Persistence and Evolution of “Staircase” Profiles in a Fluctuating Vortex Array

(Relevant to near-marginal systems)

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Outline

1) **Background**

2) **Fixed Cellular Array (FCA) Problem**

3) **Relaxing FCA with Fluctuating Vortex Array (FVA)**

4) **Results**
   a) Staircase Profile in the FVA
   b) Transport in the FVA

5) **Conclusion**

Background

Near-Marginal Systems

<u>Near-Marginal</u>

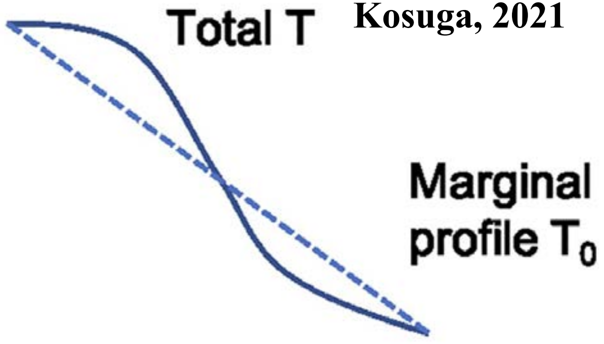
- Weak turbulence
  - $\boldsymbol{E} \times \boldsymbol{B}$ convective cells and magnetic islands excited but not strongly overlapping.

→ Instabilities are excited but not so strong as to produce large transport.

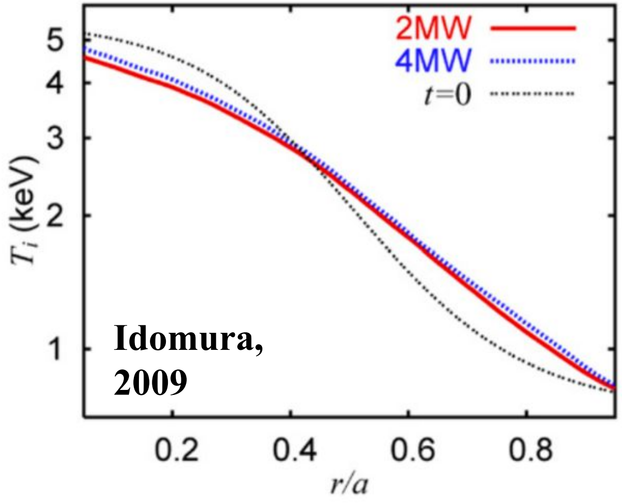
Characteristic of **<u>Stiff profiles</u>**

- I.e., Profiles that adopt roughly the same shape regardless of the applied heating and fueling profiles

Near-marginal plasmas can sometimes naturally evolve towards a **globally organized** critical state of micro-barriers and strong avalanche-like transport.



Kosuga, 2021

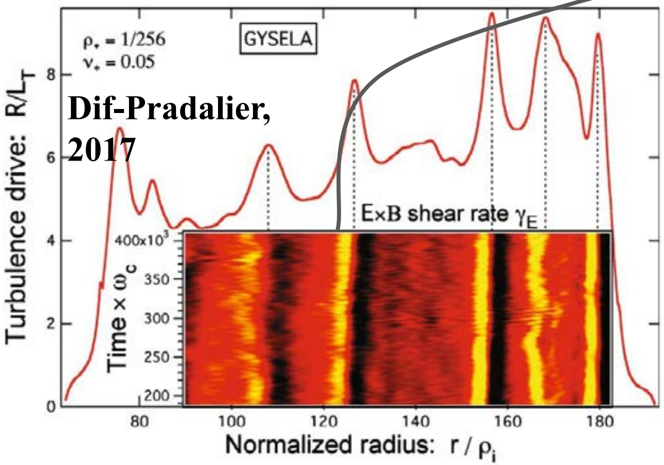


Idomura, 2009

E × *B* Staircase

E* × *B **<u>staircase</u> current subject in M.F.E**

Dif-Pradalier, 2017



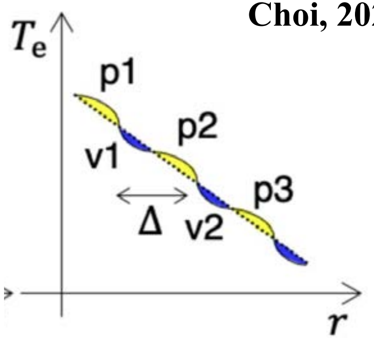
<u>Context</u>: Flat spots of high transport and nearly vertical layers acting as mini-barriers coexist. In plasmas, avalanches happen in flat spots and shear layers due to zonal flows occur in the areas of mini-barriers.

Suggested ideas:

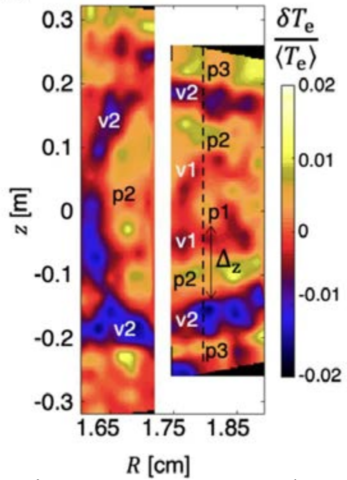
- ***E* × *B*** shear feedback, predator-prey
  - Zonal flows predator and turbulence intensity prey
- Jams

Yellow and black colors are a rapid transition of the direction of flows around peaks in turbulence drive.

Choi, 2022



KSTAR

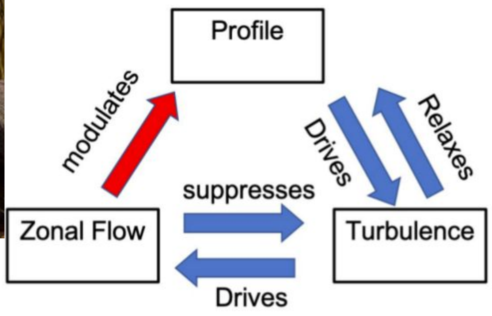


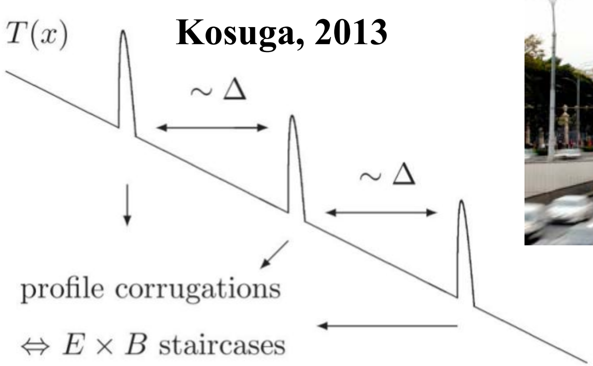
Jet like patterns in δT/<T> image correspond to staircase corrugations

Some Questions

- How does staircase beat homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?

Conventional Wisdom





Two ideas from self-organization that can explain the ***E*×*B*** staircase are ***E*×*B*** shear feedback and jams.

Useful to view DW turbulence and ZF as separate populations, which interact via a "predator-prey" feedback loop:

- DW (the prey) grow due to the gradient (instability) drive, while ZF (the predator) "feed" upon the DW population by Reynold stresses.

For **jams**, it is useful to draw inspiration from traffic flow theory (Key element: time delay)

- A driver's early reaction maintains smooth traffic, while **longer reaction triggers jams**.
- For flux driven turbulence, heat flux jams occur when there is sufficient time delay between temperature modulations and local heat flux.
  - Leads to growth of shock trains…

But… is there an even **simpler** physical mechanism that can produce **layering?**
Answer: Yes (e.g., pattern of cells)

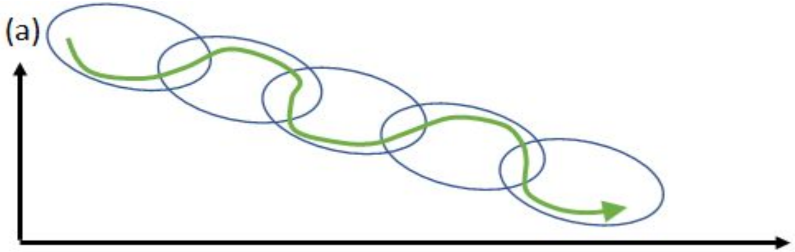
FCA Problem

(another way to get a Staircase)

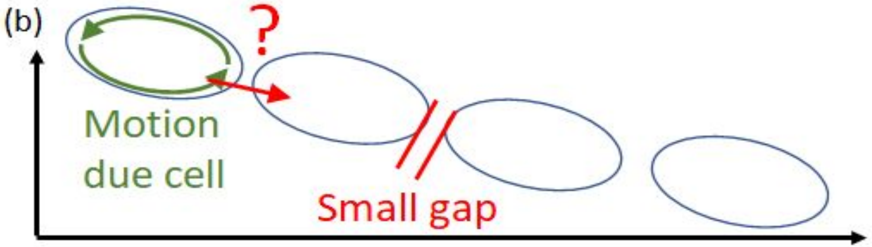
FCA Problem (similar to $E \times B$ convection)

Transport of particle between non-overlapping or marginally overlapping cells (**characteristic of near marginal**) is an important topic in fusion plasma.

Overlapping case: particles can transport directly from cell to cell, wandering along streamlines



Nearly-overlapping case (cells sit at near overlap): transport is a synergy of motion due to cells and random kicks (Collisional diffusion, ambient scattering) thru gap regions.



→ Characteristic of **near marginal**.

→ The transport over gap is random kicks (ambient diffusion): collisions, micro-turbulence.

→ **Coexistence** of:
~ **Fast transport** - Mixing in cell
~ **Slow transport** - Kicks between cells

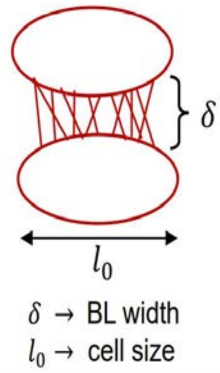
N.B.: “Profile stiffness” → Cells near overlap
→ Rapid increase in transport prevents strong overlap

What of Interest?

- **Relevant to key question of "near marginal stability"**

→ Representative of state in <u>marginal stability</u>.
- Stiff systems hovering near threshold (relevant question)

→ **Natural candidate to near marginal stability!**
- Zonal (mean) flows
- similarities SOC (fronts, spreading,...)
- **<u>Staircases</u>**



<u>Back-of-Envelope Calculation</u>

$D^* \approx f_{active}((\Delta x)^2 / \Delta t)$;

$f_{active} \equiv$ active fraction $\sim \delta / \ell_o$

$\Delta t \sim \ell_o / v_o \rightarrow$ cell circulation time

So, $\delta^2 \sim D\,\Delta t \sim D\,\ell_o / v_o$

$D^* \sim [(D\,\ell_o / v_o)^{1/2}\, 1 / \ell_o]\,(\ell_o^2 / \ell_o)\,v_o \sim [D\,D_{cell}]^{1/2}$

$\sim D\,Pe^{1/2}$

Consider a **<u>general</u>** case of a system of eddies not overlapping but tangent → **<u>Staircase</u>**

Transport? <u>Answer</u>: $Deff \sim D\,Pe^{1/2}$ {**<u>Not a simple addition of process!</u>**}

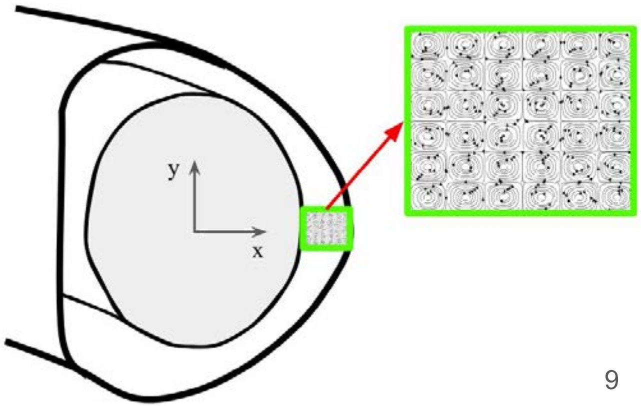
→ Two time rates: $\tau_H = d / v$ (fast), $\tau_D = d^2 / D$ (slow)

→ $Pe = v\,d / D \;>>\; 1$

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D\nabla^2 n,$$

Profile?

Consider concentration of injected dye (passive scalar transport in eddys) → profile

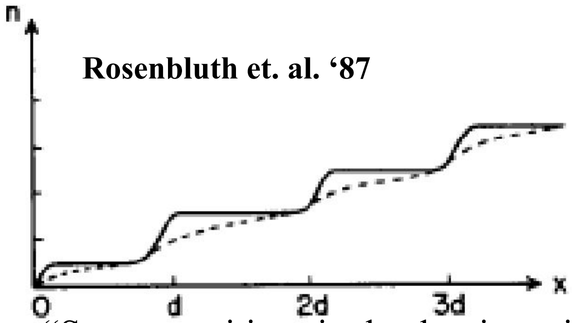


FCA → Staircase!

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D \nabla^2 n,$$

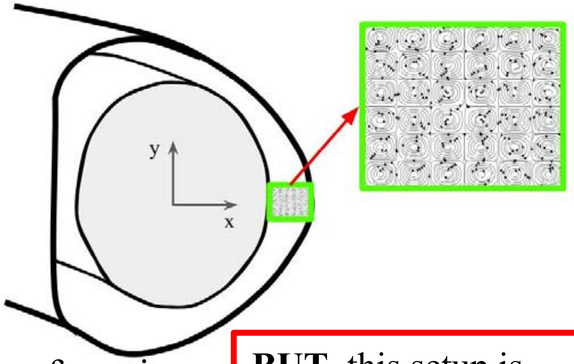
Profile?

Consider concentration of injected dye (passive scalar transport in eddys) → profile



"Steep transitions in the density exist between each cell."

→ Layering!
→ **Simple** consequence of **two rates**
→ "Rosenbluth Staircase"



Important:

- **Staircase** arises in stationary array of passive eddies (Note that there is no FEEDBACK)
- Global transport hybrid:
  - → <u>fast</u> rotation in cell
  - → <u>slow</u> diffusion in boundary layer
- Irreversibility localized to inter-cell boundary.

<u>BUT</u>, this setup is contrived, NOT self-organized!!! Cellular array is severely constrained!

Relevant to key question of "near marginal stability"

Staircase arises in an array of stationary eddies!

What about the dynamics of a **less constrained** cell array (i.e., vortex array with fluctuations) ?

Relaxing FCA with FVA

Consider a Broader Approach

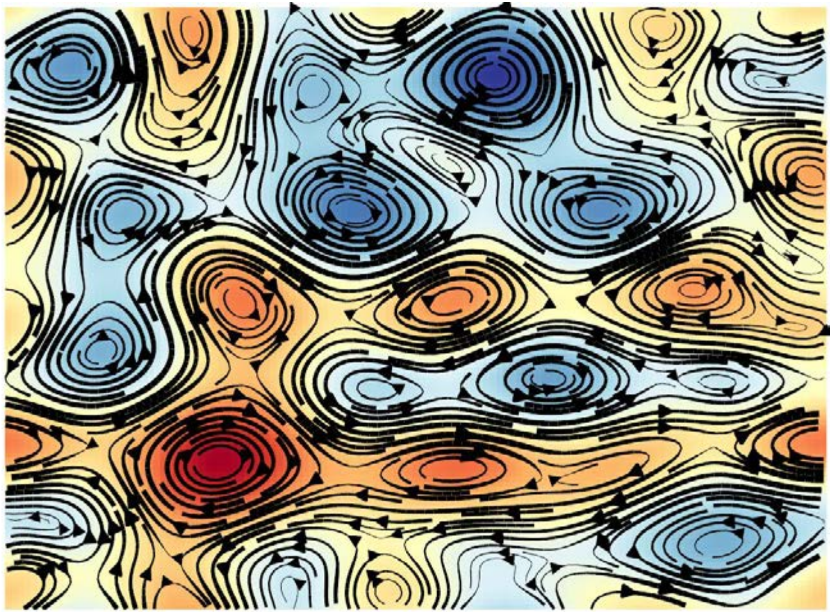
- We want to study a much more **general** and **less constrained** version of the cell array.
  - Consider a vortex array with fluctuations; jitters.
- How **resilient** is the staircase in the presence of these small variations to a fixed vortex array?

In the process of studying the **resilience** of the staircase, we aim to answer the following:

1. What happens to interspersed regions of strong scalar concentration mixing as cells relax? What about general cell interactions/behavior?
2. What is the behavior of the scalar trajectory through the VA?
3. How does the increase of scattering in the VA affect the transport of scalar concentration?

To answer these questions, we use the idea of a **Melting Vortex Crystal**…

Example of **less constrained** cell array



Fluctuating Vortex Array

Why are we doing this? We know that a system with two disparate time scales forms a staircase!

- Now consider fluctuations… → Will staircase survive?

Vortex array is an alternative way to view convection cells!

→ We begin with the 2D NS equation that can be written in nondimensional form (Perlekar and Pandit 2010),

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

$$Re = \frac{(\mathbf{u}'\cdot\nabla')\,\mathbf{u}'}{\nu\nabla'^2\mathbf{u}'} = \frac{F_{\rm amp}}{\nu^2 k^3}$$

→ The "vortex array" is simply the array of cells and "fluctuation" is related to turbulence induced variability in the structure. The fluctuating vortex array (FVA) allows us to study a **less constrained** version of the array! **Improved model of cells near marginality.**

→ The fluctuating flow structure is created by **slowly increasing the Reynolds number** in the NS equation

$$\Omega \equiv nRe$$

→ By increasing the Reynolds number this modifies the forcing and drag term, thus, **scattering** the vortex array. The **resilience** of the staircase is studied by **increasing disorder** in the vortex crystal through F_ω

$$F_\omega \equiv -n^3\left[\cos\left(nx\right) + \cos\left(ny\right)\right]/\Omega$$

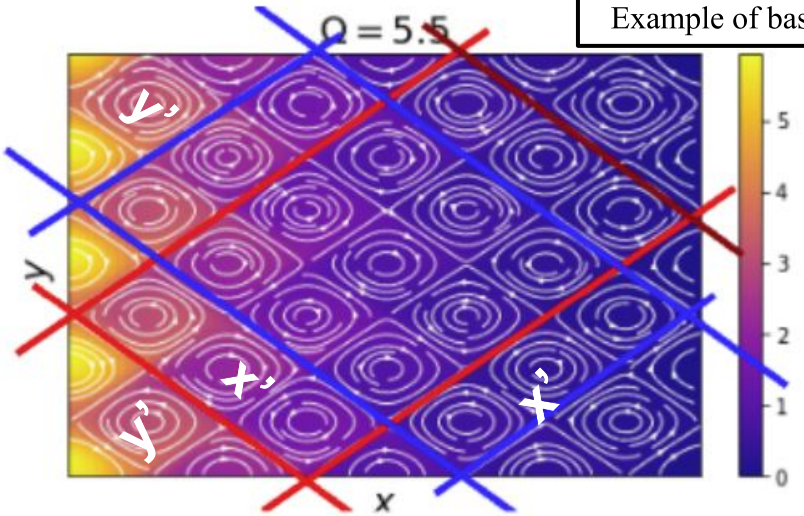
The streamfunction, ψ, at different evolutionary stages of the "fluctuating" vortex array is inserted into the passive scalar equation to study the resilience of the staircase structure.

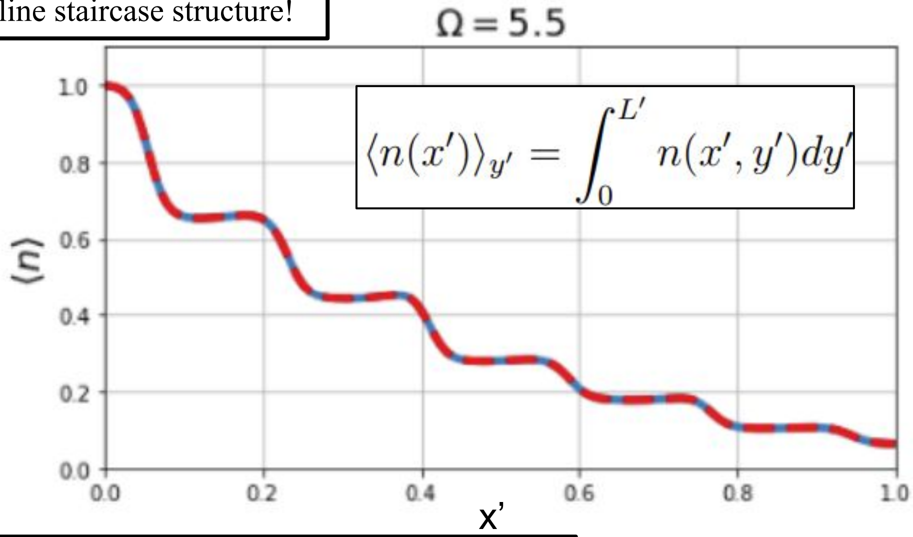
What Happens to Staircase?

	Vortex Field	Drift-Wave Turbulence (tokamak)
Inhomogeneity (free energy source)	$\boldsymbol{\nabla} n$	B_0, $\boldsymbol{\nabla} n$, and $\boldsymbol{\nabla} T$
Reynolds number (Re)	$Re = 1 - 10$	$Re = 10^1 - 10^2$ (Landau Damping)
Flux	Scalar	Turbulent Heat
Zonal Flow	Boundary layer between cells	$\mathbf{E} \times \mathbf{B}$ shear flow (poloidal)

The Staircase

Example of baseline staircase structure!



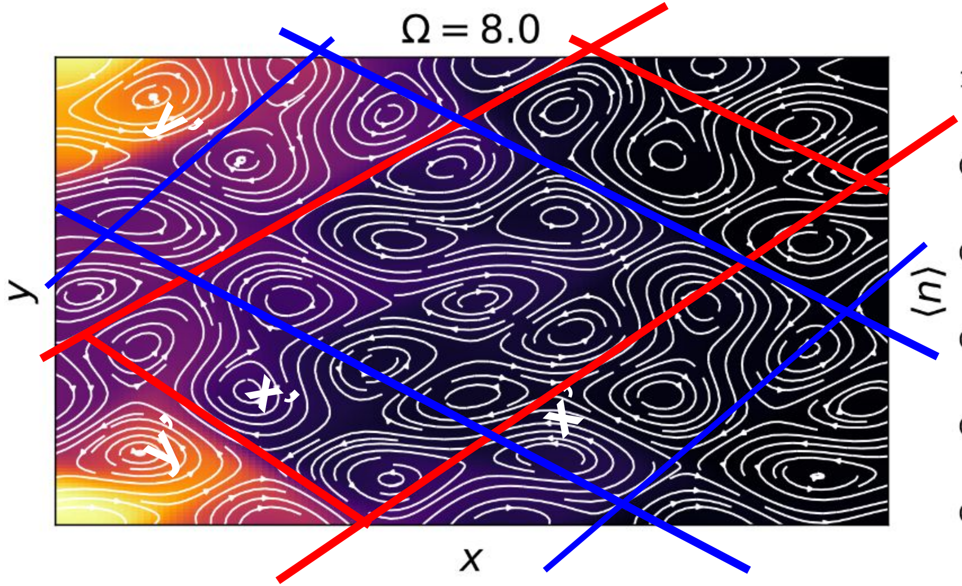


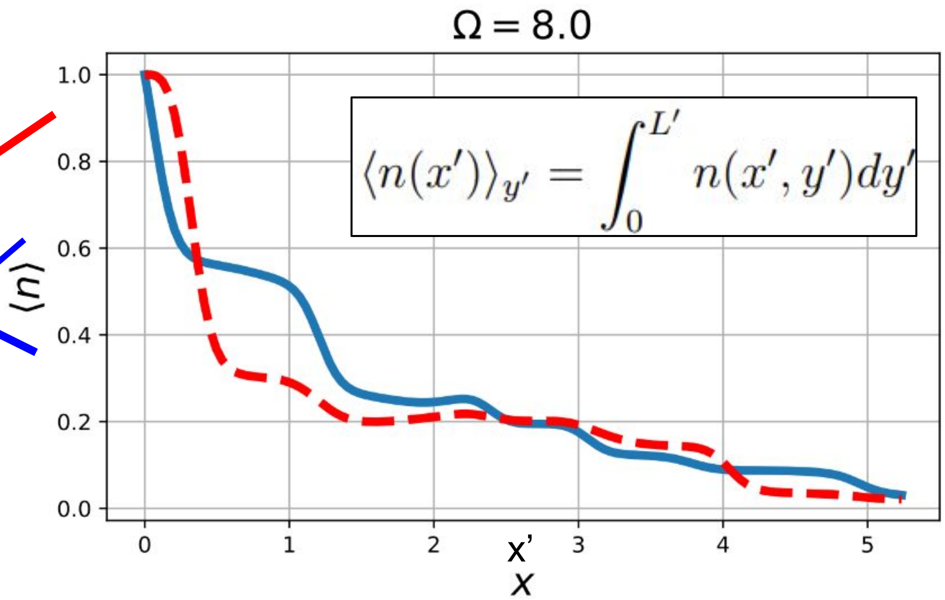
$$\langle n(x') \rangle_{y'} = \int_0^{L'} n(x', y') dy'$$

- For a weakly FVA we get a **<u>baseline staircase</u>** structure.
- On the left figure the blue and red box correspond to the blue and red plot line on the right. Note that **steps** are **evenly** spaced!
  - Both blue and red average scalar concentration have the same profile in stable stage.

So what happens to the staircase if we increase the Reynolds number in the VA?

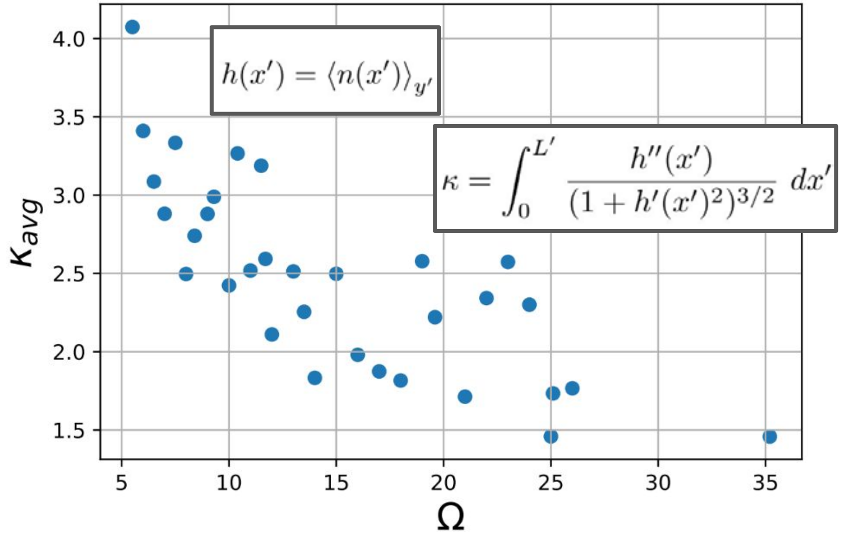
Staircase Resiliency to Fluctuations

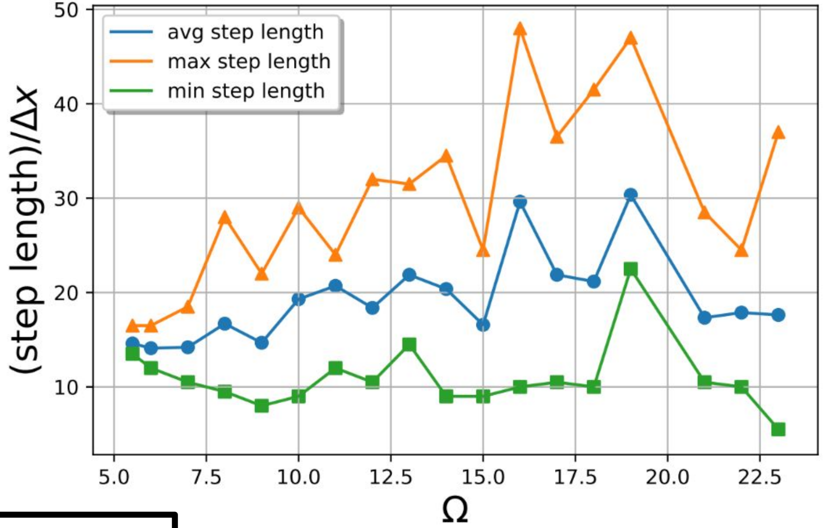




- As we **increase fluctuations in VA** through *Ω*, we can see **merger/connections** of vortex structures in the flow.
- These **vortex mergers** are shown in the scalar profile plot as **mergers in steps**.
  → As we increase jittering, staircase steps merge together.

Behaviour of Staircase as Cells Fluctuate





- To quantify the different stages of the fluctuating process, we look at the **curvature & step length** in scalar concentration.
- In general, as we **increase *Ω***, the **curvature decreases**.
  - Steps are starting to merge together as we increase *Ω*, thus scalar profile has less curvature.

Main Point: Despite that vortex array becoming more turbulent, the staircase structure does not collapse.

- Staircase steps become **less regular**. They merge into longer steps.

The Scalar Field

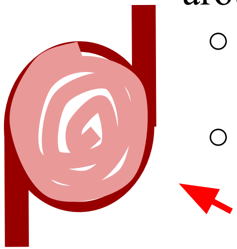
(transport in the FVA)

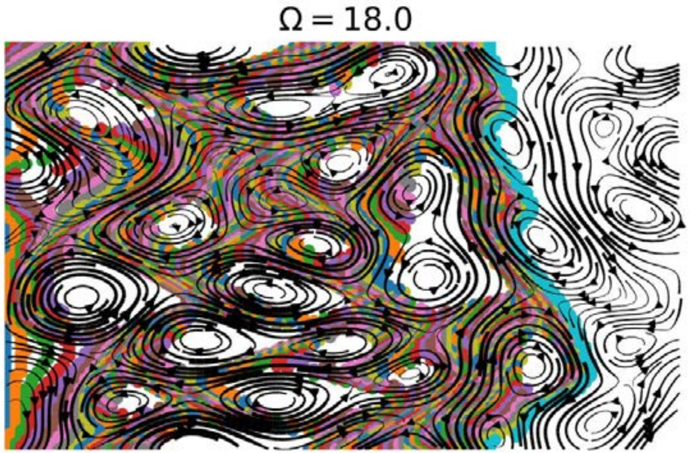
The Web

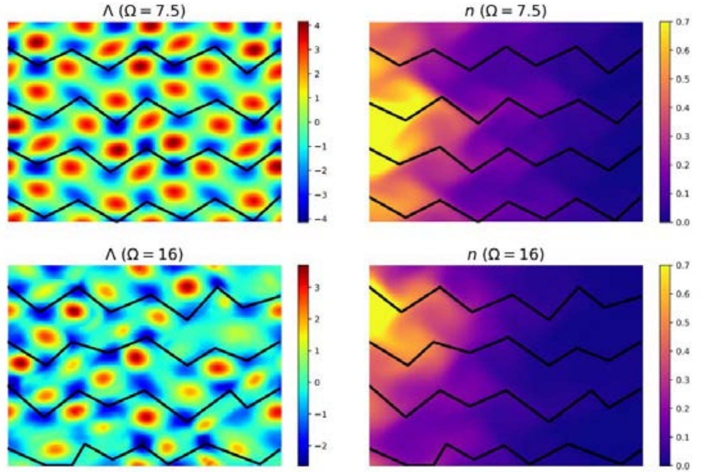
Λ = mean sq. vorticity - mean sq. shear

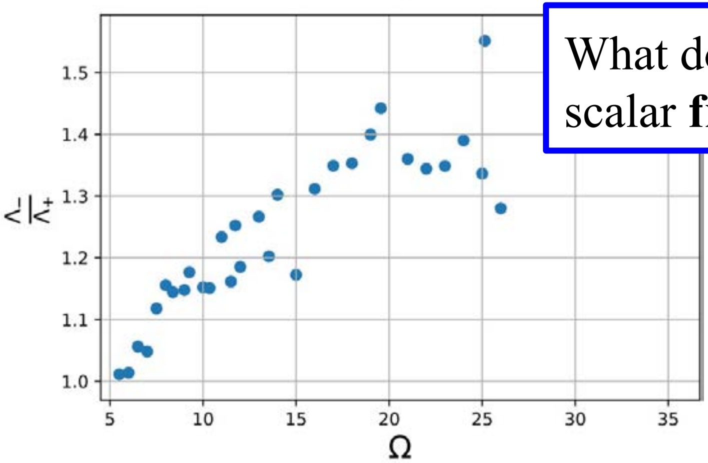
Before the **staircase** structure forms, scalar concentration field forms a "**web**":

- Scalar flows **quickly in regions of strong shear** and around vortices!
  - Staircase **barriers form first**! Scalar travels along cell boundaries.
  - Overtime, vortex **entrains** scalar by a kind of "**homogenization**" process via the synergy of differential rotation and diffusion.





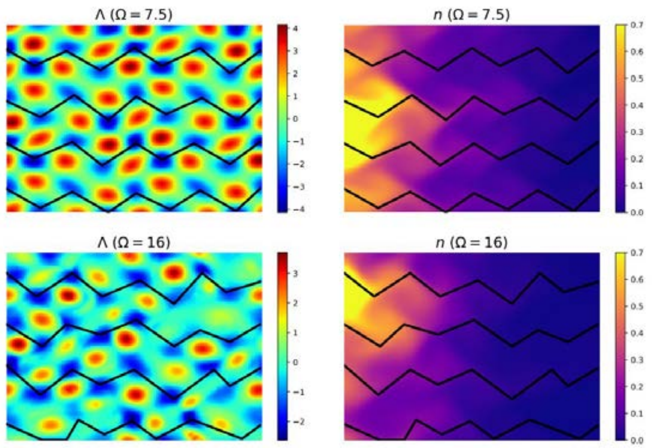




What does this mean for scalar **front propagation**?

General idea:
Imaging of turbulence in near-marginal state

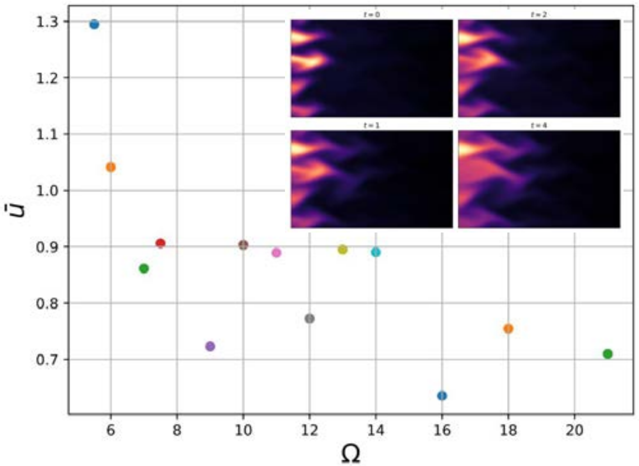
Trajectory in Scattered VA → How Avalanches Propagate



Scalar concentration **travels fast along** areas of **strong shear** ($\Lambda<0$)

- Using Okubo-Weiss field, we can **connect** regions of strong shear to their **nearest strong shear neighbor**.
- Path can be **mapped** to scalar concentration contour to show that indeed scalar travels along areas of strong shear.
  - Distance travel can be quantified.

Idea relevant here is the **<u>least time criterion</u>**. As the **vortex array fluctuates**, the **path of least time would increase in length**.



In addition to distance travel, we also **quantify the time** scalar takes to travel from one end to the other using a **<u>pulse train</u>**.

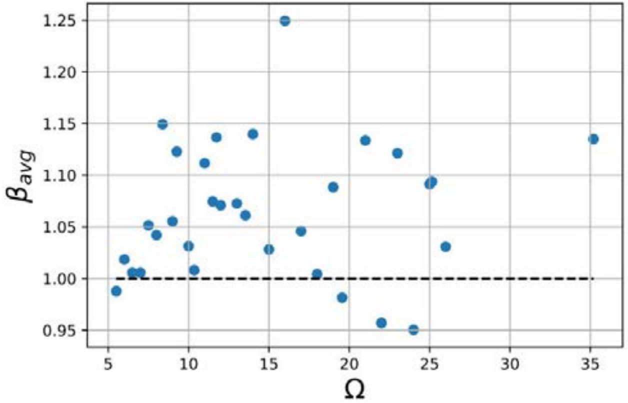
- As the scalar concentration gets injected into the flow, a **<u>flamelet</u> <u>network pattern</u>** forms (Pocheau 2008).
  - **Fingers propagate through array. Over time, the scalar slowly enters the vortex structures.**

The **scattering of vortices** leads to an overall **decrease** in scalar concentration **velocity**! Agrees with least time criterion (similar idea to scattered path of light in atmosphere).

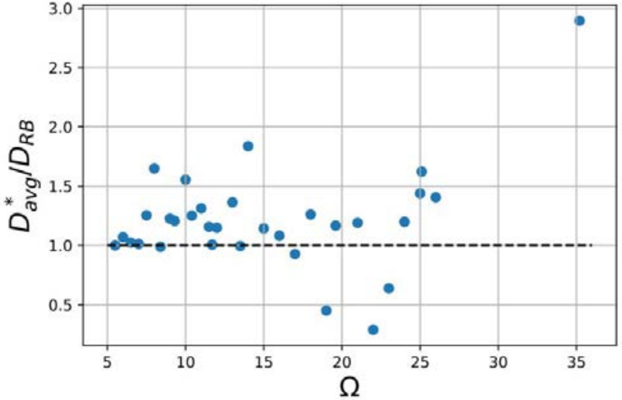
- **Staircase curvature** and **scalar velocity** are **proportional**.

Transport in FVA

$$D^* \propto D\sqrt{Pe}$$



$$D^* \propto \sqrt{d_x \beta}$$



As cells fluctuate, the **effective diffusivity** deviates but **remains close** to the Rosenbluth effective diffusivity.

- **<u>Note:</u>** we fix flow velocity and background diffusivity.
  - Only **dimensions** of cells **affect transport**.

This **suggests** that the Rosenbluth effective diffusivity is a **good approximation** even if **<u>cells are irregular</u>**!

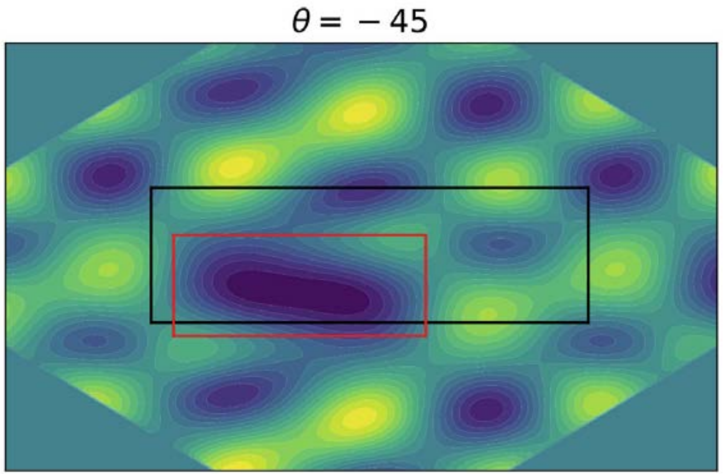
We find that as long as the **boundaries** and **speed** of the cells are **maintained**, the effective diffusivity and transport **does not change**.

- Since effective diffusivity is **proportional** to $\beta = d_x/d_y$, only through **geometric properties** of the cells does **<u>transport change</u>**!

Effective diffusivity **increases/decreases** if the cells length along the gradient (d_x) increases/decreases compared to the length perpendicular to the gradient (d_y).

Transport in FVA

$$D^* \propto D\sqrt{Pe}$$



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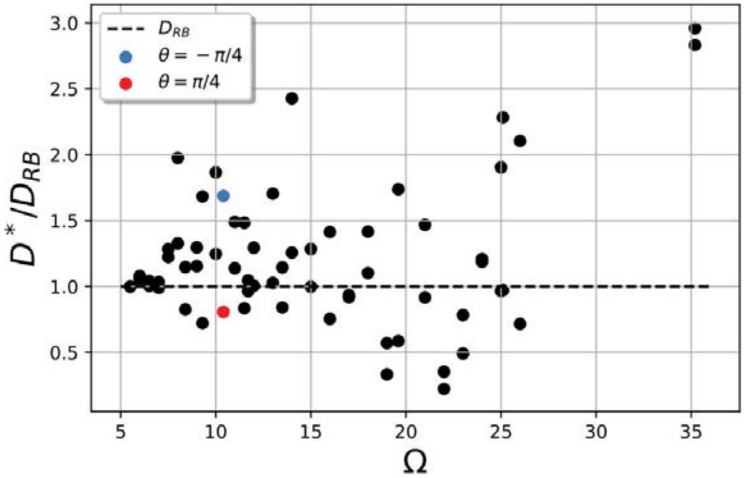


Effective diffusivity **increases/decreases** if the cells length along the gradient (d_x) increases/decreases compared to the length perpendicular to the gradient (d_y).

- Cells on average remain around $\beta \sim 1$, but there are cells that are larger in size due to cell mergers which cause the deviation of the effective diffusivity.

Summary

- Staircase form and are **resilient** and **persistent** to increasing Reynolds number (i.e., fluctuating vortex array).

- Scalar concentration **travels along** regions of **strong shear** creating a "**web**" structure.
    - IMPORTANT: Staircase barriers form first! Vortex "homogenizes" scalar at a later time!

- The scattering of vortices leads to an overall decrease in scalar concentration velocity.
    - Agrees with **least time criterion**.

- If flow velocity and background diffusion are kept fixed, only **cell geometric properties** affect the effective diffusivity! ($D^* \propto D\, Pe^{1/2}$)
    - Effective diffusivity of the perturbed VA **does not deviate** significantly!

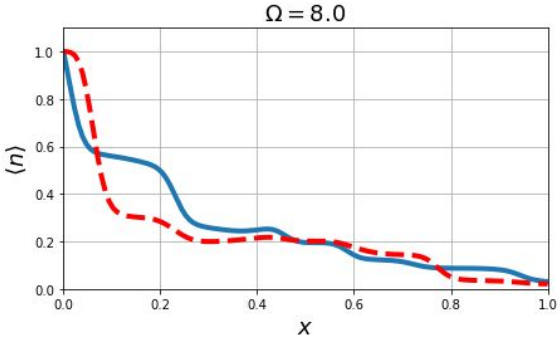
Why would a fusion experimentalist care about this?

These results have interesting implications for experiment and theory:

1. Effective diffusivity derived by Rosenbluth *et al* (for fixed cellular array) is a suitable approximation for the fluctuating cellular array (**not simple addition**: $D^* = D_0 + D_{cell}$).
    - Relevant to cells touching (similar to what we find near-marginal stability).

2. Staircase structure is resilient in the regime of low-modest Reynolds numbers (this regime is relevant to drift-wave turbulence).
    - Structures/Profiles are not exotic.
        - Staircase profile structure does not require special tuning.

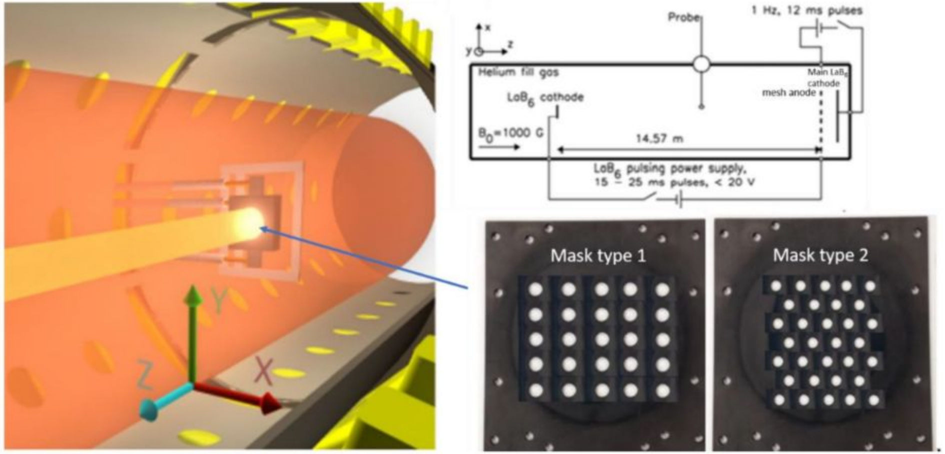
3. Geometry of streamlines is important. If more saddles than close vortices, Heat avalanches will first form the staircase barrier.
    - Fluctuating cellular flow hinders avalanche propagation.

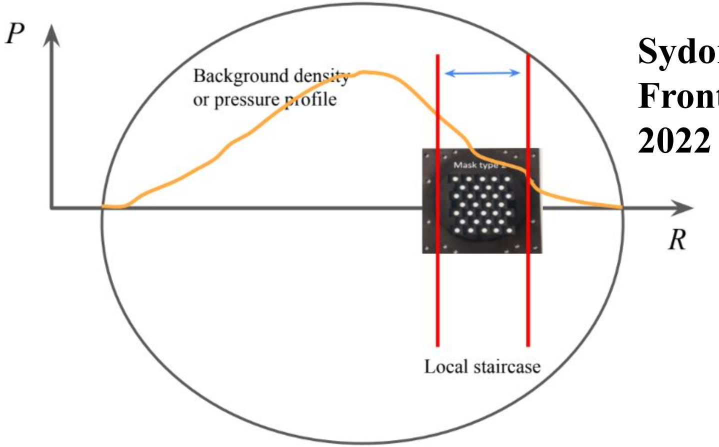


IMPORTANT: We can test the theory presented here with actual experimental data.

LAPD Experiment

Experiment happening July 2023!





Sydora, Frontiers Proposal 2022

A vortex array can be created in the large linear magnetized plasma device (LAPD)

- Modification of a cathode plasma source with designer masks that form multiple current channels in a cellular pattern → form staircase!
    - Experiment will be conducted in the afterglow phase of the main discharge.
- Staircase structure can be subject to controllable amount of of low frequency density fluctuations, which act as a noise source.
    - Allow us to test hypotheses and models of staircase resiliency!

Results of experiment will yield a unique set of observations that can be used to test staircase models.

Thank you!