# Dynamics of Turbulence Entrainment: A Comparative Study

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**N.B. "Turbulence Spreading" ≅ Entrainment**



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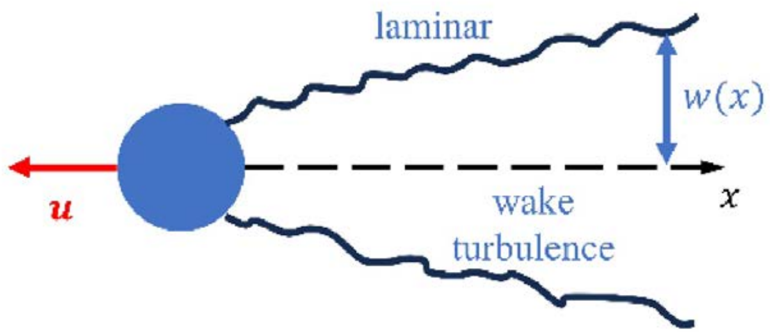
# <u>Wake-Classic Example of Turbulence Spreading</u>

⇨




Similarity Theory
Mixing Length Theory }

$W \sim (F_d/\rho U^2)^{1/3} X^{1/3}$,

$F_d \sim C_D \rho U^2 A_s$

$C_D$ independent of viscosity at high Re

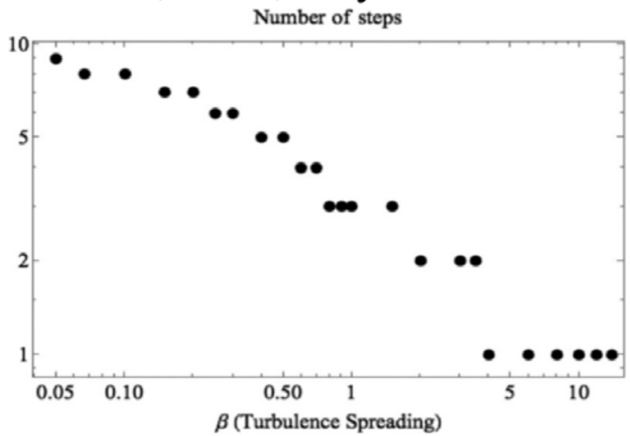
⇨ Physics: Entrainment of laminar region by expanding turbulent region.
Key is <u>turbulent mixing</u>. ⇨ Wake expands

⇨ Townsend '49:

— Distinction between momentum transport — eddy viscosity—and fluctuation energy transport
— Failure of eddy viscosity to parametrize spreading
— Jet Velocity: $V = \frac{<V_{perp} * V^2>}{<V^2>}$ ⇨ spreading flux FOM

# Why Study Spreading?

⇨ Spreading strength sets staircase step size via intensity scattering. See also F. Ramirez, P.D., Phys Rev E 2024

⇨ 


from A. Ashourvan, P.D.
(in spirit of BLY, for drift wave turbulence)

⇨ Spreading potentially significant in determining

— Physical turbulence profiles
— Non-locality phenomena

⇨ It's observed!
— M. Kobayashi + 2022
— T. Long, T. Wu (2021, 2023)
— Estrada + (2011)

# Spreading in MFE Theory

⇨ Numerous gyrokinetic simulations

N.B. Basic studies absent ...

*i.e.*

$$\partial_t \xi = \gamma \xi (1 - \xi) + \partial_x D(\xi) \partial_x \xi + D_0 \partial_x{}^2 \xi$$

⇨ Diagnosis primarily by:
- color VG
- tracking of “Front”

$\gamma \sim O(\varepsilon)$

⇨ Theory
- ⇨ Nonlinear Intensity diffusion models
- ⇨ Reaction-Diffusion Equations - especially Fisher + NL diffusion
- ⇨ Continuum DP Models - Later......

Recently:

⇨ Renewed interest in context of $\lambda_q$ broadening problem, cf. P. Diamond, Z. Li, Xu Chu

⇨ Simulations measure correlation of spreading $\langle \tilde{V}_r \tilde{p} \tilde{p} \rangle$ with $\lambda_q$ broadening (Nami Li, P.D., Xu NF 2023 )

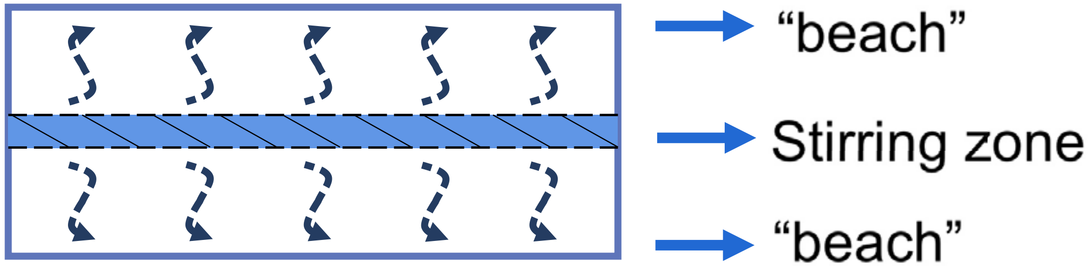
⇨ Intermittency effects T. Wu, P. D. + 2023, A. Sladkomedova 2024,

⇕

Especially blobs, voids

# **Spreading Studies** - Numerical Experiments

⇨ 2D Box, Localized Stirring Zone




⇨ Comparison of:

| **System** | **Features** |
|---|---|
| 2D Fluid | Selective Decay, Vortices<br>How to Measure Spreading? |
| 2D MHD with weak $\underline{B}_0$ perp. | Alfvenization, Vortex<br>Bursting, Zeldovich number |
| Forced Hasegawa-Mima with Zonal Flow | Waves + Eddies + ZF<br>Multiple regimes and Mechanisms |

N.B. Clear distinction between "spreading" and "avalanching"

# Numerics: 2D Dedalus simulation

**Box Characteristics:**

- Dedalus Framework analogous to BOUT++

- Grid Size: 512×512
- Doubly Periodic boundary condition, beach regulates expansion

**Forcing Characteristics:**

- Superposition of Sinusoidal Forcing, vorticity
- Spectrum: Constant E(k), ensuring uniform energy distribution across wave numbers.
- Correlation Length: Approximately 1/10 of the box scale, some room for dual cascade.
- Localized through a Heaviside step function.
- Phase of forcing randomized every typical eddy turnover time

# 2D Fluid

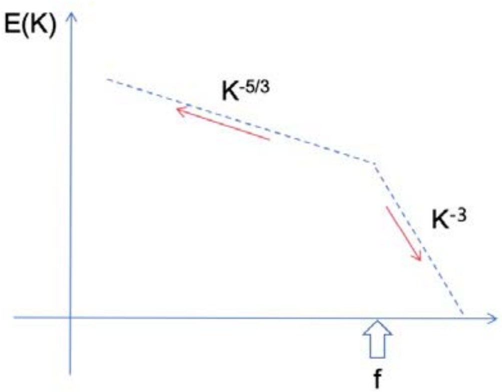
# 2D Fluid - the prototype

Vorticity Equation: $\frac{D\omega}{Dt} = \nu\nabla^2\omega - a\omega$

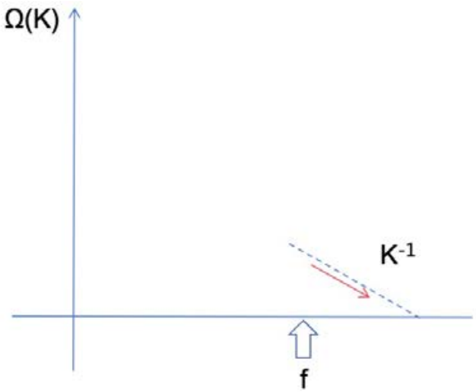
Key Physics:

- Inviscid, unforced invariants → Energy $E = \int d^2x(\nabla\varphi)^2/2$ ; Enstrophy $\Omega = \int d^2x(\nabla^2\varphi)^2/2$

⇨ Dual Cascade                Kraichnan




(3/2 law proved) Robust




("cascade" dubious)

# 2D Fluid, Cont'd

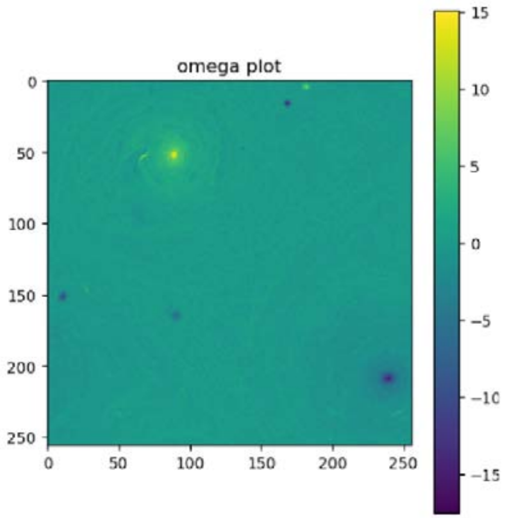
⇨ Selective Decay

Forward 'Cascade' enstrophy → Senses viscosity
Inverse 'Cascade' energy → Senses drag

For Final State of Decay:

$$\delta(\Omega + \lambda E) = 0$$ Bretherton + Haidvogel

⇨ Role Coherent Structures (Vortices) cf: B. Gallet, recent




- emergence isolated coherent vortices → survive decay

- $\frac{d}{dt}\nabla\omega = (s^2 - \omega^2)^{1/2}$

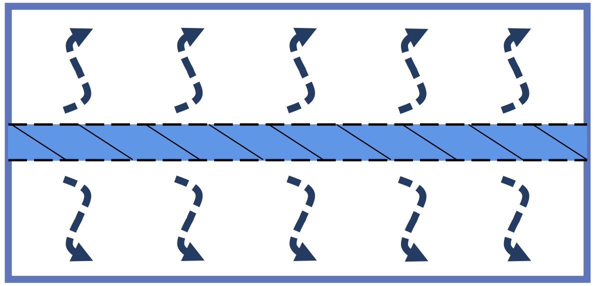
$\omega = \nabla^2\varphi$ → vorticity

$s = \partial^2_{xy}\varphi$ → shear

- Dipole vortices emerge, also

# 2D Fluid

⇨ Realize:




- Most of system in state of Selective Decay !
- Need Consider / Compare :

$\langle V_y (\nabla^2 \varphi)^2/2 \rangle$ → Enstrophy Flux

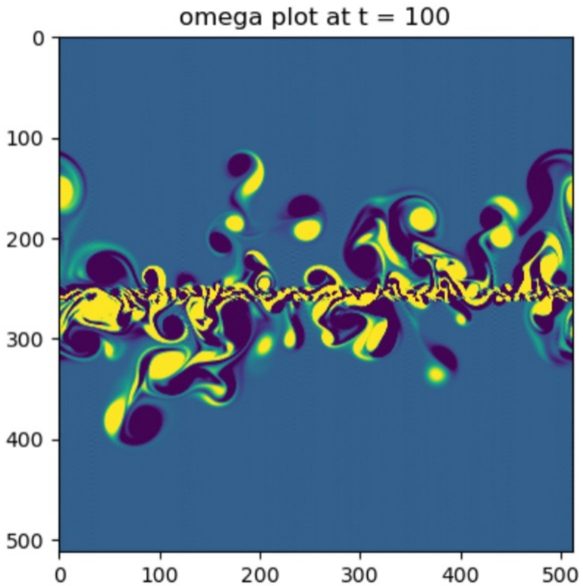
⇨ Physical Measures of Spreading

$\langle V_y (\nabla \varphi)^2/2 \rangle$ → Energy Flux

as measures of "intensity spreading". ⇨ Selective decay suggests these are radically different.

# What Happens ?

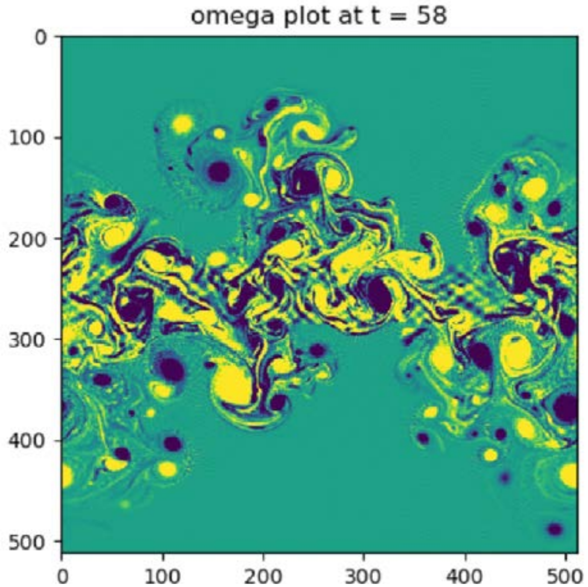
In Far Field, away from Forcing layer




Vorticity snapshot at Re~100

⇨ { Dipoles emerge
Spreading intermittent }

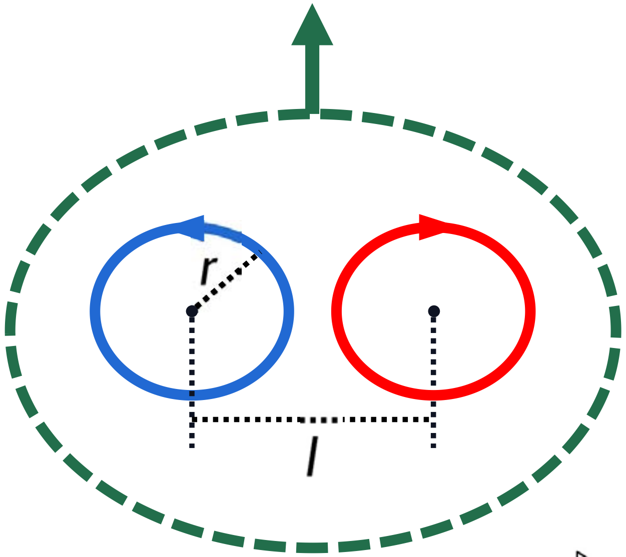
No apparent "Front"




Vorticity snapshot at Re~2000

- Dipoles, filaments, cluster
- Fractalized front

## ⇒ N.B. Dipole Vortex




— Uniform speed due to mutual induction

— $C = \frac{\Gamma}{l} = \frac{v r}{l}$

⇨ Dipole Vortices propagate at constant speed, "free flyers"

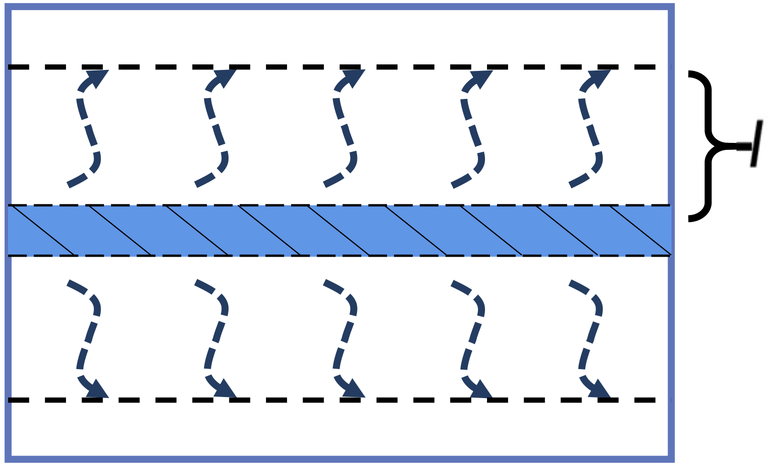
⇨ Physical origin of "ballistic spreading" ? !

i.e. ensemble dipoles expands linearly in time

c.f. Zaslavskii comment circa 2000.

# On Keeping Score

⇨ Loosely, interested in scaling of expansion of turbulent region with time



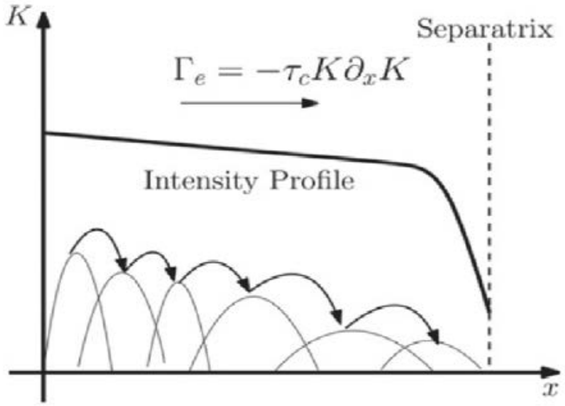

$l \sim t^a$

$a$ ?

N.B. Contrast DP ⇨ critical single site

⇨ Many approaches to $l$…

MFE favorite :




Track footprint of $|\varphi|^2$
Plot vs time,
1D projection

# Keeping Score, cont'd

⇨ Approaches

Table 1: Table describing various velocity and transport parameters.

| Parameter | Symbol | Equation | Description |
|---|---|---|---|
| RMS Velocity | $V_{rms}$ | $V_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2}$ | Root-mean-square velocity of turbulence, also known as turbulence intensity. This can either be measured near the forcing zone and averaged horizontally for a characteristic velocity as a basis of comparison, or measured globally to obtain global energy. |
| Quantity-Weighted RMS Distance | $X_{W-rms}$ | $X_{W-rms} = \sqrt{\frac{\int \lvert\delta(x)\rvert^2 \lvert Q(x)\rvert\, dx}{\int \lvert Q(x)\rvert\, dx}}$ | Quantity-weighted root-mean-square position represents the location of the quantity of interest, typically energy or enstrophy. One value is generated for each time. The quantity Q is usually energy or enstrophy. |
| Quantity-Weighted RMS Spreading Velocity | $V_{W-rms}$ | $V_{W-rms}$ is the slope of $X_{W-rms}$ plotted against time | Quantity-Weighted RMS Spreading Velocity represents the bulk motion. This is more comprehensive than the front velocity. |

N.B. :

— Quantity weighting can differ; depending on quantity

— RMS velocity sensitive to how computed

# Keeping Score, cont'd

⇨ Approaches, cont'd

— Front velocity is MFE favorite sensitive to 1D projection, definition

— Transport Flux $\langle V_y E \rangle$, $\langle V_y \Omega \rangle$, most physical, clearest connection to dynamics of 2D Fluid

but: Sensitive to viscosity and selective decay dynamics

— Jet velocity very sensitive to viscosity, field chosen

| | | | |
|---|---|---|---|
| Front Velocity | $V_{front}$ | $V_{front}$ is the slope obtained from tracking the outermost turbulent patch | This is usually comparable to $V_{W-rms}$, although front doesn't exist for low Reynolds number. |
| Transport Flux Density of certain quantity | $\Phi_Q$ | $\Phi_Q = < QV_\perp >$ | The amount of certain quantity passing through a unit length per unit time; flux is the integral of flux density through the horizontal surface, which bounds half of the region and can be related to the rate of change of the quantity in that region. |
| Transport "jet" Velocity | $V_Q$ | $V_Q = \frac{<QV_\perp>}{<Q>}$ | Also known as normalized flux density. Average is usually taken horizontally. This velocity is separately obtained for each time. |

# Keeping Score, cont'd

**Observation:**

— Lower Re → Significant speed, 'front' fluctuations due to variability in dipole population

— Transport velocities quite sensitive to viscosity and selective decay

i.e. $\langle V_y \Omega \rangle$ drops

jet velocity $\langle V_y \Omega \rangle / \langle \Omega \rangle$ rises

} especially for higher viscosity, Due selective decay

— Formation of dipoles follows decay of enstrophy

— Dipoles ultimately determine spreading

# Results

Re ~ 5000

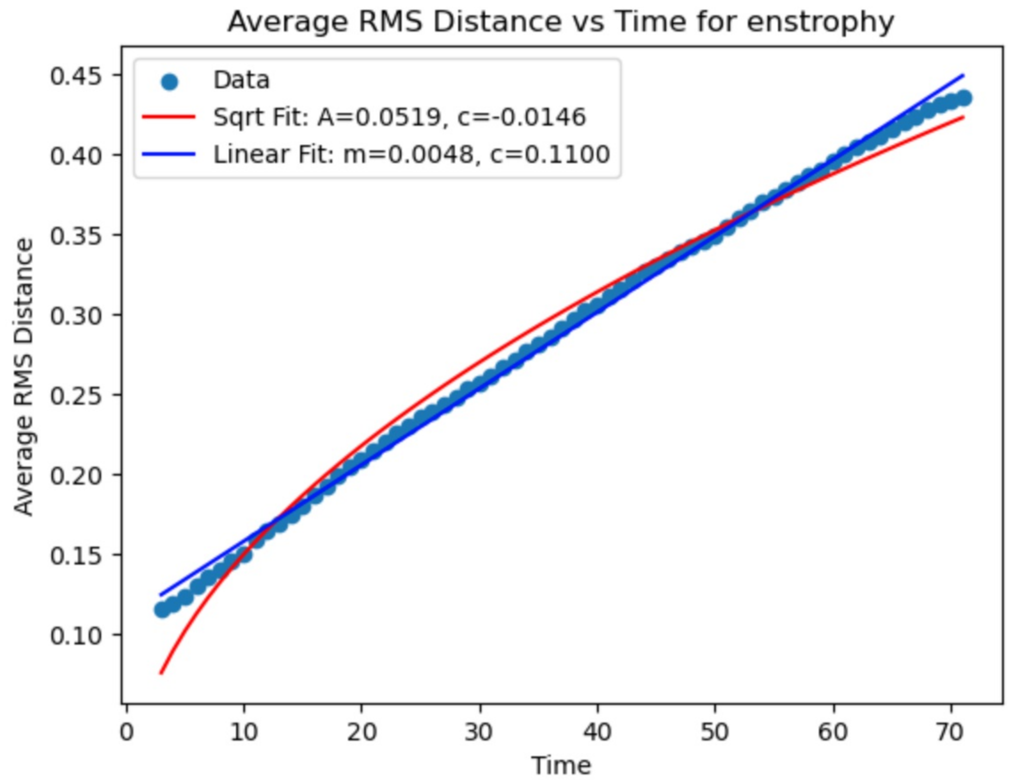
Ω–weighted
rms distance

—Constant spreading speed for enstrophy, i.e., $l \sim ct$

$\underline{\alpha = 1}$

—$c/V_{rms} \sim 0.1$

—Consistent with picture of dipole vortices carrying spreading flux



# Results, cont'd

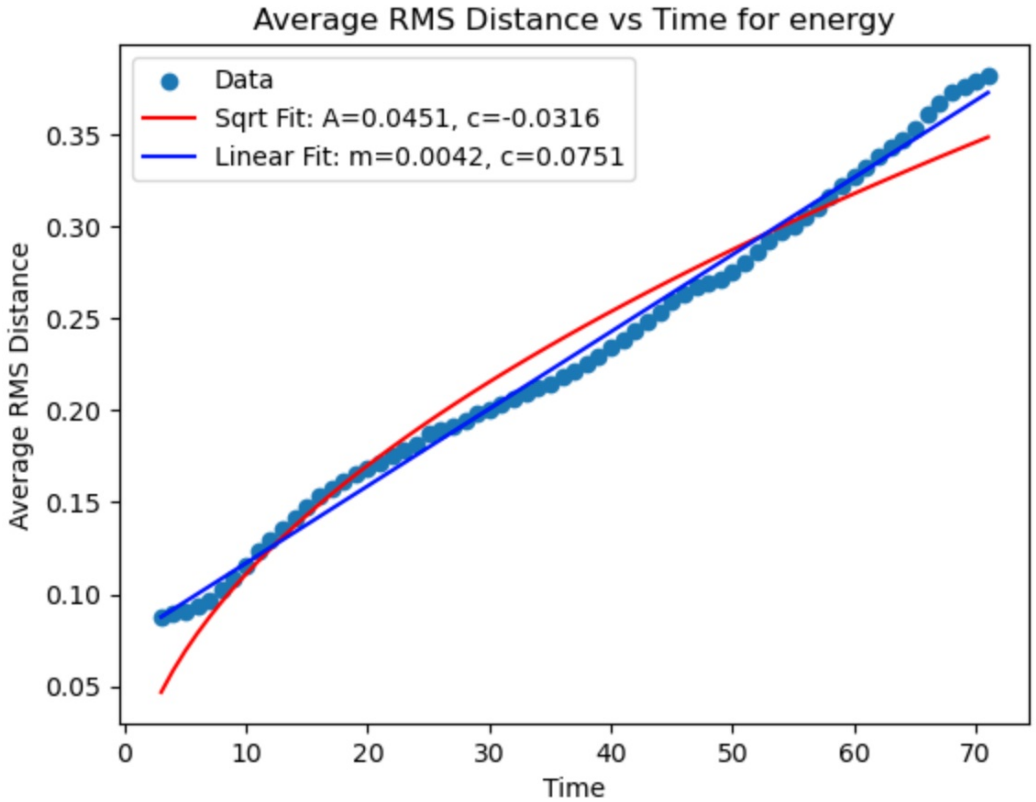
Re ~ 5000

$E$–weighted rms distance

— Constant spreading speed for energy, i.e., $a \simeq 1$

— $c/V_{rms} \sim 0.1$

— Lager dipoles ↔ more energy → increases fluctuations relative to enstrophy case



# Summary - 2D Fluid

— Coherent structures - Dipole vortices -
mediate spreading of turbulent region → free flyers

— Mixed region expands as $w \sim t$, consistent with dipoles.

— No discernable "Front", spreading is intermittent. (space+time)

— Spreading PDF is non-trivial. Requires further study.

⇨ — Turbulence spreading non-diffusive.

# 2D MHD + Weak $\underline{B_0}$

# 2D MHD

- The equations: $\frac{d}{dt}(\nabla^2\varphi) = \nu\nabla^2\nabla^2\varphi + \nabla A \times \hat{\boldsymbol{z}} \cdot \nabla\nabla^2 A + \tilde{f}$

$$\frac{d}{dt}A = \eta\nabla^2 A$$

$$\frac{d}{dt} = \partial_t + \nabla\varphi \times \hat{\boldsymbol{z}} \cdot \nabla$$

- Inviscid Invariants: $E = \langle V^2 + B^2 \rangle,\ H = \langle A^2 \rangle,\ H_c = \langle \vec{V} \cdot \vec{B} \rangle$ ⇨ 0, hereafter

  Conservation of $H$ is Key !

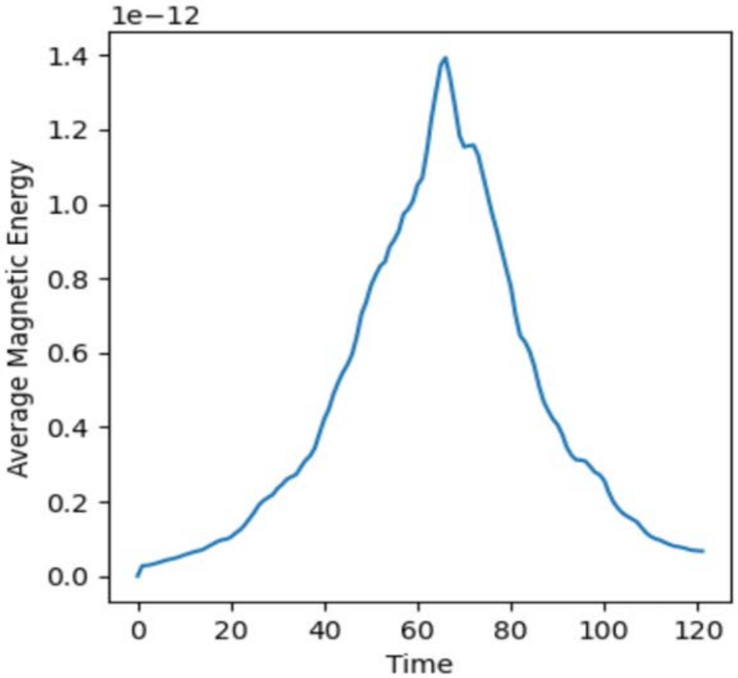
- Consider weak mean magnetic field: $B = B_0(y)\hat{x}$

  $B_0(y) \sim B_0 \sin(y) \Rightarrow$ initial imposed field

- As before, localized forcing region, effectively unmagnetized

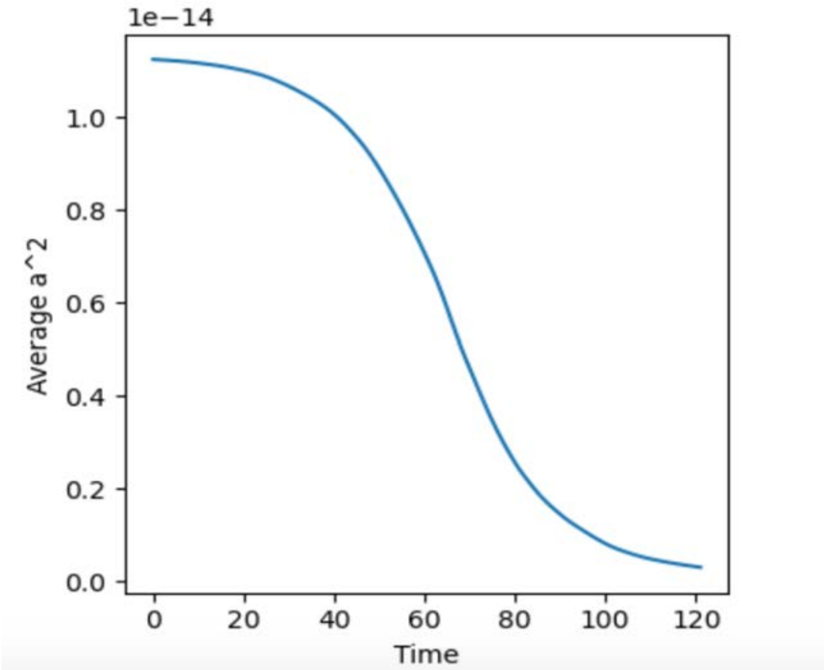
# ⇒ 2D MHD

- Zeldovich Theorem: No dynamo in 2D




⇨ Field ultimately decays

- Consequence of decay $\langle A^2 \rangle$




$$\frac{d}{dt}\langle A^2 \rangle = -\eta \langle B^2 \rangle$$

$$\int_0^t \langle B^2 \rangle dt \leq \frac{\langle A(0)^2 \rangle}{\eta}, \therefore \langle B^2 \rangle \text{ decays}$$

# Key Physics of 2D MHD

N. B. "Z" ⇨ Zeldovich

- Lorentz force suppresses inverse kinetic energy cascade.
  Inverse cascade $\langle A^2 \rangle$ develops

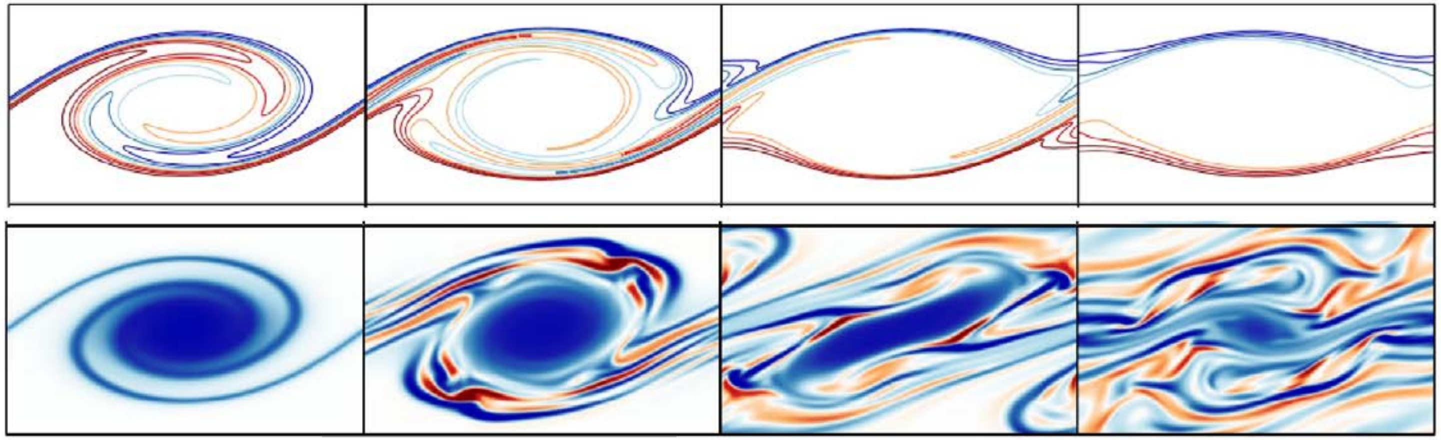
- Single Eddy: Expulsion (Weiss'66) vs. Vortex Disruption (Mak et. al 2017)

Key Parameter: $Z = Rm \frac{V_{A0}^2}{V_E^2}$

$Z \sim 1$ bounds the two regimes

Expulsion:

Vortex bursting:



from Mak et. al 2017

See also: Gilbert, Mason, Tobias 2016

# Key Physics of 2D MHD, cont'd

- Turbulent Diffusion: ( Cattaneo + Vainshtein '92;
  Gruzinov + P.D. '94 )

  Closure + $\langle A^2 \rangle$ conservation ⇨ Quenched Diffusion of $B$ - field

  From: $D_t \sim \eta_{anom} \sim \langle \tilde{V}^2 \rangle \tau_c$

  To: $D_t \sim \eta_{anom} \sim \langle \tilde{V}^2 \rangle \tau_c / [1 + R_m V_{A0}^2 / \langle \tilde{V}^2 \rangle] \sim D_{Kin} / (1 + Z)$

  ⇧

- Once again,

  Key Parameter: $Z = R_m \frac{V_{A0}^2}{<\tilde{V}^2>}$

  $< \tilde{V}^2 > \;\; vs \;\; V_E^2$

  N.B.: - $V_{A0}$ is initial weak mean magnetic field

  - $R_m$ large...

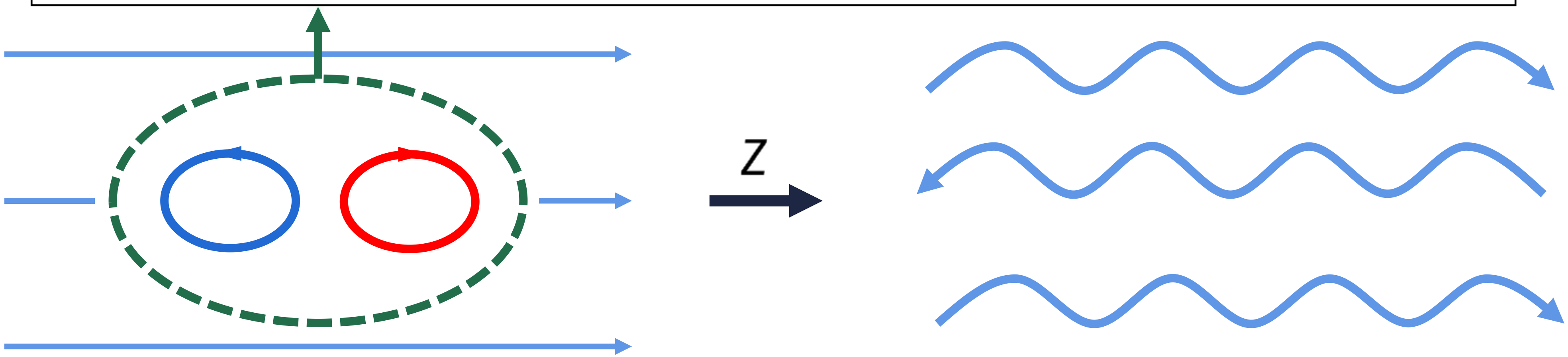
- Physics is simply $\underline{V} \cdot \nabla \omega$ vs $\underline{B} \cdot \nabla$ J and stretching

# Crux of the Issue!?

⇨ Hydrodynamics: Dipole vortex 'Carries' turbulence energy ⇨ spreading

⇨ But... weak $B_0$ can 'burst' vortices ⇨

Converts dipole kinetic energy to Alfven waves, propagating laterally, and to dissipation.

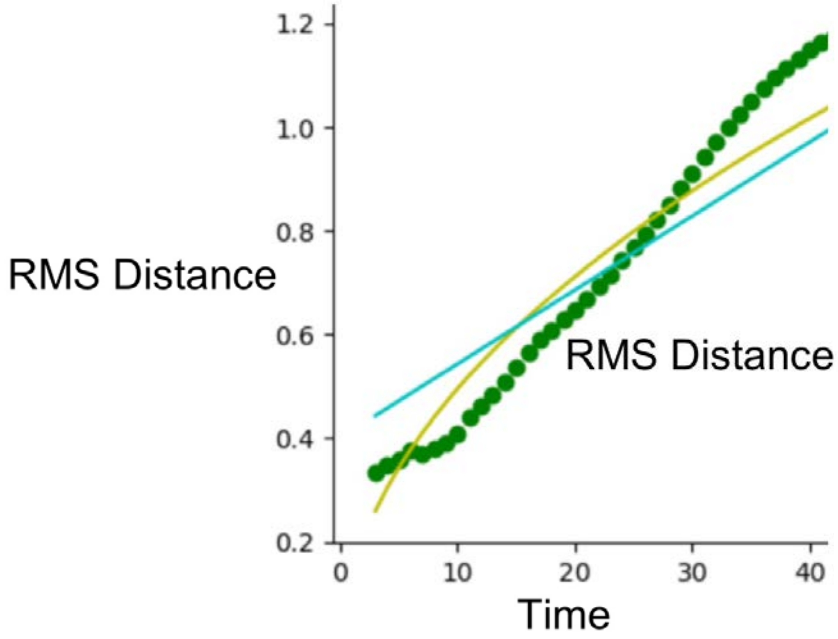



⇨ So, can a weak $B_0$ block spreading in 2D MHD ! ?

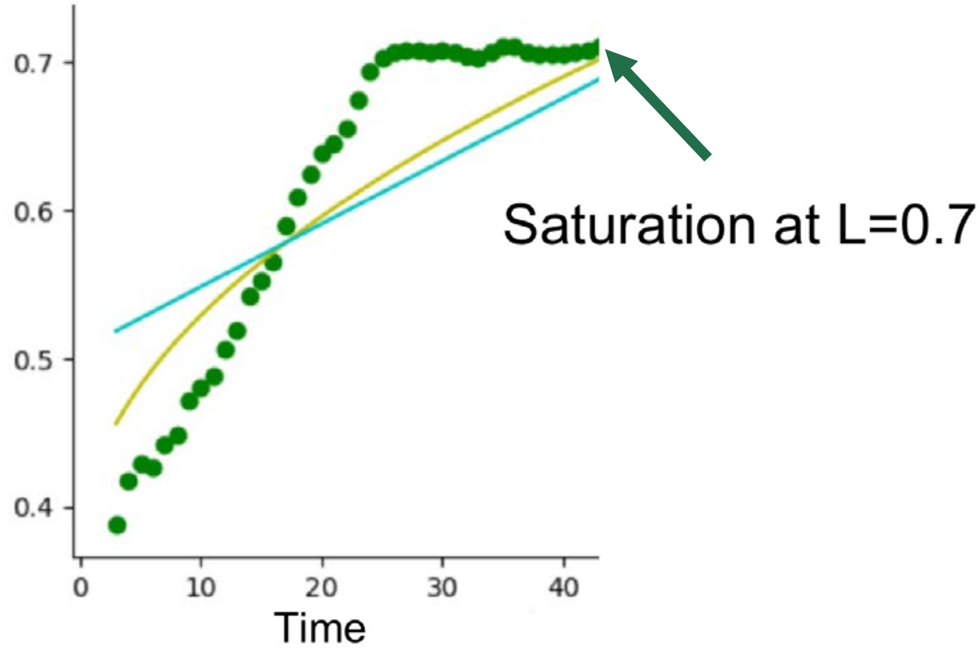
N.B. Perp Alfven waves observed

# ⇨ Time evolution of Spreading

Hydro regime: $Rm = 100,\ Bo = 0.001,\ Z = 0.01$




MHD:$Rm = 100,\ Bo = 0.01, Z = 1$




⇨ Hydro case spreads linearly

⇨ Z=1 Case saturates.
(dipoles disrupted)

# ⇨ Spreading vs. Z - Turbulence

- Now consider turbulence:

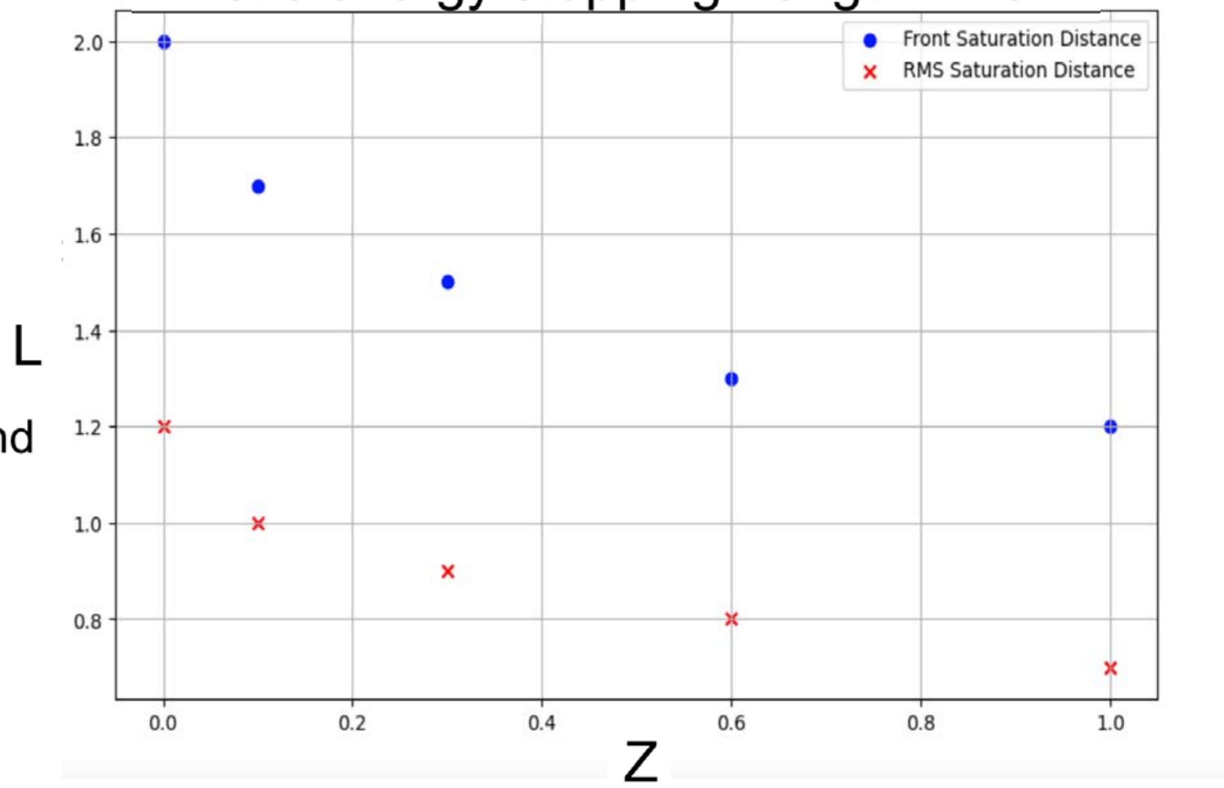
- Kinetic Energy Stopping length decreases with increasing $Z = R_m \frac{V_{A0}^2}{<V_{rms}^2>}$

  N.B. Z reflects both $R_m$ and $B_0$

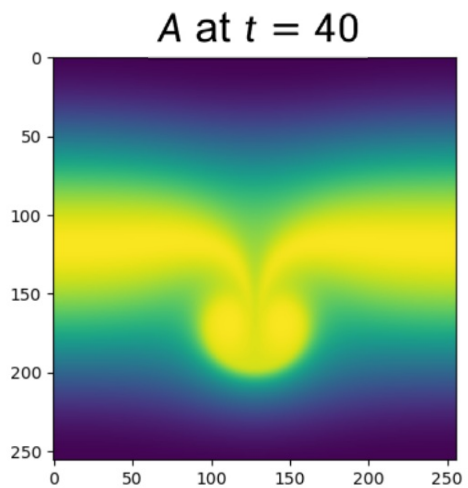
- Systematic difference between Front and RMS saturation evident, trends match

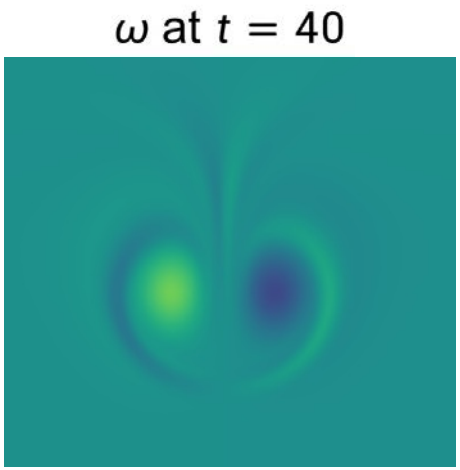
⇨ Insight from vortex studies useful

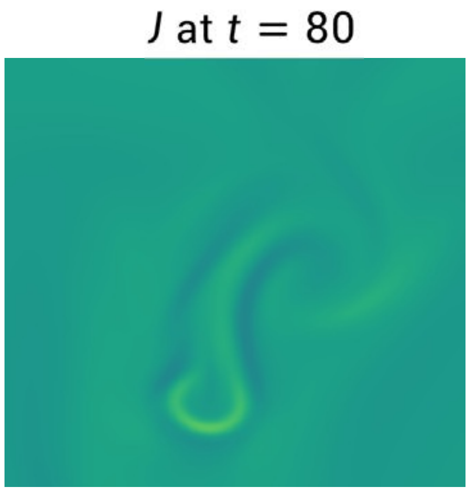
Kinetic energy stopping Length L vs. Z



## ⇒ Single Dipole in weak $B_0$



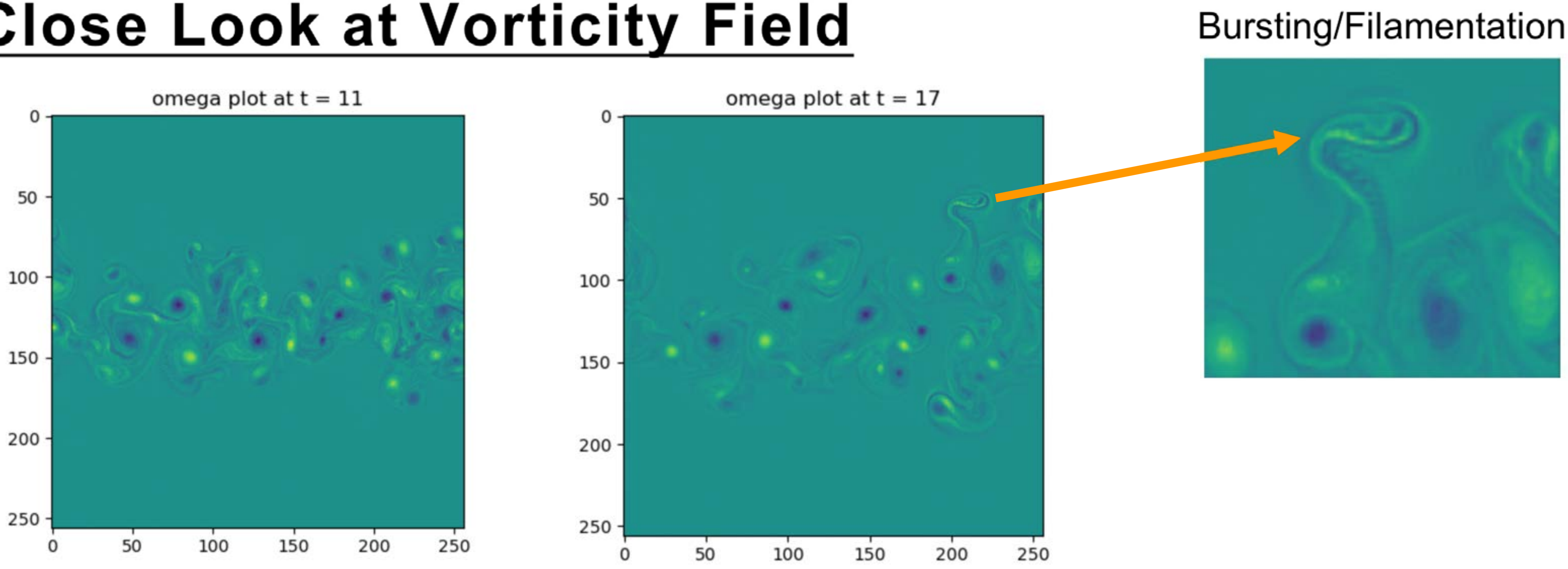




Note wrapping filament tends to cancel and push on dipole, so it distorts and ultimately bursts

Filament and vortex bursting. Concentration of energy at small scale ⟹ fast dissipation

Connection: vortex busting ⟺ MHD cascade singularity?!

## ⇨ Close Look at Vorticity Field




- Z=3, Rm≈50, Re≈500, B=0.01
- Dipoles evident at early times, but encounter stronger field as migrate
- Vortex bursting occurs at later times ⇨ Spreading halted.

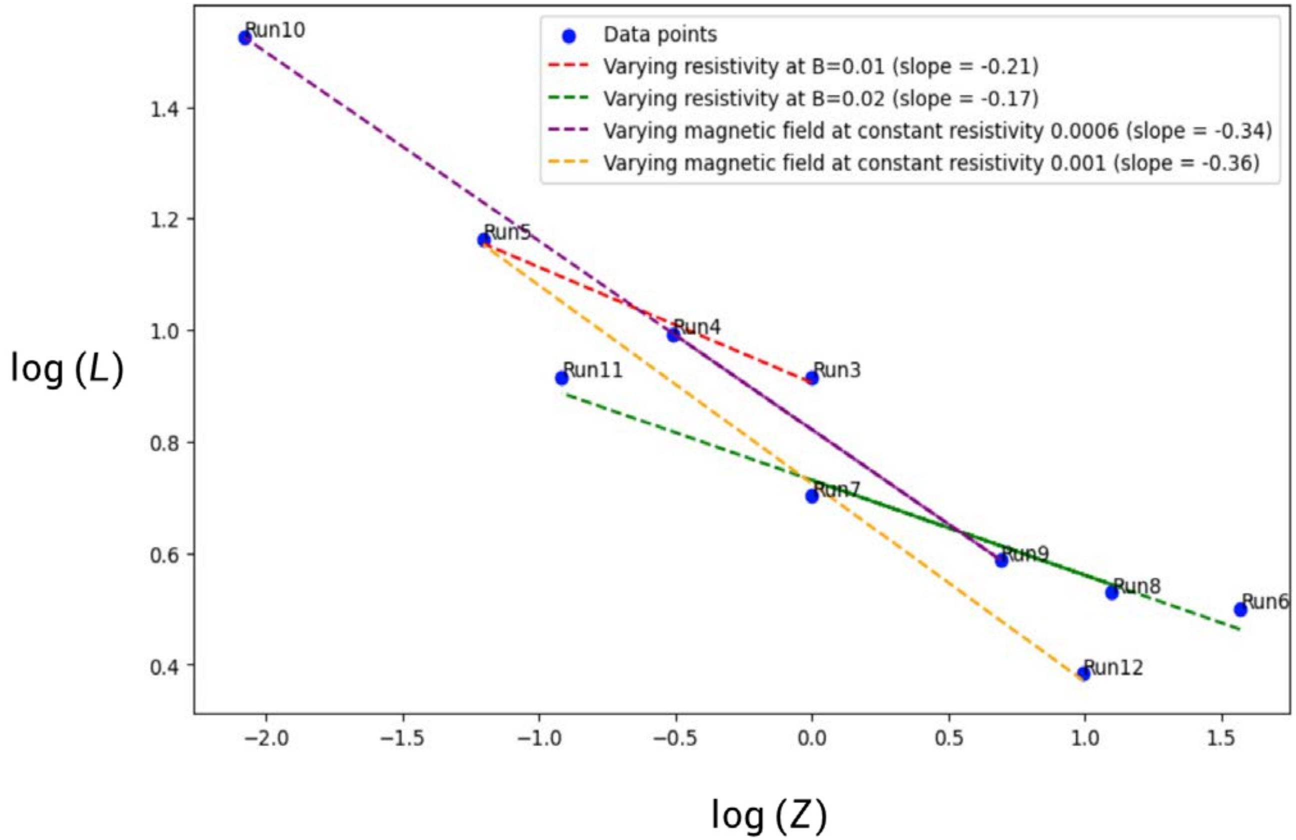
## ⇨ Single Dipole Penetration

- Dipole penetration decreases with increasing Z

- Evidence that varying $B_0$ and $R_m$ impact penetration.

But Z is not the full story… $P_m$ dependance?

Log-Log Plot of L against Z



# ⇨ 2D MHD: Summary

- Weak $B_0$ enables vortex disruption

  Dipole bursting ⇨ Saturates spreading

  ⇩

- Weak $\underline{B_0}$ blocks advance of kinetic energy
- Process: Conversion dipole KE to Alfven waves, laterally propagating

- $Z = R_m \frac{V_{A0}^2}{<V_{rms}^2>}$ as critical parameter

⇨

- Reinforces notion of "free flyer dipoles" as critical to spreading

# Forced Hasegawa – Mima + Zonal Flows

# H-M + Zonal Flow System

– System:

PV forced ⇓

$$\frac{d}{dt}(\tilde{\phi} - \rho_s^2 \nabla_\perp^2 \tilde{\phi}) + v_* \frac{\partial \tilde{\phi}}{\partial y} + v_{*u} \frac{\partial \tilde{\phi}}{\partial y} = \frac{\partial}{\partial r} \rho_s^2 \langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle + \nu \nabla^2 \nabla^2 (\tilde{\phi}) + \tilde{F}$$ -Waves, Eddys

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{v}_z \frac{\partial}{\partial y} - \nabla \tilde{\phi} \times \hat{\boldsymbol{z}} \cdot \underline{\nabla}$$

$$\frac{\partial}{\partial t} \nabla_x^2 \bar{\phi}_z + \frac{\partial}{\partial r} \langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle + \mu \nabla_x^2 \bar{\phi}_z = 0$$ -Zonal Flow (Axisymmetric)

N.B. $\bar{\phi}_z = \bar{\phi}_z(x)$, only.

N.B. : Electrons Boltzmann for waves, not for Zonal Flow

– viscosity controls small scales

– drag controls zonal flow - $\mu$

– conserved:

Energy → $\langle \tilde{\phi}^2 + \rho_s^2 (\nabla \tilde{\phi})^2 \rangle + < \rho_s^2 (\nabla \phi_z)^2 >$

Potential Enstrophy → $\langle (\tilde{\phi} - \rho_s^2 \nabla^2 \tilde{\phi})^2 \rangle + < (\rho_s^2 \nabla^2 \phi_z)^2 >$

↓ Waves ↓ ZF



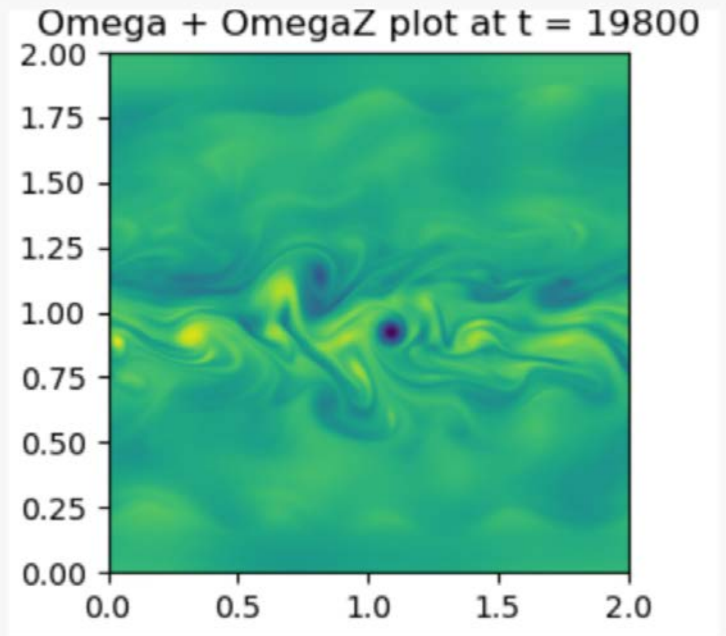
N.B. Energy, Pot Enstr. exchange between Waves and ZF possible.

# Typical saturated snapshot(Kubo 0.2)

— Dipoles disappear

— Large coherent vortex

N.B. Density gradient
precludes dipoles.




Total vorticity snapshot at the end. Steady state; Turbulent in the center only. Dipole isn't a steady structure in this system; instead, we get single vortex that looks like Jupiter's eye, which is not gonna move by itself

Total Vorticity: $\nabla^2(\tilde{\phi} + \phi_z)$

# H-M + Zonal Flow System, cont'd

→ Now: *waves* $\omega = \omega_* / (1 + k_\perp^2 \rho_s^2)$, $\underline{v_{gr}}$

eddies $\tilde{v}$

zonal mode (symmetry)

$\left\{ \begin{array}{l} \tilde{v} \text{ vs } v_* \rightarrow \\ \text{mixing length} \end{array} \right.$

i.e. ⇒ Energy Flux has two components: $\left\{ \begin{array}{l} \sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}} \rightarrow 2^{nd} \text{ order in } e\tilde{\phi}/T \\ \langle \tilde{v}_r \xi \rangle \rightarrow 3^{rd} \text{ order in } e\tilde{\phi}/T \end{array} \right.$

N.B. 2 channels for "turbulence spreading" → Waves/Wave transport; → Turbulent mixing

-Branching ratio, vs. Ku number ?

**For clarity; Contrast:**

⇨ **Spreading in presence of fixed, externally prescribed shear layer**

⇨ **Here:** → Forcing → $\left\{\begin{matrix}\text{Waves}\\ \text{Eddies}\end{matrix}\right\}$ → Zonal flow (self-generated)

∴ forcing ($\tilde{v}_{rms}$, $Re$) + drag ⇒ control parameters

⇨ "weak" and "strong" Turbulence Regimes

$$v_{gr} \text{ vs } v_r \rightarrow \frac{\langle \tilde{v}_r \xi \rangle}{\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) \xi_{\boldsymbol{k}}} \rightarrow \frac{\tilde{v}_r \tau_c f}{\Delta_c} \rightarrow Ku$$

⟺ 2nd vs 3rd order energy flux

coherency factor

$\Delta_c \sim v_{gr} \tau_c$

⇨ $Ku < 1$ → wave dominated spreading

$Ku > 1$ → mixing dominated spreading ⇨ ~ 2D fluid

# H-M + Zonal Flow System, cont'd

→ Enter the Zonal Flow…

- Multiple channels for NL interaction
- But with ZF ⟷ eddy, wave coupling to ZF dominant
- ZF is the mode of minimal inertia, damping, transport

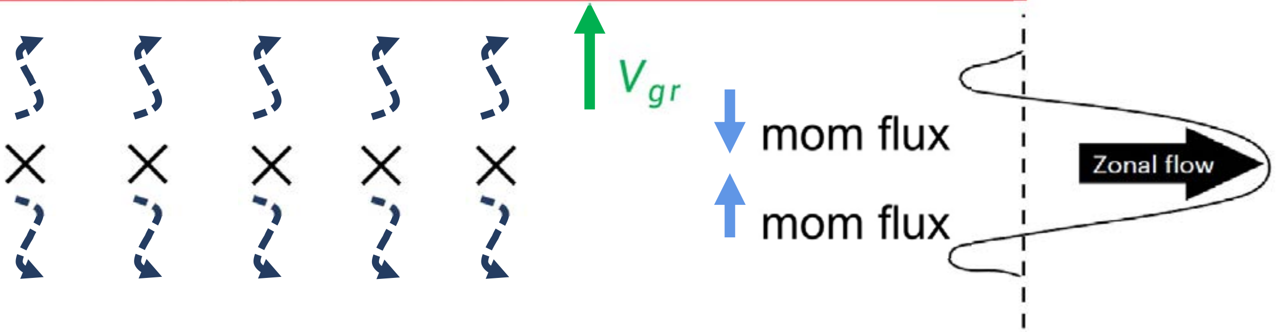
Waves:

$$\frac{\partial}{\partial t}(1 + k_\perp^2 \rho_s^2)\tilde{\phi} = \ldots\ldots$$

ZF:

$$\frac{\partial}{\partial t}(k_r^2 \rho_s^2)\bar{\phi}_z = \ldots\ldots$$

⇒ energy coupled to ZF ($\tilde{v}_r = 0$) cannot "spread", unless recoupled to waves

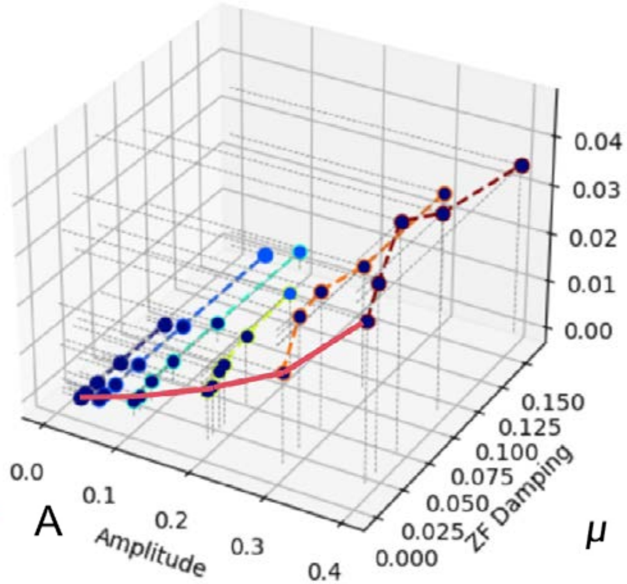



→ Degradation of ZF (back transfer) is crucial to spreading

→ ∴ $\mu$ must regulate spreading. What of $\mu \to 0$ regimes?

# Results

Potential Enstrophy Flux




Dimits Regime

⇨ averaged spatially and temporally

— Potential enstrophy flux generally increases as drag increases. “Dimits regime” for turbulence spreading. Spreading diminishes as power coupled to Z.F. (Fixed, spatially)

— Self-generated barrier to spreading.

— For A increasing, PE flux rises sharply, even for weak ZF damping. Fate of ZF?

— “KH-type” mechanism loss of Dimits regime at higher A? Characterization??

N.B. “Dimits Regime”= Condensation of energy into ZF for weaker forcing.

# Results, Cont'd
# Wave Energy Flux

Wave Energy Flux $< -\frac{\partial \phi}{\partial t} \nabla \phi >$ ⟷ $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) E_{\boldsymbol{k}}$

for drift waves

− Dimits regime at low forcing and ZF damping

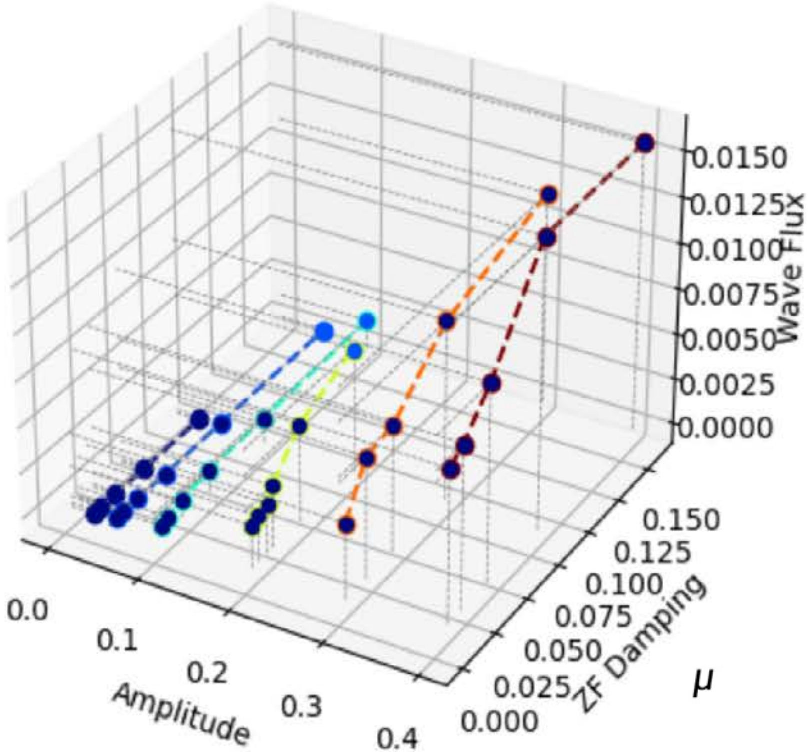
−Increases with ZF damping and forcing amplitude

− Dominant $K_x$ increases due ZF decorrelation

− Spectrum condensation towards low k with inverse cascade

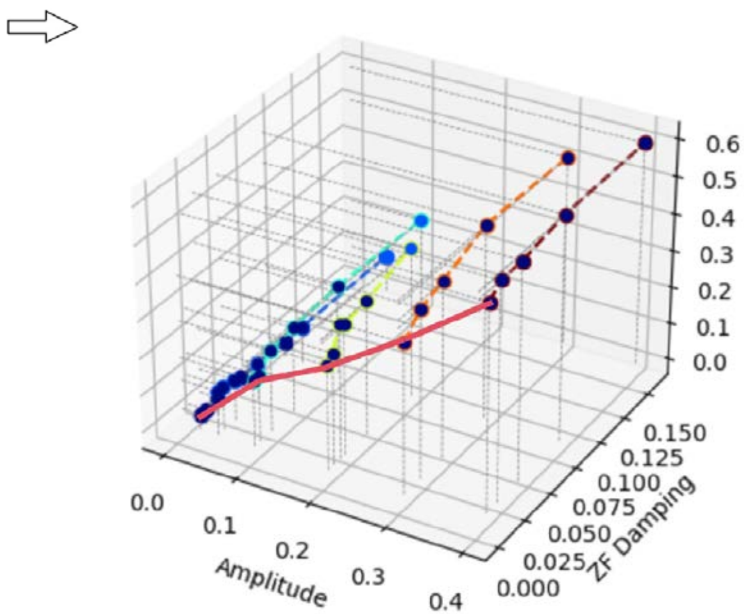
implication for $v_{gr}$ and $\sum_{\boldsymbol{k}} v_{gr}(\boldsymbol{k}) E_{\boldsymbol{k}}$

− Take note of increasing W.E.flux as $\mu \to 0$,
A increases.

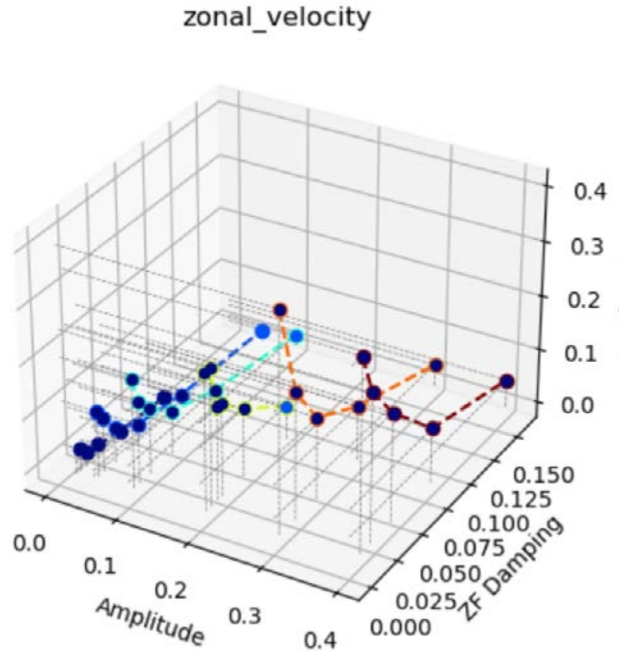


# Results, Cont'd

$\frac{\tilde{v_r}\tau_c f}{\Delta_{c_c}}$ where $\Delta_c \sim \langle K_x^2 \rangle^{-1/2}$

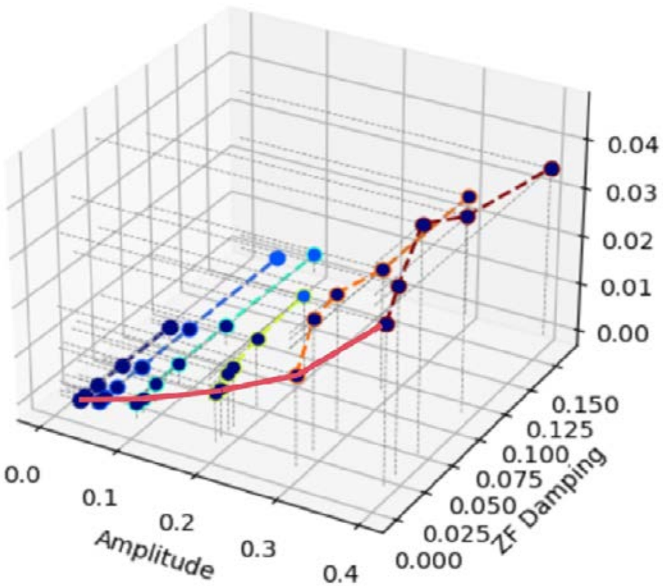



Fluctuation intensity increases as drag increases




Zonal velocity decreases with increasing drag (clear)

# →Spreading and Fate of Zonal Flows




→ Spreading rises for increased forcing, even for $\mu \to 0$

→ Dimits regime destroyed. How?

$\underline{\Rightarrow}$ Seems necessary for spreading in systems with ZF

→ Animal Hunt for linear instabilities(KH, Tertiary …) seems pointless in turbulence

→ Instead, $P_{\mathrm{Re}} = -\langle \widetilde{V_x} \widetilde{V_y} \rangle \cdot \dfrac{\partial \overline{V}_y}{\partial x}$ Power transfer [fluctuations → flow]

$P_{Re} < 0$ : Wave → ZF transfer

$P_{Re} > 0$ : ZF → Wave transfer ⇒ ZF decay

# Quantifying Wave-ZF Power transfer

$$1/2 * \frac{\partial \overline{V}_y^2}{\partial t} = \omega_Z < \widetilde{v_x}\widetilde{v_y} > -drag * \overline{V}_y$$

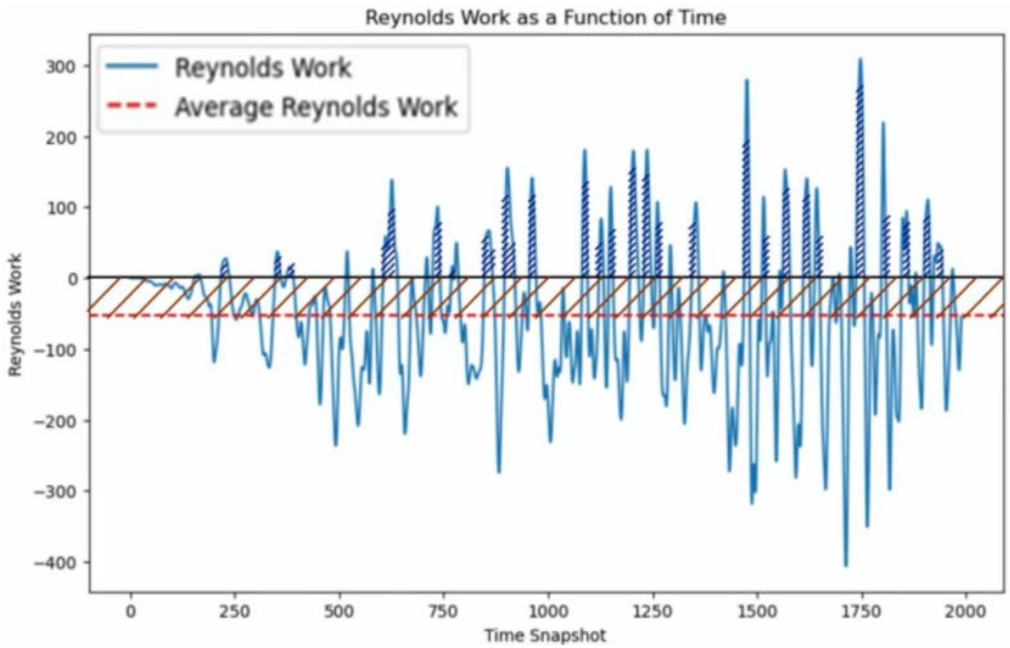
Reynolds power

We quantify ZF → Waves Power Transfer as the ratio of the area above the axis to mean work done on the zonal flow.

N.B.:

$$P_{\mathrm{Re}} = -\langle \widetilde{V_x}\widetilde{V_y} \rangle \cdot \frac{\partial \overline{V}_y}{\partial x} \rightarrow D_t(\partial V_y^{\square}/\partial x)^2?$$

Mixing length model fails capture 2 signs




Reynolds power vs time

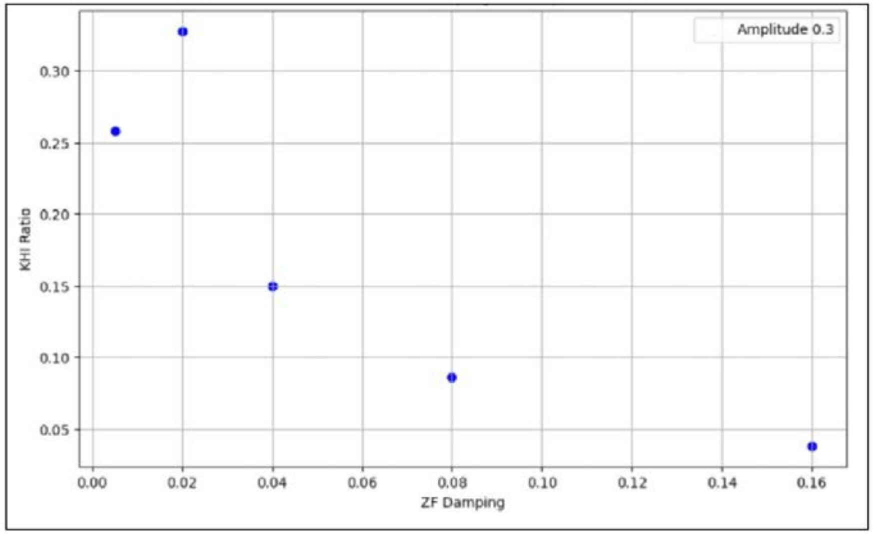
$P_{Re} < 0 \Rightarrow$ Wave → ZF transfer

$P_{Re} > 0 \Rightarrow$ ZF → Wave transfer

# Results, Cont’d

$P_{Re}$ *ratio vs ZF damping*

Dimits Regime

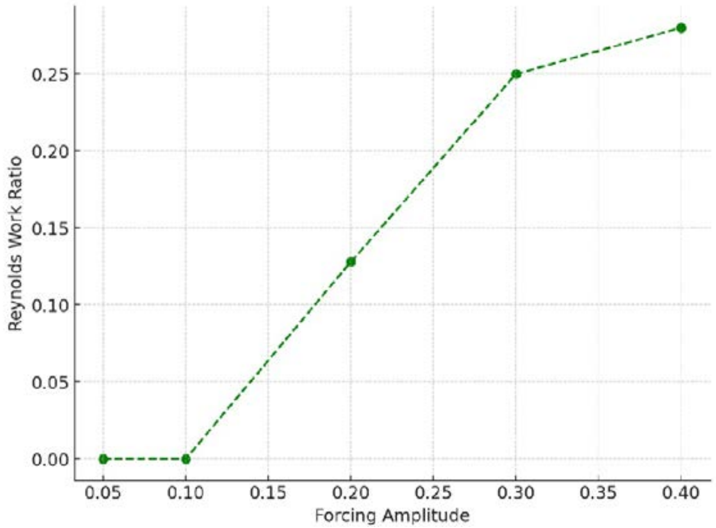



— The ratio generally decreases as a function of ZF damping

⟺ Damped Zonal Flow More Stable.

# Results, Cont'd, $P_{Re}$ Ratio vs Forcing Strength

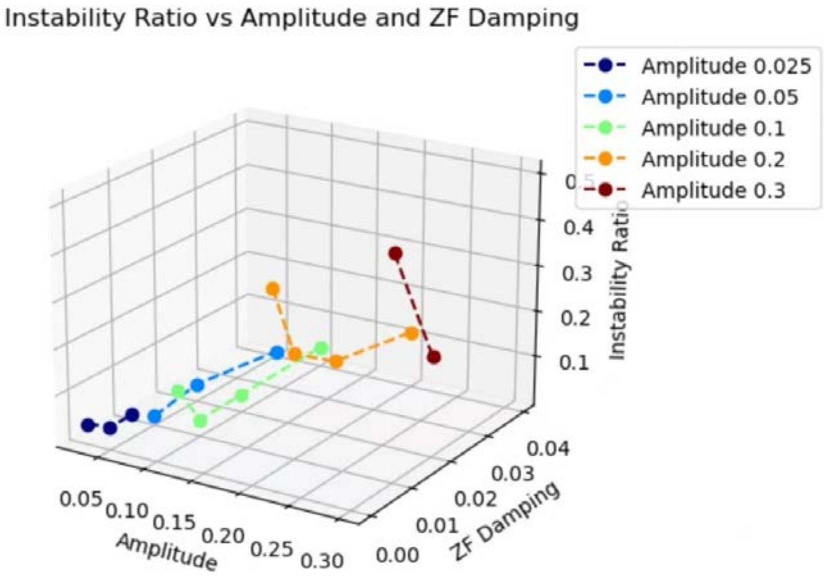
$P_{Re}\ ratio\ vs\ forcing\ amplitude$




Preliminary
→ Explore other FOMs

— The ratio decreases as a function of forcing strength
— Indicates that re-coupling of ZF energy to turbulence increases for stronger forcing
— This approach avoids instability morass.

# $P_{Re}$ Ratio vs A, $\mu$



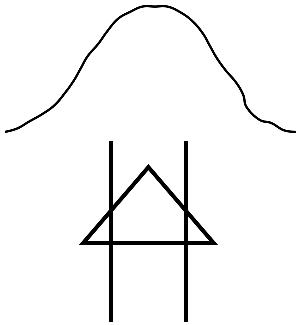

- $P_{Re}$ back transfer increases with forcing, and as $\mu$ decreases

- Further analysis required

- Is vortex shedding the mechanism of turbulence propagation?

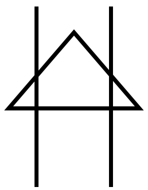
# Related Problem: Jet Migration(Laura Cope)

i.e. - Here:



- Symmetry broken by forcing region
- Turbulence patch propagates, drags ZF/Jet along
- Zonon breaks symmetry

\- There:



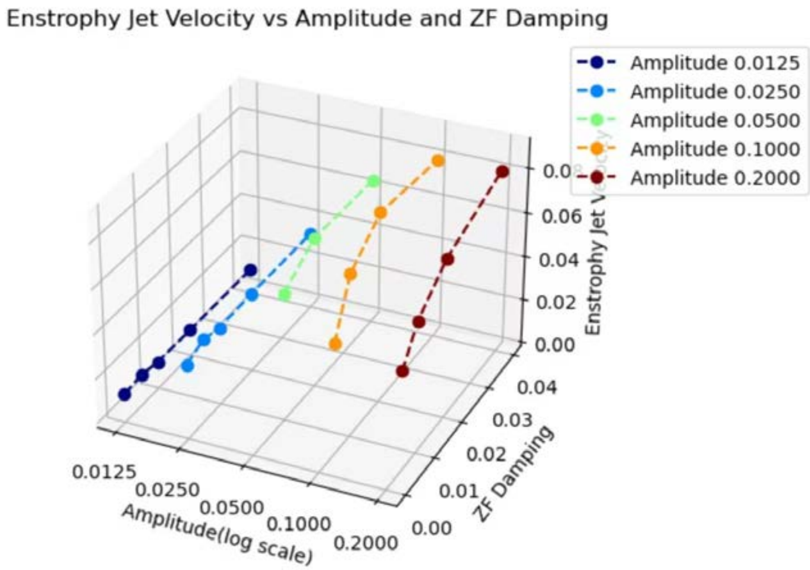
- Jet migrates
  but Migration enabled by dynamics of fluctuation field, via zonon
- How does zonon modify turbulence field?

# So Jet Velocity !?

→ As waves/eddys drag along zonal flow, Jet velocity(ala' Townsend) is related to Jet Migration.

so

→ Enstrophy Jet Velocity?!




- Now familiar trends

- Seems semi-quantitatively consistent with Cope results.

# Summary - Drift Wave Turbulence

→ Spreading fluxes mapped in forcing, ZF damping parameter space

→ Dominant mechanism ⟷ Ku (waves vs mixing) , Both waves and mixings in play.

→ Dimits-like regime discovered. Fixed ZF pattern.

→ ZF quenching intimately linked to spreading

→ $P_{Re} > 0$ bursts track breakdown of Dimits regime and onset turbulent mixing
Spreading increases.

## →General Summary

→ Coherent structures dipoles frequently mediate spreading

←→ underpin "ballistic scaling"

→ Spreading dynamics non-diffusive; Conventional wisdom misleading, or worse.

→ In DWT, wave propagation and turbulent mixing both drive spreading

→ ZF quenching critical to spreading in DWT. Power coupling most useful to describe ZF quench.

→ Closely related to jet migration.

# →Future Plans

— High resolution studies
— Understand ZF quenching physics and calculate power recoupling-general case, GK formulation?
— What is physics of $P_{Re}$>0 bursts? - shedding?

— Spreading in Avalanching. Relative Efficiency? Spreading and Transport? Flux-driven H-W System. Potential Enstrophy Flux!?

More general:
— Is spreading mechanism universal? Seems unlikely
— Towards a model, models… Ku~1 is an interesting challenge
— Relation/connection of DW+ZF spreading and Jet Migration (L. Cope)
— Is Directed Percolation of any use in this?
Ideas, Approaches-yes?! Details-??