Cosmic Ray Confinement and Transport Models for Probing their Putative Sources

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Outline

1. Two regimes of CR transport along magnetic field:
   -collectively self-impeded
   -test particle transport in background-turbulence
2. CR self-confinement and escape
   - Equation for CR Self-Confinement
   - Self-Similar Solution
   - γ-ray Spectra From CR-Illuminated Molecular Clouds
   - Environmental vs Transport-Induced Spectral Breaks
   - Summary and Conclusions
3. CR transport in background wave field
   - Equation for Transport Driven by Pitch-Angle Scattering
   - CR Transport Equation and its Asymptotic Reduction
   - Chapman-Enskog expansion
   - conclusions (CR test particle transport)

Why need two different approaches to CR transport?

simple facts about CRs in SNR and in ISM

- $D_{\|} \simeq D_{Bohm}\,(B/\delta B)^2 \rightarrow D_{\|}$ varies between $\sim D_B$ (in CR source, e.g. SNR) and $D_{ISM} \sim 10^5\,D_{Bohm}$ ("quiet" ISM).
- Away from the sources (SNR), CR energy is comparable to magnetic energy, thermal plasma $\gtrsim$ star light and CMB all $\sim eV/cc$
- Therefore, in and around a CR source (SNR) $P_{CR} \gg B^2/8\pi$
- Therefore, CRs cannot escape the source without driving strong MHD waves, so
- need to evolve CR from strong self-confinement regime $D \sim D_{Bohm}$ in and around the source to their dissolution into the ambient ISM, $D \sim D_{ISM} \ll D_{Bohm}$

  $\rightarrow$Interface Problem

Problems addressed under self-confinement/free diffusion regime

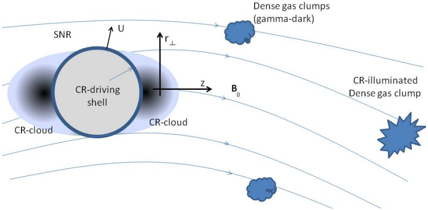
Self-confinement

- acceleration (Bell 77)
- propagation (Voelk, Wentzel, rev. ca 73-74)
- illumination of adjacent MC (e.g., Aharonian, Drury, Voelk)
- source morphology
- source calorimetry...

"Free" propagation

- CR background spectra
- CR chemical composition, spectral anomalies
- p/He Pamela (Adriani 2011)
- Explanations: Drury; Ohira; Blasi; Ptuskin; MM, Diamond, Sagdeev 2012
- CR anisotropy, particularly sharp $\sim 10^{\circ}$ Milagro (Abdo 2008)
- Interpretations: Drury&Aharonian; Desiati&Lazarian; MM, Diamond, Drury and Sagdeev (ApJ 2010), Giacinti, Ahlers (coh. turb. realization, statistical effects), (MM, PoP 2015)

CR escape from SNR/Problem setting/geometry



CR escape along MF from two polar cusps of SNR

- CR diffuse along MF (⊥-propagation may be important, but too many issues, e.g. Kirk, Duffy and Gallant 96)
- generate Alfvén waves that suppress diffusion
- to obtain CR distribution both processes are to be treated self-consistently
- result will determine MC emissivity

Equations

- CR propagation in self-excited waves

$$\frac{d}{dt}P_{\mathrm{CR}}\left(p\right)=\frac{\partial}{\partial z}\frac{\kappa_{\mathrm{B}}}{I}\frac{\partial P_{\mathrm{CR}}}{\partial z}$$

$P_{\mathrm{CR}}\left(p\right)$ -partial pressure, $I\left(p\right)$ -wave energy, resonance $kp=eB_0/c$,
$d/dt=\left[\partial/\partial t+\left(U+C_{\mathrm{A}}\right)\partial/\partial z\right]$, $\kappa_B\sim cr_g\left(p\right)$

- Wave generation by ∇P_{CR} associated with the CR pitch-angle anisotropy

$$\frac{d}{dt}I=-C_{\mathrm{A}}\frac{\partial P_{\mathrm{CR}}}{\partial z}-\Gamma I$$

- QL integral (Sagdeev et al '61)

$$P_{\mathrm{CR}}\left(z,t\right)=P_{\mathrm{CR}0}\left(z'\right)-\frac{\kappa_{\mathrm{B}}}{C_{\mathrm{A}}}\frac{\partial}{\partial z}\ln\frac{I\left(z,t\right)}{I_0\left(z'\right)}$$

$z'=z-\left(U+C_{\mathrm{A}}\right)t$

Equations

- CR/Alfven wave coupling

$$\frac{\partial W}{\partial t}-\frac{\partial}{\partial z}\frac{1}{W}\frac{\partial W}{\partial z}=-\frac{\partial}{\partial z}\mathcal{P}_0\left(z\right)$$

$W=\frac{C_{\rm A}a(p)}{\kappa_{\rm B}(p)}I$ -dimensionless wave energy, $d/dt\approx\partial/\partial t$, $\mathcal{P}_0$-initial CR distribution, $|z/a|<1$

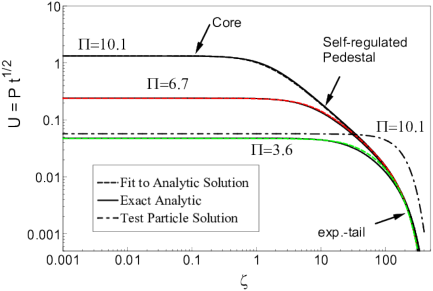
- Self-similar solution in variable $\zeta=z/\sqrt{t}$, $W\left(z,t\right)=w\left(\zeta\right)$ for $|z|>a$, outside initial CR cloud

$$\frac{d}{d\zeta}\frac{1}{w}\frac{dw}{d\zeta}+\frac{\zeta}{2}\frac{dw}{d\zeta}=0$$

- solution depends on a background turbulence level $W_0\ll1$, and integrated CR pressure in the cloud:

$$\Pi=\int_0^1\mathcal{P}_0dz\gg1$$

CR self-similar distribution



Self-confinement vs test-particle escape, $\sqrt{t}\mathcal{P}_{CR}$ vs $z/\sqrt{t}$ for different values of CR pressure Π (from MM, P. Diamond, R. Sagdeev, F. Aharonian and I. Moskalenko ApJ 2013)

Comparing and Contrasting with conventional TP predictions:

$$\Pi \simeq 3\frac{C_{\mathrm{A}}}{c}\frac{a(p)}{r_{\mathrm{g}}(p)}\frac{\bar{P}_{\mathrm{CR}}(p)}{B_0^2/8\pi} \gg 1$$

- considerable delay of CR escape
- narrower spatial distribution of CR cloud
- extended self-similar, $\mathcal{P} \propto 1/z$ region

Results/CR pressure distribution

- CR partial pressure (found in closed but implicit form) is well approximated by:

$$\sqrt{t}\mathcal{P} = 2\left[\zeta^{5/3} + (D_{\mathrm{NL}})^{5/6}\right]^{-3/5} e^{-W_0\zeta^2/4}, \quad \zeta = z/\sqrt{t}$$

- particle diffusivity is strongly suppressed by **self-confinement** effect:

$$D_{\mathrm{NL}} \sim D_{\mathrm{ISM}} e^{-\Pi},$$

- integrated CR pressure parameter is typically large:

$$\Pi \simeq 3\frac{C_{\mathrm{A}}}{c}\frac{a(p)}{r_{\mathrm{g}}(p)}\frac{\bar{P}_{\mathrm{CR}}(p)}{B_0^2/8\pi} \gg 1$$

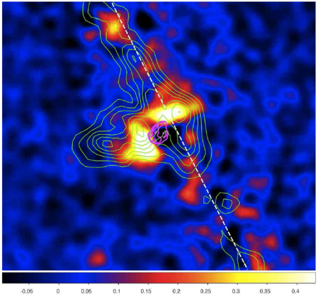
Breaks in Spectra of Escaping CRs

- normalized partial pressure $\mathcal{P}(p)$ approximation

$$\mathcal{P} \approx 2\left\{z^{5/3} + \left[D_{\rm NL}(p)\,t\right]^{5/6}\right\}^{-3/5}$$

- for $D_{\rm NL}(p) < z^2/t$ momentum independent (DSA TP $f \sim p^{-4}$)
- at $p = p_{\rm br}$, $\quad D_{\rm NL}(p_{\rm br}) = z^2/t$, $\quad D_{\rm NL} \propto p^{\delta}$, $\quad$ break index $= \delta/2$
- δ from $D_{\rm ISM}(p)$ and CR pressure $\Pi(p)$
- if $\exp(-\Pi) \propto p^{-\sigma}$ and $D_{\rm ISM} \propto p^{\lambda}$ at $p \sim p_{\rm br}$, so $\delta = \lambda - \sigma$, then
- $\mathcal{P}$ is flat at $p < p_{\rm br}$ for $\delta > 0$ and steepens to $p^{-\delta/2}$ at $p = p_{\rm br}$.
- if $\delta < 0$, $\mathcal{P}$ raises with p as $p^{-\delta/2}$ at $p < p_{\rm br}$ and it levels off at $p > p_{\rm br}$

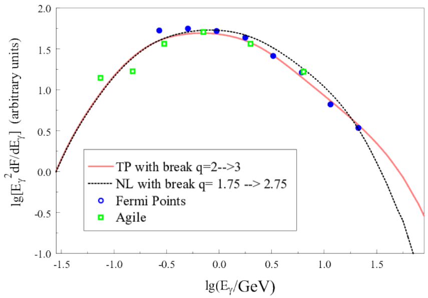
Morphological Signatures of CR Self-Confinement



Fermi-LAT γ-image of SNR W44, *Uchiyama et al 2012*

- central source (magenta radio image) emission is masked
- bi-polar morphology of escaping CR is clearly seen
- not everywhere correlated with the dense gas (green contours) distribution: strong γ-flux is expected from overlapping regions of CR and gas density
- strong indication of field aligned propagation
- CR diffusivity is suppressed by up to a factor of ten (*e.g. Uchiyama et al 2012*)

Spectral Signatures of the DSA and subsequent escape from W 44



γ -emission from MC near SNR W44

- presumably from dense MC illuminated by CR
- best fit is given by a TP source spectrum E^{-q}, $q = 2$, no cut-off required within
- CR subjected to propagation in evanescent Alfven waves inside MC
- wave evanescence and damping are due to ion-neutral collisions in MC
- $\rightarrow$break in the CR spectrum of index unity $E^{-q} \rightarrow E^{-q-1}$

Escape Summary and Conclusions

- escape of CR from accelerator is treated **self-consistently with self-generated Alfven waves**
- resulting CR distribution is obtained in a closed form
- strong **self-confinement** of CR is demonstrated for $P_{CR} \gg B^2/8\pi$
- results are consistent with recent observations of W44 by *Fermi*-LAT
- escape spectra are roughly DSA-like power laws with breaks (not peaked at $E_{\max}$)
- environment (target for pp reactions) is equally important to interpret observed emission from adjacent MC

Test Particle Transport: Basic Equation

CR transport driven by pitch-angle scattering, gyro-phase averaged

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left(1 - \mu^2\right) D\left(\mu\right) \frac{\partial f}{\partial \mu}$$

z -along $\mathbf{B}$; μ -cosine of CR pitch angle

- need evolution equation for

$$f_0\left(t, z\right) \equiv \left\langle f \right\rangle \left(t, z\right) \equiv \frac{1}{2} \int_{-1}^{1} f\left(\mu, t, z\right) d\mu.$$

Answer deems well known (e.g., Parker 65, Jokipii 66): average and expand in $1/D$:

$$\frac{\partial f_0}{\partial t} = -\frac{v}{2} \frac{\partial}{\partial z} \left\langle \left(1 - \mu^2\right) \frac{\partial f}{\partial \mu} \right\rangle, \quad \frac{\partial f}{\partial \mu} \simeq -\frac{v}{2D} \frac{\partial f_0}{\partial z}$$

“Master” Equation

- equation for f_0

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial z}\kappa\frac{\partial f_0}{\partial z} \quad \kappa = \frac{v^2}{4}\left\langle\frac{1-\mu^2}{D}\right\rangle$$

- Critical step: $\partial f/\partial t$ is neglected compared to $v\partial f/\partial z$
- Justification: for $Dt \gtrsim 1$ anisotropic part $\tilde{f} = f - f_0$ must largely decay $\propto e^{-\lambda_1 Dt}$
- higher orders → suggest retaining $\partial f/\partial t \rightarrow \partial^2 f_0/\partial t^2$ and higher derivative terms in “master” equation, e.g. Earl 1973, Litvinenko & Schlickeiser 2013
- arrive at telegrapher’s equation:

$$\frac{\partial f_0}{\partial t} - \frac{\partial}{\partial z}\kappa\frac{\partial f_0}{\partial z} + \tau\frac{\partial^2 f_0}{\partial t^2} = 0$$

where $\tau \sim 1/D$, $\kappa \sim v^2/D$

Pros and Cons of telegraph equation

$$\frac{\partial f_0}{\partial t} - \frac{\partial}{\partial z}\kappa\frac{\partial f_0}{\partial z} + \tau\frac{\partial^2 f_0}{\partial t^2} = 0$$

Pros

- ameliorates the major defect of diffusive approximation, infinite propagation velocity (UHECR: Aloisio $ Berezinsky, 2013)
- allows for a wave-like transport of CR clouds
- keeps the transport description simple

Cons

- parabolic equation becomes hyperbolic
- no longer an evolution equation
- cannot be solved with no recourse to lower-level equation (needed to compute $\partial f_0/\partial t$ at $t = 0$)
- infinite sequence of $\partial^n f_0/\partial t^n$ -terms will result from iterations
- smells of not properly handled (eliminated) secular terms

Looking ahead: Is the telegraph term real? TE: $\frac{\partial f_0}{\partial t} - \frac{\partial}{\partial z}\kappa\frac{\partial f_0}{\partial z} + \tau\frac{\partial^2 f_0}{\partial t^2} = 0$

In a sense... yes, BUT!

- coefficient τ at $\partial^2 f_0/\partial t^2$-term obtained by simple iteration in $1/D$, is numerically incorrect
- neglected $\sim \partial^4 f_0/\partial z^4$-term contributes to the same order
- $\sim \partial^3 f_0/\partial z^3$ contribute to even lower order unless $D(\mu)$ is symmetric and the term zeroes out
- being converted into $\partial^2 f_0/\partial t^2$ term the term $\sim \partial^4 f_0/\partial z^4$ alters τ
- τ-term is subdominant compared to the other two for $Dt \gtrsim 1$ and may largely be ignored altogether in the long time transport

Basic Equation with magnetic focusing

- gyro-phase averaged equation with magnetic focusing

$$\frac{\partial f}{\partial t}+v\mu\frac{\partial f}{\partial z}+v\frac{\sigma}{2}\left(1-\mu^2\right)\frac{\partial f}{\partial \mu}=\frac{\partial}{\partial \mu}\nu D\left(\mu\right)\left(1-\mu^2\right)\frac{\partial f}{\partial \mu}$$

$\sigma=-B^{-1}\partial B/\partial z$ - magnetic mirror inverse scale; ν -pitch angle scattering rate, while $D\left(\mu\right)\sim 1$

- small parameter

$$\varepsilon=\frac{v}{l\nu}=\frac{\lambda}{l}=\frac{CR\ m.f.p.}{problem\ scale}\ll 1$$

l - scale of the problem;
$\tau\nu\rightarrow\tau$; $z/l\rightarrow z$; $\sigma l\rightarrow\sigma\sim 1$

$$\frac{\partial f}{\partial t}-\frac{\partial}{\partial \mu}D\left(\mu\right)\left(1-\mu^2\right)\frac{\partial f}{\partial \mu}=-\varepsilon\left(\mu\frac{\partial f}{\partial z}+\frac{\sigma}{2}\left(1-\mu^2\right)\frac{\partial f}{\partial \mu}\right)$$

Formal Expansion in $\varepsilon \ll 1$

$$f = f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + \cdots \equiv f_0 + \tilde{f}$$

where

$$\langle f \rangle = f_0, \quad \text{with} \quad \langle \cdot \rangle = \frac{1}{2} \int_{-1}^{1} (\cdot)\, d\mu$$

"Master" equation

$$\frac{\partial f_0}{\partial t} = -\varepsilon \left(\frac{\partial}{\partial z} + \sigma \right) \left\langle \mu \tilde{f} \right\rangle$$

$$= \frac{\varepsilon^2}{2} \left(\frac{\partial}{\partial z} + \sigma \right) \sum_{n=1}^{\infty} \varepsilon^{n-1} \left\langle \left(1 - \mu^2\right) \frac{\partial f_n}{\partial \mu} \right\rangle$$

- fixed evolutionary structure
- similarly to Lorenz's gas (Gurevich 61, Kruskal &Bernstein) f_0 depends on "slow time" $t_2 = \varepsilon^2 t$ rather than on t.

Regular Expansion

- slow time in f_0 evolution suggests to attribute time derivative term to higher orders, so term $\propto \partial^2 f_0/\partial t^2$ appears, converting the convection - diffusion equation into a "telegraph" equation.
- However, $\tilde{f}$ does depend on t (as on the "fast" time)
- Ordering should be different ($f = f_0 + \varepsilon f_1 + \cdots \equiv f_0 + \tilde{f}$):

$$\frac{\partial f_n}{\partial t} - \frac{\partial}{\partial \mu} D(\mu)\left(1-\mu^2\right)\frac{\partial f_n}{\partial \mu}$$

$$= -\mu \frac{\partial f_{n-1}}{\partial z} - \frac{\sigma}{2}\left(1-\mu^2\right)\frac{\partial f_{n-1}}{\partial \mu} \equiv \Phi_{n-1}(t,\mu,z)$$

Chapman-Enskog analysis

- Solubility condition for f_2:
  $\langle \Phi_1 \rangle = \frac{1}{2} \left(\partial/\partial z + \sigma \right) \left\langle \left(1 - \mu^2\right)/D \right\rangle \partial f_0/\partial z = 0$
- Too strong restriction...
- However, f_0 depends on slow time t_2 suggesting multi-time expansion:
- CE: $\partial/\partial t = \left(\partial/\partial t\right)_0 + \varepsilon \left(\partial/\partial t\right)_1 + \dots$
- more customary is a hierarchy of independent variables $t \to t_0, \ t_1, \dots$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon \frac{\partial}{\partial t_1} + \varepsilon^2 \frac{\partial}{\partial t_2} \dots$$

$$\frac{\partial f_n}{\partial t_0} - \frac{\partial}{\partial \mu} D(\mu) \left(1 - \mu^2\right) \frac{\partial f_n}{\partial \mu} = \mathcal{L}_{n-1}[f]\left(t_0, \dots, t_n; \mu, z\right)$$

$$\equiv -\mu \frac{\partial f_{n-1}}{\partial z} - \frac{\sigma}{2} \left(1 - \mu^2\right) \frac{\partial f_{n-1}}{\partial \mu} - \sum_{k=1}^{n} \frac{\partial f_{n-k}}{\partial t_k}$$

Chapman-Enskog analysis

Solution can be written as

$$f_n = \overline{f}_n\left(t_2,\ldots;\mu\right) + \tilde{f}_n\left(t_0,\ldots;\mu\right)$$

where $\tilde{f}_n$ and $\overline{f}_n$ satisfy, respectively, the following equations:

$$\frac{\partial \tilde{f}_n}{\partial t_0} - \frac{\partial}{\partial \mu} D\left(\mu\right)\left(1-\mu^2\right)\frac{\partial \tilde{f}_n}{\partial \mu} = \mathcal{L}_{n-1}\left[\tilde{f}\right]\left(t_0,\ldots,t_n;\mu,z\right)$$

and

$$-\frac{\partial}{\partial \mu} D\left(\mu\right)\left(1-\mu^2\right)\frac{\partial \bar{f}_n}{\partial \mu} = \mathcal{L}_{n-1}\left[\bar{f}\right]\left(t_2,\ldots,t_n;\mu,z\right)$$

The solution for $\tilde{f}_n$ takes the form

Chapman-Enskog analysis

$$\tilde{f}_n = \sum_{k=1}^{\infty} C_k^{(n)}(t)\, e^{-\lambda_k t_0} \psi_k(\mu)$$

can be easily found for any n using eigenfunctions of diffusion operator

$$-\frac{\partial}{\partial \mu} D(\mu)\left(1-\mu^2\right)\frac{\partial \psi_k}{\partial \mu} = \lambda_k \psi_k,$$

For $D = 1$, for example, ψ_k are the Legendre polynomials with $\lambda_k = k(k+1)$, $k = 0, 1, \ldots$.

- constants $C_k^{(n)}$ are determined by initial conditions for $\tilde{f}_n$
- all $\tilde{f}_n$ exponentially decay in time for $t \gtrsim 1$
- Starting from $n = 0$

$$\frac{\partial \bar{f}_0}{\partial t_0} = 0$$

Chapman-Enskog analysis

- solubility condition for $\bar{f}_1$

$$\frac{\partial \bar{f}_0}{\partial t_1} = 0.$$

$$\bar{f}_1 = -\frac{1}{2} W \frac{\partial f_0}{\partial z}$$

where

$$\frac{\partial W}{\partial \mu} = \frac{1}{D}, \quad \langle W \rangle = 0.$$

- solubility condition for f_2 yields nontrivial result for master eq., the leading term of the $\partial f_0 / \partial t$ expansion in $\varepsilon \ll 1$

$$\frac{\partial f_0}{\partial t_2} = \frac{1}{4} \left(\frac{\partial}{\partial z} + \sigma \right) m \frac{\partial f_0}{\partial z}, \quad m = \left\langle \frac{(1 - \mu^2)}{D} \right\rangle .$$

Chapman-Enskog analysis

- solubility condition for f_3 and $f_4, \ldots$

$$\frac{\partial f_0}{\partial t_3} = -\frac{1}{4}\left(\frac{\partial}{\partial z}+\sigma\right)\left(\frac{\partial}{\partial z}+\frac{\sigma}{2}\right)\left\langle \mu W^2\right\rangle \frac{\partial f_0}{\partial z}$$

$$\frac{\partial f_0}{\partial t_4} = \frac{1}{8}\left(\frac{\partial}{\partial z}+\sigma\right)\times$$

$$\left\{\left(\frac{\partial}{\partial z}+\frac{\sigma}{2}\right)^2\left\langle W^2\left(U'-m\right)\right\rangle+\left(\frac{\partial}{\partial z}+\sigma\right)\frac{\partial}{\partial z}\left\langle\frac{\left[m\left(1-\mu\right)+U\right]^2}{2D\left(1-\mu^2\right)}\right\rangle\right\}\frac{\partial f_0}{\partial z}$$

where

$$U \equiv \int_{-1}^{\mu}\frac{1-\mu^2}{D}d\mu, \quad U' = \partial U/\partial\mu$$

Chapman-Enskog analysis

- process may be continued *ad infinitum* since terms containing $\left\langle \left(1-\mu^2\right) \partial f_n / \partial \mu \right\rangle$ can be expressed through $f_{n-1},\ f_{n-2}, \ldots$
- interested in evolving f_0 on time scales $t_2 \gtrsim 1$ or $t \gtrsim \varepsilon^{-2}$ neglect contributions of $\tilde{f}_n$ and retain only $\bar{f}_n$'s:
- master equation up to ε^4 :

$$\frac{\partial f_0}{\partial t} = \frac{\varepsilon^2}{4} \partial'_z \left\{ m - \varepsilon \partial''_z \left\langle \mu W^2 \right\rangle + \right.$$

$$\left. \frac{\varepsilon^2}{2} \left[\left(\partial''_z\right)^2 \left\langle W^2 \left(U' - m\right) \right\rangle + \partial'_z \partial_z \left\langle \frac{\left[m\left(1-\mu\right) + U\right]^2}{2D\left(1-\mu^2\right)} \right\rangle \right] \right\} \frac{\partial f_0}{\partial z}$$

here $\partial'_z = \partial_z + \sigma$ and $\partial''_z = \partial_z + \sigma/2$.

Is Telegraph Equation recoverable?

The structure of ME (magnetic focusing, asymmetry dropped)

$$\partial_t f = \varepsilon^2 \partial_z^2 \left(1 - \tau \varepsilon^2 \partial_z^2\right) f$$

or, to the leading order in ε, "formally"

$$\varepsilon^2 \partial_z^2 f \approx \partial_t f, \;\; \varepsilon^4 \partial_z^4 f \approx \partial_t^2 f$$

$$\frac{\partial f_0}{\partial t} - \varepsilon^2 \frac{\partial^2 f_0}{\partial z^2} + \tau \frac{\partial^2 f_0}{\partial t^2} = 0$$

- τ-term belongs to the fast time part of the CE reduction scheme
- associated with the anisotropic part of f, $\varepsilon \to 0$:

$$(1 + \tau \partial_t) \partial_t f = \mathcal{O}\left(\varepsilon^2\right)$$

- First solution: transient phenomenon, decays at $t \gtrsim \tau$
- Second solution: long time evolution $f = const$ for $\varepsilon = 0$

Conclusions (Test Particle CR transport)

- CR transport, constrained by scattering on magnetic irregularities revisited
- Chapman-Enskog approach revealed convective terms arising from the magnetic focusing effect, only
- no “telegrapher” (second order time derivative) term emerges in any order of the proper asymptotic expansion
- the telegraph $\partial^2/\partial t^2$ -term may formally be *back-converted* from the fourth order of expansion, (usually not entertained in the literature) as well as higher time derivative terms but they do not play any significant role in the CR transport within the method’s validity range