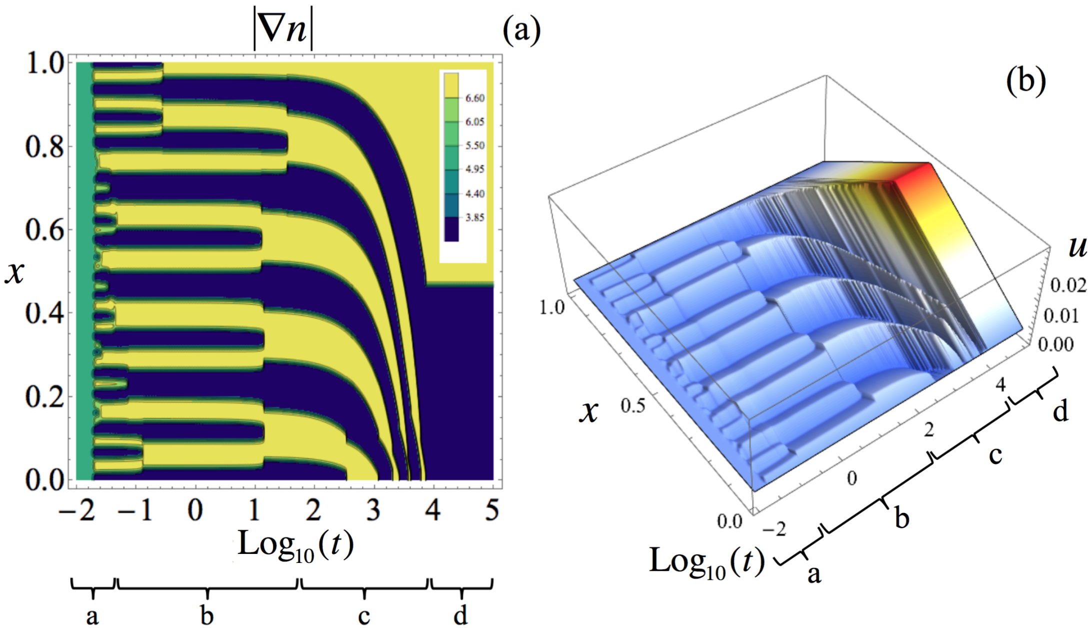
Negative Viscosity Models-

Zonal Scale Selection, Staircases and Dynamical Symmetry Breaking



I.) Scales and Staircases

- Theory of zonal flow scale selection and staircase formation
- Model reveals migration and condensation of staircase steps to form macro barrier layers
- Novel mechanism for nonlocality, via 'escalator mode'

Fig: Stages of evolution: a) Micro-steps merge into meso-steps. b) Meso-steps to barriers. c) Barriers condense at boundaries. d) Stationary profile.

II.) Dynamical Symmetry Breaking

- New dynamical symmetry breaking mechanism amplifies toroidal shear flows in electron drift wave turbulence, significant in weak shear plasmas
- Shear amplification enhances residual stress effect on flow profile gradient

Some Questions:

See: A. Ashourvan, P.H. Diamond
- Phys. Rev. E 'Rapid Comm', 2016
- Submitted, 2016

M. Malkov, , P.H. Diamond
- Submitted, 2016

Related: Z.B. Guo, P.H. Diamond, PRL 2016

- Key to self-regulation of drift wave turbulence is zonal flow (Diamond et al., 2005).
  BUT:

→ Gyro-Bohm breaking observed (McKee, 2006) and related to long range tail (Hennequin, 2015).

→ Zonal flow scale selection and saturation determine degree of GB-breaking. Scale selection-how?

→ ExB staircase is *hint* as to pattern formation scenario.

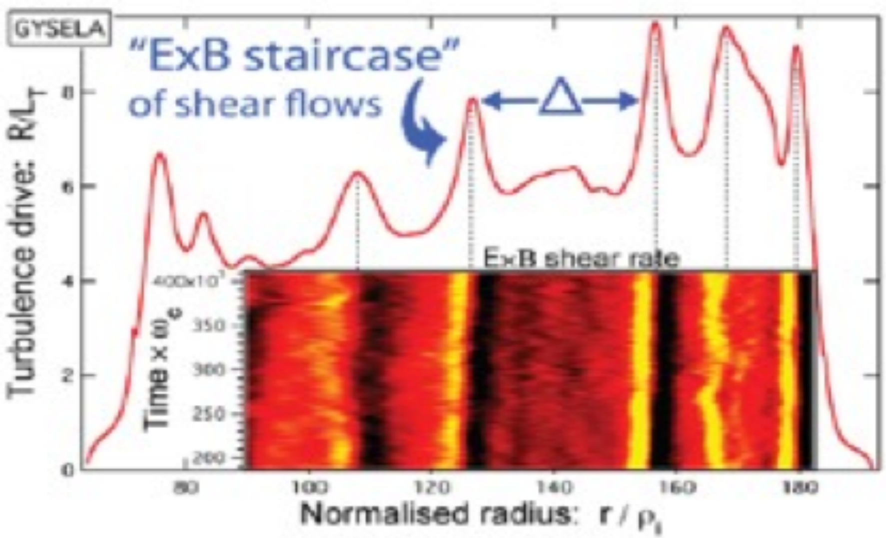
→ But previous limitation to simulation precludes understanding. *Model* needed to step beyond color VG's.

I.) Observation: Coherent ZF Pattern ⟺ ExB staircase

- ExB flows often observed to self-organize in magnetized plasmas

  eg. mean sheared flows, zonal flows, ...

- `ExB staircase'-coherent pattern-is observed (G. Dif-Pradalier, P.D. et al. Phys. Rev. E. '10)



- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases

- Questions:
  - What controls scale selection?
  - How does staircase form and evolve?
  - Nonlinear evolution of modulations?
  - Why coherent?

⇨ How understand coherent pattern selection in drift wave turbulence?

Beyond Color VG: The Reduced 1D Model

Reduced system is obtained from Hasegawa-Wakatani system for DW

Variables:

$u = \partial_x V_y$ Zonal shearing field

Reduced density: $\log(N/N_0) = n(x,t) + \tilde{n}(x,y,t)$, **Vorticity:** $\rho_s^2 \nabla_\perp^2 (e\varphi/T_e) = u(x,t) + \tilde{u}(x,y,t)$

Potential Vorticity (PV): $q = n - u$, **Turbulent Potential Enstrophy (PE):** $\varepsilon = \frac{1}{2}\langle(\tilde{n} - \tilde{u})^2\rangle$

Two components

Mean field equations:

density $\partial_t n = -\partial_x \Gamma_n + \partial_x [D_c \partial_x n]$, $\Gamma_n = \langle \tilde{v}_x \tilde{n} \rangle = -D_n \partial_x n$ → **Reflect instability**

Taylor ID: $\Pi_u = \langle \tilde{v}_x \tilde{u} \rangle = \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle$

vorticity $\partial_t u = -\partial_x \Pi_u + \partial_x [\mu_c \partial_x u]$, $\Pi_u = \langle \tilde{v}_x \tilde{u} \rangle = (\chi - D_n)\partial_x n - \chi \partial_x u$

Residual vort. flux — **Turb. viscosity**

From closure

Turbulence evolution: (Potential Enstrophy)

$$\partial_t \varepsilon = \partial_x [D_\varepsilon \partial_x \varepsilon] - (\Gamma_n - \Gamma_u)[\partial_x (n - u)] - \varepsilon_c^{-1} \varepsilon^{3/2} + P$$

Turbulence spreading — Internal production — dissipation — External production $\sim \gamma\varepsilon$

Two fluxes Γ_n, Γ_u set model ! $\Leftrightarrow$ Vorticit flux sets flow evolution. Contains residual stress driven by ∇n.

What are the Key Points in this model?

○In this model PE conservation is a central feature.

○Mixing of Potential Vorticity (PV) is the fundamental effect regulating the interaction between turbulence and mean fields. Mixing inhomogeneous, via intensity feedback.

○Dimensional and physical arguments used to obtain functional forms for the turbulent diffusion coefficients. From the flux relations for HW system we obtain

$$D_n \cong l^2 \frac{\varepsilon}{\alpha} \qquad \chi \cong c_\chi l^2 \frac{\varepsilon}{\sqrt{\alpha^2 + a_u u^2}}$$

* l Dynamic mixing length → Set by Rhines scale

α Parallel diffusion rate

○*Inhomogeneous mixing of PV results in the sharpening of density and vorticity gradients in some regions and weakening them in other regions, leading to shear lattice and density staircase formation.*

Key Element: Mixing Scale <-> Tied to Rhines Scale

- $l = l_0/(1 + l_0^2[\partial_x(n-u)]^2/\epsilon)^{k/2} = l_0/\left(1 + l_0^2/l_{Rh}^2\right)^{k/2}$
- $l_{Rh} = \text{Rhines Length} = \sqrt{\epsilon}/|\partial_x q|$

  From: $\omega \approx k_\theta v_*/(1 + k_\perp^2 \rho_s^2) \approx l_{Rh}\sqrt{q}$
- $D_q \approx l_0^2 \epsilon^{1/2}/[1 + l_0^2(\langle q \rangle')^2/\epsilon]$

PV mixing exhibits quench with ∇q
Note: No KH/tertiary. Feedback on gradient drive assures saturation

$\Longrightarrow$ Robust, generic mechanism

- Staircase Structure via Mudulation

Snapshots of evolving profiles at t=1 (non-dimensional time)

(Boundary value problem)

Initial conditions: $n = g_0(1-x), \quad u = 0, \quad \varepsilon = \varepsilon_0$

Boundary conditions: $n(0,t) = g_0,\ n(1,t) = 0; \quad u(0,1;t) = 0; \quad \partial_x \varepsilon(0,1;t) = 0$

Structures:

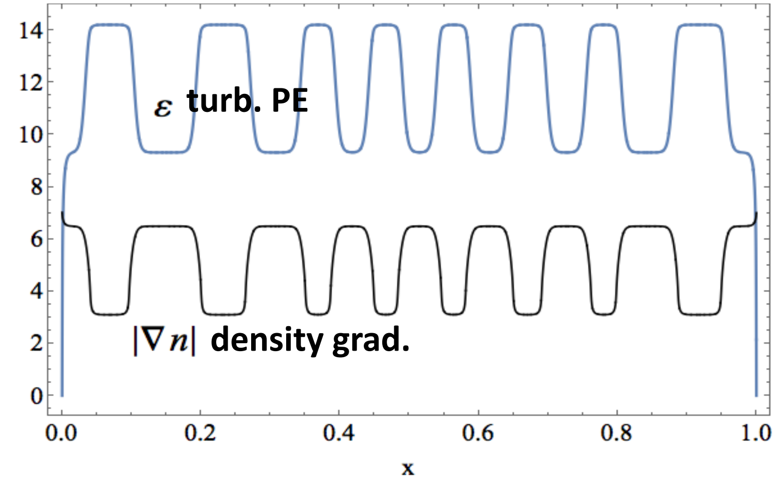
○**Staircase in density profile:**

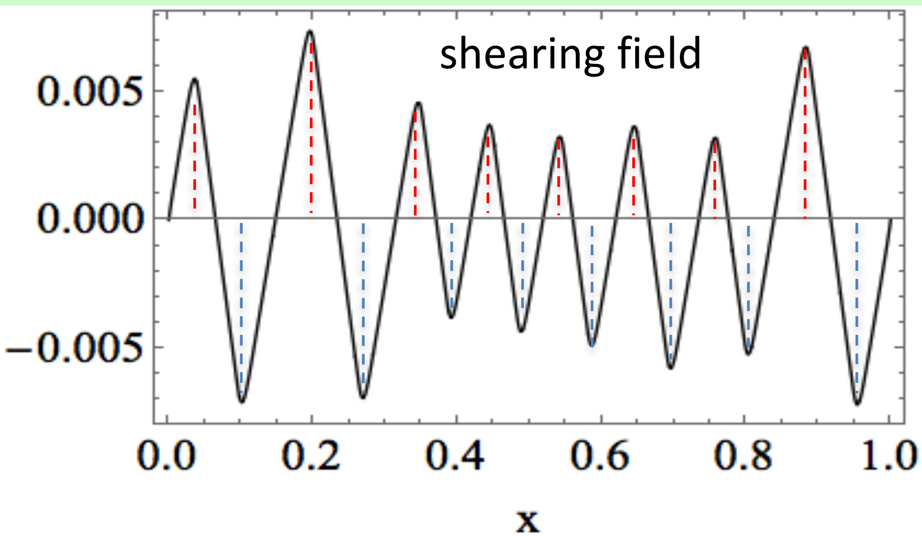
jumps → regions of steepening

steps → regions of flattening

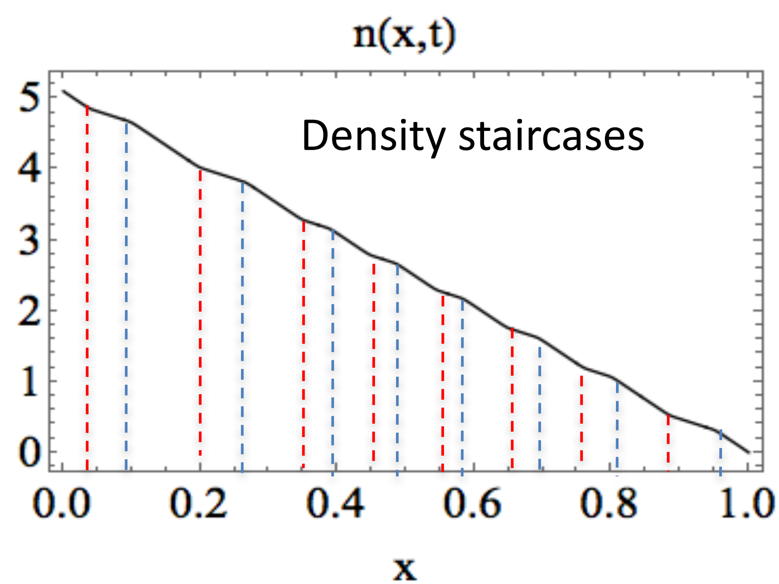
○**At the jump locations, turbulent PE is suppressed.**

○**At the jump locations, vorticity gradient is positive**





Density staircase + Vorticity lattice

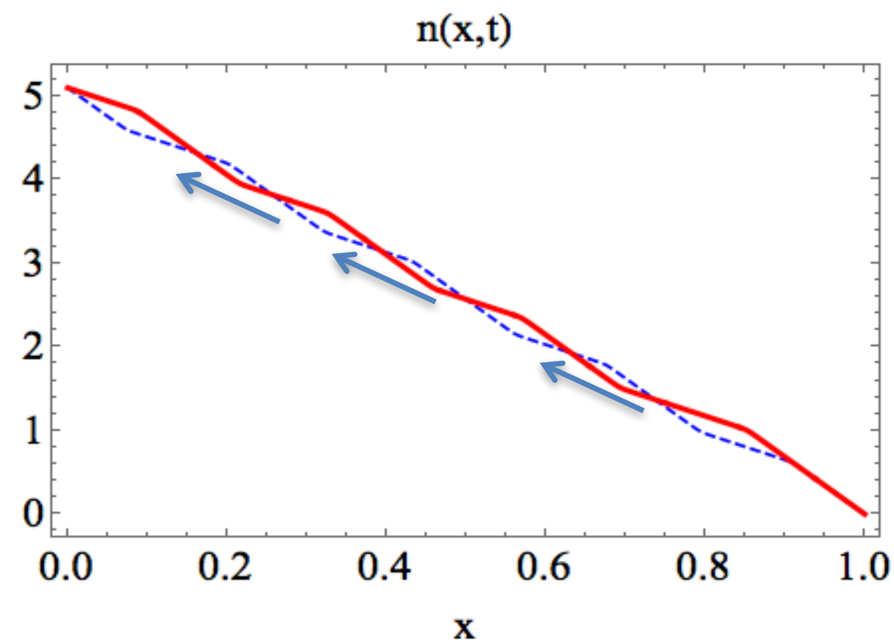


- Staircases are Dynamic

○Barriers in density profile move upward in an "Escalator-like" motion.

➔ Macroscopic Profile Re-structuring

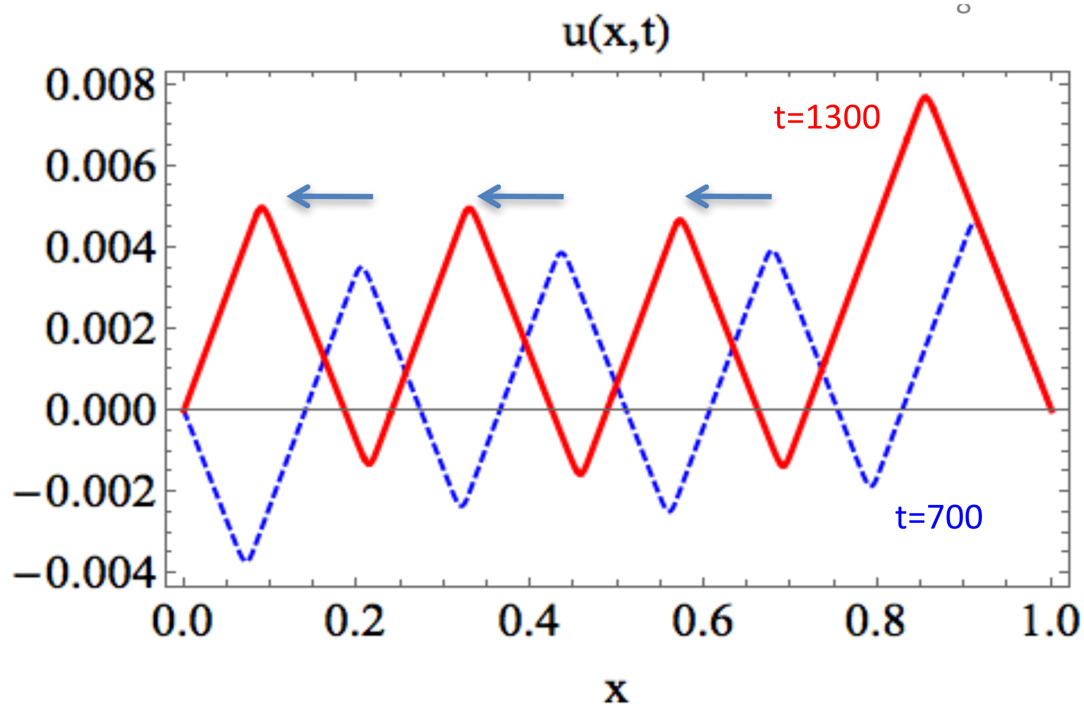
Novel Mechanism for 'Non-locality' via profiles and boundary conditions.



○Shear pattern detaches and delocalizes from its initial position of formation.

○Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at x=0.

○Shear lattice propagation takes place over much longer times. From t~O(10) to t~(10⁴).



- Profile Evolution: Staircase Coalescence and Condensation to Barrier

(a) Fast merger of micro-scale SC. Formation of meso-SC.

↓

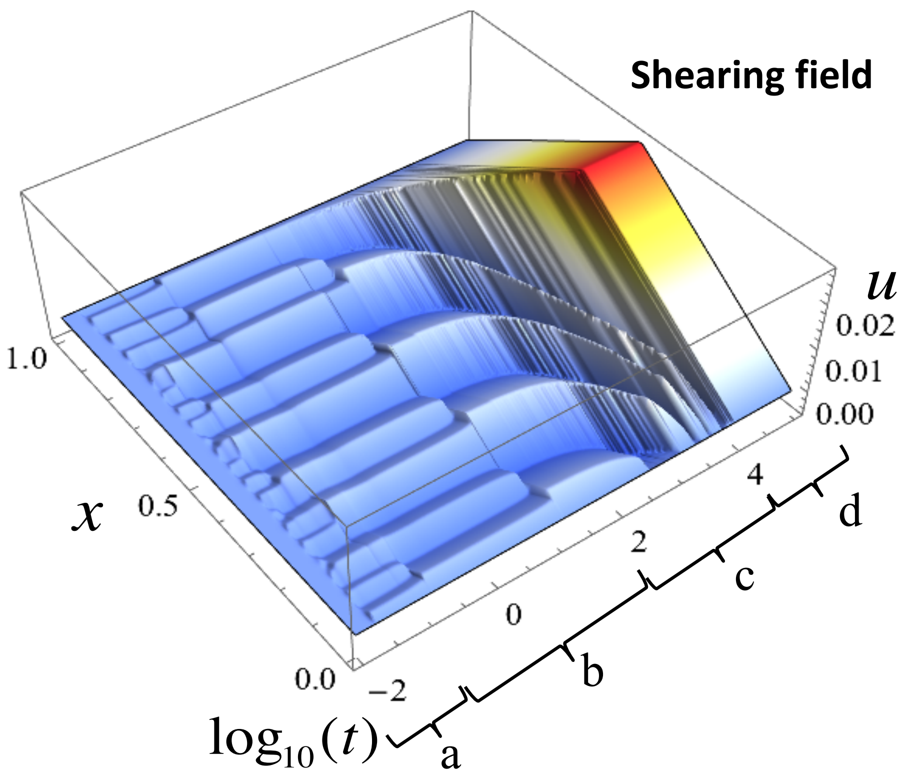
(b) Meso-SC *coalesce* to barriers

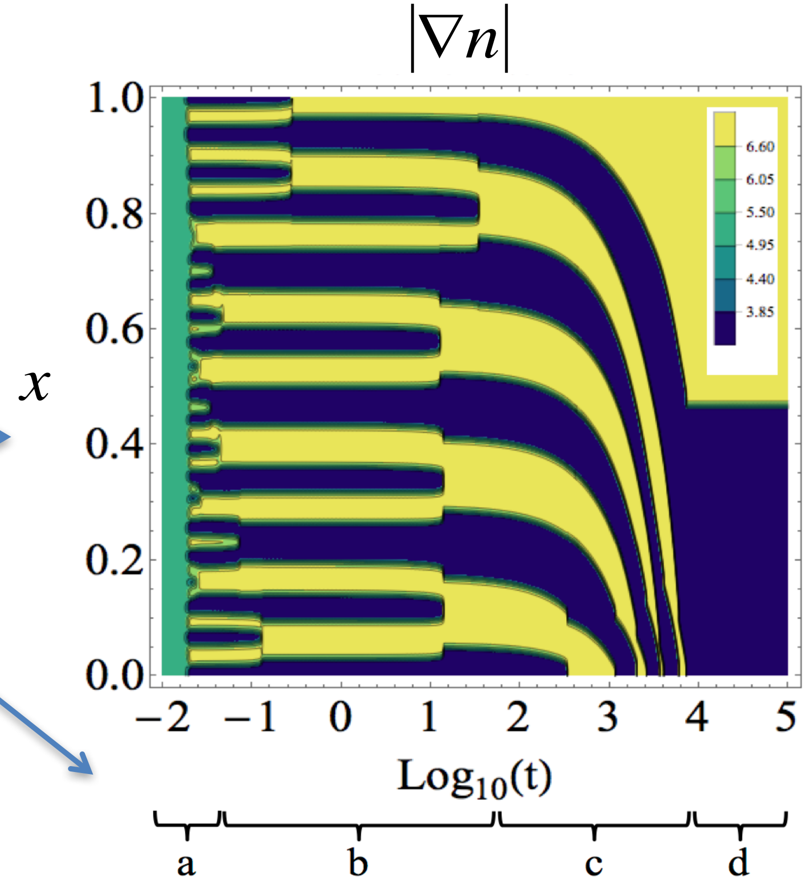
↓

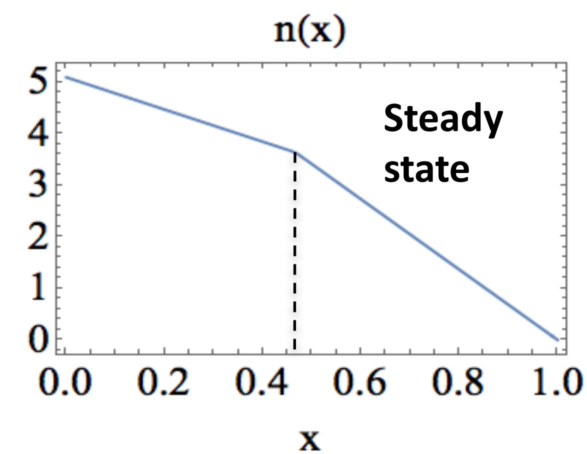
(c) Barriers propagate along gradient, condense at boundaries

↓

(d) Macro-scale stationary profile







- Flux driven evolution

Add an external particle flux drive to the density Eq., use its amplitude Γ_0 as a control parameter to study:

- **What profile structure emerges from this dynamics?**
- **Variation of the macroscopic steady state profiles with Γ_0. (shearing, density, turbulence, and flux).**
- **Transport bifurcation of the steady state (macroscopic)**
- **Particle flux-density gradient landscape.**

$$\partial_t n = -\partial_x \Gamma - \partial_x \Gamma_{dr}(x,t)$$

➔ **Source as $\nabla \cdot \Pi_{ex}$**

External particle flux (drive) $\Gamma_{dr}(x,t) = \Gamma_0(t)\exp[-x/\Delta_{dr}]$

Internal particle flux (turb. + col.) $\Gamma = -[D_n(\varepsilon,\partial_x q) + D_{col}]\partial_x n$

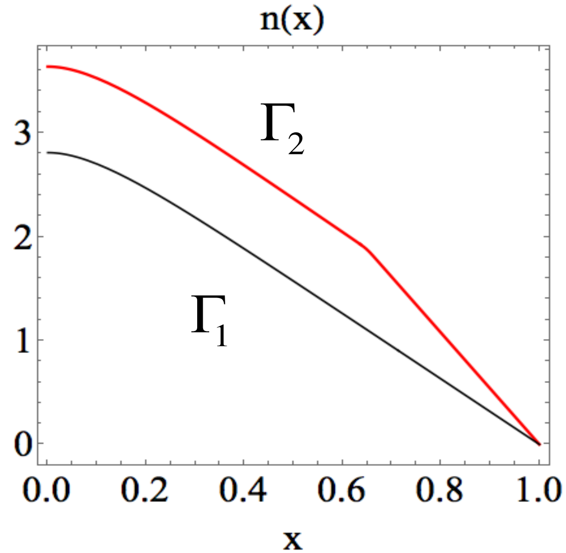
- Transitions to Globally Enhanced Confinement Occur by Staircase Evolution

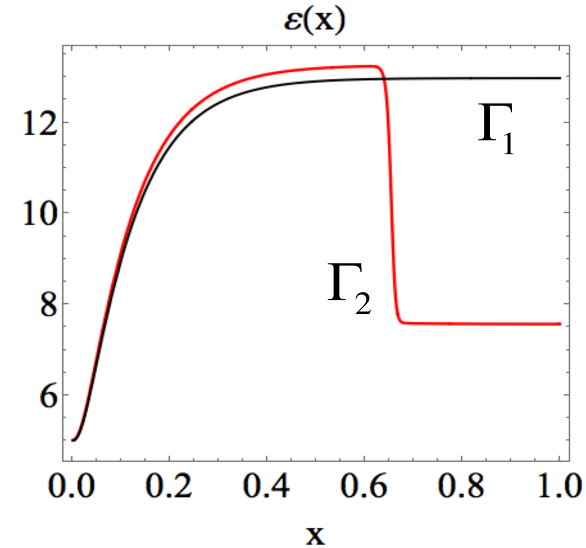
Steady state solution for the system undergoes a transport bifurcation as the flux drive amplitude Γ_0 is raised above a threshold Γ_{th}.

$$\Gamma_1 < \Gamma_{th} < \Gamma_2$$

$$\Gamma_0 = \Gamma_1 \rightarrow \text{Normal Conf. (NC)}$$

$$\Gamma_0 = \Gamma_2 \rightarrow \text{Enhanced Conf. (EC)}$$

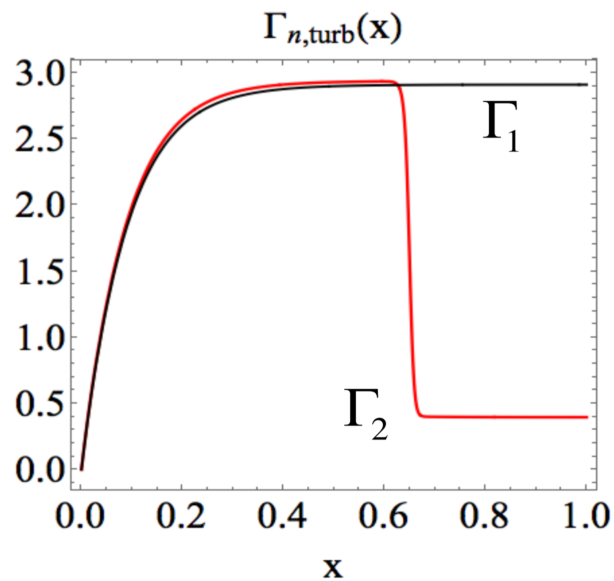


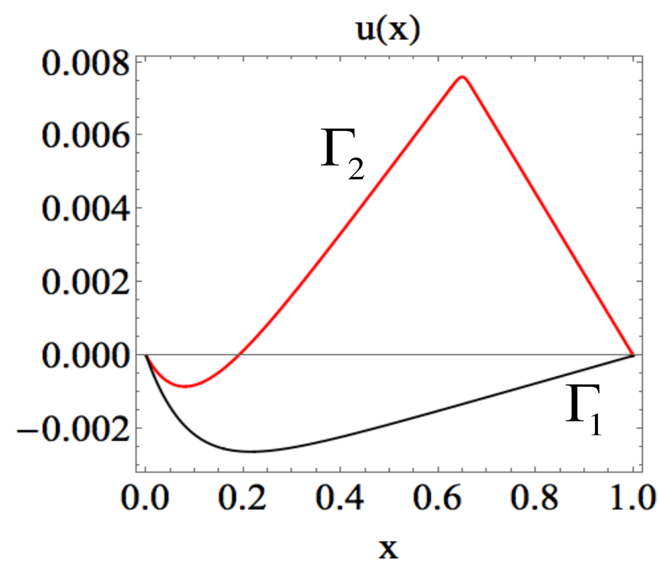


With NC to EC transition we observe:

- Rise in density level
- Drop in turb. PE and turb. particle flux beyond the barrier position
- Enhancement and sign reversal of vorticity (shearing field)

N.B.: *Macro* transition occurs via staircase evolution





- Flux Landscape in $(x, \nabla n)$ Forms from Staircase Condensation

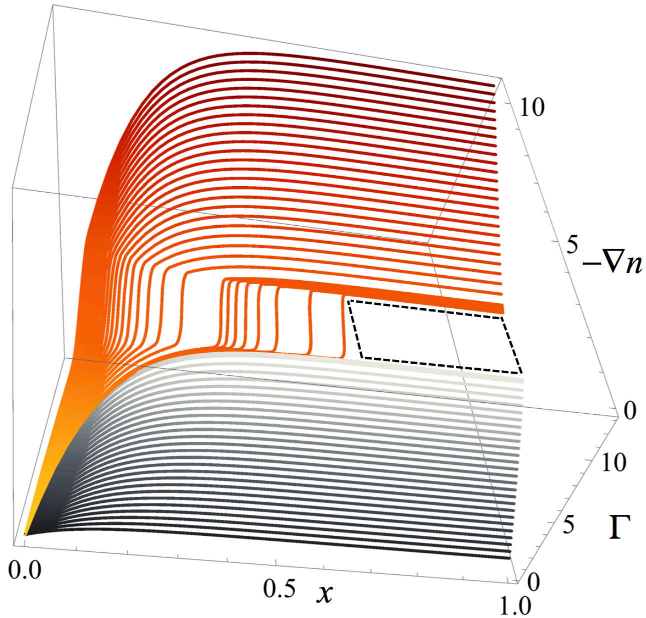


Fig: Flux landscape of the local $\Gamma(x)$ vs $-\partial_x n$ vs x for $g_i = 4.5$. Shades of red are for the enhanced confinement state s (EC) and gray scale is for normal confinement state (NC).

Hysteresis evident in the GLOBAL flux-gradient relation

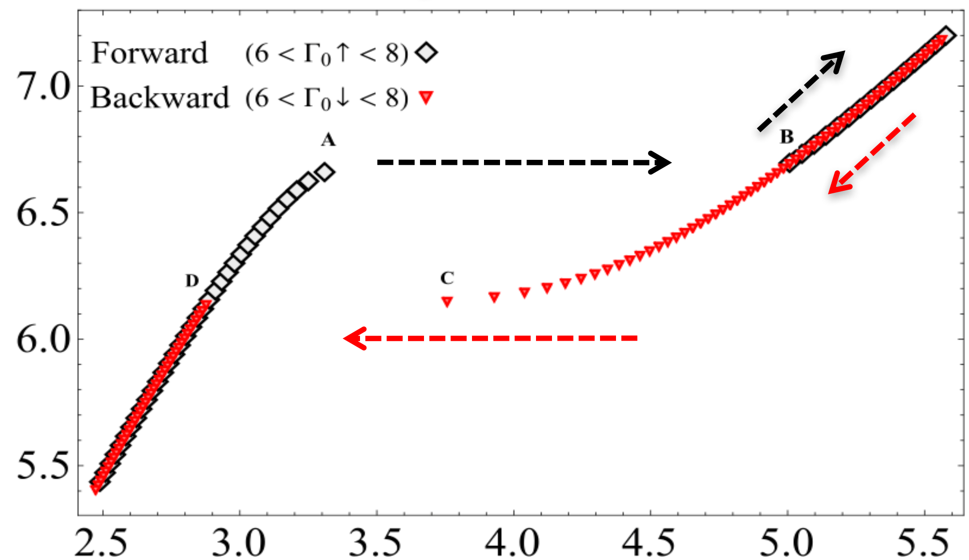
In one run from initially flat density profile, Γ_0 is adiabatically raised and lowered.

Forward Transition:
Abrupt transition from NC to EC (from A to B).

From B to C:
Barrier moves to the right with lowering the density gradient.

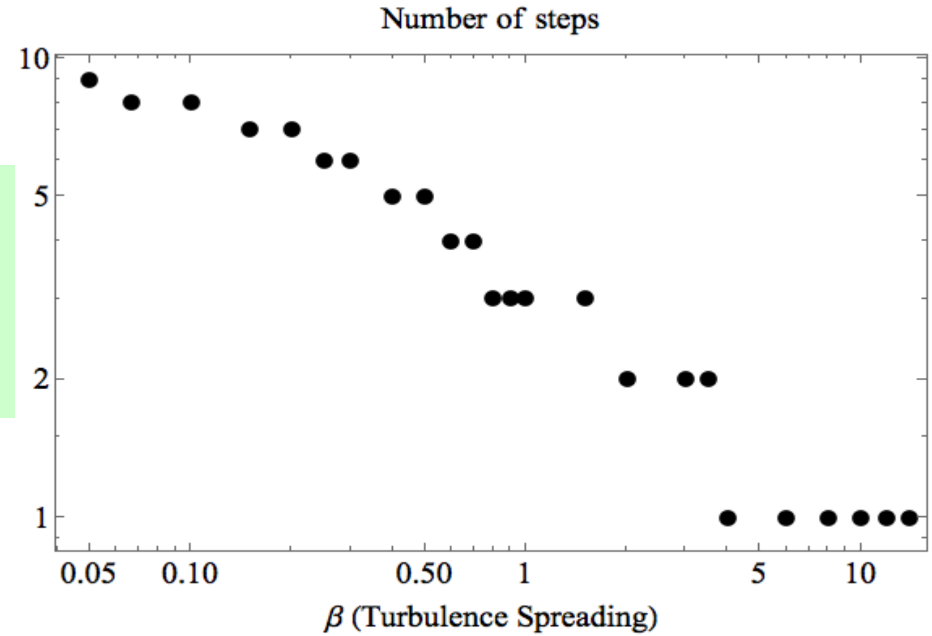
Backward Transition:
Abrupt transition from EC to NC (from C to D). Barrier moves rapidly to the right boundary and disappears.

$\langle\Gamma\rangle$ Global



$\langle -\nabla n \rangle$

- Role of Turbulence Spreading?

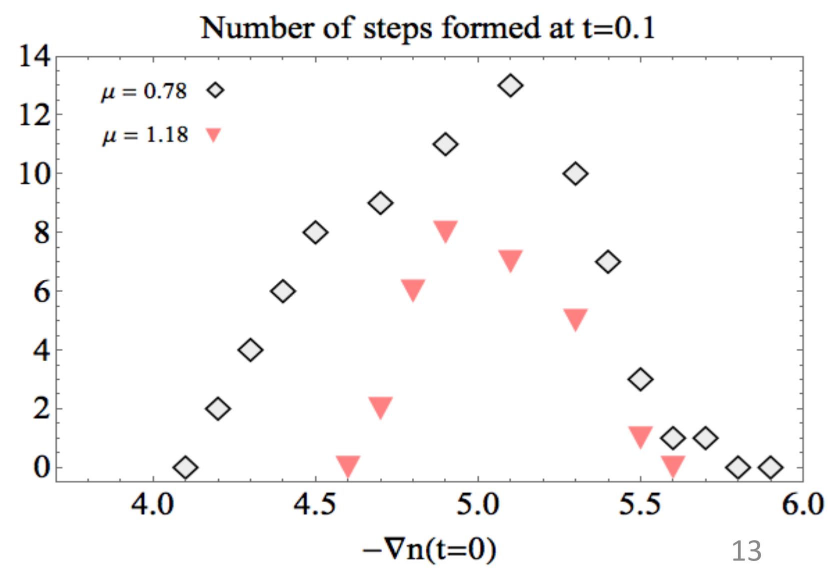


○ **Large turbulence spreading wipes out features on smaller spatial scales in the mean field profiles, resulting in the formation of fewer density and vorticity jumps.**

$$\partial_t \varepsilon = \beta \partial_x [(l^2 \varepsilon^{1/2}) \partial_x \varepsilon] + \ldots$$

- $\beta \to 0$, excessive profile roughness

Initial condition dependence:



○**Solutions are not sensitive to initial value of turbulent PE.**

○**Initial density gradient is the parameter influencing the subsequent evolution in the system.**

○**At lower viscosity more steps form.**

○**Width of density jumps grows with the initial density gradient.**

Lessons I.)

- A) Coherent ZF structures evolve from modulations
  - "Staircase" is 'natural upshot' of modulation in bistable/multi-stable system
  - Bistability is a consequence of generic mixing scale dependence on gradients, and intensity ←→ define feedback process
  - *Mergers* result from accommodation between boundary condition, drive(L), and initial secondary instability
  - Scale selection for ZF layers is intrinsically *global* → responds to boundaries $\Longrightarrow$ Nonlocality mechanism.
- B) ZF Patterns are Dynamic (not previously appreciated)
  - Mergers occur, jumps/steps migrate. B.C.'s, drive *all* essential.
  - Condensation of mesoscale staircase jumps into macroscopic transport barriers occurs.
  - *Global* 1st order transition, with macroscopic hysteresis occurs from staircase evolution, condensation.
  - Flux drive + B.C. effectively constrain system states.

J. Li, P.D. et al, PoP, 2016
J. Li, P.D. et al, submitted 2016

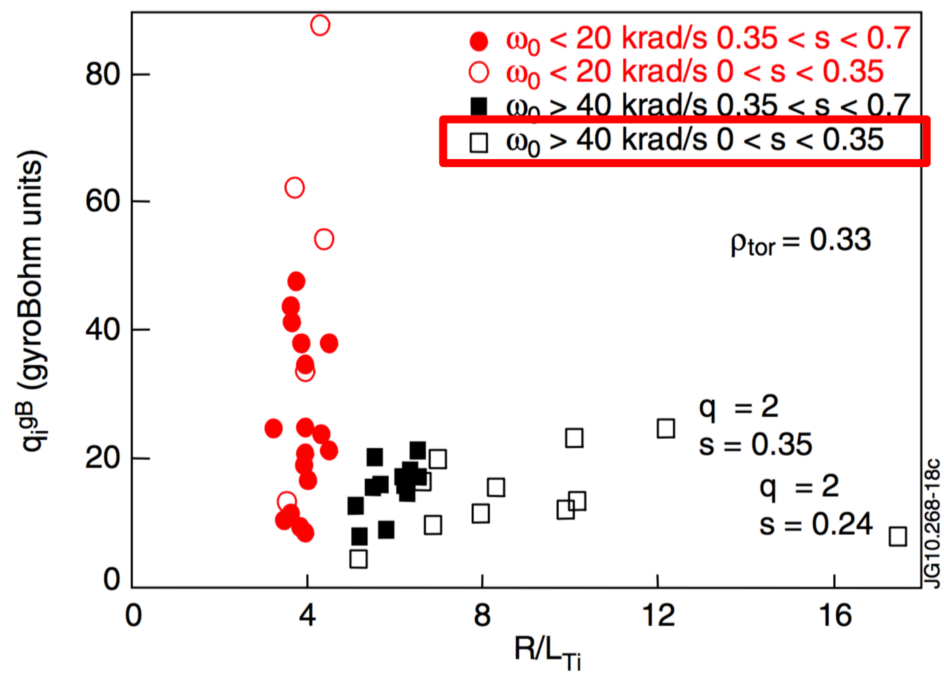
II.) Intrinsic Rotation in Weak Shear

- JET: Weak shear **AND** Rotation → Enhanced confinement
- But external torque limited in ITER
- Need understand: ***Intrinsic rotation in weak shear regimes***

- Important for:
  - Total effective torque

    $$\tau = \tau_{ext} + \tau_{intr}$$

  - Contribution to $V'_{E\times B}$



[P. Mantica, PRL, 2011;
Rice, PRL, 2013]

FIG. 4 (color online). q_i^{GB} vs R/L_{T_i} at $\rho_{tor} = 0.33$ for similar plasmas with different rotation and s values.

Recall: Conventional Wisdom of Intrinsic Rotation

- Self-acceleration by intrinsic torque due to residual stress ($\tau_{intr} = -\nabla \cdot \Pi^{Res}$)

$$\langle \tilde{v}_r \tilde{v}_\parallel \rangle = -\chi_\phi \frac{d\langle v_\parallel \rangle}{dr} + V_P \langle v_\parallel \rangle + \Pi_{r\parallel}^{Res}$$

- Residual stress $\Pi_{r\parallel}^{Res}$
  - Driven by turbulence, i.e. $\Pi_{r\parallel}^{Res} \sim \nabla P, \nabla T, \nabla n_0$
- $\Pi_{r\parallel}^{Res} \sim \langle k_\theta k_\parallel \rangle$ etc. requires symmetry breaking in *k* space
- Relevance in weak shear dubious!
- Symmetry breaking usually relies on magnetic shear
- Rotation builds up from edge, driven by $\Pi_{r\parallel}^{Res}$ at edge

[W.X. Wang, PRL, 2009]

Intrinsic Rotation in Weak Shear

- Weak shear ($q' \to 0$)
- External torque $\cong 0$

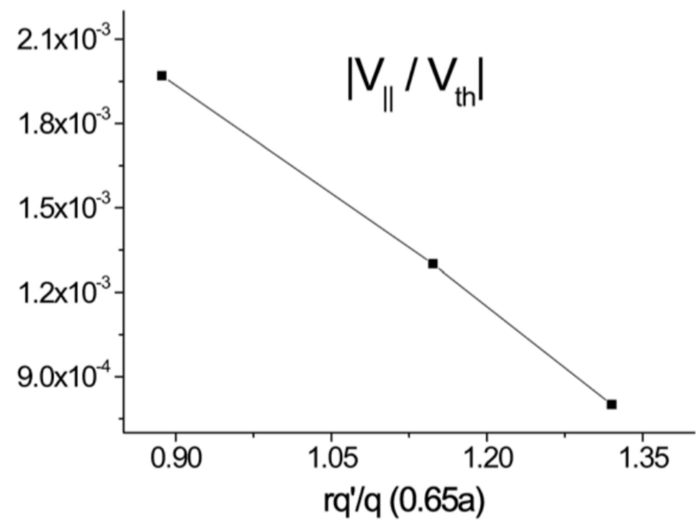
→ **Intrinsic Rotation?**

↓

- Beneficial for confinement and stability.

- Results
  - GK Simulation: stronger intrinsic rotation at weaker magnetic shear
- Problems:
  - Intrinsic rotation requires symmetry breaking
  - Most involve magnetic shear
  - Conventional symmetry breaking models fail
  - But weak shear
    → non-resonant mode structure!
  - Need re-visit fundamentals of intrinsic torque, absent shear. (J. Li et al., 2016)

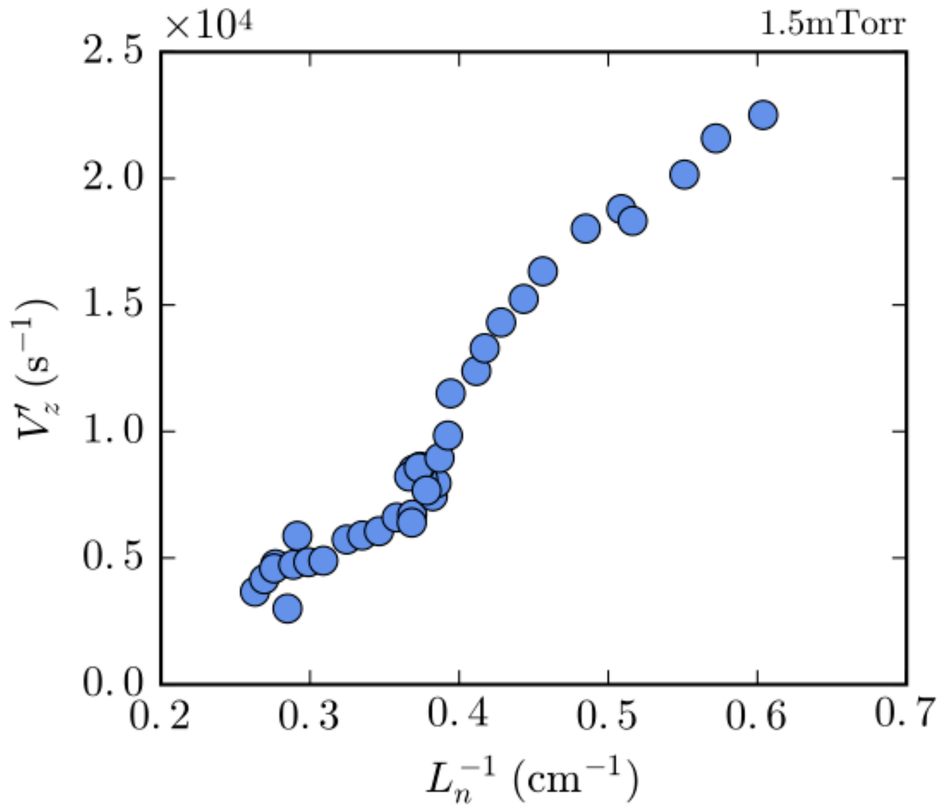
Status:



[Kwon, NF (2012);
Z.X. Lu, NF&PoP, 2015]

Intrinsic $\nabla\langle v_z\rangle$ in Drift Wave Turbulence

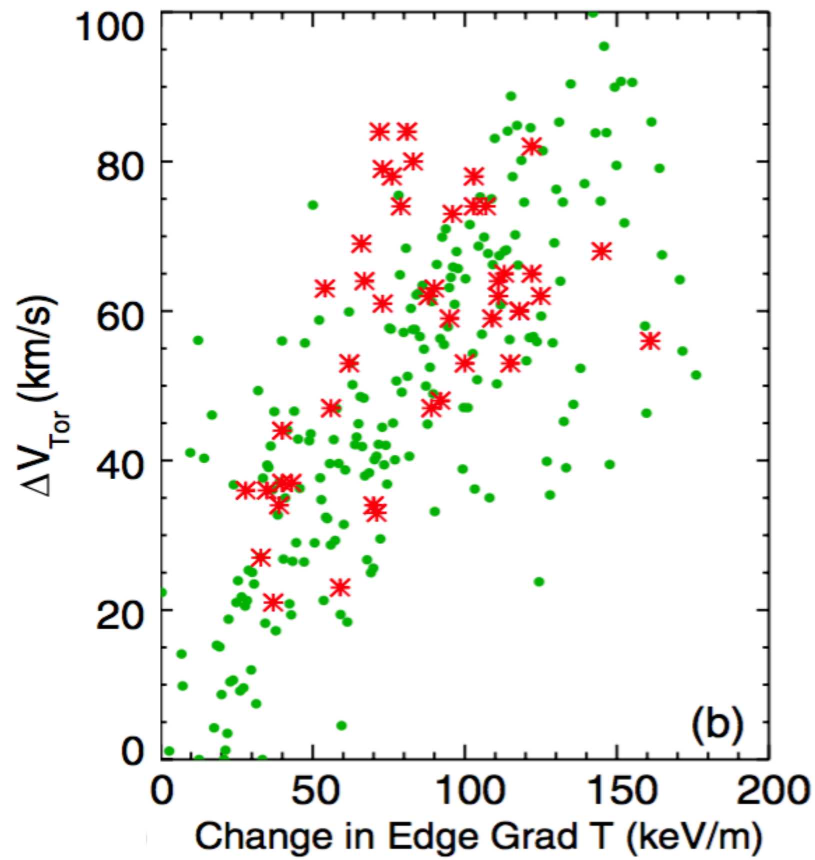
- Axial flow in CSDX:
- ∇n_0 is free energy source
- $\langle v_z\rangle' \sim \frac{1}{n_0}\nabla n_0$



(Zero magnetic shear)

- Compare:
- Intrinsic $\nabla\langle v_z\rangle$ in C-Mod pedestal:
- $\Delta\langle v_\phi\rangle \sim \nabla T$

[Rice, PRL, 2011]



(Standard shear)

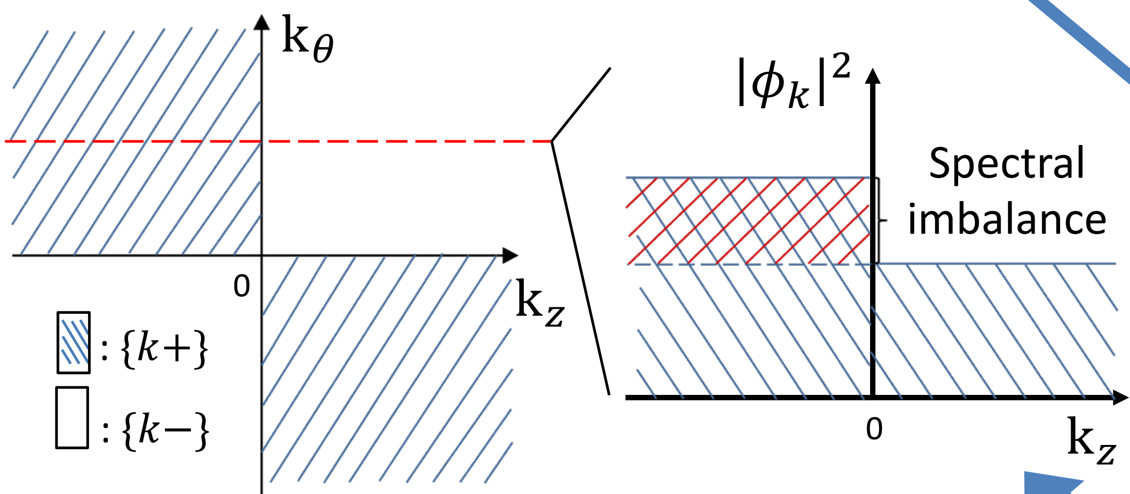
Resolution: Dynamical Symmetry Breaking

Key: $\delta\langle v_z\rangle' \rightarrow$ Frequency shift $\rightarrow$ Change in $\omega_k - \omega_*$

- Growth rate <-> frequency shift:

$$\omega_k \cong \frac{\omega_*}{1+k_\perp^2\rho_s^2} - \frac{k_\theta k_z \rho_s c_s \langle v_z\rangle'}{\omega_*}$$

$$\gamma_k \cong \frac{\nu_{ei}}{k_z^2 v_{The}^2}\frac{\omega_*^2}{(1+k_\perp^2\rho_s^2)^2}\left(\frac{k_\perp^2\rho_s^2}{1+k_\perp^2\rho_s^2} + \frac{k_\theta k_z \rho_s c_s \langle v_z\rangle'}{\omega_*^2}\right)$$



$\{k\pm\}$: Domains where modes grow faster/slower

Spectral imbalance

Operates through electron growth

- Spectral imbalance:

Infinitesimal test axial flow shear, e.g. $\delta\langle v_z\rangle' < 0$

Modes with $k_\theta k_z < 0$ grow faster than others

$\gamma_k|_{k_\theta k_z<0} > \gamma_k|_{k_\theta k_z>0}$

Spectral imbalance in $k_\theta k_z$ space

$\langle k_\theta k_z\rangle < 0 \rightarrow \Pi_{rz}^{Res} \neq 0$

Negative Viscosity *Increment*

- Reynolds stress: $\langle \tilde{v}_r \tilde{v}_z \rangle = -\chi_\phi \langle v_z \rangle' + \Pi_{rz}^{\mathrm{Res}}$

  residual

- Turbulent momentum diffusivity:

$$\chi_\phi = \sum_k \frac{\nu_{\mathrm{ei}}}{k_z^2 v_{\mathrm{The}}^2} \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} k_\theta^2 \rho_s^2 |\phi_k|^2$$

- Residual stress → Negative viscosity ***increment***

  → Mechanism resembles modulational instability: seed + feedback

- $\delta \Pi^{Res} = \left| \chi_\phi^{Inc} \right| \delta \langle v_z \rangle'$ [Li et al, PoP, 2016]

$$\delta \Pi_{rz}^{\mathrm{Res}} = \frac{\nu_{\mathrm{ei}} L_n^2}{v_{\mathrm{The}}^2} \sum_k (1 + k_\perp^2 \rho_s^2)(4 + k_\perp^2 \rho_s^2) |\phi_k|^2 \delta \langle v_z \rangle'$$

Modulational Enhancement of $\delta\langle v_z\rangle'$

- $\delta\langle v_z\rangle' \rightarrow \Pi^{Res} \rightarrow \chi_\phi^{tot} = \chi_\phi - |\chi_\phi^{Inc}|$
- Dynamics of $\delta\langle v_z\rangle'$:

$$\frac{\partial}{\partial t}\delta\langle v_z\rangle' + \frac{\partial^2}{\partial r^2}\left(\delta\Pi_{rz}^{Res} - \chi_\phi \delta\langle v_z\rangle'\right) = 0$$

- Growth rate of flow shear modulation

$$\gamma_q = -q_r^2\left(\chi_\phi - |\chi_\phi^{Inc}|\right)$$

- $\chi_\phi^{tot} < 0 \rightarrow$ Modulational growth of $\delta\langle v_z\rangle'$
- Feedback loop: $\delta\langle v_z\rangle' \rightarrow \Pi^{Res} \rightarrow -|\chi_\phi^{Inc}|$

Upper Range of $\langle v_z \rangle'$ Limited by PSFI

- Parallel shear flow instability (PSFI) driven by $\nabla\langle v_z \rangle$, negative compressibility

$$\gamma_k^{PSFI} \cong \sqrt{\frac{k_\theta k_z \rho_s c_s \left(\langle v_z \rangle' - \langle v_z \rangle'_{crit}\right)}{1 + k_\perp^2 \rho_s^2}}$$

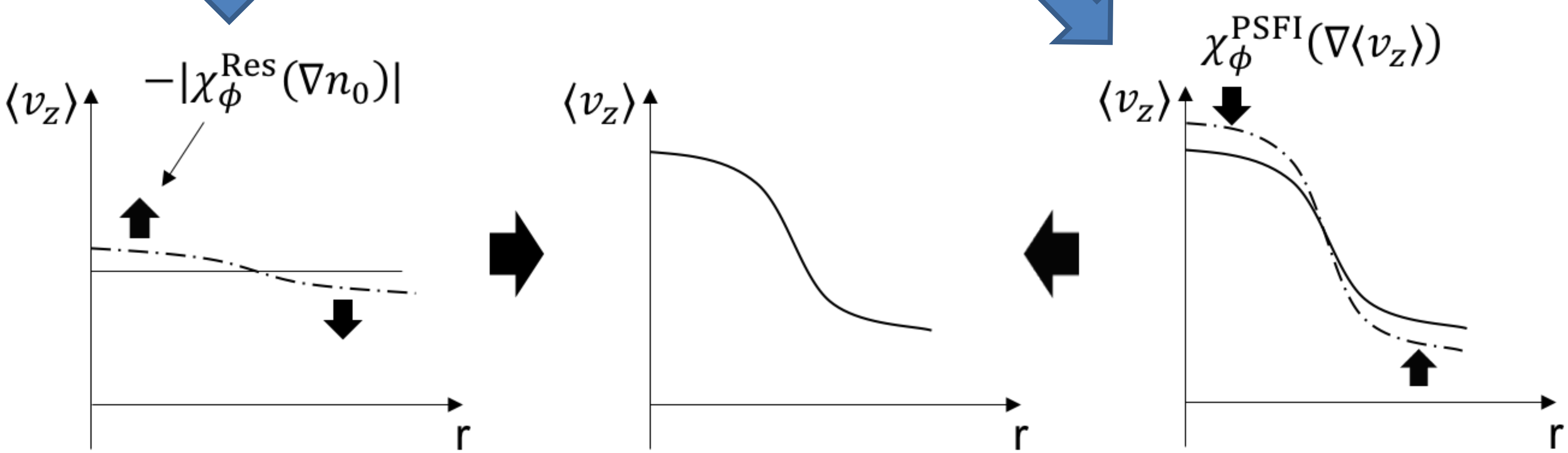
$$\chi_\phi^{PSFI} \cong \sum_k |\phi_k|^2 k_\theta^2 \rho_s^2 \frac{4(1 + k_\perp^2 \rho_s^2)^2}{\omega_*^2} \sqrt{\frac{k_\theta k_z \rho_s c_s \left(\langle v_z \rangle' - \langle v_z \rangle'_{crit}\right)}{1 + k_\perp^2 \rho_s^2}}$$

→ Nonlinear in $\nabla\langle v_z \rangle$

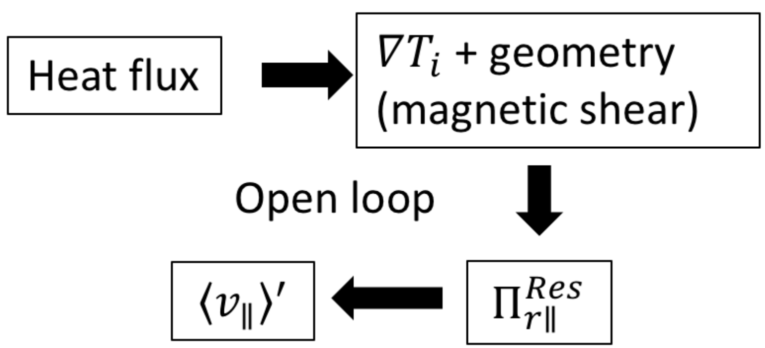
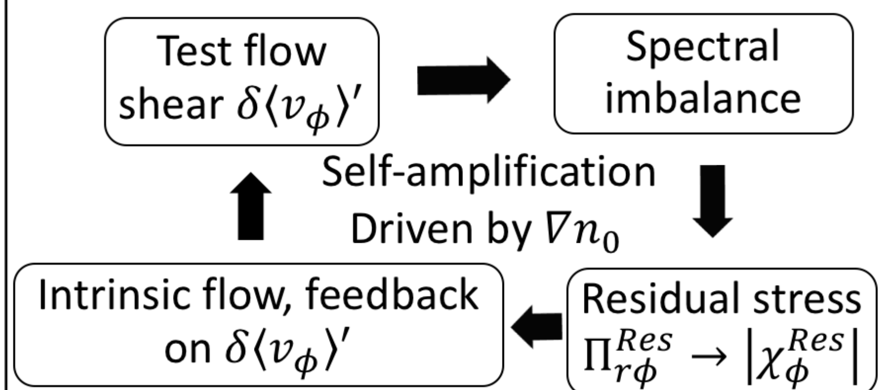
- Hit PSFI threshold → χ_ϕ^{PSFI} nonlinear in $\nabla\langle v_z \rangle$ → $\chi_\phi^{tot} > 0$
- $\delta\langle v_z \rangle'$ → Π^{Res} → $\delta\langle v_z \rangle'$ growth ← Saturated by PSFI

$$\chi_\phi^{tot} = \chi_\phi^{DW} - \left|\chi_\phi^{Inc}\right| < 0$$

$$\chi_\phi^{tot} = \chi_\phi^{DW} + \chi_\phi^{PSFI} - \left|\chi_\phi^{Inc}\right| > 0$$



Comparing Symmetry Breaking Mechanisms

	Standard Symmetry Breaking	Dynamical Symmetry Breaking
Free energy	$\nabla T_i, \nabla T_e, \nabla n_0, \ldots$	∇n_0, ∇T_e--electron drift waves
Symmetry breaker	$E_r', I(x)', \ldots$ All tied to magnetic field configuration	Test toroidal flow shear, $\delta\langle v_\phi\rangle'$; No requirement for shear of $\boldsymbol{B}$ structure.
Effect on flow	Intrinsic torque, $-\partial_r \Pi_{r\parallel}^{Res}$	Negative viscosity, $-\lvert\chi_\phi^{Res}\rvert$ driven by ∇n_0
Flow profile	$\langle v_\parallel\rangle' = \dfrac{\Pi_{r\parallel}^{Res}}{\chi_\phi}$	$\langle v_\phi\rangle' = \dfrac{\text{Flow drive (e.g. } \Pi_{r\phi}^{Res}, \Delta P_i)}{\chi_\phi(\nabla n_0, \nabla\langle v_\phi\rangle) - \lvert\chi_\phi^{Res}\rvert}$
Feedback loop		

Summary (II.)

- Dynamical symmetry breaking mechanism
- Negative viscosity increment induced by Π^{Res}
  - $\delta\Pi^{Res} = \left|\chi_\phi^{Inc}\right| \delta\langle v_z\rangle'$
  - Total viscosity: $\chi_\phi^{tot} = \chi_\phi - \left|\chi_\phi^{Inc}\right|$
  - $\chi_\phi^{tot} < 0$ → Modulational growth of $\delta\langle v_z\rangle'$
- Broader lesson for tokamaks
  - Synergy of $\langle v_\phi\rangle'$ self-amplification and Π^{Res}
  - $\langle v_\phi\rangle'$ driven by $\tau_{NBI}, \Pi^{Res}(\nabla n_0, \nabla T)$
  - $\langle v_\phi\rangle'$ enhanced by $-\left|\chi_\phi^{Inc}\right|$