THEORY OF HEAT LOAD BROADENING BY ENTRAINMENT: FORMULATING A COST-BENEFIT ANALYSIS FOR TURBULENT PEDESTALS

P.H. DIAMOND
Department of Physics, University of California, San Diego
La Jolla, United States of America
Email: pdiamond@ucsd.edu

X. CHU[1], M. CAO[2], T. WU[3], Z.B. GUO[4]
1. Department of Astrophysics, Princeton University, Princeton, United States of America
2. Department of Physics, University of California, San Diego, La Jolla, United States of America
3. Center for Fusion Science, Southwestern Institute of Physics, Chengdu, China
4. School of Physics, Peking University, Beijing, China

Abstract

Developments in the theory of heat load broadening by entrainment of the stable SOL by pedestal turbulence is presented. Turbulent and neoclassical effects add in quadrature to set λ_q. The SOL intensity is determined by matching the turbulence energy flux from the pedestal to SOL. Explicit expressions are derived for λ_q for modest and strong broadening. Scalings of λ_q with R, B_θ, T_{sep}, q and spreading flux are determined. A fundamental limit on the extent of λ_q broadening is suggested. Spreading fluxes for drift wave and ballooning mode turbulence in the pedestal are derived and used to show that interesting levels of SOL broadening can be achieved for tolerable pedestal fluctuation levels. A simple treatment of the unavoidable departures from the realm of mean field theory is proposed.

1. THE PROBLEM

Edge transport barriers are supported by strong $E \times B$ shear layers at the separatrix, and typically manifest high edge temperatures as compared to ordinary L-mode plasmas. The resulting increased edge temperatures generate strong SOL $E \times B$ shear, of the form $v_E' \sim 3T_e/\lambda^2|e|$, where λ is the SOL width ($\lambda = \lambda_q$ here) [1]. Note that edge temperature and λ_q entirely determine the SOL $E \times B$ shear strength. In turn, the strong SOL $E \times B$ shear quenches the SOL turbulence which generally defines the SOL width in L-mode. The latter is given by the familiar expression $\lambda^2 \sim 2D_\perp \tau_d$, where τ_d is the SOL 'dwell time' $\tau_d = 2Rq/c_s$ and $D_\perp$ is the perpendicular turbulent diffusivity driven by SOL modes. Quenching of turbulence generated in the SOL reveals the residual neoclassical processes. These define the Heuristic Drift expression [2] for the SOL width $\lambda_q \sim v_D \tau_d$, and its counterpart for the conductive regime. Here $v_D \sim \rho_s c_s/R$ is the magnetic drift velocity. Alternatively put, draining the swamp of L-mode turbulence uncovers Goldston HD scaling. In addition, the strong $E \times B$ shears at the separatrix inhibits the penetration of pedestal turbulence into the SOL, though such turbulence spreading is still possible. Indeed, even in L-mode, turbulence spreading and intermittency have been shown to impact SOL widths [3]. The upshot is the HD model—a neoclassical scaling for λ_q. This prediction has the dubious distinction of being both exceedingly pessimistic ($\lambda \sim \epsilon_T \rho_{\theta_i}$) and remarkably successful. Thus, the heat load problem and the need to find a way to broaden λ_q have emerged as high priority issues for MFE. The theoretical problem is ultimately one of determining the scaling trends for the broadened λ_q. Limits on broadening are of particular interest.

2. A SOLUTION

Spreading of turbulence from the pedestal to the SOL is a possible solution of the heat load problem [4]. 'Turbulence spreading' is the transport of turbulence intensity by nonlinear scattering, and may be thought of as the real space transfer counterpart of the familiar story of cascading in k-space [5]. Turbulence spreading is sometimes referred to as 'entrainment'. A simple, familiar example of spreading is the cross-track expansion of a turbulent wake, downstream of a moving object. The wake expansion is a consequence of the invasion of laminar fluid by turbulence generated because of the object's passage. Spreading is energized by the object's motion against drag. This process is sketched in Fig. 1.

In the case of pedestal→SOL spreading, the *turbulence energy flux at the separatrix is of central importance.* It must be strong enough to overcome the edge barrier, and sufficient to energize the SOL against strong damping

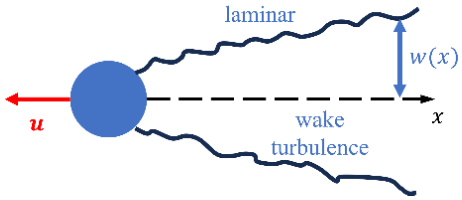

FIG. 1. Sketch of wake expansion due to turbulence spreading. The width of the turbulent region $w(x)$ expands with downstream distance x.

due to $E \times B$ shear and sheath effects. These requirements constrain the class of viable turbulent pedestals. And they highlight the critical issue of the trade-off between SOL broadening and confinement degradation.

While a few simulation papers have alluded to turbulence spreading effects on λ_q, none have progressed beyond the level of 'proof by color pictures'. No detailed theoretical or computational analysis of the relevant physics has been offered until after the publication in ref. [4] of the theoretical analysis described here.

In this paper we present the theory of SOL broadening by pedestal turbulence. The analysis proceeds in three stages:

(a) Part 1: Calculating λ_q for a turbulent SOL—in which we unite fluctuations and the neoclassical processes of the HD model.
(b) Part 2: Calculating SOL turbulence driven by spreading—in which we link SOL turbulence and λ_q to the spreading flux, which originates from the separatrix.
(c) Part 3: Relating the spreading flux to pedestal properties—in which we link the resulting λ_q to turbulence properties and turbulence structure.

2.1. Part 1: λ_q for a Turbulent SOL

To calculate the width of a stable SOL, consider the equation of motion for a particle undergoing magnetic drift v_D and turbulent 'kicks' ($\tilde{v}_r$), so

$$\frac{dr}{dt} = v_D + \tilde{v}_r. \tag{1a}$$

A simple calculation or a Fokker-Planck analysis then gives δ^2—the mean square excursion—as:

$$\delta^2 = v_D^2\tau_d^2 + \langle\tilde{v}_r^2\rangle\tau_c\tau_d, \tag{1b}$$

where $\tau_c = \int_0^\infty \tilde{v}_r(0)\tilde{v}(\tau)d\tau\,/|\tilde{v}_r(0)|^2$ is the turbulent correlation time. Since $\tau_c > \tau_d$ is unphysical and $\tau_c < \tau_d$ suggests very strong turbulence—which is irrelevant to this study—we take $\tau_c \sim \tau_d$. Thus

$$\begin{aligned}\delta^2 &= v_D^2\tau_d^2 + \langle\tilde{v}_r^2\rangle\tau_d^2 \\ &= v_D^2\tau_d^2 + e\tau_d^2\end{aligned}. \tag{1c}$$

So:

$$\lambda_q^2 \equiv \lambda_{HD}^2 + \lambda_T^2, \tag{1d}$$

where $\lambda_T^2 = e\tau_d^2$ is the square of the turbulent width and e is the SOL turbulence energy density. Note Eqn. (1d) is the outcome of a simple random walk argument, with the recognition that $\tau_c \leq \tau_d$. At this point, the expression for λ_q is formal, since we have yet to calculate e, the SOL turbulence field. In the next section, we relate e to the spreading flux from the pedestal.

2.2. Part 2: Calculating the SOL Turbulence *(e)* and λ_q Driven by Spreading.

We now express SOL fluctuation levels in terms of the influx of pedestal turbulence into the SOL. Employing a one-dimensional $K - \epsilon$ type model for the turbulence energy field e gives:

$$\partial_t e = \gamma e - \sigma e^{1+\kappa} - \partial_x \Gamma_e. \tag{2}$$

Here γ is the growth rate, so $\gamma < 0$ in the stable SOL, due to $E \times B$ shear and sheath effects. $\sigma e^{1+\kappa}$ is a model of nonlinear-dissipation, with σ a coefficient set by the underlying turbulence model. Here $\kappa \sim 1$ for weak turbulence while $\kappa \sim 1/2$ for strong turbulence. Γ_e refers to the flux of turbulence energy—i.e., the spreading flux. Note that Γ_e is simply an energy flux, and is not approximated by a Fickian form. Of course, the model of Eqn. (2) is minimal, and could be extended to 2D (in radius and along field line), with improved treatment of production, nonlinear dissipation, etc.

Integrating Eqn. (2) from the separatrix to the turbulence energy field width (i.e., the layer width for e—which is simply λ_T), and taking e as constant in the thin SOL layer gives, for $\partial e/\partial t = 0$,

$$\Gamma_{e,0} + \lambda_T \gamma e = \lambda_T \sigma e^{1+\kappa}. \tag{3}$$

Here $\Gamma_{e,0} = \Gamma_0$ hereafter is the flux of turbulence energy across the separatrix, from the pedestal to the SOL. Beyond the SOL, $\Gamma_e \to 0$, so there is no spreading outflow from the SOL. Γ_0 is the "spreading drive", and is the single control parameter in the theory characterizing the process of turbulence spreading. Obviously Γ_0 is a functional of pedestal profiles and parameters. Note that Γ_0 will be determined by pedestal turbulence localized slightly inside the separatrix. Eqn. (3) states that nonlinear damping in the SOL balances local growth and spreading drive. Thus, for the relevant case of $\gamma < 0$ (stable SOL):

$$\Gamma_0 = \lambda_T |\gamma| e + \sigma \lambda_T e^{1+\kappa}, \tag{4}$$

so spreading drive is ultimately balanced by linear damping (due damped SOL modes!) plus nonlinear dissipation. Of course, we will have:

$$\lambda_q^2 = \lambda_{HD}^2 + e\tau_d^2 = \lambda_{HD}^2 + \lambda_T^2. \tag{5}$$

Equations (4,5) constitute a simple, closed minimal model of turbulent SOL broadening.

Equations (4,5) may be solved analytically in the limits where either linear or nonlinear damping predominate. These correspond to weak and strong broadening, respectively.

For linear damping dominant, Eqn. (4) gives $\Gamma_0 \approx |\gamma| \lambda_T e$. But $\lambda_T^2 = e\tau_d^2$, so eliminating e gives

$$\lambda_T^3 = \Gamma_0 \tau_d^2 / |\gamma|, \tag{6a}$$

and

$$\lambda_q = \left[\lambda_{HD}^2 + \left(\frac{\Gamma_0 \tau_d^2}{|\gamma|} \right)^{2/3} \right]^{1/2}, \tag{6b}$$

which is the broadened heat load width. Here $|\gamma|$ is set by $E \times B$ shearing and sheath effects. Note that the increment to λ_q^2 induced by spreading scales as $(\Gamma_0/|\gamma|)^{2/3}$—i.e., increasing with Γ_0 and decreasing with SOL damping. This is intuitively plausible. For modest broadening, $|\gamma| \sim T_{sep}/|e|\lambda_{HD}^2$. So high T_{sep} and high current inhibit SOL broadening. The current scaling of λ_T differs from that of λ_{HD}. $\lambda_T \sim \Gamma_0^{1/3}$, so the dependence of the broadening upon spreading drive is modest.

The cross over from the HD to broadened width regime occurs when $(\Gamma_0 \tau_d^2/|\gamma|)^{2/3} = \lambda_T^2 > \lambda_{HD}^2$. This is equivalent to:

$$\left[\left(\frac{\Gamma_0}{|\gamma|} \right)^{2/3} \left(\frac{B_T}{B_\theta} \right)^{4/3} \frac{r^{4/3}}{c_s^{4/3}} \right] / \epsilon_T^2 \rho_\theta^2 > 1. \tag{7a}$$

The LHS of Eqn. (7a) scales as

$$LHS \sim \Gamma_0^{2/3} R^{4/3} B_\theta^{2/3} / T_{e,sep}^2, \tag{7b}$$

so

$$\lambda_T / \lambda_{HD} \sim \Gamma_0^{1/3} R^{2/3} B_\theta^{1/3} / T_{e,sep}. \tag{7c}$$

$T_{e,sep}$ can be related to P_{SOL}. Thus, larger machine size favors a broadened SOL, while higher separatrix temperature works against it. Sensitivity to spreading drive is modest. Higher current somewhat favors broadening beyond the HD prediction by squeezing λ_{HD} relative to λ_T. Γ_0 can be expected to contain implicit ρ_* scaling, which may off-set some of the explicit size scaling. Further study of the scalings of Γ_0 is required. In particular, the ρ_* scaling of the spreading flux remains a critical issue.

For the limit where nonlinear damping is dominant, $\Gamma_0 \approx \lambda_T \sigma e^{1+\kappa}$. Since once again $\lambda_T^2 = e\tau_d^2$ and $\lambda_q^2 = \lambda_{HD}^2 + \lambda_T^2$, a similar calculation to that above gives the heat load width

$$\lambda_q = \left[\lambda_{HD}^2 + (\Gamma_0/\sigma)^{\frac{2}{3+4\kappa}} \tau_d^{\frac{4(1+\kappa)}{3+2\kappa}}\right]^{1/2}, \quad (8a)$$

for the nonlinearly damped limit. For $\kappa = 1/2$ (strong turbulence):

$$\lambda_q = \left[\lambda_{HD}^2 + (\Gamma_0/\sigma)^{2/5} \tau_d^{3/2}\right]^{1/2}. \quad (8b)$$

For $\kappa = 1$ (weak turbulence):

$$\lambda_q = \left[\lambda_{HD}^2 + (\Gamma_0/\sigma)^{2/7} \tau_d^{8/5}\right]^{1/2}. \quad (8c)$$

Finally, note that in the limit of strong broadening, the (likely appropriate) choice $\kappa = 1/2$ gives:

$$\lambda_q \sim (\Gamma_0/\sigma)^{1/5} (Rq/c_s)^{3/4}, \quad (8d)$$

so $\lambda_q \sim R^{3/4} B_\theta^{-3/4} T_{e,sep}^{-3/8} (\Gamma_0/\sigma)^{1/5}$. Once again, we find off-setting trends in size and separatrix temperature, and weak dependence on turbulence spreading flux. Indeed, the scaling of λ_q with Γ_0 tends toward saturation as the layer broadens. This is consistent with expectations for strong turbulence regimes.

It is perhaps appropriate to point out that while SOL broadening is clearly desirable for heat load management, too much broadening can be counter-productive, as it may induce an H→L back transition. To see this, recall that SOL $E \times B$ shear scales as $v_E' \sim T_{e,sep}/|e|\lambda^2$ where $\lambda \sim \lambda_q$ is the SOL width. Thus, increasing λ weakens $E \times B$ shear. SOL stability to interchange modes is maintained by $E \times B$ shear, with the marginality condition set by the balance of interchange drive with $E \times B$ shearing $c_s/\sqrt{R\lambda} \approx T_{e,sep}/|e|\lambda^2$. Thus for $\lambda_q > \left(T_{e,sep}/|e|c_s\right)^{2/3} R^{1/3}$, SOL interchange turbulence can be excited. Brown and Goldston have suggested that for λ_q broadening by collisions (i.e., as for conductive heat transport at high density), SOL interchange destabilization can trigger an H→L back transition following invasion of the pedestal turbulence. The associated interchange marginality condition correlates well with the H-mode density limit, which is initiated by such an H→L back transition. *However, the proposed mechanism is far more general, and suggests a fundamental limit on the broadening of* λ_q *by turbulence spreading.* Using Eqn. (8d) for λ_q, interchange stability requires $(\Gamma_0/\sigma)^{1/5}(Rq/c_s)^{3/4} \le \left(T_{e,sep}/|e|c_s\right)^{2/3} R^{1/3}$. This bounds $\Gamma_0 < \Gamma_{max}$, where

$$\Gamma_{max} \cong (T_e/|e|c_s)^{10/3} R^{5/3} / (Rq/c_s)^{15/4} \sigma, \quad (9a)$$

so

$$\Gamma_{max} \sim T_{e,sep}^{85/24} / R^{15/4} q^{15/4}. \quad (9b)$$

Interestingly, Γ_{max} decreases with R and q, and increases with $T_{e,sep}$. *Γ_{max} constitutes a fundamental limit on the spreading flux which can maintain a stable SOL.* It is likely an upper bound on the flux, since spreading into a weakly damped or marginal region can be expected to result in strong excitation of turbulence. Clearly, the two-sided implications of strong SOL broadening merit further study. Fig.2 shows a plot λ_q/λ_{HD} vs. $\Gamma_{e0} = \Gamma_0$, the spreading flux. The plot shows results with both linear and nonlinear damping, and with nonlinear damping alone. For the relevant first case, linear and nonlinear regimes are apparent, with a cross-over regime connecting them. Note that the tendency of λ/λ_{HD} to approach saturation for large Γ_0 is evident. Fig. 2 shows the crucial role of Γ_0 as a control parameter for the SOL width.

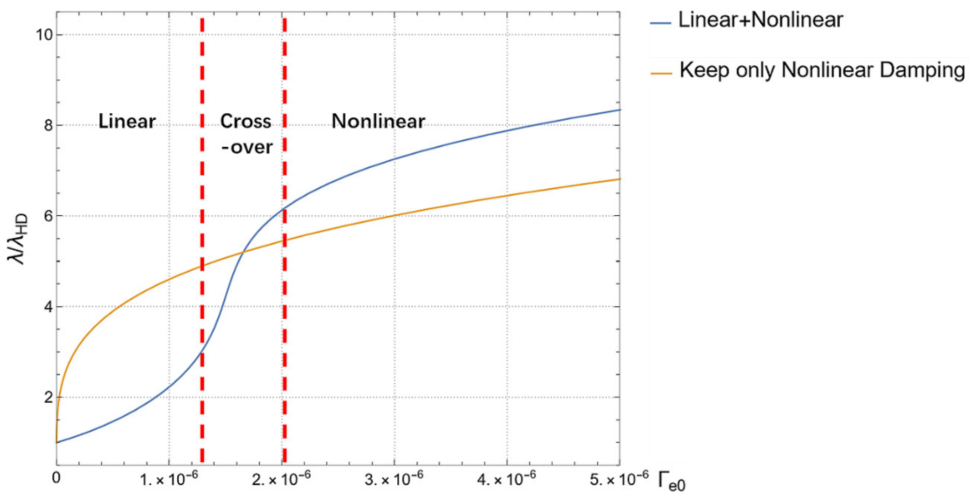

FIG. 2. λ_q/λ_{HD} plotted vs. spreading flux Γ_0 from the pedestal for $q = 4$, $\beta = 0.001$, $\kappa = 1/2$, $\sigma = 0.6$.

2.3. Part 3: Relating the Spreading Flux to Pedestal Properties

Here, we estimate the spreading flux in terms of pedestal parameters and structure. This addresses the question of what exactly is Γ_0. An additional concrete output of this analysis is the scaling of the pedestal fluctuation level needed to broaden the SOL. Some general comments are in order before proceeding to the calculation.

(a) Pedestal turbulence located close to the separatrix necessarily has the greatest impact on SOL broadening.
(b) The edge transport barrier (shear layer) will necessarily tend to inhibit spreading through the separatrix.
(c) Pedestal turbulence with larger mixing length will be more effective for turbulence spreading. This favors larger scale modes.
(d) Turbulence spreading into the SOL from the pedestal is almost certainly both convective and diffusive (i.e., driven by intensity gradient), and partly mediated by the dynamics of structures, such as blobs and voids. However, our understanding of how to actually *calculate* the non-diffusive flux is still developing. Hence this analysis is limited to a diffusive model of spreading.

The mean flux of turbulence kinetic energy from the pedestal to the SOL is given by

$$\Gamma_0 = -\tau_c e \partial_r e \approx \tau_c e^2 / w_{ped}. \tag{10}$$

Here τ_c is the pedestal turbulence correlation time and w_{ped} is the pedestal width, which serves as an estimate of the scale of pedestal turbulence intensity. Fig. 7 of ref. [4] shows a sketch of diffusive turbulence spreading in the pedestal. Equation (10) can be simplified by considering diffusive scattering in the presence of shearing so as to elucidate τ_c. The Kubo formalism gives:

$$D = \int_0^\infty \langle v(0)v(\tau)\rangle d\tau = \int_0^\infty d\tau \sum_{\boldsymbol{k}} |v_{\boldsymbol{k}}|^2 \exp[-k_\theta^2 \omega_s^2 D\tau^3 - k^2 D\tau]\,. \tag{11}$$

Here ω_s is the shearing frequency. In the relevant strong shear limit, $\tau_c^{-1} = k^2 D(1+\omega_s^2\tau_c^2)$ and $D \sim |v|^{3/2} k^{-1/2} \omega_s^{-1/2}$. The pieces may then be assembled to yield

$$\Gamma_0 \cong \tau_k^{0.5} \omega_s^{-0.5} e \partial_r e \cong \tau_k^{0.5} \omega_s^{-0.5} e^2 / w_{ped}, \tag{12a}$$

where

$$\omega_s = \partial_r \nabla p / n|e| \sim \left(\rho_i^2 / w_{ped}^2\right) \Omega_i. \tag{12b}$$

Here τ_k is the eddy turn-over rate $1/k|v|$. We have taken the $E \times B$ shear to be due to ∇p (i.e., neglected rotation). Note that $\Gamma_0 \sim \tau_{eff} e^2 / w_{ped}$, where $\tau_{eff} \sim (\tau_k/\omega_s)^{1/2}$, the geometric mean of the two time scales. ρ_i is the ion gyro-radius and Ω_i the ion cyclotron frequency. Equations (12a, b) give a general expression for the spreading flux through the ETB shear layer to the SOL, for the case of electrostatic turbulence. Note also that ρ_* dependence can enter both via the pedestal turbulence level and via the shearing frequency.

At this point, Eqns. (12a, b) can be coupled to a pedestal turbulence model and used to calculate Γ_0, which may then be inserted into Eqns. (6b) and (8a) to obtain λ_q. We use the expressions for λ_q and Γ_0 to determine the level of pedestal turbulence required to render $\lambda/\lambda_{HD} \geq 1$—i.e., the level required to broaden the layer beyond HD. For illustrative purposes, we consider both microturbulence (collisional drift waves) and ideal ballooning modes (representative of grassy ELMs, pedestal with large scale turbulence, etc.). The outcomes are formulated in terms of the fluctuation level required to broaden the layer. This result is a simple cost-benefit analysis of turbulent SOL broadening. Modest required fluctuation levels means that the cost is acceptable. High means that the cost is prohibitive.

For drift waves, one expects the basic correlation time scaling $\tau_{c,0} v_* \sim \rho_i$, where v_* is the diamagnetic velocity. Then using Eqns. (10, 12a, 12b) and taking the mixing length as scaling with ρ_i, we find the critical required fluctuation level scales as:

$$\tilde{v}_r / c_s \sim \frac{|e|\hat{\phi}}{T} \sim \left(\frac{\rho_i}{R}\right)^{1/2} q^{-1/4} \tag{13a}$$

for the weak $E \times B$ shear limit. For the strong $E \times B$ shear limit

$$\tilde{v}_r/c_s \sim \frac{|e|\hat{\phi}}{T} \sim \left(\frac{\rho_i}{R}\right)^{1/2} q^{-1/4}\left(w_{ped}/\rho_i\right)^{-1/8}. \tag{13b}$$

Note that in both expressions, the key factor is $(\rho_i/R)^{1/2}$. This indicates that the minimal fluctuation level is not excessive. The $(\rho_i/R)^{1/2}$ scaling also suggests that the critical levels of broadening can be achieved in larger devices, with higher toroidal field. Fig. 3 plots λ_q/λ_{HD} vs. pedestal drift wave $|e|\hat{\phi}/T$ in cases of linear damping only, nonlinear damping only and combined damping. Of course, the linear damping only model fails for larger levels of $|e|\hat{\phi}/T$. The combined model clearly shows a weak fluctuation, linear regime, a cross-over regime, and a strong broadening regime. The cross-over regime is of greatest interest and relevance. Results indicate that broadenings of λ_q/λ_{HD} from $3 \rightarrow 5$ are possible for modest fluctuation levels $\left(|e|\hat{\phi}/T \leq 0.05\right)$. These are understood to be for the level at the separatrix. Note this figure is a defacto 'price list' for the cost of SOL broadening. It tells the shopper what he must 'pay' in pedestal fluctuations and transport to 'purchase' a desired λ/λ_{HD}.

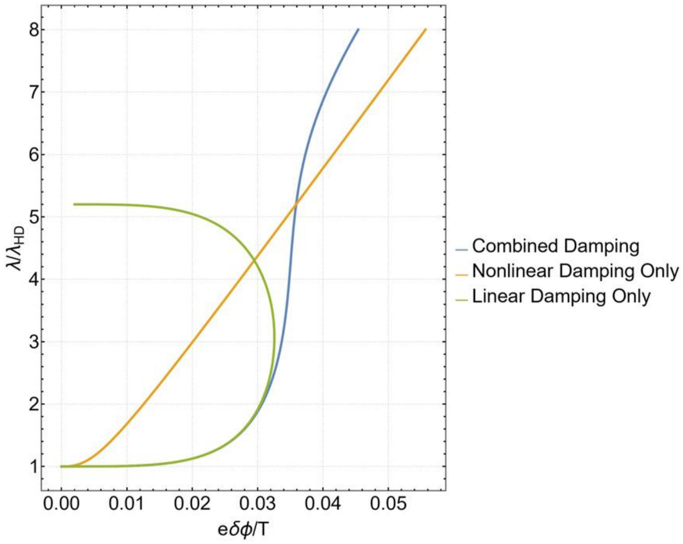

FIG. 3. A typical case for DW: the normalized pedestal width λ_q/λ_{HD} plotted against the normalized pedestal fluctuation level $|e|\hat{\phi}/T$.

We now turn to the case of pedestal MHD turbulence, as found in Grassy ELMs or Turbulent QH-mode. For simplicity, we consider the case of weak ideal ballooning turbulence. Of course, the mode growth rate is

$$\gamma_b^2 \sim \omega_A^2\left(L_{pc}/L_p - 1\right), \tag{14}$$

where $\omega_A = v_A/Rq$, L_p is the pressure gradient scale length, and L_{pc} is the critical gradient. Then $\tau_{c,0} \sim 1/\gamma_b$ gives

$$\omega_s \tau_{c,0} = \sqrt{\beta} qR\rho/w_{ped}^2\left(L_{pc}/L_p - 1\right)^{1/2}. \tag{15}$$

Turbulence levels are simply $\tilde{v} \sim \gamma\Delta$, where Δ is the radial displacement associated with the ballooning mode. Results are formulated as the degree of super-criticality required to achieve broadening.

For Grassy ELMs, this margin should be small. The results are:

$$\text{weak } E \times B \text{ shear:} \quad \frac{L_{pc}}{L_p} - 1 \sim \left(q\frac{\rho_i}{R}\right)^{4/3} \frac{R^2}{w_{ped}^2}\left(\frac{w_{ped}}{\Delta}\right)^{8/3} \beta, \tag{16a}$$

$$\text{strong } E \times B \text{ shear:} \quad \frac{L_{pc}}{L_p} - 1 \sim \left(q\frac{\rho_i}{R}\right)^{10/7} \left(\frac{R}{w_{ped}}\right)^{16/7} \left(\frac{w_{ped}}{\Delta_r}\right)^{16/7} \beta. \tag{16b}$$

In both limits, the required supercriticality scales with $(\rho_i/R)^{\alpha}$, $\alpha > 1$ and with β, and thus is seen to be quite modest. We see that a state of quasi-marginal ballooning turbulence is sufficient to achieve the cross-over level for SOL width broadening. And once again we note a ρ/R scaling which is favorable for future larger devices.

Fig. 10a, b of ref. [4] shows λ/λ_{HD} vs. the margin of supercriticality for weak and strong shear cases. Note broader modes (Δ/L_p larger) are more effective at spreading out the heatload.

A few comments are in order here, to conclude section 2.

(a) Sensitivity analysis reveals that those results are more sensitive to linear damping in the SOL than to the details of nonlinear scattering.

(b) There is little difference between cases of weak and strong $E \times B$ shear. This is due to off-setting trends in τ_c and w_{ped} in the expression for $\Gamma_{e,0}$.

(c) Larger scale turbulence near the separatrix is more effective at SOL broadening.

(d) The calculation of the spreading flux needs to be revisited, so as to incorporate intermittency effects. Recent simulations [6,7] and experiments [8] indicate that the spreading flux is strongly skewed, with skewness vanishing at a radius close to the separatrix. The turbulence exhibits spatio-temporal intermittency, and thus is a challenge to model. The turbulence exhibits spatio-temporal intermittency. Turbulence spreading into the SOL thus consists of positively skewed fluctuations, which may be thought of as 'blobs'. The effects of these structures are not addressed by the diffusive spreading flux model employed here. Clearly, the Fickian model of the spreading flux is inadequate. A complete model of spreading remains an unfulfilled goal, and an important one.

3. BEYOND MEAN FIELD THEORY

Until now, the SOL broadening model has been formulated as a mean field theory. The central role of the mean spreading flux Γ_0 as the key control parameter is the most evident symptom of this restriction. However, edge turbulence is intermittent, so the spreading flux can be expected to exhibit strong fluctuations. Thus, $\Gamma_e = \langle \Gamma_e \rangle + \tilde{\Gamma}_e$, with $|\tilde{\Gamma}_e| \geq |\langle \Gamma_e \rangle|$—i.e., large fluctuations. Indeed, it's likely that the $pdf(\tilde{\Gamma}_e)$ is strongly non-Gaussian, with divergent second moment. Pulses or avalanches [9] then emerge as a natural description of the dynamics. Indeed, wake expansion has long been thought of as due to an ensemble of localized jets, leading to a rippled interface [10]. Here, we consider pressure fluctuations δp, with $\delta p \sim \nabla p - \nabla p_{crit}$, and with $\delta p \sim \delta e$, so intensity pulses track avalanches. $\delta p > 0$ corresponds to 'blobs', and $\delta p < 0$ to 'voids'. The theory of continuum avalanching is built upon a Burgers model, derived from considerations of a conserved order parameter and joint reflection symmetry. In its simplest form, the equation the equation for δp is

$$\partial_t \delta p + \alpha \delta p \partial_x \delta p - \nu \partial_x^2 \delta p = \tilde{s}. \tag{17}$$

Here the nonlinear term represents steepening and $\tilde{s}$ is the driving noise source. α is a model dependent coefficient. δp is ultimately realized as an ensemble of shock trains.

Applicability to the SOL problem requires some additional elements in the model. Most important is that finite SOL dwell time $\tau_d = \tau$ introduces dissipation and breaks order parameter conservation, and so is represented by a Krook damping term. Second, magnetic drift velocity must be retained, to recover HD physics for weak fluctuation levels. Noise is replaced by boundary flux fluctuations in time. These act as a source. The pulse model for the SOL then becomes:

$$\partial_t \delta p + v_D \partial_x \delta p + \alpha \delta \partial_x \delta p - D_0 \partial_x^2 \delta p + \delta p/\tau = 0, \tag{18a}$$

with

$$\delta p(0,t) \sim \tilde{\Gamma}_{e,sep}(t). \tag{18b}$$

D_0 is added for regularization. In the limit of weak fluctuations, Eqn. (18a) becomes $v_D \partial_x \delta p + \delta p/\tau \sim 0$ which sets a scale $\lambda \sim v_D \tau \sim \lambda_{HD}$. Thus, HD scalings are recovered. For scattering to matter, $\alpha \delta p > v_D$ is required. The structure of Eqn. (18a) is that of Burgers + Krook model, hereafter referred to as a 'Krooked Burgers' (KB) equation. A striking feature of the KB Eqn. (18a) is that due to Krook damping, a critical perturbation gradient at the boundary is required to form shocks, which survive penetration. Recall that for

$$dp/dt = -\delta p/\tau \tag{19a}$$

with characteristic equation

$$dx/dt = \alpha \delta p. \tag{19b}$$

Solution by method of characteristics gives:

$$x = \alpha \left[z + \frac{\left(1 - e^{-\epsilon/\tau}\right)}{1/\tau} f(z) \right], \tag{19c}$$

where $f(z)$ is set by boundary data. Shocks occur for $f'(z) < -1/\tau$. This condition states that the initial perturbation slope (i.e., slope at the separatrix) must be sufficiently steep so as δp will shock before damping, in a dwell time. This sets a pulse formation criterion $\alpha \partial \delta p/\partial x|_{sep} < -1/\tau$, which is defined by the (very variable and intermittent!) size of the perturbation gradient at the separatrix. Perturbations which satisfy the criterion will propagate as coherent structures (shocks) into the SOL with a finite penetration length. Perturbations which do not satisfy the criterion will rapidly decay and evaporate, with only weak penetration. SOL broadening will be determined by the population of penetrating coherent structures. The aim of the theory, then, is to characterize the statistics of the pulses and penetration depth distribution in terms of the $Pdf(\partial_x \delta p)$ at the separatrix. Utility demands that the latter must ultimately be related to macroscopics. Note that the penetration depth distribution is analogous to the calculation of λ_T in terms of Γ_0 in mean field theory. Likewise, relating the $Pdf(\partial_x \delta p)$ to macroscopics is analogous to the calculation of the spreading flux Γ_0 in terms of pedestal parameters and properties. In both formulations the latter part of the problem is more difficult. Characterizing the statistics is thus a challenging problem, requiring significant future effort. $E \times B$ shear can be included by formulating a 2D version of the theory—a double Krooked Burgers model. $E \times B$ shear is well known to impact intermittency [11], spreading and structure populations. Finally we remark that the incidence of spreading and avalanching driving SOL turbulence suggests that SOL profiles are likely *not* pure exponentials [12].

ACKNOWLEDGEMENTS

We thank Zeyu Li, Nami Li, Rongjie Hong, Ting Long, Xi Chen, G. Tynan, Filipp Khabanov, Zheng Yan, G. McKee, X-Q Xu, and Jose Boedo for many interesting discussions and for collaboration on related works. This research was supported by DOE under Grant No. DE-FG02-04ER54738.

REFERENCES

[1] BROWN, A.O., GOLDSTON, R.J., Generalization of the Heuristic Drift SOL model for finite collisionality and effect on flow shearing rate vs. interchange growth rate, Nucl. Mater. Energy **27** (2021) 101002.

[2] GOLDSTON, R.J., Heuristic drift-based model of the power scrape-off width in low-gas-puff H-mode tokamaks, Nucl. Fusion **52** 1 (2011) 013009.

[3] WU, T., DIAMOND, P.H., et al., How turbulent transport broadens the heat flux width: local SOL production or edge turbulence spreading? Nucl. Fusion, **63** 12 (2023) 126001.

[4] CHU, X., DIAMOND, P.H., GUO, Z.B., SOL width broadening by spreading of pedestal turbulence, Nucl. Fusion, **62** 6 (2022) 066021.

[5] HAHM, T.S., DIAMOND, P.H., Mesoscopic transport events and the breakdown of Fick's law for turbulent fluxes, J Korean Phys Soc **73** (2018) 747-792.

[6] LI, Z., et al., Nat. Commun., submitted (2023).

[7] Li, N., et al., Nucl. Fusion, submitted (2023).

[8] KHABANOV, F., et al., "Statistical properties of turbulence at the edge of DIII-D L-mode positive and negative triangularity plasmas", presented at US Transport Task Force on Boundary/SOL, Madison, 2023.

[9] DIAMOND, P.H., HAHM, T.S., On the dynamics of turbulent transport near marginal stability, Phys. Plasmas **2** 10 (1995) 3640-3649.

[10] TOWNSEND, A.A., Momentum and energy diffusion in the turbulent wake of a cylinder, Proc. R. Soc. A: Math. Phys. Eng. Sci. **197** 1048 (1949) 124-140.

[11] BOEDO, J.A., RUDAKOV, D.L., et al., Transport by intermittency in the boundary of the DIII-D tokamak, Phys. Plasmas **10** 5 (2003) 1670-1677.

[12] DIAMOND, P.H., "The SOL as a Turbulence-Driven Boundary Layer: Implications for Heat Load Scalings", CO06.00004, presented at APS-DPP on Divertor & Scrape-Off-Layer, remote, 2020.