How Phase Patterns Define Zonal Flow Structure and Avalanche Scale

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Outline

I) Motivation

II) From linear coupled phase lattice to global phase continuum

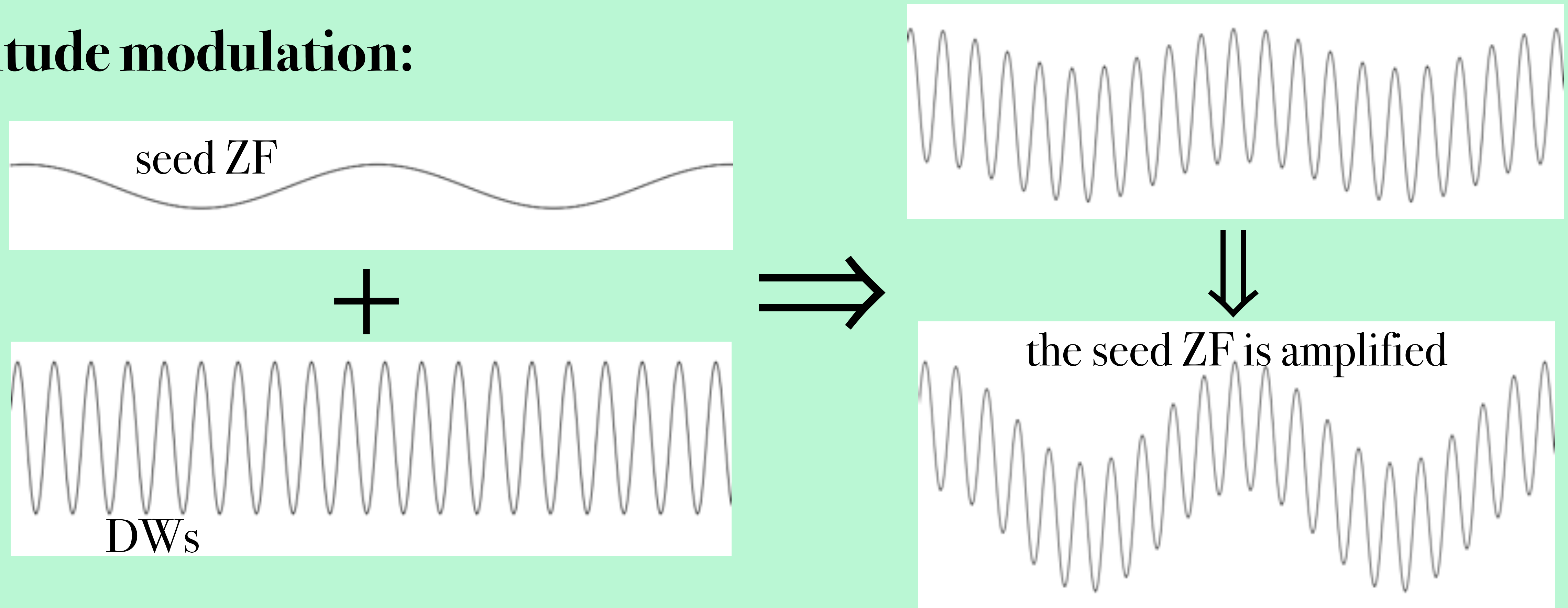
III) Linear stage:
roughening of the phase-gradient profile and ZF generation

IV) Nonlinear stage: an expanded Predator-Prey loop: phase gradient-ZF

V) Summary

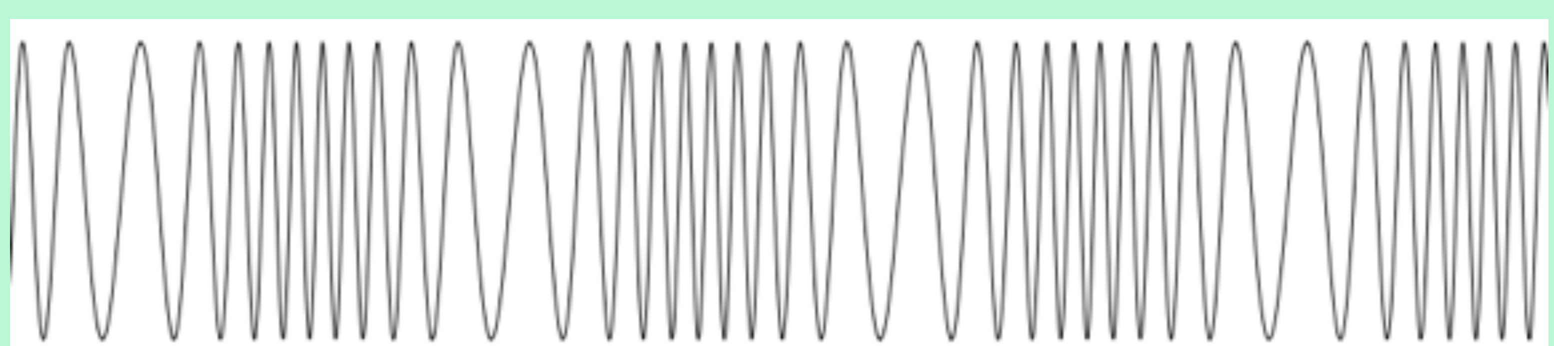
I. Motivation

Amplitude modulation:



Frequency modulation:

?



I. Motivation

There is an apparent drawback of amplitude modulation analysis:

*The structure of the generated ZF is sensitive to the seed ZF(i.e., initial condition).

**So far such models have not explained the spatial distribution of ZF, which is crucial to understanding avalanche dynamics.

In other words, a deeper understanding of ZF physics in Tokamak requires an expanded framework that can describe the global dynamical process of the ZF generation.

$\Downarrow$

ZF generation based on global phase modulation mechanism can overcome the drawbacks of amplitude modulation models.

I. Motivation

The general logic:

Zonal flow is a meso-scale structure, while drift wave is a micro-scale structure.

⬇

An essential step of generating zonal flow by drift waves is the **global coupling of these micro-structures**.

⬇

Toroidal coupling provides a mechanism of global coupling of the local structures!

A natural question: how toroidal coupling induces macro/meso-scale dynamics of the local structures?

Answer in this work: via **phase coupling**!

note: In modulational analysis, it is the seed ZF that induces the nonlocal(in space) coherence of the local structures, which in turn amplifies the seed ZF. Thus, the long range coherence is not induced in an intrinsic way.

II. Reynolds' force driven by global phase curvature

How does phase patterning drives ZF??

Zonal flow evolution

'spiky' distribution of the local structures. At each rational surface, we only keep the resonance mode.

$$\frac{\partial}{\partial t}\langle V\rangle = \underbrace{\sum_m \partial_x^2\phi_m \partial_y \phi_m^*}_{\text{vorticity flux}} - \gamma_d \langle V\rangle \simeq \partial_x^{\,2}\Phi\partial_y\Phi^* - \gamma_d\langle V\rangle$$

$-\gamma_d\langle V\rangle$ represents a ZF friction term.

Note: ZF is driven by radial coherence of the micro-structures, we replaced ϕ by its envelope Φ.

$$\Phi = |\Phi| e^{iS} \quad \Longrightarrow \quad \langle \partial_x^{\,2}\Phi\partial_y\Phi^*\rangle = \underbrace{k_y \partial_x S \partial_x |\Phi|^2}_{\text{turbulence-intensity inhomogeneity}} + \underbrace{k_y |\Phi|^2 \partial_x^2 S}_{\text{phase curvature}}$$

$$\Rightarrow \quad \frac{\partial}{\partial t}\langle V\rangle = k_y \partial_x S \partial_x |\Phi|^2 + k_y |\Phi|^2 \partial_x^{\,2} S - \gamma_d \langle V\rangle$$

$\Downarrow$

Phase curvature can drive a net vorticity flux!

III. From linear coupled phase lattice to global phase continuum

Eikonal equation of the phase

$$\frac{\partial}{\partial t}S = -\omega - \mathbf{k}\cdot\tilde{v}$$

stochastic Doppler shift

$$\omega_k + 2\hat{\omega}_{\mathrm{De}} + k_y\langle V\rangle$$

eigenfrequency magnetic drift coherent Doppler shift

$$2\hat{\omega}_{de}\phi_m = \frac{\rho_s c_s}{R_0}\left[k_y\left(\phi_{m+1}+\phi_{m-1}\right)+k_x\left(\phi_{m+1}-\phi_{m-1}\right)\right]$$

with $\phi_m \simeq |\Phi_m| e^{iS(m,x,t)}$

strong coupling approximation

$\Downarrow$

eigenvalue of magnetic drift operator

$$2\hat{\omega}_{\mathrm{De}}\phi_m \simeq \left[2k_yV_D - k_yV_D\Delta^2\left(\frac{\partial S}{\partial x}\right)^2 + 2k_xV_D\Delta\frac{\partial}{\partial x}S\right]\phi_m$$

III. From linear coupled phase lattice to global phase continuum

Global phase evolution equation

$$\frac{\partial}{\partial t}\bar{S} \simeq -k_y\langle V\rangle - 2k_x V_D \Delta \frac{\partial}{\partial x}\bar{S} + k_y V_D \Delta^2 \left(\frac{\partial \bar{S}}{\partial x}\right)^2 + D_s \frac{\partial^2}{\partial x^2}\bar{S}$$

convective motion of the global phase pattern

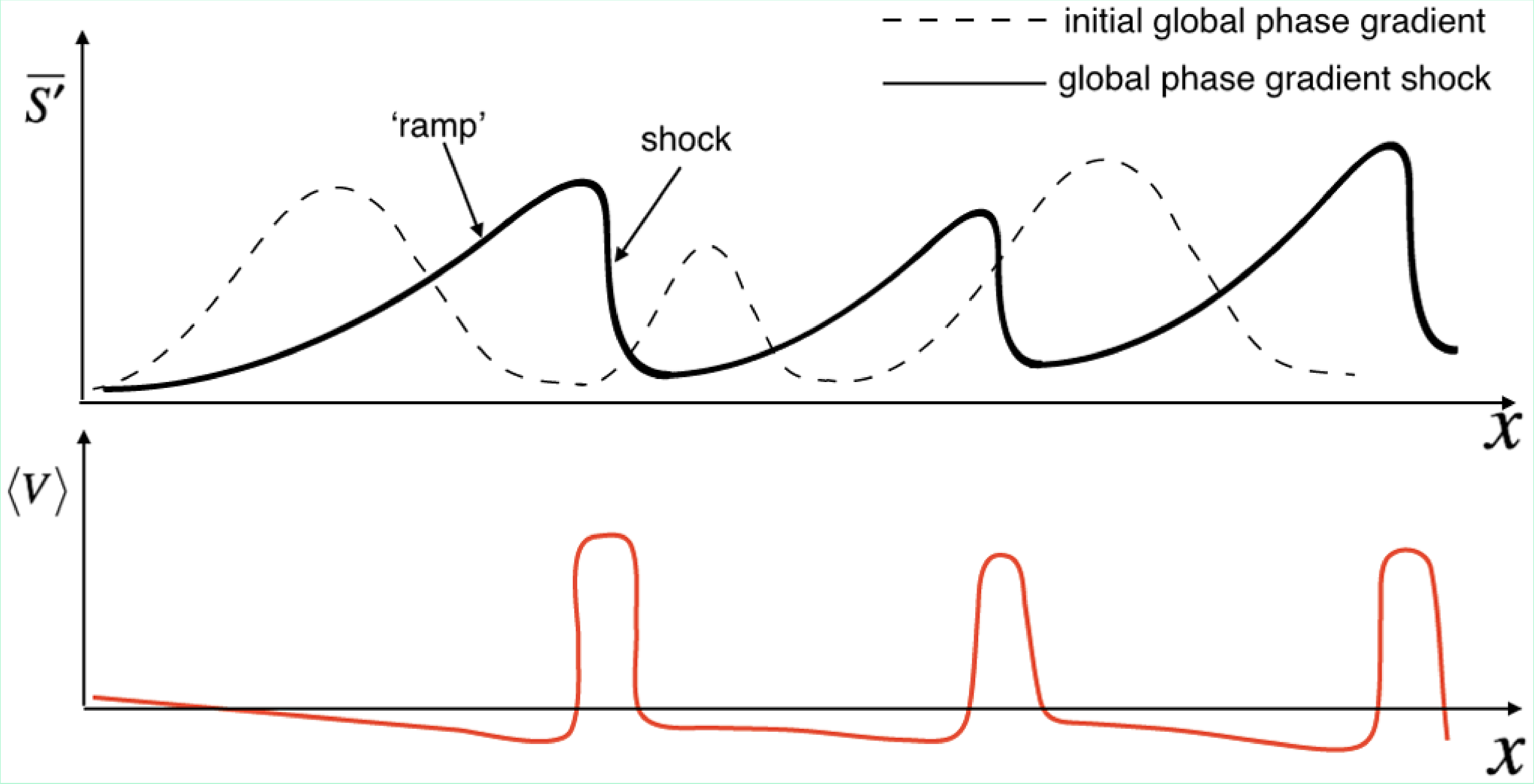
nonlinear steepening

phase diffusion associated with stochastic Doppler shift

Global phase gradient evolution equation

$$\frac{\partial}{\partial t}\bar{S}' = -k_y\langle V\rangle' - 2k_x V_D \Delta \frac{\partial}{\partial x}\bar{S}' + 2k_x V_D \Delta^2 \bar{S}' \frac{\partial}{\partial x}\bar{S}' + D_s \frac{\partial^2}{\partial x^2}\bar{S}',$$

Global phase-gradient shocks and ZF patterning



Shock layer ⟶ large global phase curvature ⟶ ZF layer

Width of ZF layer

Balance of steepening $k_y V_D \Delta^2 \left(\frac{\partial \bar{S}}{\partial x}\right)^2$ and broadening $D_s \frac{\partial^2}{\partial x^2}\bar{S}$,

⇛ $$2k_y V_D \Delta^2 |\delta \bar{S}'| / L_{\mathrm{ZF}} \simeq D_s / L_{\mathrm{ZF}}^2$$

$|\delta \bar{S}'| \simeq 1/\Delta$

$D_s \simeq \rho_s c_s \rho_s / a$

↓

$$L_{ZF} \simeq \frac{R}{a}\rho_s$$ –a meso-length scale

PDF of ZF layers

PDF of ZF layers is determined by the PDF of global phase gradient shocks

$$\frac{\partial}{\partial t}\bar{S}' = 2k_y V_D \Delta^2 \bar{S}' \frac{\partial}{\partial X}\bar{S}' + D_s \frac{\partial^2}{\partial X^2}\bar{S}' + F(X,t),$$

driving 'force'

homogeneous noise

(A. chekhov&V. Yakhot1995) $P(\delta\bar{S}' < 0) \sim |\delta\bar{S}'|^{-4}$

power law tail by the intermittency of shocks

PDF of ZF layer width $P(L_{\mathrm{ZF}}) \sim L_{\mathrm{ZF}}^4$

IV. Feedback of ZF shear on global phase evolution

ZF feedback effect is most prominent at the 'shoulders' of phase shocks, where ZF shearing is strong.

Then, global phase gradient evolution is governed by

$$\frac{\partial^2}{\partial t^2}\bar{S}' - \left(D_s\frac{\partial^2}{\partial X^2} - \gamma_d\right)\frac{\partial}{\partial t}\bar{S}' = (D_s\gamma_d - 2k_y^2 I)\frac{\partial^2}{\partial X^2}\bar{S}'$$

$$(\partial_t \to \gamma_{\mathcal{K}},\ \partial_x \to i\mathcal{K}),$$

$$\gamma_{\mathcal{K}} = \frac{\sqrt{(D_s\mathcal{K}^2 - \gamma_d)^2 + 8k_y^2 I\mathcal{K}^2} - (D_s\mathcal{K}^2 + \gamma_d)}{2}$$

$$\gamma_{\mathrm{K}} > 0 \Leftrightarrow 2k_y{}^2 I > D_s\gamma_d$$

i.e., distortion effect by ZF shear (measured by $2k_y^2 I\mathcal{K}^2$) should exceed flattening effects by diffusion ($D_s\mathcal{K}^2$) and damping by ZF friction (γ_d).

V. Towards to an expanded Predator-Prey system

ZF: $$\frac{\partial}{\partial t}\langle V\rangle \simeq 2k_y k_x \frac{\partial}{\partial x} I + 2k_y \frac{\partial}{\partial x} I \frac{\partial}{\partial x}\bar{S} + 2k_y I \frac{\partial^2}{\partial x^2}\bar{S} - \gamma_d \langle V\rangle$$

Global phase gradient: $$\frac{\partial}{\partial t}\bar{S}' = -k_y \langle V\rangle' - 2k_x V_D \Delta \frac{\partial}{\partial x}\bar{S}' + 2k_x V_D \Delta^2 \bar{S}' \frac{\partial}{\partial x}\bar{S}' + D_s \frac{\partial^2}{\partial x^2}\bar{S}'$$

Turbulence intensity: $$\frac{\partial}{\partial t} I = \gamma_l I + 2k_y I \bar{S}' \langle V\rangle' + \frac{\partial}{\partial x}\left(D_T I \frac{\partial}{\partial x} I\right) - \gamma_{nl} I^2$$

Summary

★ Global phase curvature can drive a ZF in the absence of turbulence intensity inhomogeneity.

★ Width of phase-curvature driven ZF scales as $L_{ZF} \simeq \frac{R}{a}\rho_s$ and its PDF $P(L_{\mathrm{ZF}}) \sim L_{\mathrm{ZF}}^4$

★ Including global phase evolution, an expanded PP system is reached: ZF, turbulence intensity&phase.