PPCF 2022 (Special issue on L-H transition) in press

AAPPS-DPP2022 on-line e-conference
Oct. 9 - 14, 2022

Theory of mean E × B shear in a stochastic magnetic field: ambipolarity breaking and radial current

Weixin Guo[1*] , Min Jiang[2*], P. H. Diamond[2, 3] , C.-C. Chen[3]

M. Y. Cao[3], Hanhui Li[1], Ting Long[2]

[1] HUST, Wuhan, China
[2] SWIP, Chengdu, China
[3] UCSD, San Diego, USA

Jiang* and Guo* are joint first authors.

Acknowledgement: 10th festival in Aix (work was initiated) 2019

Xavier Garbet, Lu Wang, Lothar Schmitz, Minjun Choi, George Mckee, Zheng Yan, T S Hahm and Yunfeng Liang

Outline

- ☐ Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues (L→H transition with RMP, island, stellarator, etc)
- ☐ Mean field model $\langle E_r \rangle$—follow radial force balance
  - Key fundamentals:
    - — *Break ambipolarity*
    - — *Mean radial current density*
  - Turbulent transport
    - — *Momentum transport (poloidal and toroidal rotation)*
    - — *Particle transport*
    - — *Ion heat transport*
- ☐ Applications
  - L-H transition with $\tilde{b}^2$, 0D at present
- ☐ Implications and future work

Outline

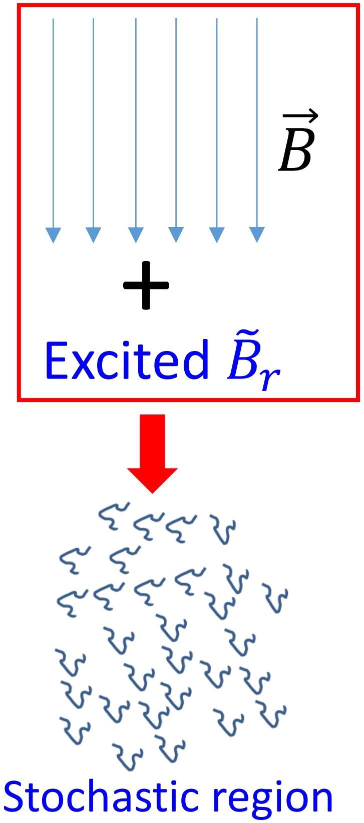
- ☐ Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues (L→H transition with RMP, island, stellarator, etc)
- ☐ Mean field model $\langle E_r \rangle$—follow radial force balance
  - Key fundamentals:
    - — *Break ambipolarity*
    - — *Mean radial current density*
  - Turbulent transport
    - — *Momentum transport (poloidal and toroidal rotation)*
    - — *Particle transport*
    - — *Ion heat transport*
- ☐ Applications
  - L-H transition with $\tilde{b}^2$, 0D at present
- ☐ Implications and future work

Motivation

- 40 years ago, H-mode with enhanced confinement makes great sense! While, attention has shifted to control it for now (←ELM)
  - ✓ need to reconcile good power handling with good confinement
- Resonant magnetic perturbations (RMPs) suppress or mitigate ELM, and induces stochastic layer
  - ✓ Stochastic occurs when separation of magnetic field lines grows exponentially
  - ✓ good working criterion for stochasticity: magnetic island overlapping
  - ✓ degree of stochastization: auto-correlation length to the scattering length l_{ac}/l_c, where

$$l_{ac} = 1/\left|\Delta k_\parallel\right|,\ \ l_c = \left(\frac{k_\theta^2 D_M}{3L_s^2}\right)^{-1/3}$$

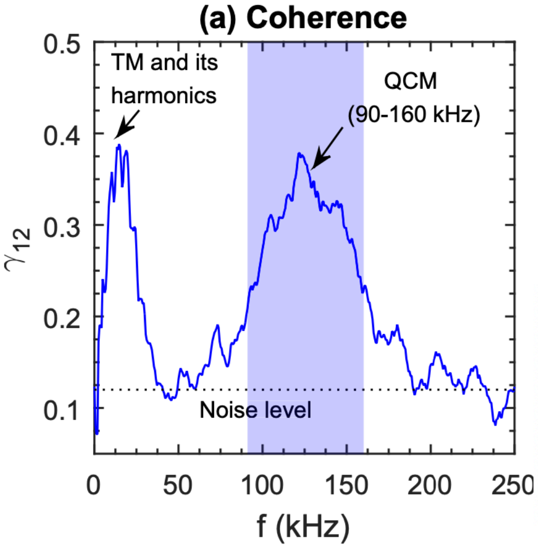
with stochastic magnetic diffusivity $D_M = \sum_{\boldsymbol{k}} \left|\tilde{b}_{r,\boldsymbol{k}}^2\right| \pi\delta(k_\parallel)$



Operation with RMP encounter challenge : L → H transition with a pre-existing, thin stochastic layer at the boundary

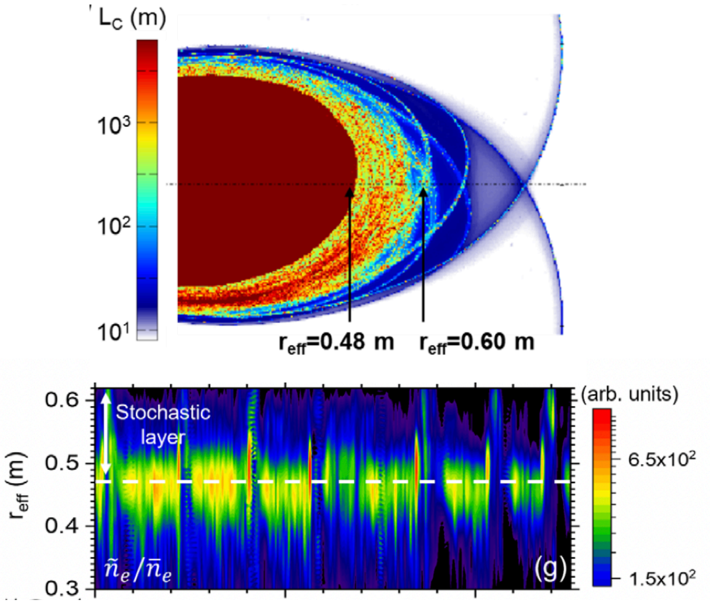
- **Interestingly, interaction and co-existence** of stochastic magnetic field and turbulence

Modulation of QCM (i.e., TEM) by 2/1 TM @ HL-2A



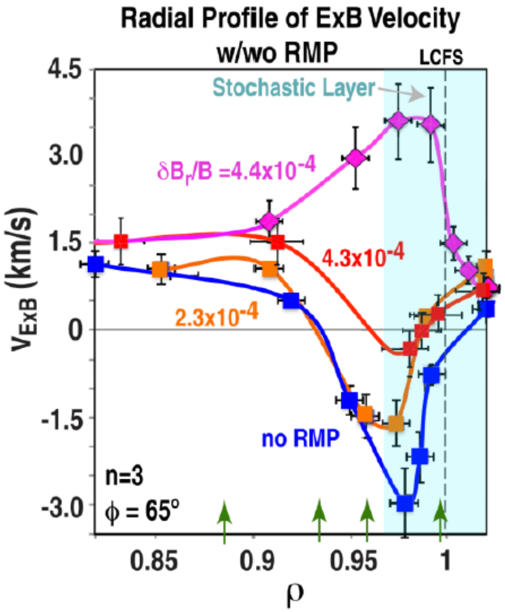
Min Jiang, NF 2020

Turbulence in stochastic region @LHD



Kobayashi, PRL 2022

L→H occurs precisely in stochastic layer, outboard midplane separatrix @DIII-D

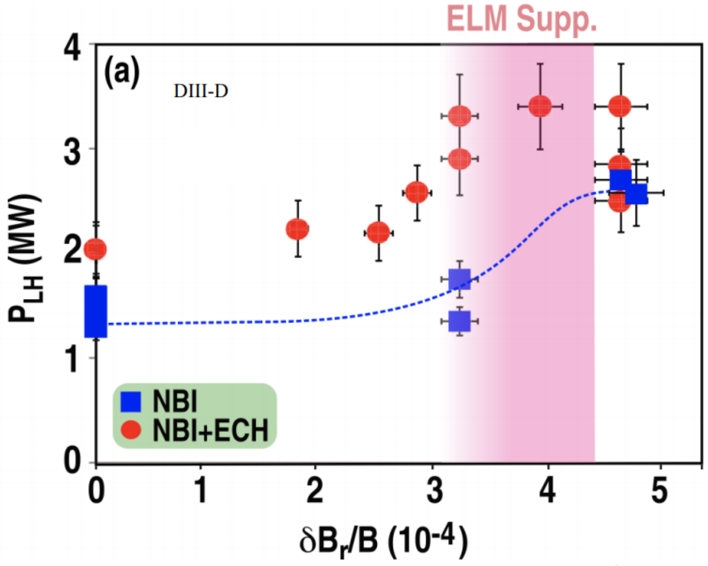


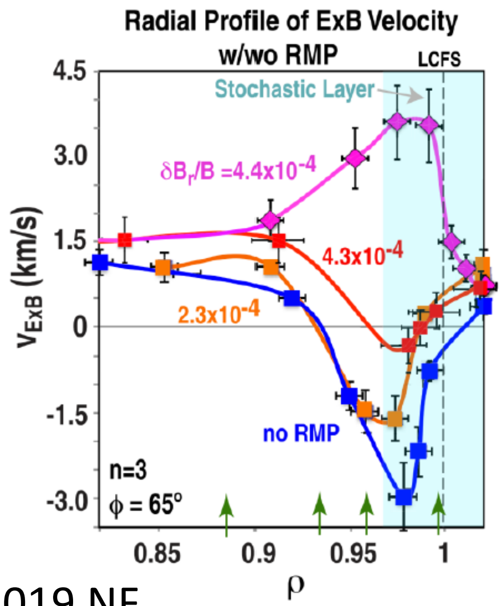
L. Schmitz et al, NF 2019

- ✓ Doubt?! But indeed localized precisely where the L-H transition is trigged
- ✓ Edge turbulence and flows evolve in stochastic layer → physics of L-H transition conflated with RMP pump-out and Reynolds stress decoherence

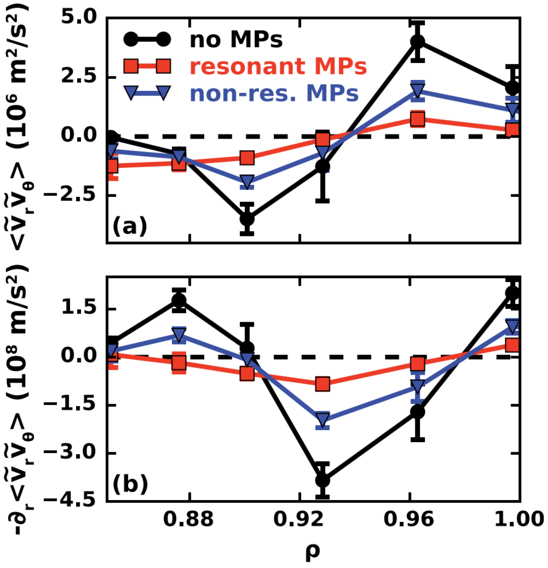
Interactions among the plasma profiles, flows and turbulence preceding the L-H transition with RMP application are not clear

- Increase in the power threshold added new challenges to the understanding of the L → H transition
- An explanation: edge stochasticity and a resonant electromagnetic torque → reduction of the E × B flow shear → increase in the turbulent transport and a reduction of the Reynold stress driven poloidal flow





Schmitz 2019 NF



Kriete 2020 PoP

It is of prime importance to understand the physics of E × B shear layer structure in a stochastic magnetic field

Our goal – understand effects of stochastic field on $\langle E_r \rangle$ and $\langle v_E' \rangle$

- In this work, we present a mean field theory for E × B shear in an ambient stochastic layer

- Novel approach: revisit mean radial electric field (radial force balance Eq.)

$$\underbrace{\langle E_r \rangle}_{\langle v_E' \rangle} = \underbrace{\frac{\langle \nabla P_i \rangle}{e n_i}}_{\text{Heat, particles}} - \underbrace{\langle V_\theta \rangle B_\phi + \langle V_\phi \rangle B_\theta}_{\perp, \parallel \text{ flows} \to \text{momentum}} .$$

✓ Study turbulence, particle, momentum and heat transport to ascertain change of $\langle E_r \rangle$ due to stochastic B field.

✓ Goal is towards $\langle \boldsymbol{J_r} \rangle$ ↔️ $\langle \boldsymbol{E_r} \rangle$ relation— effective “Ohm’s law”

☐ Stochastic B-field, externally excited but self-consistent within plasma (Ampere’s law), enters $\langle \boldsymbol{J_r} \rangle$

☐ $\langle V_\theta \rangle$, $\langle V_\phi \rangle$, $\nabla \langle P_i \rangle$ and $\langle n_i \rangle$ are all are modified by the mean radial current density $\langle J_r \rangle$ induced by $\langle \tilde{J}_\parallel \tilde{b}_r \rangle$

Outline

- ☐ Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues (L→H transition with RMP, island, stellarator, etc)
- ☐ Mean field model $\langle E_r \rangle$—follow radial force balance
  - Key fundamentals:
    - — *Break ambipolarity*
    - — *Mean radial current density*
  - Turbulent transport
    - — *Particle transport*
    - — *Momentum transport (poloidal and toroidal)*
    - — *Ion heat transport*
- ☐ Applications
  - L-H transition with $\tilde{b}^2$, 0D at present
- ☐ Implications and future work

Ambipolarity breaking $\Rightarrow \langle J_r \rangle$

- Most general expression of the net mean radial current density $\langle J_r \rangle$ from $\nabla \cdot \mathbf{J} = 0$

$$\frac{\partial \langle J_r \rangle}{\partial r} = \frac{\partial \langle \tilde{V}_r \tilde{\rho}_{pol} \rangle}{\partial r} + \frac{\partial \langle \tilde{b}_r \tilde{J}_\parallel \rangle}{\partial r}$$

with evident counterpart

$$\langle J_r \rangle = \langle \tilde{V}_r \tilde{\rho}_{pol} \rangle + \langle \tilde{b}_r \tilde{J}_\parallel \rangle$$

Both can break ambipolarity, thus influence $\langle E_r \rangle$

✓ flux of polarization charge preferentially transports ions due to greater ion inertia:

$\tilde{\rho}_{pol}$ → fluctuation vorticity $\nabla_\perp^2 \tilde{\phi}$ → vorticity flux, i.e., Reynolds stress

✓ net flow along radially tilted field lines preferentially transports electrons:

How this relates to? Maxwell stress !!!

Ambipolarity breaking $\Rightarrow \langle J_r \rangle$

- Ambipolarity breaking due to stochastic field $\Rightarrow \langle J_r \rangle$

projection

$$\langle J_r \rangle = \langle \vec{J}_\parallel \cdot \vec{e}_r \rangle = \frac{\langle \tilde{J}_\parallel \tilde{B}_r \rangle}{B} \qquad \langle J_\parallel \rangle = \langle J_{\parallel,e} \rangle + \langle J_{\parallel,i} \rangle$$

- From Ampere law: $\tilde{J}_\parallel = -\frac{c}{4\pi} \nabla^2 \tilde{A}_\parallel$

(Self- consistency)

Stochastic field produces currents in plasmas

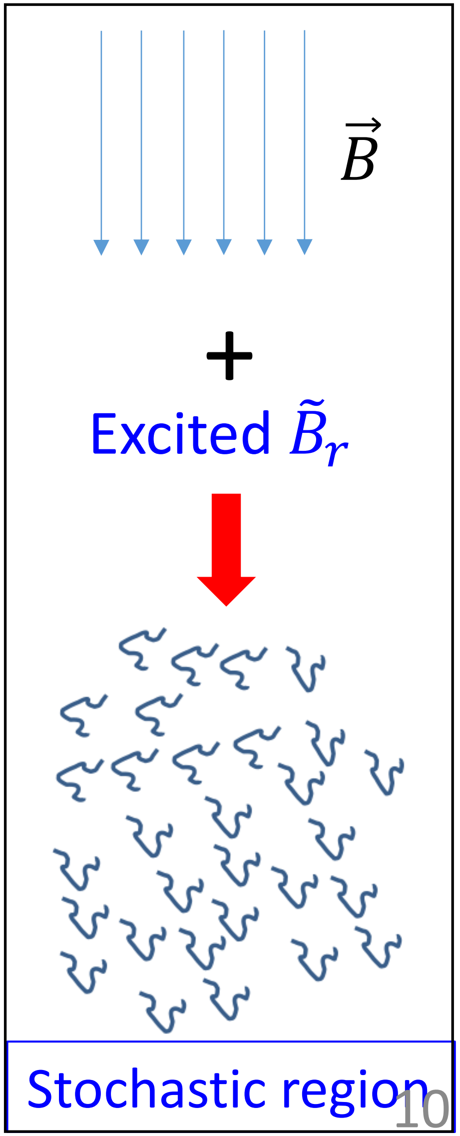
$$\langle J_r \rangle = \frac{\langle \tilde{J}_\parallel \tilde{B}_r \rangle}{B} = -\frac{c}{4\pi B} \left\langle \frac{\partial}{\partial y} \tilde{A}_\parallel \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \tilde{A}_\parallel \right\rangle$$

$$= -\frac{c}{4\pi B} \frac{\partial}{\partial x} \left\langle \left(\frac{\partial}{\partial x} \tilde{A}_\parallel \right) \left(\frac{\partial}{\partial y} \tilde{A}_\parallel \right) \right\rangle = \frac{c}{4\pi B} \frac{\partial}{\partial x} \langle \tilde{B}_x \tilde{B}_y \rangle$$

$$= \frac{cB}{4\pi} \frac{\partial}{\partial x} \langle \tilde{b}_x \tilde{b}_y \rangle \Rightarrow \frac{cB_0}{4\pi} \frac{\partial}{\partial r} \boxed{\langle \tilde{b}_r \tilde{b}_\theta \rangle}$$

Maxwell stress!!!

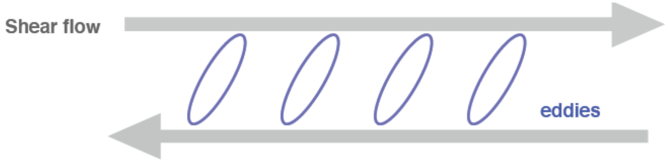
Note: (1) Stochasticity excited externally (RMP) but Ampere's law must be satisfied in plasma.
(2) Further progress is determined by cross phase



Cross Phase in Maxwell stress

- $$\boxed{\begin{array}{c}\dfrac{\partial A}{\partial t}+V\cdot\nabla A=\mu\,\boldsymbol{J}\\ \dfrac{\partial A}{\partial t}+\underset{\substack{\uparrow\\ \text{Shear flow}}}{V_E'}\dfrac{\partial A}{\partial y}+\tilde{V}\cdot\underset{\substack{\uparrow\\ \text{Fluctuation scattering}}}{\nabla} A=\mu\,\boldsymbol{J}\end{array}}$$

fluctuating magnetic potential $\tilde{A}_k$ tilted by developing $\boldsymbol{E}\times\boldsymbol{B}$ flow, scattered by fluctuation.



- Maxwell stress: $\langle\tilde{b}_r\tilde{b}_\theta\rangle=\dfrac{1}{B^2}\sum_k|\tilde{A}_k|^2\langle k_r k_\theta\rangle$

✓ phase set by $\langle k_r k_\theta\rangle$

- Evolution of radial wave number k_r:

$$\frac{dk_r}{dt}=-\frac{\partial}{\partial x}(\omega+k_\theta\langle V_E\rangle)\rightarrow k_r=k_r^0-k_\theta V_E'\tau_c$$

→ →

$$\boxed{\langle\tilde{b}_r\tilde{b}_\theta\rangle=\langle\tilde{b}_r\tilde{b}_\theta\rangle_0+\frac{1}{B_0^2}\langle V_E\rangle'\sum_{\boldsymbol{k}}\left(|\tilde{A}_k|^2k_\theta^2\tau_c\right)}$$

✓ coherence time of MP

$$\tau_c=\left(\frac{k_\theta^2V_E'^2D_T}{3}\right)^{-1/3}$$

✓ $\langle\tilde{b}_r\tilde{b}_\theta\rangle_0$ defined by k_r^0, differs dramatically from often-discussed test particle approach → Discuss the detail later

✓ $\boldsymbol{E}\times\boldsymbol{B}$ shear aligns phases, regardless of mechanism.

✓ Tilting will tend to align turbulent RS and stochastic Maxwell stress.

Expression of $\langle J_r \rangle$ and its implication

- Ultimately $\langle J_r \rangle = \frac{cB_0}{4\pi}\frac{\partial}{\partial r}\left[\langle \tilde{b}_r \tilde{b}_\theta \rangle_0 - \langle V_E \rangle' \sum_{\boldsymbol{k}} \left(\left| \tilde{b}^2_{r,\boldsymbol{k}} \right| \tau_{c,b} \right) \right]$

Notable features (1):

✓ No explicit dependence on electron inertia, $\mathbf{k} \cdot \mathbf{B_0}$ resonance … of test particle transport by magnetic stochasticity. Still successful

[Payan, NF 1995
Ashourvan, NF 2022]

✓ Our analysis → a more solid fundamental foundation

— uses exact Ampere's law to relate $\tilde{J}_\parallel$ to stochastic fields $\tilde{b}_r$,

— entails no linearization, and not require $\langle J_r \rangle = 0$

✓ How resolve discrepancy? → k_r^0 is key!!

Recall $\langle J_r \rangle \sim |\tilde{A}_k|^2 \, \langle k_\theta k_r \rangle$ with
$k_r = k_r^0 - k_\theta \langle V_E \rangle' \tau_c$

from Ampere's law: $-(k_r^2 + k_\theta^2)\tilde{A}_k = \frac{4\pi}{c}\int n_0 |e| v_\parallel \tilde{f}_k d^3 v$

for $k_r^2 > k_\theta^2$, linear response for $\tilde{f}_k$ to $\tilde{A}_k$ → $k_r^2 = -\frac{4\pi n_0 |e|}{c}\int v_\parallel \left(-i\frac{k_\theta}{k}\right)\frac{\delta f_k}{\delta A_k} d^3 v$

Here, $\frac{\delta f_k}{\delta A_k}$ relates $\tilde{f}_k$, and thus $\tilde{J}_\parallel$, to $\tilde{A}_k$ by the linear response.

— $k_r^0 k_\theta$ capture stochastic field effects on electrons

— $k_\theta^2 \langle V_E \rangle' \tau_c$ is due mainly to ion effects.

- Detailed analysis is left for future

Expression of $\langle J_r \rangle$ and its implication

- Ultimately $\langle J_r \rangle = \frac{cB_0}{4\pi} \frac{\partial}{\partial r} \left[\langle \tilde{b}_r \tilde{b}_\theta \rangle_0 - \langle V_E \rangle' \sum_{\boldsymbol{k}} \left(\left| \tilde{b}_{r,\boldsymbol{k}}^2 \right| \tau_{c,b} \right) \right]$

Notable features (2):

✓ $\langle J_r \rangle \sim \frac{\partial}{\partial r} \left(\left| \tilde{b}_r^2 \right| \right)$ → Dependence on magnetic fluctuation envelope structure

✓ Thus, introduce novel scale,the radial envelope or spectral scale,

$\ell_{env}^{-1} = \left| \tilde{b}_r^2 \right|^{-1} \frac{\partial}{\partial r} \left(\left| \tilde{b}_r^2 \right| \right)$ i.e., characteristic scale of the fluctuation intensity profile

— ℓ_{env} is quite small, only a few centimeters in radial extension

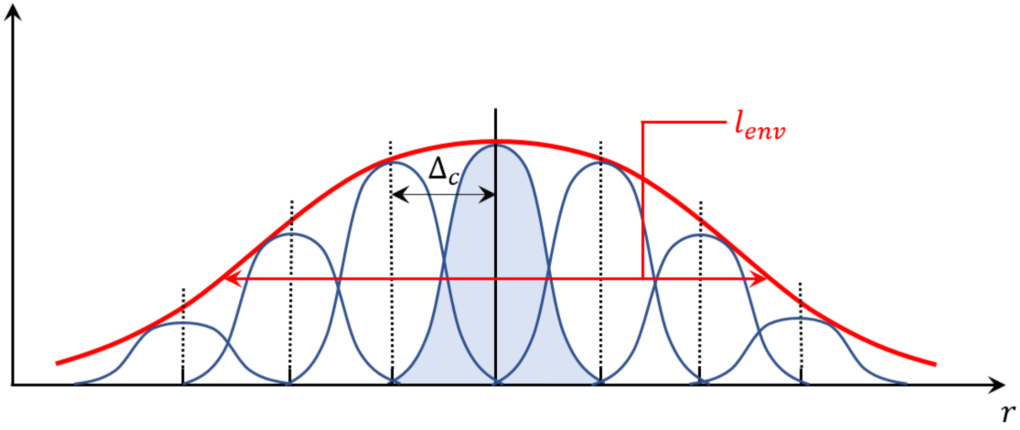
— rather modest $\left| \tilde{b}_r^2 \right|$ can produce significant effects near separatrix

— $\langle J_r \rangle$ is not necessarily “small”, even in the absence of electron inertia

Notable features (3):

✓ multi-scale character

Δ_c (mode width) is set by drift wave propagation



Expression of mean radial electron current density $\langle J_{r,e}\rangle$

- Analysis of the L→H transition, requires $\langle J_{r,e}\rangle = \langle J_r\rangle - n_0|e|\langle \tilde{b}_r \tilde{V}_{\parallel,i}\rangle$
- Net ion flow along the tilted lines $\langle \tilde{b}_r \tilde{V}_{\parallel,i}\rangle$ by [Chen C.C et al PPCF2021]

— weak turbulence ($\omega < k_\parallel c_s$)

$$\langle \tilde{b}_r \tilde{V}_{\parallel,i}\rangle \cong -D_M \frac{\partial \langle V_{\parallel,i}\rangle}{\partial r}$$

with $D_M = \sum_{\boldsymbol{k}} \left|\tilde{b}^2_{r,\boldsymbol{k}}\right| \pi\delta(k_\parallel)$

$$\langle J_{r,e}\rangle = \frac{cB_0}{4\pi|e|}\frac{\partial}{\partial r}\sum_{\boldsymbol{k}}\left[\langle \tilde{b}_r \tilde{b}_\theta\rangle_0 - \left|\tilde{b}^2_{r,\boldsymbol{k}}\right|\langle V_E\rangle' \tau_{c,b}\right] + n_0|e|\, D_M \frac{\partial \langle V_{\parallel,i}\rangle}{\partial r}$$

— strong turbulence ($\omega > k_\parallel c_s$)

$$\langle \tilde{b}_r \tilde{V}_{\parallel,i}\rangle \cong -\frac{1}{\rho c_s^2} D_{st} \frac{\partial \langle P_i\rangle}{\partial r}$$

with $D_{st} = c_s^2 \left|\tilde{b}^2_{r,\boldsymbol{k}}\right| \tau_c$

$$\langle J_{r,e}\rangle = \frac{cB_0}{4\pi|e|}\frac{\partial}{\partial r}\sum_{\boldsymbol{k}}\left[\langle \tilde{b}_r \tilde{b}_\theta\rangle_0 - \left|\tilde{b}^2_{r,\boldsymbol{k}}\right|\langle V_E\rangle' \tau_{c,b}\right] + n_0|e|\frac{D_{st}}{\rho c_s^2} \frac{\partial \langle P_i\rangle}{\partial r}$$

✓ $\langle J_{r,e}\rangle$ cannot be written as the divergence of a flux

✓ $\langle J_{r,e}\rangle$ is proportional to $\left|\tilde{b}^2_{r,k}\right|$, driven by magnetic fluctuation intensity gradient as well as by $\frac{\partial \langle V_{\parallel,i}\rangle}{\partial r}$ and $\frac{\partial \langle P_i\rangle}{\partial r}$

Flow 1: Stochastic B-field affects $\langle V_\theta \rangle$

- Poloidal momentum balance

Turbulence Reynold stress

Maxwell stress of stochastic field perturbation

$$\frac{\partial \langle V_\theta \rangle}{\partial t} = -\mu(\langle V_\theta \rangle - V_{\theta,neo}) - \frac{\partial}{\partial r}\left(\langle \tilde{V}_\theta \tilde{V}_r \rangle - \frac{1}{4\pi\rho} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right)$$

$B_\phi \langle J_r \rangle$

- For SS: $\langle V_\theta \rangle = \langle V_\theta \rangle_{neo} + \Delta V_\theta$

$$\Delta V_\theta = -\frac{1}{\mu}\frac{\partial}{\partial r}\left[\langle \tilde{V}_\theta \tilde{V}_r \rangle - V_A^2 \langle \tilde{b}_r \tilde{b}_\theta \rangle \right] \qquad \text{Competition!!!}$$

$$= \frac{1}{\mu}\frac{\partial}{\partial r} \langle V_E \rangle' \left[\sum_{\boldsymbol{k}} \left| \tilde{V}_{r,\boldsymbol{k}} \right|^2 \tau_{c,\phi} - V_A^2 \left| \widetilde{\boldsymbol{b}}^2_{r,\boldsymbol{k}} \right| \tau_{c,b} \right]$$

☐ V_E' phasing via tilt tends to align turbulence and stochastic B-field, which counteracts the spin-up of $\langle V_\theta \rangle$.

☐ $\frac{\partial}{\partial r} |\tilde{b}_r|^2$, i.e., profile of stochastic enters → introduce stochastic layer width as novel scale

Flow 2: Stochastic B-field affects $\langle V_\phi \rangle$

- For V_ϕ: $\frac{\partial \langle V_\phi \rangle}{\partial t} + \nabla \cdot \langle \tilde{V}_r \tilde{V}_\phi \rangle = \frac{1}{\rho c} \langle J_r \rangle B_\theta + S_\phi$

$$\langle \tilde{V}_r \tilde{V}_\phi \rangle = -\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle, \quad \chi_\phi = \chi_T = \frac{\rho_s^2 C_s}{L_T}, \quad S_\phi = S_a \exp(-\frac{r^2}{2L_{M,dep}^2})$$

Only consider diffusive term.

$B_\theta \langle J_r \rangle$

$$\Rightarrow \frac{\partial \langle V_\phi \rangle}{\partial t} = \frac{\partial}{\partial r}\left(\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle\right) + \frac{1}{4\pi\rho} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \tilde{B}_r \tilde{B}_\theta \rangle + S_\phi$$

- For SS: $\frac{\partial}{\partial r}\left(\chi_\phi \frac{\partial}{\partial r} \langle V_\phi \rangle\right) = -\frac{V_{Ti}^2}{\beta} \frac{B_\theta}{B} \frac{\partial}{\partial r} \langle \tilde{b}_r \tilde{b}_\theta \rangle - S_\phi$

Stochasticity affects edge toroidal velocity, shear

$$\frac{\partial}{\partial r} \langle V_\phi \rangle |_{r_{sep}} = -\frac{1}{\chi_\phi}\left[\int_0^{r_{sep}} S_\phi dr + \frac{V_{Ti}^2}{\beta \chi_\phi} \frac{B_\theta}{B} \langle \tilde{b}_r \tilde{b}_\theta \rangle |_{r_{sep}}\right]$$

Integrated external torque

with $\langle \tilde{b}_r \tilde{b}_\theta \rangle = V_E' \tau_c' |\tilde{b}_r|^2$

✓ Force through radial current across separatrix.

Note: $\langle V_\phi \rangle'$ proportional to $|\tilde{b}_r|^2 / \chi_\phi$. Quenched χ_ϕ → stronger $\langle V_\phi \rangle'$ effects!

Stochasticity contribution to particle flux: two ingredients

- For electron density : $\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_{\mathrm{e}}) = S_p$

with $\Gamma_e = \boxed{-(D_{neo} + D_T)\frac{\partial n}{\partial r}} + \boxed{\Gamma_{e,\,stoch}} \longleftarrow$

- $D_{neo} = (m_e/m_i)^{1/2}\chi_{i,neo}$
- $D_T \sim bD_{GB}$ with b<1

$$S_p = \Gamma_a \frac{a - r + d_a}{L_{dep}^2} \exp(-\frac{(a + d_a - r)^2}{2L_{dep}^2})$$

- The stochastic field can induce particle flux ($n_e = n_i$):

$$\Gamma_{e,\,stoch} = \frac{c}{4\pi eB}\langle\tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel\rangle + n\langle\tilde{V}_{\parallel,i}\tilde{b}_r\rangle$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\langle\tilde{b}_r\tilde{J}_\parallel\rangle \qquad \langle\tilde{b}_r\tilde{J}_{\parallel,i}\rangle$$

with

$$\frac{c}{4\pi eB}\langle\tilde{b}_r \nabla_\perp^2 \tilde{A}_\parallel\rangle = -\frac{cB}{4\pi e}\frac{\partial}{\partial r}\langle\tilde{b}_r\tilde{b}_\theta\rangle$$

✓ $\langle\tilde{b}_r\tilde{b}_\theta\rangle$ phasing via V_E' tilt.

$n\langle\tilde{V}_{\parallel,i}\tilde{b}_r\rangle$: parallel ion flow along tilted field lines

- RMP induced density "pump-out"?
- Kinetic stress $\langle\tilde{b}_r\delta P\rangle$

Stochasticity contribution to particle flux: feature

- After integrating in the absence of source

$$\frac{\partial \langle n_e \rangle}{\partial t} = \frac{1}{|e|}\frac{\partial}{\partial r}\langle J_{r,\mathrm{e}} \rangle \rightarrow \boxed{\frac{\partial N_e}{\partial t} = A_{sep}\frac{1}{|e|}\langle J_{r,\,e} \rangle|_{r_{sep}} + C'_{sep}.}$$

$\langle J_{r,e} \rangle$ is proportional to $|\tilde{b}^2_{r,k}|$

- N_e is the total electron number
- A_{sep} is the surface area of the plasma at the separatrix.
- C'_{sep} is an integration constant, set by SOL physics.

✓ Determines "pump out" caused by RMP induced stochasticity

✓ Mean quasi-neutrality is maintained during pump-out by the ion polarization charge flux, → balances against $\langle J_{r,e} \rangle$ to maintain constant net charge → → competition contributes to determining $\langle E_r \rangle$

✓ Particle transport is very sensitive to the envelope scale of the magnetic perturbations

$$\frac{\partial \langle n_e \rangle}{\partial t} \sim |\tilde{b}^2_r| / l^2_{env}$$

A narrow edge stochastic layer can trigger a significant change in density, even for modest $|\tilde{b}^2_r|$

Ion heat flux with stochastic field

- Mean ion pressure evolves

$$\frac{\partial \langle P_i \rangle}{\partial t} + \frac{\partial}{\partial r}\langle \tilde{V}_r \tilde{P}_i \rangle = -\rho c_s^2 \frac{\partial}{\partial r}\langle \tilde{b}_r \tilde{V}_{\parallel,i} \rangle$$

✓ principal magnetic effect is due to the divergence of the ion flow along tilted magnetic field lines

— weak turbulence ($\omega < k_\parallel c_s$)

$$\langle \tilde{b}_r \tilde{V}_{\parallel,i} \rangle \cong -D_M \frac{\partial \langle V_{\parallel,i} \rangle}{\partial r}$$

with $D_M = \sum_{\boldsymbol{k}} \left| \tilde{b}_{r,\boldsymbol{k}}^2 \right| \pi \delta(k_\parallel)$

— strong turbulence ($\omega > k_\parallel c_s$)

$$\langle \tilde{b}_r \tilde{V}_{\parallel,i} \rangle \cong -\frac{1}{\rho c_s^2} D_{st} \frac{\partial \langle P_i \rangle}{\partial r}$$

with $D_{st} = c_s^2 \left| \tilde{b}_{r,\boldsymbol{k}}^2 \right| \tau_c$

✓ the radial current density $\langle J_r \rangle$ does not enter to determine $\langle P_i \rangle$

✓ for typical $\left| \tilde{b}_r^2 \right|$, direct effects on ion heat transport are rather modest.

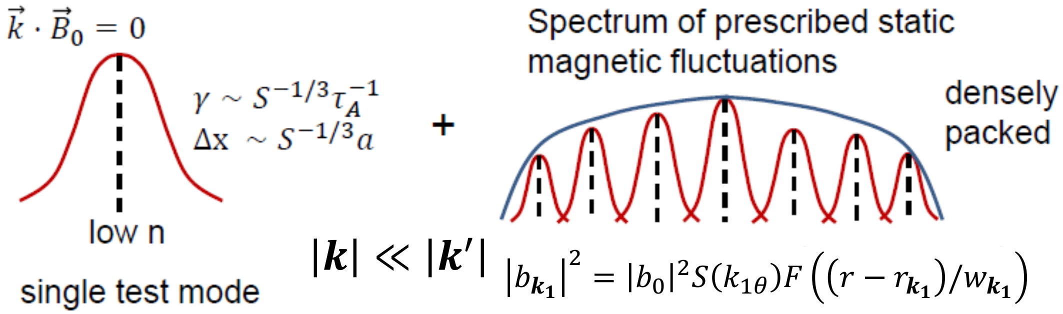
- Ion sound propagation (c_s) sets the fundamental speed for ion heat transport along wandering magnetic field lines.

$$\chi_i = c_s \sum_k \left| \tilde{b}_{r,\boldsymbol{k}} \right|^2 \delta(k_\parallel) \quad \text{much smaller than} \quad \chi_e = v_{the} \sum_k \left| \tilde{b}_{r,\boldsymbol{k}} \right|^2 \delta(k_\parallel)$$

Effects of stochastic magnetic fields on turbulence

- Common theme: a simple model $\nabla \cdot \boldsymbol{J} = 0$ (Kadomtsev and Pogutse ’78)

  A low-k single test mode + a high-k stochastic magnetic field



$|\boldsymbol{k}| \ll |\boldsymbol{k}'|$ $\quad |b_{\boldsymbol{k}_1}|^2 = |b_0|^2 S(k_{1\theta}) F\left((r - r_{\boldsymbol{k}_1})/w_{\boldsymbol{k}_1}\right)$

$$\frac{\partial}{\partial t} \nabla_\perp^2 \bar{\varphi} = -\frac{S}{\tau_A} \left(\nabla_\parallel^{(0)} + \widetilde{\boldsymbol{b}} \cdot \nabla_\perp \right)^2 \bar{\varphi} - \frac{g B_0}{\rho_0} \frac{\partial \bar{p}}{\partial y}$$

If only $\widetilde{\boldsymbol{b}}$ and $\bar{\varphi}$, $\nabla \cdot \boldsymbol{J} = 0$

is not guaranteed!

What is missing?

Analogy	Kadomtsev and Pogutse ’78	This model
Base state	$\langle T(r) \rangle$	$\bar{\varphi}_{\boldsymbol{k}}$
External fluctuation	$\widetilde{\boldsymbol{b}}$	$\widetilde{\boldsymbol{b}}$
Constraint	$\nabla \cdot \boldsymbol{q} = 0$	$\nabla \cdot \boldsymbol{J} = 0$
Resulting fluctuation	$\tilde{T}$	$\tilde{\varphi}$

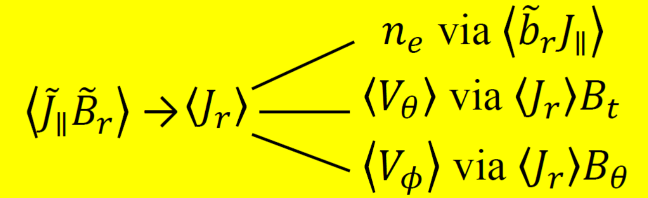
electrostatic potential fluctuation induced by $\widetilde{\boldsymbol{b}}_{\boldsymbol{r}}$

See more details: M. Y. Cao, F-I8, this meeting

Towards an expression for $\langle V_E \rangle'$

- From Ohm's law, $\langle E_r \rangle$ and $\langle J_r \rangle$ are related :

$$\boxed{E_r = \frac{1}{enB}\frac{\partial}{\partial r}P_{\mathrm{i}} + v_\phi B_\theta - v_\theta B_\phi .}$$



$\langle \tilde{J}_\parallel \tilde{B}_r \rangle \rightarrow \langle J_r \rangle$

n_e via $\langle \tilde{b}_r J_\parallel \rangle$

$\langle V_\theta \rangle$ via $\langle J_r \rangle B_t$

$\langle V_\phi \rangle$ via $\langle J_r \rangle B_\theta$

- Elements for ***E***×***B*** shear:
  - ✓ Poloidal flow → a modification in the $\langle V_\theta \rangle$ since $\langle \tilde{V}_\theta \tilde{V}_r \rangle - V_A^2 \langle \tilde{b}_r \tilde{b}_\theta \rangle$

$$\Delta V_\theta = -\frac{1}{\mu}\frac{\partial}{\partial r}\left[\langle \tilde{V}_\theta \tilde{V}_{\mathrm{r}} \rangle - V_{\mathrm{A}}^2 \left\langle \tilde{b}_{\mathrm{r}} \tilde{b}_\theta \right\rangle_0 + \langle V_E \rangle' V_{\mathrm{A}}^2 \sum_{\boldsymbol{k}} \left(\left| \tilde{b}_{\mathrm{r},\boldsymbol{k}}^2 \right| \tau_{c,b} \right) \right],$$

  - ✓ Toroidal flow → induce a "intrinsic torque"

$$\frac{\partial}{\partial r}\langle V_\phi \rangle |_{r_{\mathrm{sep}}} = -\frac{1}{\chi_\phi}\left[\int_0^{r_{\mathrm{sep}}} S_\phi \mathrm{d}r - + \frac{v_{\mathrm{thi}}^2}{\beta}\frac{B_\theta}{B_0}\left[\left\langle \tilde{b}_{\mathrm{r}} \tilde{b}_\theta \right\rangle_0 - \langle V_E \rangle' \sum_{\boldsymbol{k}} \left(\left| \tilde{b}_{\mathrm{r},\boldsymbol{k}}^2 \right| \tau_{c,b} \right) \right] |_{r_{\mathrm{sep}}} + C_{\mathrm{sep}} \right]$$

  - ✓ Density →necessarily also closely linked to RMP pump-out

$$\frac{\partial \langle n_{\mathrm{e}} \rangle}{\partial t} = \frac{1}{|e|}\frac{\partial}{\partial r}\langle J_{\mathrm{r,\,e}} \rangle , \quad \langle J_{\mathrm{r,\,e}} \rangle = \frac{cB_0}{4\pi |e|}\frac{\partial}{\partial r}\left\langle \tilde{b}_{\mathrm{r}} \tilde{b}_\theta \right\rangle - n_0 |e| \left\langle \tilde{b}_{\mathrm{r}} \tilde{V}_{\|,\,\mathrm{i}} \right\rangle .$$

  - ✓ χ_i is modified by magnetic effects, albeit rather slightly

Five different radial scales in the model

- Additional comments

① $\langle \tilde{V}_\theta \tilde{V}_r \rangle - V_A^2 \langle \tilde{b}_r \tilde{b}_\theta \rangle$, competition appears not due to Alfvenization since the static magnetic fluctuations $\tilde{b}_r$ are externally driven, while the drift waves are dynamic, and heat flux driven and evolving

② The problem is multi-scale, even in its most simple manifestation.

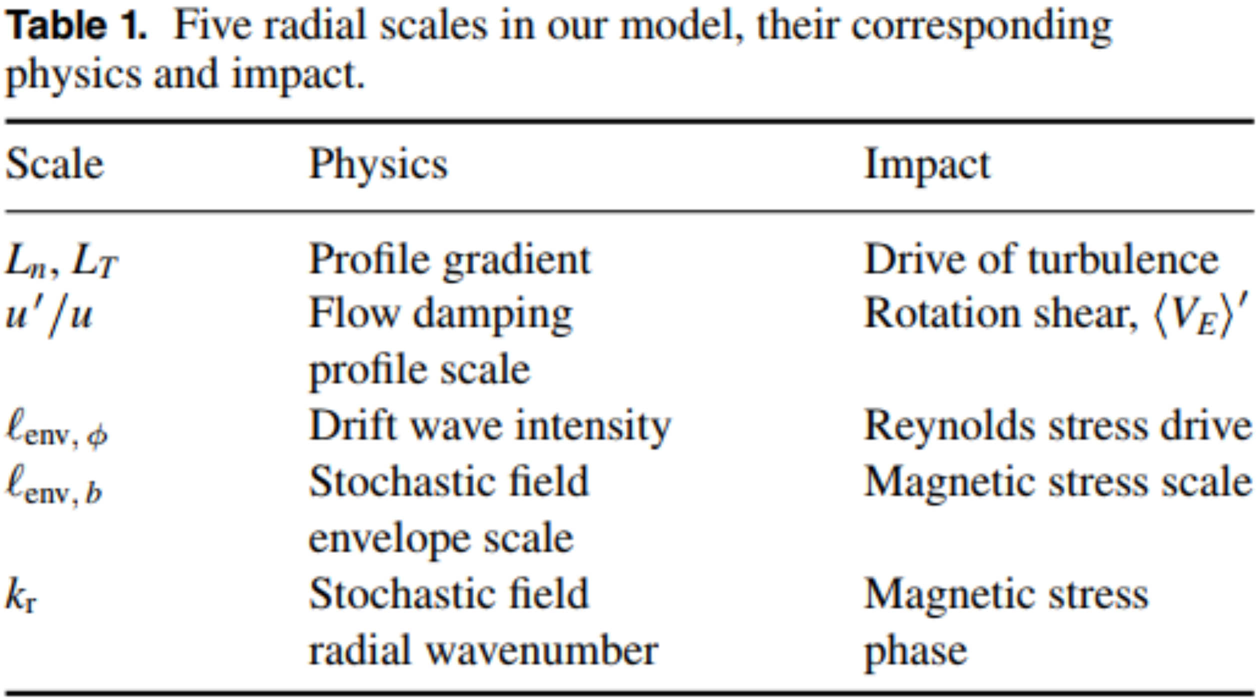


Table 1. Five radial scales in our model, their corresponding physics and impact.

Scale	Physics	Impact
L_n, L_T	Profile gradient	Drive of turbulence
u'/u	Flow damping profile scale	Rotation shear, $\langle V_E \rangle'$
$\ell_{\mathrm{env},\,\phi}$	Drift wave intensity	Reynolds stress drive
$\ell_{\mathrm{env},\,b}$	Stochastic field envelope scale	Magnetic stress scale
k_{r}	Stochastic field radial wavenumber	Magnetic stress phase

Outline

- Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues (L→H transition with RMP, island, stellarator, etc)
- Mean field model $\langle E_r \rangle$—follow radial force balance
  - Key fundamentals:

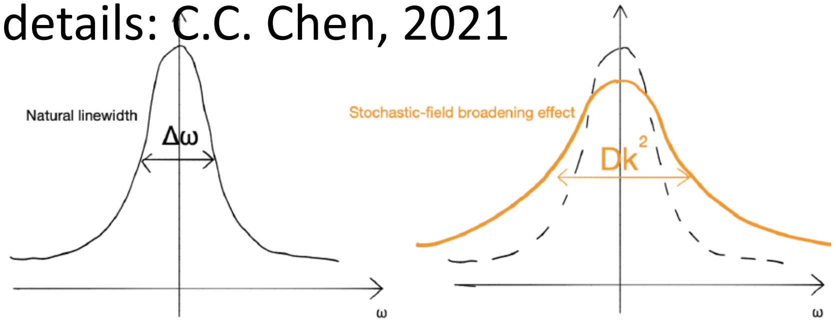
    — *phases*
    — *instability in stochastic field*
  - Turbulent transport

    — *Particle transport*
    — *Momentum transport (poloidal and toroidal)*
    — *Ion heat transport*
- Applications
  - L-H transition with $\tilde{b}^2$, 0D at present
- Implications and future work

Increment of LH threshold power

Concern: $\Delta\omega < Dk_{\perp}^{2}$, D=$v_A D_M$

More details: C.C. Chen, 2021



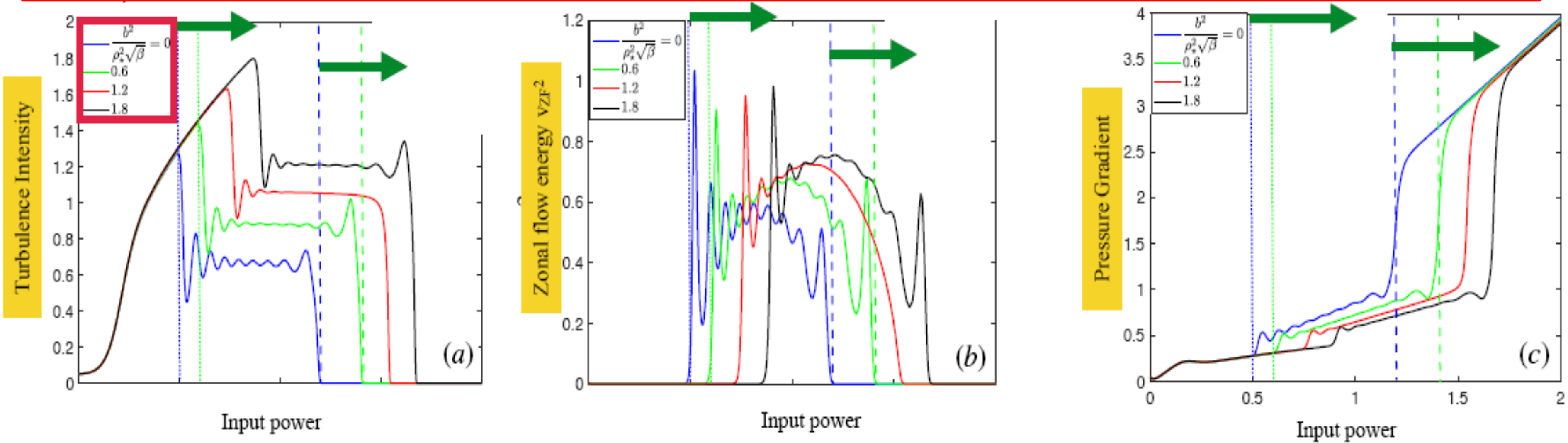
Turbulence decorrelation

$\Delta\omega \sim D_{ES} k_{\perp}^{2}$

Stochastic field induced scattering

Broadening parameter: $\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} = 0.0, 0.2, 0.4, 0.6, 0.8...., 2.0$

Kim and Diamond model, 2003 PRL, predator: zonal flow, prey: turbulence



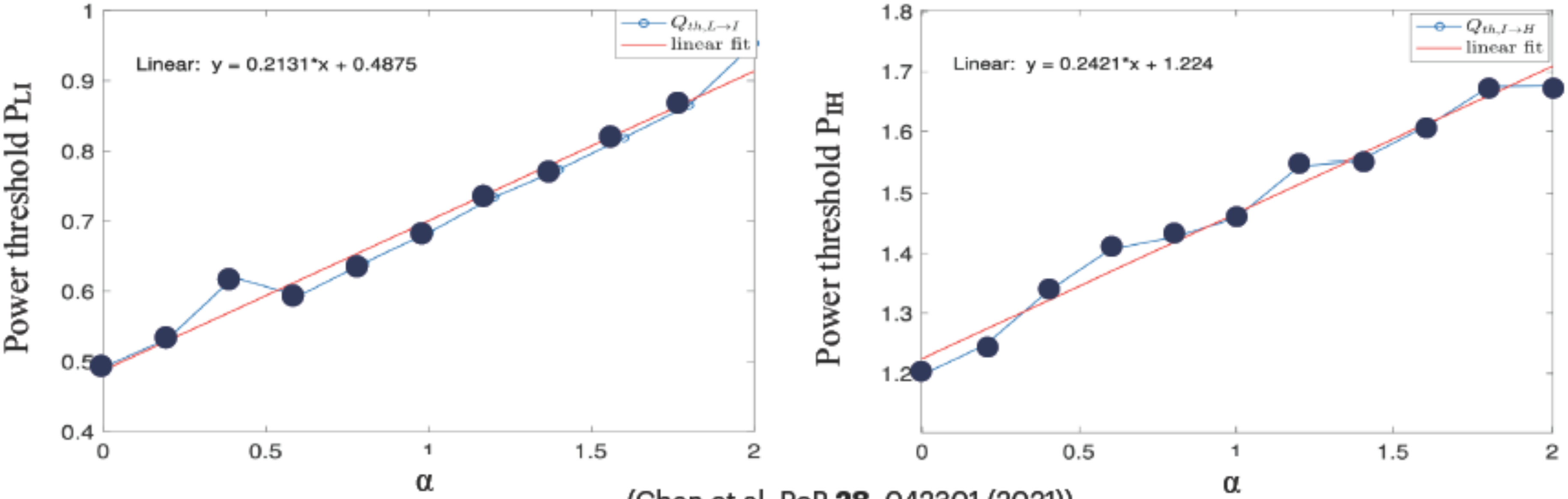
The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.

Increment of LH threshold power

$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon} = 0.0, 0.2, 0.4, 0.6, 0.8...., 2.0$$

More details: C.C. Chen, O 3.2, this meeting

P_{LI} v.s. α P_{IH} v.s. α



(Chen et al.,PoP **28**, 042301 (2021))

- ☐ Testable prediction: The threshold power increase linearly with $\alpha \sim b^2/\rho_*^2$.
- ☐ Could compare directly with $\Delta\omega$ $(k_\perp^2 v_A D_M)$

Outline

- ☐ Motivation and background
  - Why? → Interaction and co-existence of stochastic B field and turb.
  - Key issues (L→H transition with RMP, island, stellarator, etc)
- ☐ Mean field model $\langle E_r \rangle$—follow radial force balance
  - Key fundamentals:
    - — *phases*
    - — *instability in stochastic field*
  - Turbulent transport
    - — *Particle transport*
    - — *Momentum transport (poloidal and toroidal)*
    - — *Ion heat transport*
- ☐ Applications
  - L-H transition with $\tilde{b}^2$, 0D at present
- ☐ Implications and future work

Results and implications

Topic	Goal	Key physical results	prediction
Reynold stress (C C Chen)	Flow shear evolution	Dephasing when $\Delta\omega \sim v_A D_M k_\perp^2$	Critical parameter $\alpha \sim b^2/\rho_*^2\sqrt{\beta}$
Parallel flow and ion heat transport (P.H. Diamond)	Calculate kinetic stress $\langle\tilde{b}_r\delta P\rangle$ in turbulence	Physical understanding of stochasticity-turbulence interaction	Hybrid stochastic field + turbulence viscosity
Instability evolution in stochastic field (M.Y. Cao)	Understand how prescribed $\tilde{b}$ affect instability evolution	Maintaining $\nabla \cdot J = 0$ forces generation of small scale cells by $\tilde{b}$	$\langle\tilde{b}_r\tilde{\phi}\rangle \neq 0$ →turbulence "lock on" to $\tilde{b}$.
Mean field theory for $\langle E_r\rangle$ (All)	Understand electric field shear evolution	Unified model including all transport channels	$\langle V_E\rangle'$ aligns stochastic field, particle flux due to $\tilde{b}$

Future work

☐ Towards to the 1D model to study the interplay in L-H transition.

☐ Related to experiments:

① Understand the relationship between the RMP effects on power threshold and micro-physics? [stress, fluctuations, transport…]

② How does the RMP change the evolution of the shear layer (Er well) ? How it builds up?

③ How the **cross-phase** of Reynolds stress change (evolution) vs RMP current?

④ How does the RMP change the **LCO (limit cycle oscillation)**?

⑤ How **toroidal velocity** change at pedestal region?

☐ Related work in density limit, esp. effect of RMP on Er shear

Thanks for your attention!!!