How Phase Patterns Define Zonal Flow Structure and Avalanche Scale

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I. Motivation

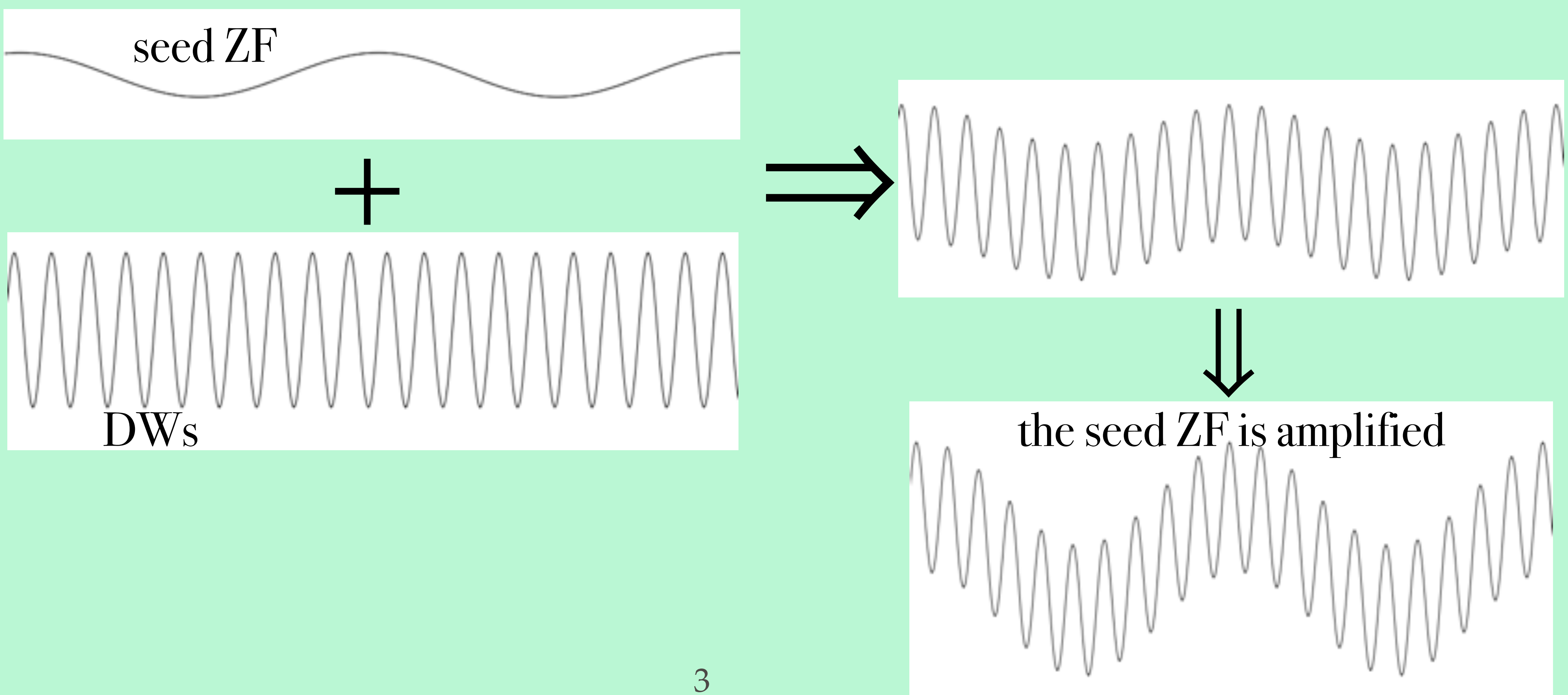
Zonal flow is an important issue in Tokamak physics:LH transition, avalanche dynamics,...

Its existence has been confirmed both by experiment and simulation,

BUT

nonlinear study of its generation and especially its saturation mechanisms(by DWs) is limited...

The most frequently involved mechanism of ZF generation is modulational instability[Diamond1998, Chen2000],



I. Motivation

HOWEVER

there is an apparent drawback of modulational analysis:

it requires a seed ZF, furthermore the structure of the generated ZF is sensitive to the seed ZF(i.e., initial condition). It should treat the intrinsic features of the system.

so far such models have not explained the spatial distribution of ZF, which is crucial to understanding avalanche dynamics.

In other words, a deeper understanding of ZF physics in Tokamak requires an expanded framework that can describe the global dynamical process of the ZF generation.

$$\Downarrow$$

In this work, we report a new ZF generation mechanism, which can overcome the drawbacks of modulation models.

I. Motivation

The general logic:

Zonal flow is a meso-scale structure, while drift wave is a micro-scale structure.

⬇

An essential step of generating zonal flow by drift waves is the **global coupling of these micro-structures**.

⬇

Toroidal coupling provides a mechanism of global coupling of the local structures!

A natural question: how toroidal coupling induces macro/meso-scale dynamics of the local structures?

Answer in this work: via **phase coupling**!

note: In modulational analysis, it is the seed ZF that induces the nonlocal(in space) coherence of the local structures, which in turn amplifies the seed ZF. Thus, the long range coherence is not induced in an intrinsic way.

II. From linear coupled phase lattice to global phase continuum

➠ linear coupled phase lattice

General form of toroidal drift wave evolution equation

$$\frac{\partial}{\partial t}\phi_m = -i\omega_m\phi_m - ik_y\langle V\rangle + \gamma_l\phi_m + i2(\omega_{de}\phi)_m + N_m$$

$\omega_m(\gamma_l)$ – linear local eigen frequency (growth rate) of the DW(i.e., ITG, TEM...)

ω_{de} – toroidicity induced drift

$$\omega_{de} = \frac{\rho_s c_s}{R_0}(k_y\cos\theta + k_x\sin\theta)$$

N_m – nonlinear interaction terms

Here the toroidal mode # is fixed.

Employing direct-interaction-approximation, *Nm* can be written as

$$N_m = -\gamma_{nl}\phi_m + F_m$$

$-\gamma_{nl}\phi_m$: coherent interaction

F_m : incoherent interaction(e.g., noise)

$2(\omega_{de}\phi)_m$ is rewritten as

$$2(\omega_{de}\phi)_m = \frac{\rho_s c_s}{R_0}\left[k_y\left(\phi_{m+1}+\phi_{m-1}\right)+k_x\left(\phi_{m+1}-\phi_{m-1}\right)\right]$$

II. From linear coupled phase lattice to global phase continuum

Assumption:

Neighboring modes are overlapped, i.e., toroidal mode number is large(strong coupling limit).

Evolution of each mode follows as:

$$\frac{\partial}{\partial t}\phi_m = -i\omega_m\phi_m - ik_y\langle V\rangle + (\gamma_l - \gamma_{nl})\phi_m + iV_D[k_y(\phi_{m+1}+\phi_{m-1}) + k_x(\phi_{m+1}-\phi_{m-1})] + F_m$$

$$V_D \equiv \frac{\rho_s c_s}{R_0}$$

$$\phi_m(x,t) = |\phi_m(t)|\, e^{iS_m(t)+ik_x(x-x_m)+im\theta}$$

S_m : phase of the mode at a certain rational surface;
x_m : location of rational surface

Global phase continuum

Introducing an envelope function: $\Phi(x,t) = |\Phi(x,t)|\, e^{iS(x,t)}$

with $|\Phi(x_m,t)| = |\phi_m(t)|, |S(x_m,t)| = S_m(t)$

Substituting it into the single mode evolution equation and taking continuous limit, one has:

II. From linear coupled phase lattice to global phase continuum

$$\frac{\partial}{\partial t}S = -\omega - k_y\langle V\rangle + 2k_yV_D + V_Dk_y\Delta^2\frac{1}{|\Phi|}\frac{\partial^2}{\partial x^2}|\Phi| + 2k_xV_D\Delta\frac{1}{|\Phi|}\frac{\partial}{\partial x}|\Phi| - k_yV_D\Delta^2\left(\frac{\partial}{\partial x}S\right)^2 + F_S \qquad \textbf{(I)}$$

$$\frac{\partial}{\partial t}|\Phi| = (\gamma_l - \gamma_{nl})|\Phi| - \left(k_yV_D\Delta^2\frac{\partial^2}{\partial x^2}S + 2k_xV_D\Delta\frac{\partial}{\partial x}S\right)|\Phi| - 2k_yV_D\Delta^2\frac{\partial}{\partial x}S\frac{\partial}{\partial x}|\Phi| + F_\Phi \qquad \textbf{(II)}$$

$\Delta = \frac{q}{nq'}$ distance between rational surfaces.

$F_S = \mathrm{Im}(-ie^{-iS_m}\frac{F_m}{|\phi_m|})$ phase scatter

$F_\phi = \mathrm{Im}(e^{-iS_m}F_{NL})$ amplitude scatter

In order to account the minimal nonlinear phase dynamics, we keep our expansion to~ $O(\Delta^2)$

(I) is equivalent to an inviscid burgers equation after taking its spatial derivative

(II) explicitly shows how the spatial structure of the envelope-intensity is modulated by the global phase patterning.

Note: (I) and (II) are not closed system. Its closure requires knowing the evolution equation of the zonal flow <V>.

III. Roughening of the phase-gradient profile and ZF generation

'spiky' distribution of the local structures. At each rational surface, we only keep the resonance mode.

How does phase patterning drives ZF??

Zonal flow evolution

$$\frac{\partial}{\partial t}\langle V\rangle = \underbrace{\sum_m \partial_x^2 \phi_m \partial_y \phi_m^*}_{\text{vorticity flux}} - \gamma_d \langle V\rangle \simeq \partial_x^{\,2}\Phi\,\partial_y \Phi^* - \gamma_d \langle V\rangle$$

$-\gamma_d\langle V\rangle$ represents a ZF friction term.

Note: ZF is driven by radial coherence of the micro-structures, we replaced ϕ by its envelope Φ.

$$\langle \partial_x^{\,2}\Phi\,\partial_y \Phi^*\rangle = \underbrace{k_y\,\partial_x S\,\partial_x|\Phi|^2}_{\text{turbulence-intensity inhomogeneity}} + \underbrace{k_y|\Phi|^2\,\partial_x^2 S}_{\text{phase curvature}}$$

Note:
k_y, S flip sign simultaneously.

To drive a ZF, inhomogeneity of turbulence intensity is not a necessary condition.

Phase curvature can drive a net vorticity flux, too!

$$\Rightarrow\quad \frac{\partial}{\partial t}\langle V\rangle = k_y\,\partial_x S\,\partial_x|\Phi|^2 + k_y|\Phi|^2\,\partial_x^{\,2} S - \gamma_d\langle V\rangle \qquad \textbf{(III)}$$

III. Roughening of the phase-gradient profile and ZF generation

Assumptions:

quasi-translation invariant of the local structures, i.e., $\partial_x |\phi| \simeq 0$

eigenfrequency of the local structures is homogeneous $\frac{\partial}{\partial x}\omega = 0$

The turbulence intensity-ZF feedback loop is not included here.

Physically, phase gradient is of interest, so we do a spatial derivative on (I).

Then, (I),(II)&(III) are significantly simplified:

$$\frac{\partial}{\partial t} S' = -k_y \langle V \rangle' - k_y V_D \Delta^2 S' \frac{\partial}{\partial x} S' + F_S' \qquad \textbf{(I')}$$

$$S' \equiv \frac{\partial}{\partial x} S$$

$$\frac{\partial}{\partial t} |\Phi| = \left[(\gamma_l - \gamma_{nl}) - \left(k_y V_D \Delta^2 \frac{\partial^2}{\partial x^2} S - 2 k_x V_D \Delta \frac{\partial}{\partial x} S \right) \right] |\Phi| + F_\Phi \qquad \textbf{(II')}$$

A Langevin equation with a dynamical friction coefficient

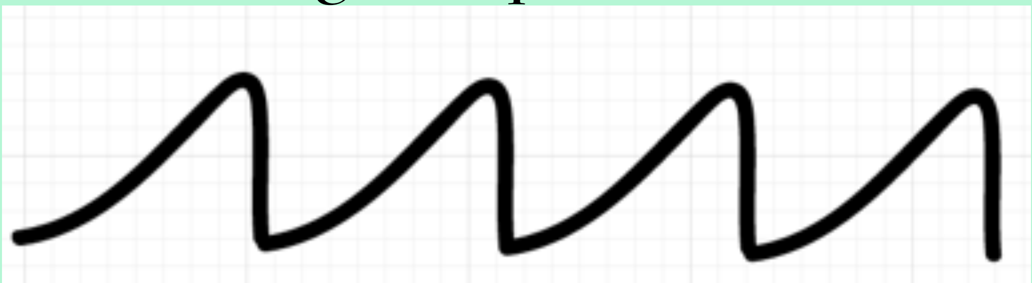
$$\frac{\partial}{\partial t} \langle V \rangle = \underline{k_y |\Phi|^2 \partial_x{}^2 S} - \gamma_d \langle V \rangle \qquad \textbf{(III')}$$

indicates phase curvature can drive a ZF from zero!

III. Roughening of the phase-gradient profile and ZF generation

In the initial stage, detuning effect by ZF shear is neglected, so that the global-phase gradient evolution follows a noisy inviscid Burgers equation:

$$\frac{\partial}{\partial t} S' = -k_y V_D \Delta^2 S' \frac{\partial}{\partial x} S' + F_S'$$



The spatial profile of S' is dominant by shock waves.

The short-short interaction among DWs is weak due to their strong dispersive property, so it is reasonable to make a weak noise assumption.

The roughness of the phase-gradient pattern is described by

κ : wave number of S'

$$\langle S'^2 \rangle = \sum_\kappa (S')_\kappa^{\;2}$$

While the "energy spectrum" of Burgers turbulence is $(S')_\kappa^{\;2} \propto \kappa^{-2}$

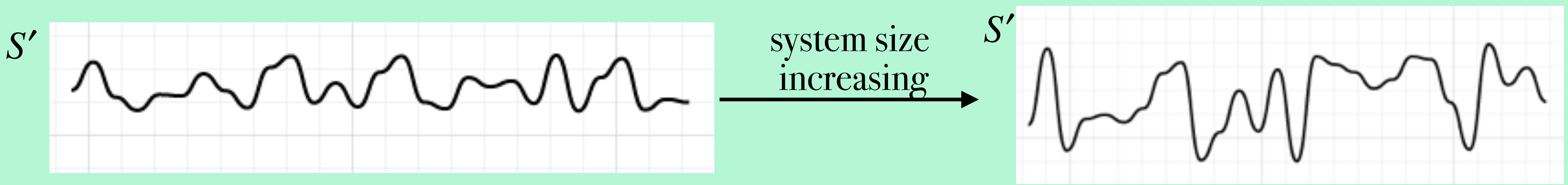
⇒ $$\langle S'^2 \rangle \propto a$$

a^{-1} is the lower limit of κ

$i.e$, a is the minor radius

The roughness of the phase-gradient pattern is proportional to the system size!

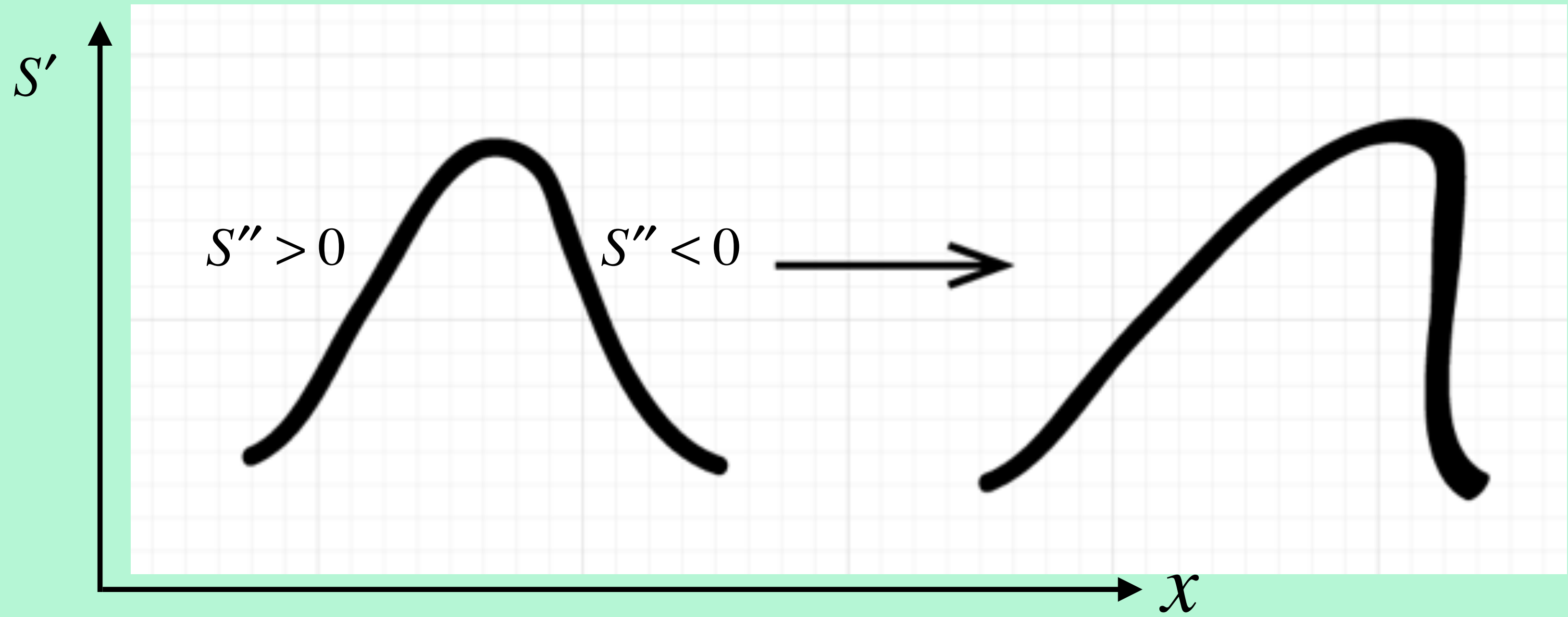
III. Roughening of the phase-gradient profile and ZF generation



The rougher the phase-gradient, the larger the phase curvature will be.

ZF is more pervasive in larger system.

It's known that the Burgers turbulence is compose of shock&ramp regions:



In shock region, $S'' < 0$: ZF generate effectively;

In ramp region, $S'' > 0$: ZF grows slowly and stops growing because of the smooth process of the ramp;

shock wave train

↓

ZF lattice(staircase)

III. Roughening of the phase-gradient profile and ZF generation

➠ shock regions($S'' < 0$) corresponds to transport barriers;

ramp regions($S'' > 0$) corresponds to avalanche regions;

Knowing the PDFs of shocks and ramps are greatly important in assessing the degree of Gyro-Bohm breaking.

Fortunately, there are enormous studies of Burgers equation, their conclusions can be used immediately.

★ PDF of shocks:

$P\left(S'' < 0\right) \propto \left(S''\right)^{-4}$:larger scale shocks are more prominent (Yakhot 1995)

valid for $S'' \ll 0$

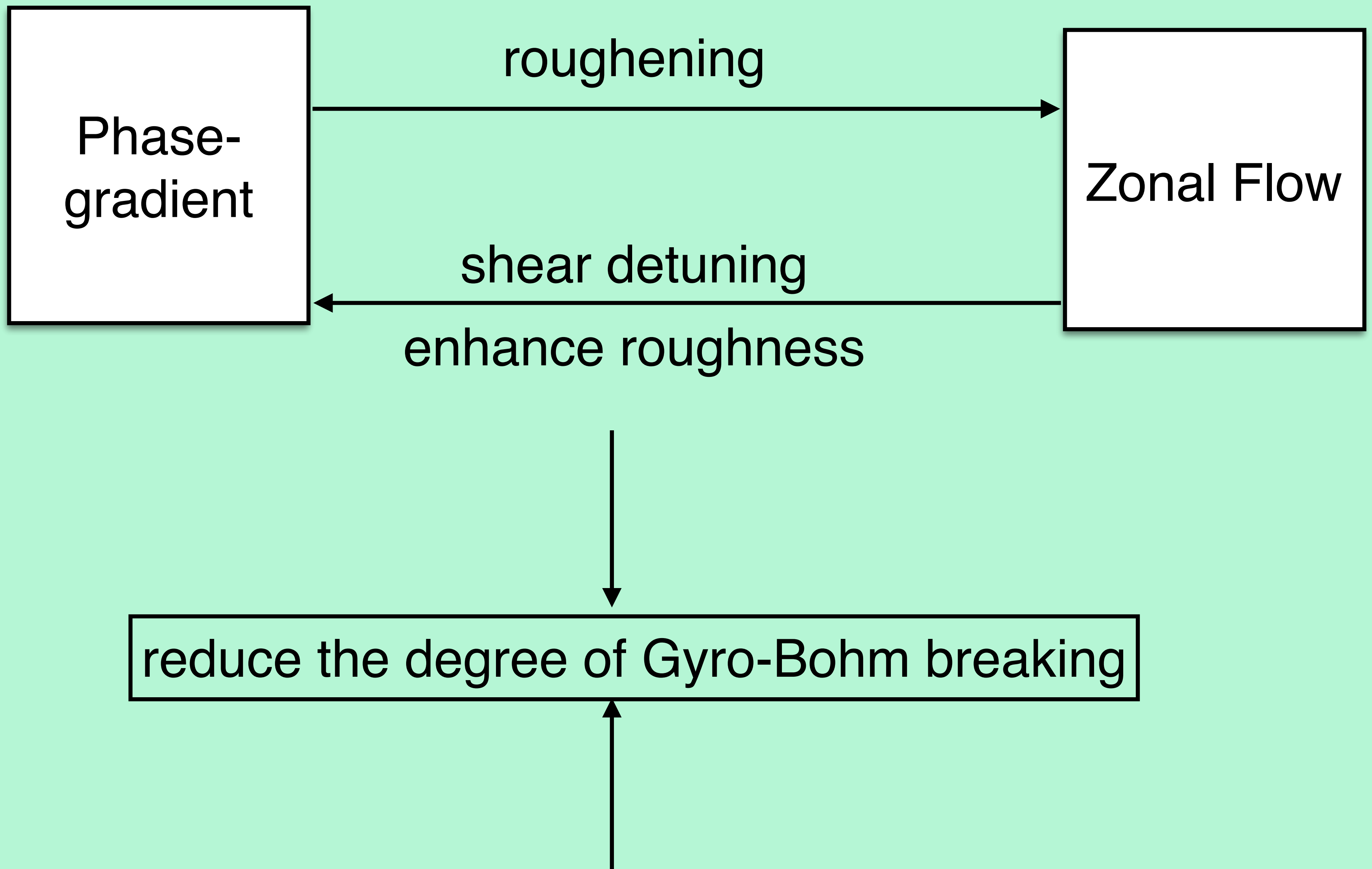
★ PDF of ramps:

$P\left(S'' > 0\right) \propto e^{-CS''^3}$:Zero curvature ramps are dominant (Guraie 1995)

IV. Nonlinear stage: an expanded feedback loop: phase gradient-ZF

With the appearance of ZF, its shearing effect tends to detuning the phase-gradient dynamics, so that

a new feedback loop forms:



This is a positive feedback, so the ZF is nonlinearly enhanced.

IV. Nonlinear stage: an expanded feedback loop: phase gradient-ZF

When the ZF approaches to a steady state, one has

$$0 = k_y |\Phi|^2 \partial_x{}^2 S - \gamma_d \langle V \rangle .$$

Substituting it into Eqn. (I') yields a Burgers equation with **negative viscosity**

$$\frac{\partial}{\partial t} S' = \underline{-k_y{}^2 |\Phi|^2} \frac{\partial^2}{\partial x^2} S' - k_y V_D \Delta^2 S' \frac{\partial}{\partial x} S' + F_s'$$

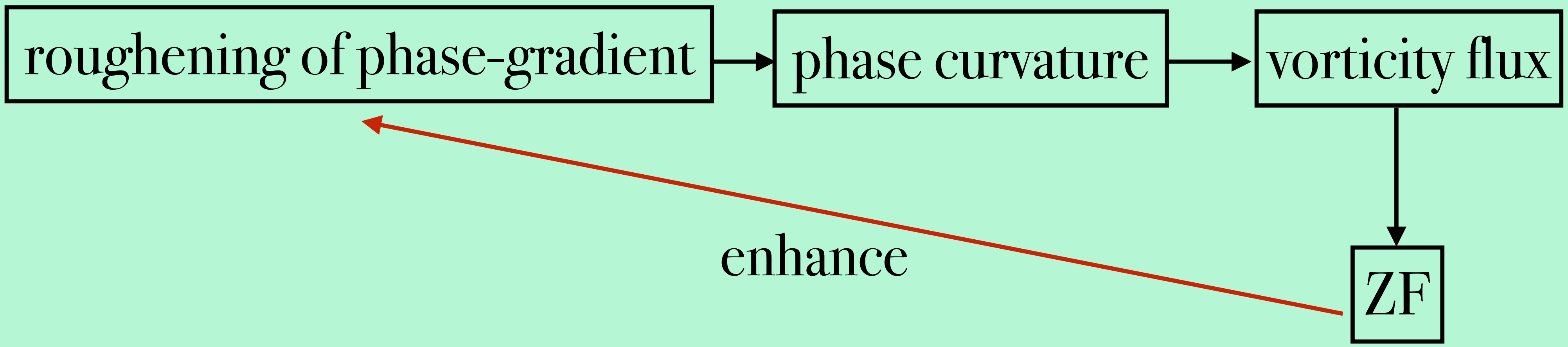
the negative viscosity

➠ **The phase-gradient profile is steeper under the impact of 'negative' diffusion, and eventually, become singular.**

Resolving of the singularity requires higher order expansion terms in the phase-gradient equation...

V. Summary

★ Global phase patterning provides a new paradigm of ZF generation:



★ ZF structures is determined by shock waves in the phase-gradient profile.

Strength of ZF is proportional to the size of the system.

★ This paradigm provides an intriguing way to understand avalanche dynamics, i.e., Gyro-Bohm breaking.