On the Physics of Intrinsic Flow in Plasmas Without Magnetic Shear

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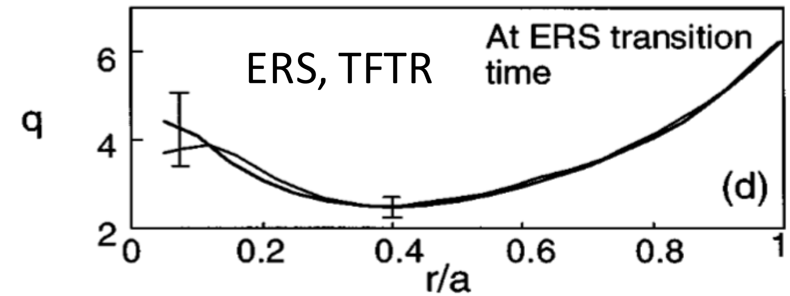
Acknowledgment: This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DE-FG02-04ER54738

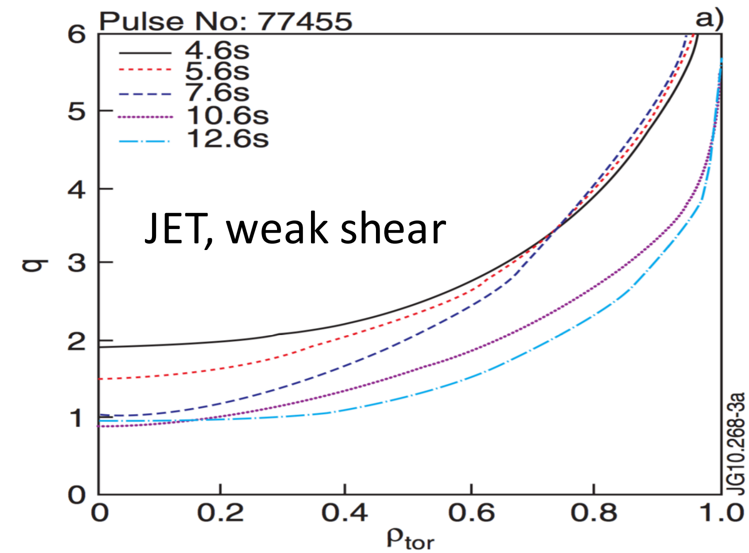
Outline

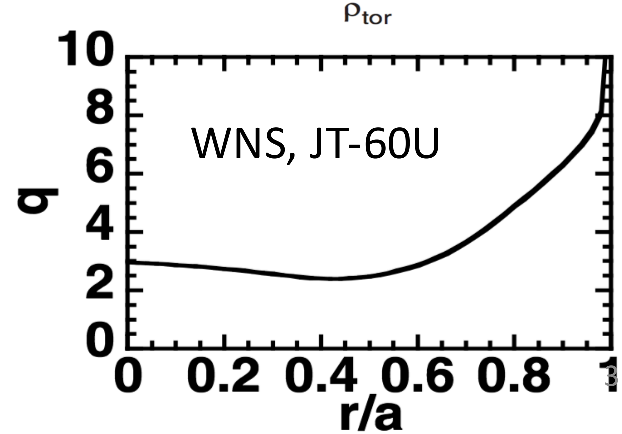
- Merits of weak magnetic shear + rotation for confinement
- Question: intrinsic rotation with weak shear?
- CSDX: a test bed for intrinsic flows in unsheared magnetic fields
  - Intrinsic axial flow
- New mechanism for intrinsic flows
  - No requirement for magnetic symmetry breaking
  - Builds on electron drift wave turbulence
- Broader lesson for tokamaks
  - Not limited to weak shear regimes
  - Outcome: enhanced $\nabla\langle v_\phi\rangle$
- Future work: flow boundary layer ↔ ion-neutral coupling

Why "weak shear" profile?

- Long known:
- weak or reversed shear beneficial for confinement (ERS, NCS, WNS, …)
  - B.W. Rice (DIII-D), PPCF, 1996;
  - E.J. Synakowski (TFTR), PoP, 1997;
  - M. Yoshida (JT-60U), NF, 2015…
- De-stiffened, ITB-like state observed for "weak shear" (JET)
  - P. Mantica, PRL, 2011;
  - C. Gormazano, PRL, 1998, etc.
- For more, see previous talks by Dr. Turco, Dr. Pan.







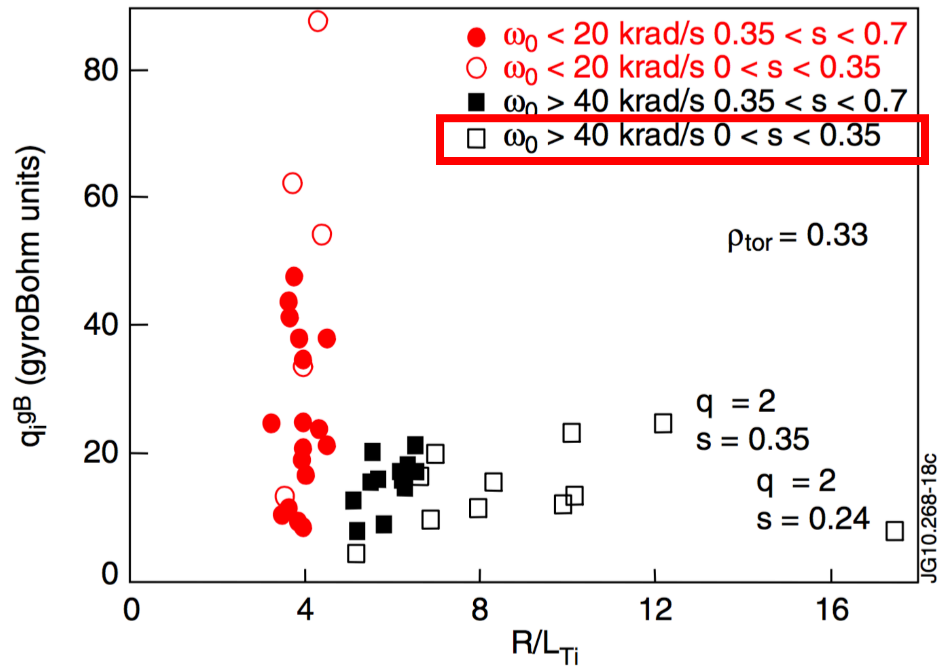
Intrinsic Rotation in Weak Shear Profiles

- JET: Weak shear **AND** Rotation → Enhanced confinement
- But external torque limited in ITER
- So need understand: Intrinsic rotation in weak shear regimes

- Important for:
  - Total effective torque

    $\tau = \tau_{ext} + \tau_{intr}$

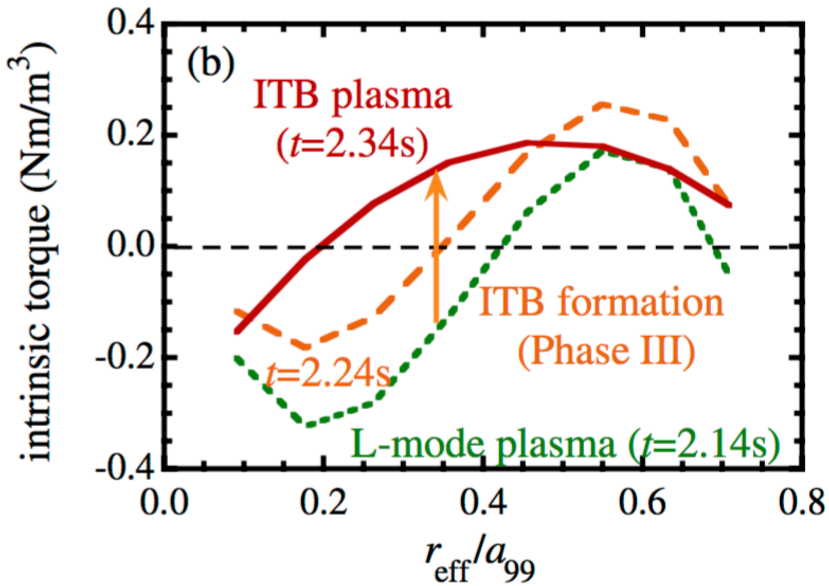
  - Contribution to $V'_{E\times B}$

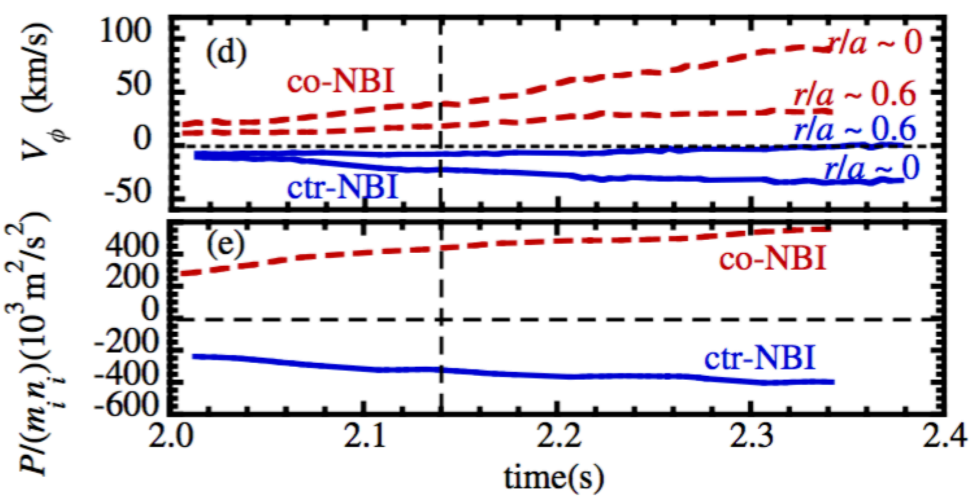


[P. Mantica, PRL, 2011; Rice, PRL, 2013]

FIG. 4 (color online). q_i^{GB} vs R/L_{T_i} at $\rho_{tor} = 0.33$ for similar plasmas with different rotation and s values.

Intrinsic Rotation in ITB





[Ida (LHD), PRL, 2013]

- Intrinsic rotation: self-accelerated toroidal rotation
- Discovered in JFT-2M, C-Mod (~ 95')
- During ITB formation:
  - τ_{intr} in the core flip sign
  - Build up from edge
- $\tau_{intr} \sim \tau_{ext}$
- Strong coupling between heat transport and momentum transport
- Consistent with

$$\Delta\langle v_\phi \rangle \sim \nabla T, \nabla P$$

Intrinsic Rotation in Weak Shear

- Weak shear ($q' \rightarrow 0$)
- External torque $\cong 0$

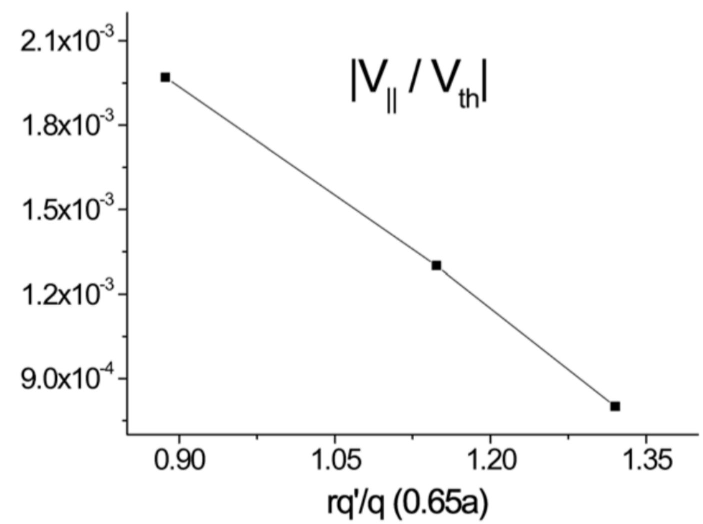
Intrinsic Rotation?

- Beneficial for confinement and stability. How much?

[G.M. Staebler and H.E. St John, NF Lett., 2006]

Status:

- Results
  - GK Simulation: stronger intrinsic rotation at weaker magnetic shear
- Problem
  - Intrinsic rotation requires symmetry breaking
  - Conventional symmetry breaking models fail
  - Most involve magnetic shear
  - But weak shear
    → non-resonant mode structure!



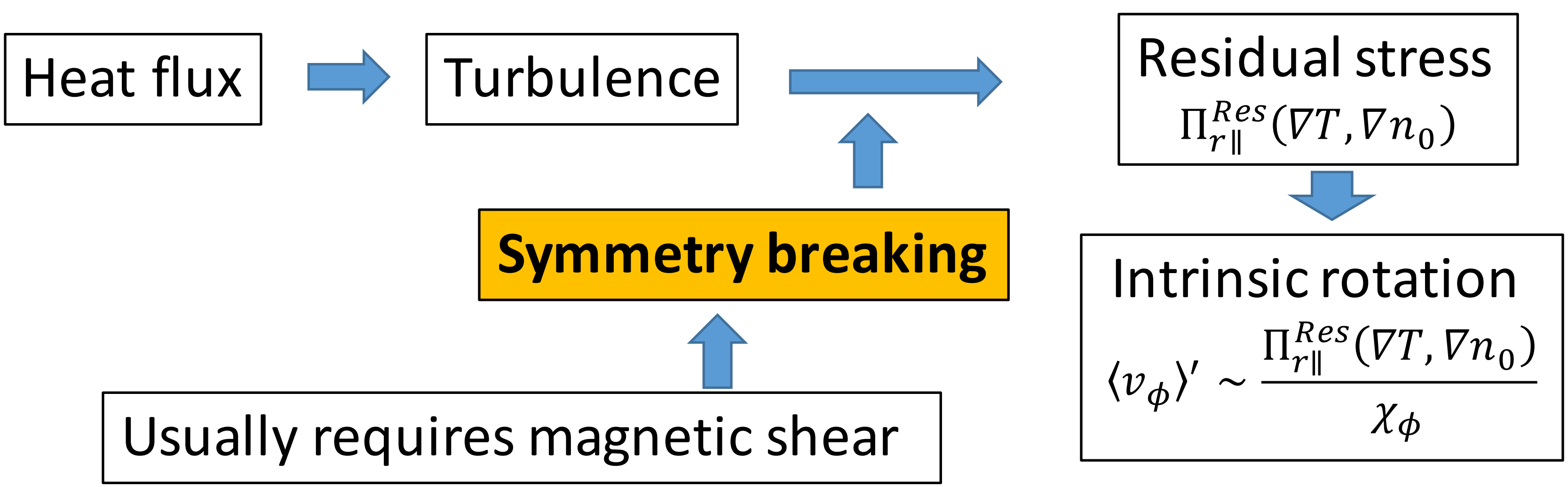
[Kwon, NF (2012);
Z.X. Lu, NF&PoP, 2015]

A conceptual Model of Intrinsic Rotation: Heat Engine

- Car motion vs plasma rotation [Kosuga, PoP, 2010]:

	Car	Intrinsic Rotation
Fuel	Gas	Heating → $\nabla T, \nabla n_0$
Conversion	Burn	∇T, ∇n_0 driven turbulence
Work	Cylinder	Symmetry breaking
Result	Wheel rotation	Flow

- Plasma rotation:



Details: Conventional Wisdom of Intrinsic Rotation

- Self-acceleration by intrinsic torque due to residual stress ($\tau_{intr} = -\nabla \cdot \Pi^{Res}$)

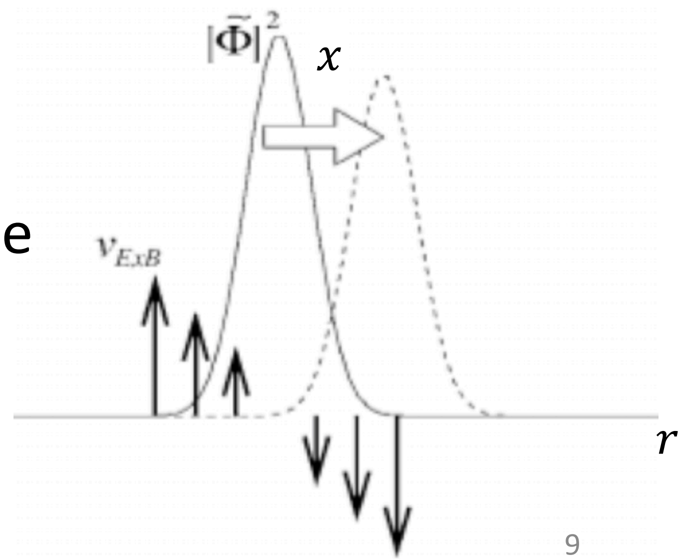
$$\langle \tilde{v}_r \tilde{v}_\parallel \rangle = -\chi_\phi \frac{d\langle v_\parallel \rangle}{dr} + V_P \langle v_\parallel \rangle + \Pi_{r\parallel}^{Res}$$

- Residual stress $\Pi_{r\parallel}^{Res}$
  - Driven by turbulence, i.e. $\Pi_{r\parallel}^{Res} \sim \nabla P, \nabla T, \nabla n_0$
- $\Pi_{r\parallel}^{Res} \sim \langle k_\theta k_\parallel \rangle$ requires symmetry breaking in *k* space
- Symmetry breaking usually relies on magnetic shear
- Rotation builds up from edge, driven by $\Pi_{r\parallel}^{Res}$ at edge

[W.X. Wang, PRL, 2009]

A Simple Example

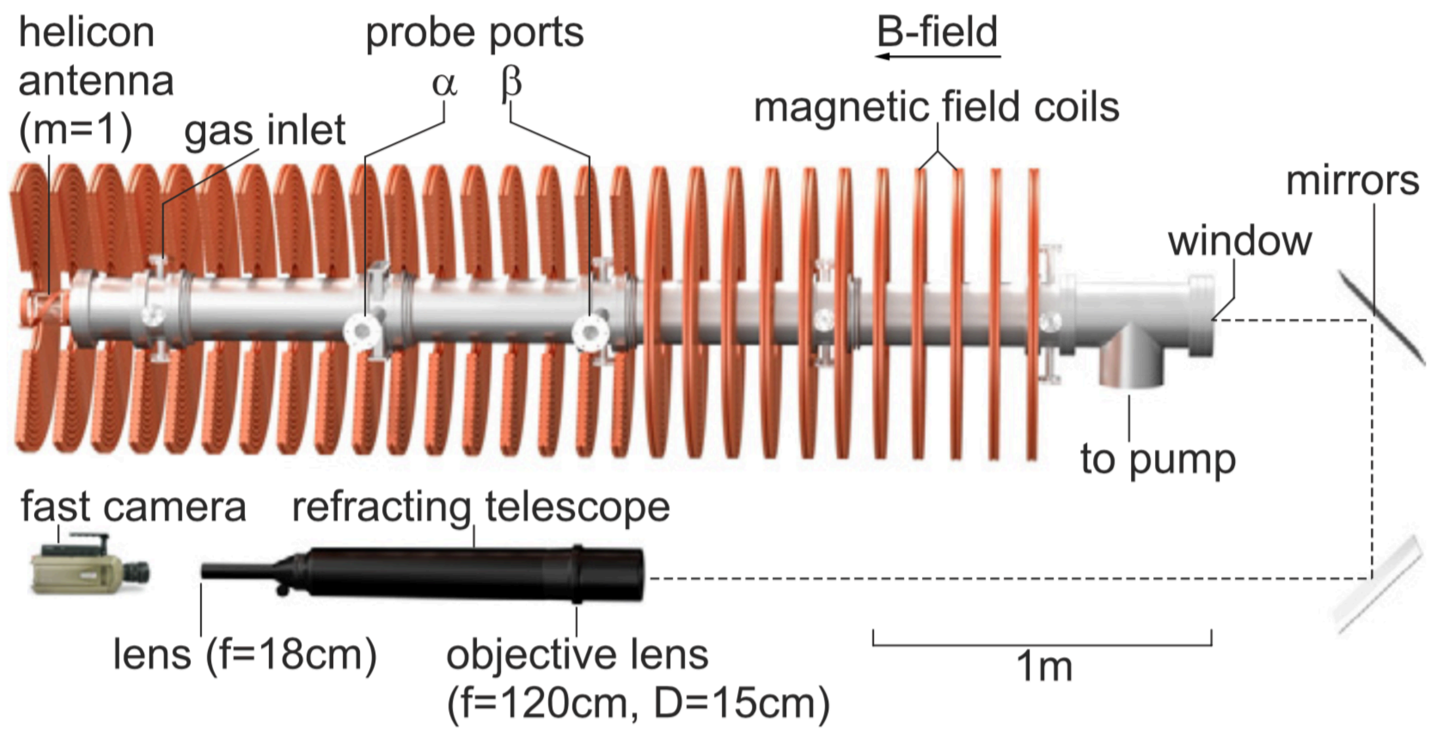
- $k_{\parallel} = k_\theta \frac{x}{L_s} \rightarrow \langle k_\theta k_{\parallel} \rangle \sim k_\theta^2 \frac{\langle x \rangle}{L_s}$
- $\langle x \rangle$: averaged distance from mode center to rational surface
- $\langle x \rangle$ set, in simple models, by:
  - E_r': centroid shift
  - $I'(x)$: spatial dispersion of envelope
- What of weak shear ($q' \rightarrow 0$)?



[Gurcan et al, PoP, 2007&2010]

Intrinsic Parallel Flow in Linear Device

- Controlled Shear Decorrelation Experiment (CSDX)
- Straight, uniform magnetic field in axial direction
- Ideal test bed for studying intrinsic flows in unsheared magnetic fields



Device of CSDX.

More Generally:
Why study linear device?

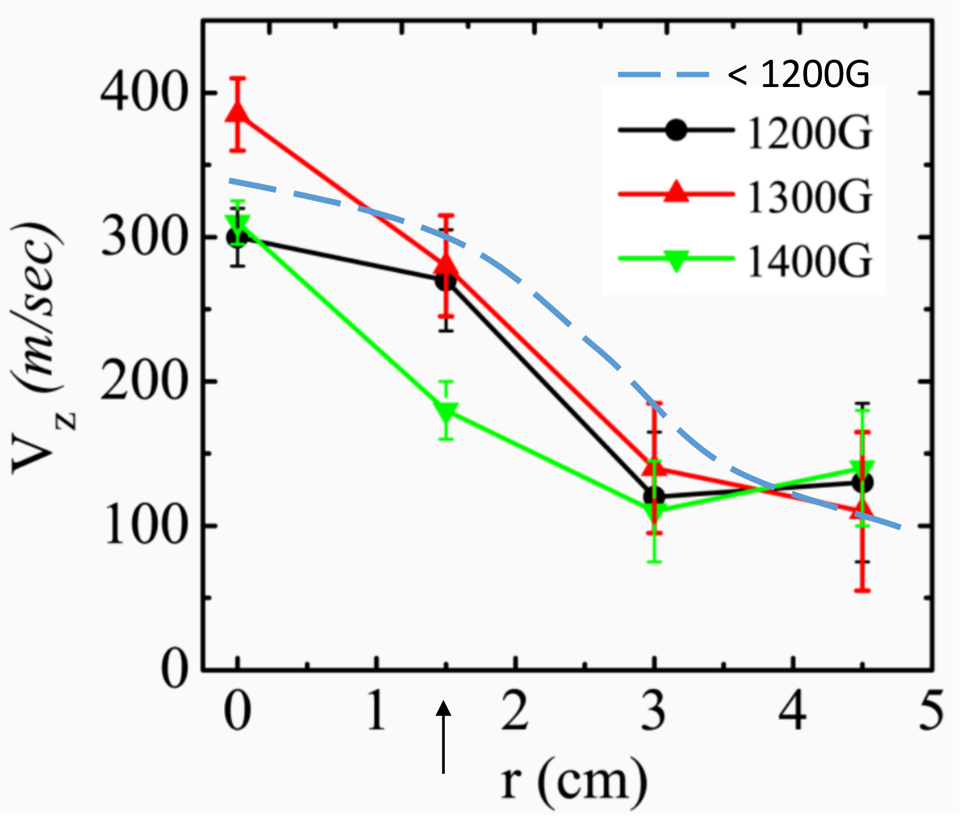
- Correspondence between CSDX and tokamaks:

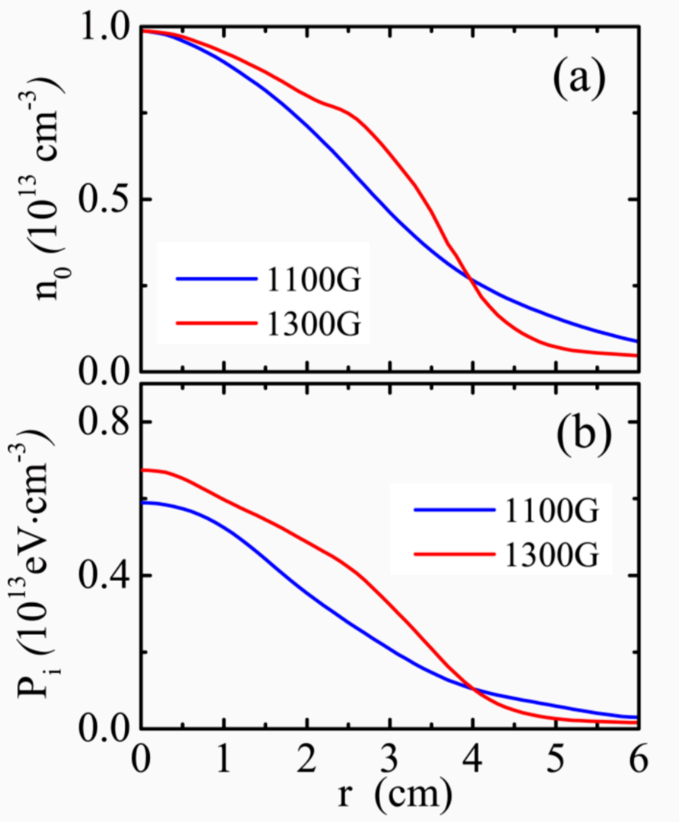
Tokamaks	CSDX
Toroidal field structure usually sheared	Uniform axial magnetic field (shear-free)
Intrinsic toroidal rotation	Intrinsic axial flow
Rotation boundary condition set by SOL	Axial flow boundary condition set by boundary neutral layer
L-H transition	Transport bifurcation driven by ∇n_0
Inward pinch; density peaking	n_0 profile steepening; localized net inward particle flux

[Cui, PoP, 2015, 2016]

Observation

- Intrinsic axial flow evident (without momentum input)
- Barrier formed when $B > B_{crit}$
- $\nabla\langle v_z\rangle$ steepening occurs with ∇n_0, ∇P_i steepening → barrier formation



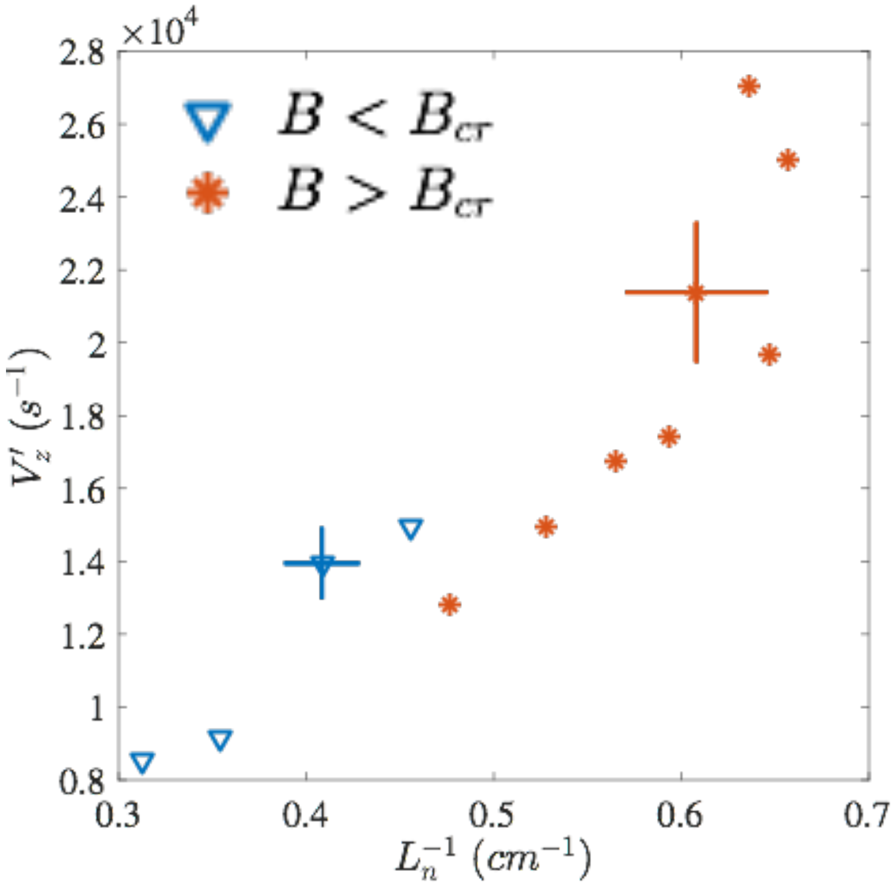


$\langle v_z\rangle$ profile evident, steepens during transition

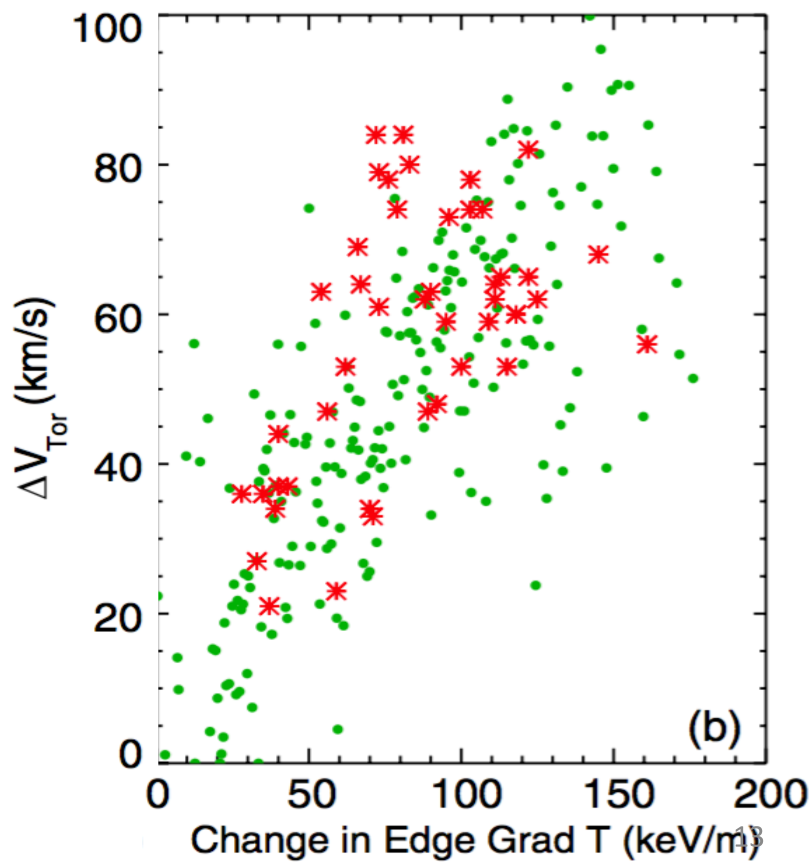
[Cui, PoP, 2015&2016; TTF talk, 2015]

Intrinsic $\nabla\langle v_z\rangle$ tracks L_n^{-1}

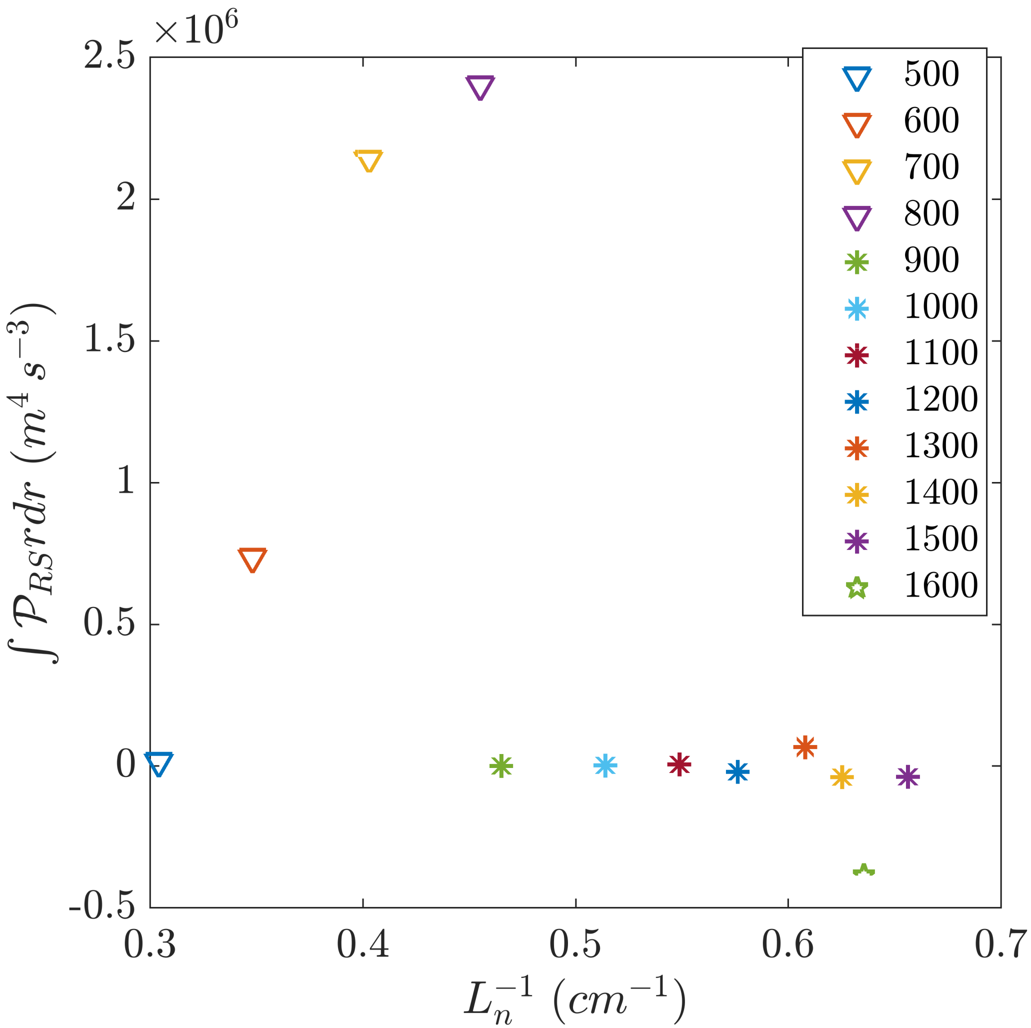
- Axial flow in CSDX:
- $\langle v_z\rangle' \sim \frac{1}{n_0}\nabla n_0$
- ∇n_0 is free energy source



- Recall
- Intrinsic rotation in C-Mod:
- $\Delta\langle v_\phi\rangle \sim \nabla T$ [Rice, PRL, 2011]



Axial Reynolds Power Tracks L_n^{-1}



- Total Reynolds power: $P_{Res}^{tot} = -\int \langle \tilde{v}_r \tilde{v}_z \rangle' \langle v_z \rangle r dr$
- Total power coupled to $\nabla \langle v_z \rangle$ from fluctuations
- Axial Reynolds power rises with $1/L_n$ (for $B < B_{crit}$)
- Evidence for direct connection of fluctuations with intrinsic flow

Theory

- Dynamical Symmetry Breaking
- No requirement on specific magnetic field structure → aspects relevant to both weak shear, and standard configurations
- Electron drift waves (CTEM → ITER relevant)

Electron Drift Wave System

- System equations:

$$\frac{D}{Dt}n_e + \frac{1}{L_n}\frac{1}{r}\frac{\partial\phi}{\partial\theta} + \frac{\partial v_{e,z}}{\partial z} = 0$$

$$\frac{D}{Dt}\nabla_\perp^2\phi = \frac{\partial}{\partial z}\left(v_z - v_{e,z}\right)$$

$$\frac{D}{Dt}v_z - \langle v_z\rangle'\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial n_e}{\partial z} \qquad \left(\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}_E\cdot\nabla\right)$$

- Non-adiabatic electrons: $n_e \cong (1 - i\delta)\phi$

$$\delta \cong \frac{\nu_{ei}(\omega_* - \omega)}{k_z^2 v_{The}^2}, \text{with } 1 < \frac{k_z^2 v_{The}^2}{\nu_{ei}\omega} < \infty$$

- Dispersion relation

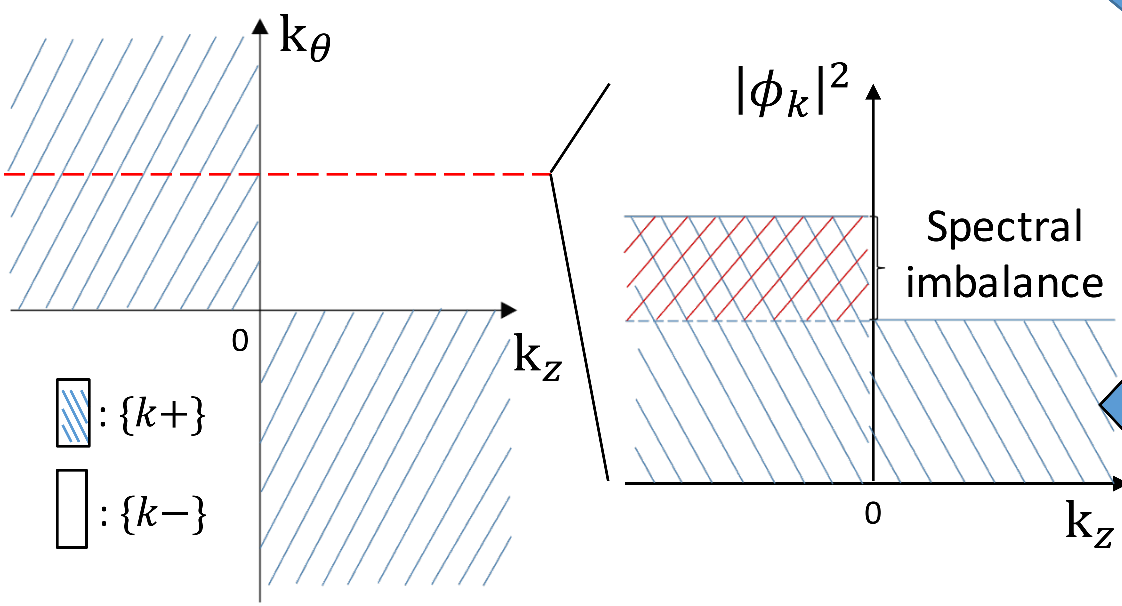
$$1 + k_\perp^2\rho_s^2 - i\delta - \frac{\omega_*}{\omega} + \frac{k_\theta k_z \rho_s c_s \langle v_z\rangle'}{\omega^2} - (1 - i\delta)\frac{k_z^2 c_s^2}{\omega^2} = 0$$

Dynamical Symmetry Breaking

- Growth rate ~ frequency shift:

$$\omega_k \cong \frac{\omega_*}{1 + k_\perp^2 \rho_s^2} - \frac{k_\theta k_z \rho_s c_s \langle v_z \rangle'}{\omega_*}$$

$$\gamma_k \cong \frac{\nu_{ei}}{k_z^2 v_{The}^2} \frac{\omega_*^2}{(1 + k_\perp^2 \rho_s^2)^2} \left(\frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} + \frac{k_\theta k_z \rho_s c_s \langle v_z \rangle'}{\omega_*^2} \right)$$



$\{k\pm\}$: Domains where modes grow faster/slower

Spectral imbalance

- Spectral imbalance:

Infinitesimal test axial flow shear, e.g. $\delta \langle v_z \rangle' < 0$

Modes with $k_\theta k_z < 0$ grow faster than other modes,

$\gamma_k |_{k_\theta k_z < 0} > \gamma_k |_{k_\theta k_z > 0}$

Spectral imbalance in $k_\theta k_z$ space

$\langle k_\theta k_z \rangle < 0 \rightarrow \Pi_{rz}^{Res} \neq 0$

Negative Viscosity *Increment*

- Reynolds stress: $\langle \tilde{v}_r \tilde{v}_z \rangle = -\chi_\phi \langle v_z \rangle' + \Pi_{rz}^{\mathrm{Res}}$

- Turbulent momentum diffusivity:

$$\chi_\phi = \sum_k \frac{\nu_{\mathrm{ei}}}{k_z^2 v_{\mathrm{The}}^2} \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} k_\theta^2 \rho_s^2 |\phi_k|^2$$

- Residual stress → Negative viscosity ***increment***

- $\delta \Pi^{Res} = \left|\chi_\phi^{Inc}\right| \delta \langle v_z \rangle'$ [Li et al, submitted to PoP, 2016]

$$\delta \Pi_{rz}^{\mathrm{Res}} = \frac{\nu_{\mathrm{ei}} L_n^2}{v_{\mathrm{The}}^2} \sum_k (1 + k_\perp^2 \rho_s^2)(4 + k_\perp^2 \rho_s^2) |\phi_k|^2 \delta \langle v_z \rangle'$$

Modulational Enhancement of $\delta\langle v_z\rangle'$

- $\delta\langle v_z\rangle' \rightarrow \Pi^{Res} \rightarrow \chi_\phi^{tot} = \chi_\phi - |\chi_\phi^{Inc}|$
- Dynamics of $\delta\langle v_z\rangle'$:

$$\frac{\partial}{\partial t}\delta\langle v_z\rangle' + \frac{\partial^2}{\partial r^2}\left(\delta\Pi_{rz}^{Res} - \chi_\phi\delta\langle v_z\rangle'\right) = 0$$

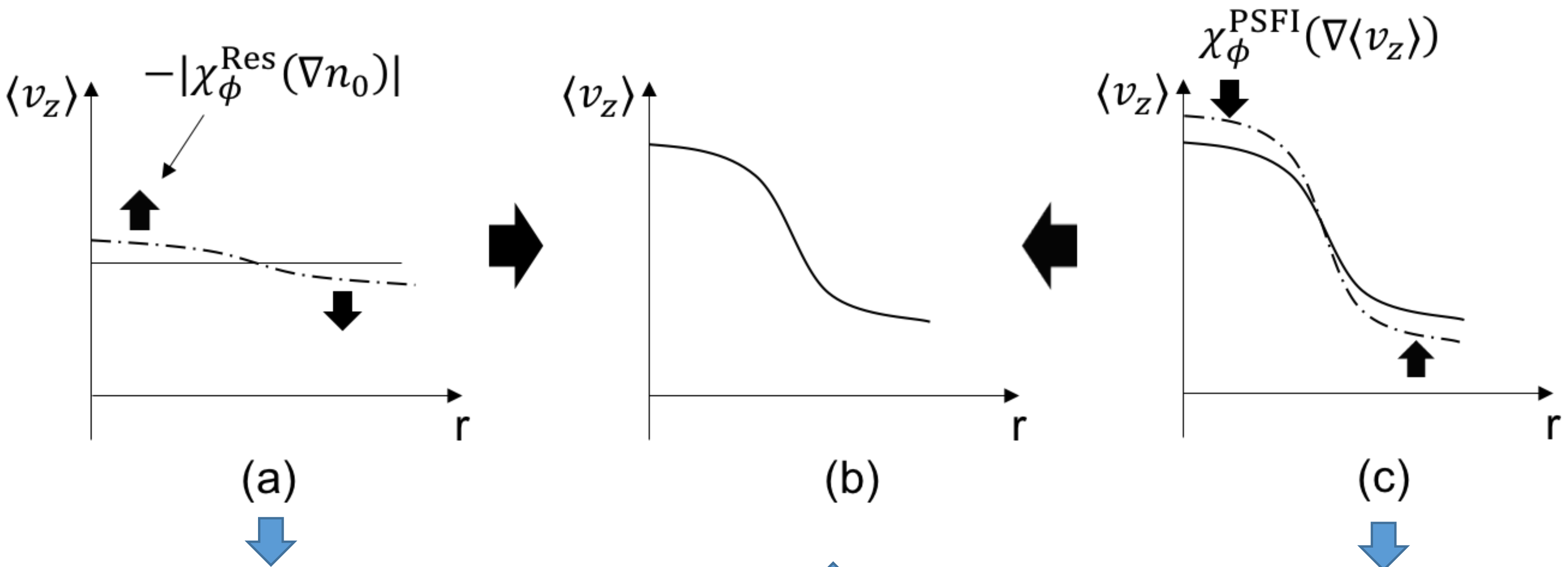
- Growth rate of flow shear modulation

$$\gamma_q = -q_r^2\left(\chi_\phi - |\chi_\phi^{Inc}|\right)$$

- $\chi_\phi^{tot} < 0 \rightarrow$ Modulational growth of $\delta\langle v_z\rangle'$
- Feedback loop: $\delta\langle v_z\rangle' \rightarrow \Pi^{Res} \rightarrow -|\chi_\phi^{Inc}|$

Upper Limit of $\langle v_z \rangle'$ Set by PSFI

- Parallel shear flow instability (PSFI)
- Driven by $\nabla\langle v_z \rangle$, negative compressibility (similar to ITG)



$$\chi_\phi^{tot} = \chi_\phi^{DW} - |\chi_\phi^{Inc}| < 0$$

$$\chi_\phi^{tot} = \chi_\phi^{DW} + \chi_\phi^{PSFI} - |\chi_\phi^{Inc}| > 0$$

$$\chi_\phi^{tot} = \chi_\phi^{DW} + \chi_\phi^{PSFI}\Theta(\langle v_z \rangle' - \langle v_z \rangle'_{crit}) - |\chi_\phi^{Inc}|$$

Parallel shear flow instability

- Growth rate and resulting turbulent momentum diffusivity:

$$\gamma_k^{PSFI} \cong \sqrt{\frac{k_\theta k_z \rho_s c_s\left(\langle v_z\rangle' - \langle v_z\rangle'_{crit}\right)}{1 + k_\perp^2 \rho_s^2}}$$

$$\chi_\phi^{PSFI} \cong \sum_k |\phi_k|^2 k_\theta^2 \rho_s^2 \frac{4(1 + k_\perp^2 \rho_s^2)^2}{\omega_*^2} \sqrt{\frac{k_\theta k_z \rho_s c_s\left(\langle v_z\rangle' - \langle v_z\rangle'_{crit}\right)}{1 + k_\perp^2 \rho_s^2}}$$

- Hit PSFI threshold → χ_ϕ^{PSFI} nonlinear in $\nabla\langle v_z\rangle$ → $\chi_\phi^{tot} > 0$
- $\delta\langle v_z\rangle'$ → Π^{Res} → $\delta\langle v_z\rangle'$ growth ← Saturated by PSFI

↑ $\chi_\phi^{tot} = \chi_\phi^{DW} - |\chi_\phi^{Inc}| < 0$

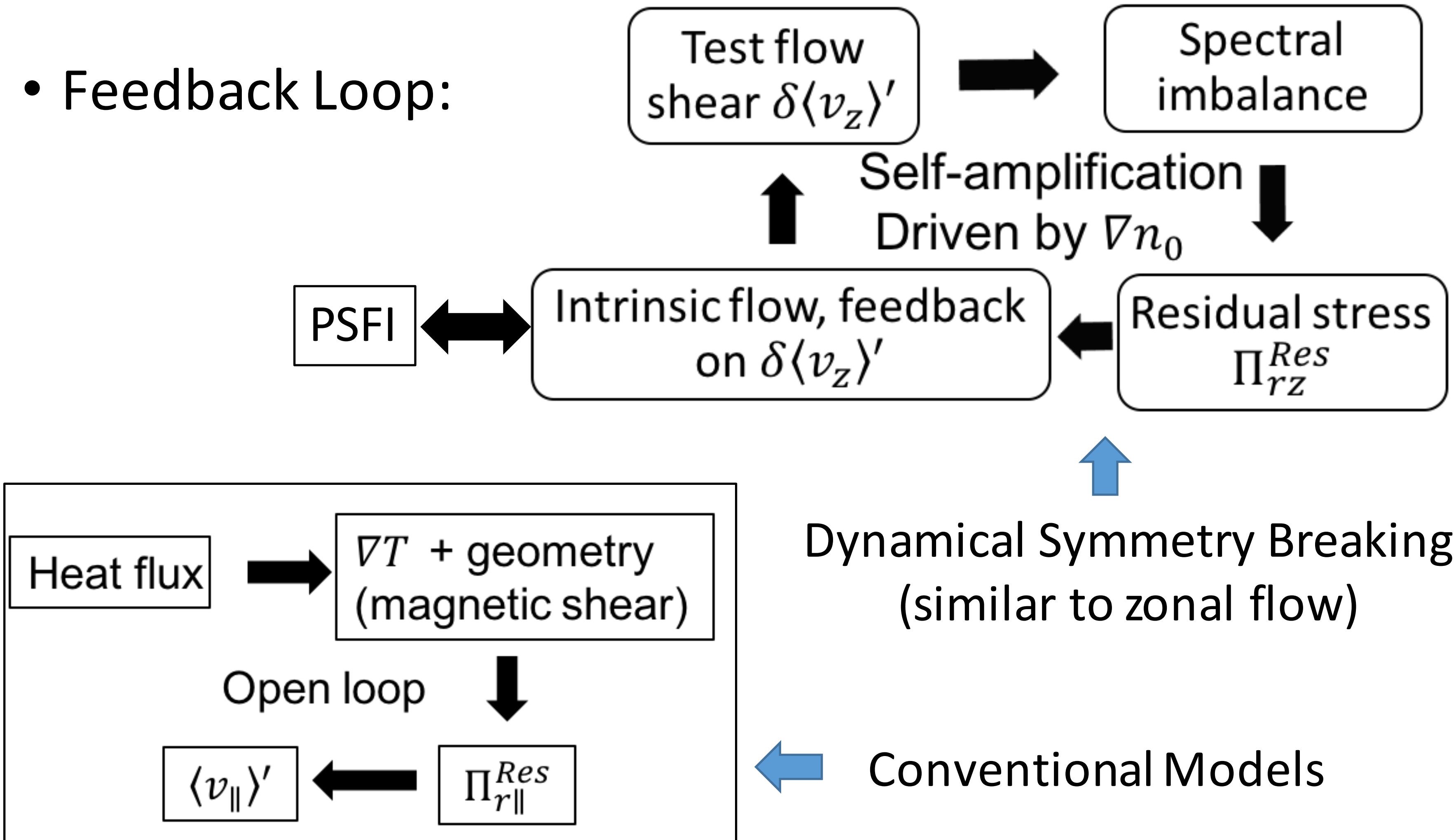
↑ $\chi_\phi^{tot} = \chi_\phi^{DW} + \chi_\phi^{PSFI} - |\chi_\phi^{Inc}| > 0$

Comparing Symmetry Breaking Mechanisms

	Standard Symmetry Breaking	Dynamical Symmetry Breaking
Free Energy	∇T, ∇n_0	∇n_0
Symmetry Breaker	E_r', $I(x)'$, etc. Linked to magnetic shear.	Test axial flow shear, $\delta\langle v_z\rangle'$; No requirement of $\boldsymbol{B}$ structure.
Effect on the Flow	Intrinsic torque, $-\partial_r \Pi_{r\parallel}^{Res}$	Negative viscosity increment, $-\lvert\chi^{Res}\rvert$ driven by ∇n_0
Flow Profile	$\langle v_\phi\rangle' = \dfrac{\Pi_{r\parallel}^{Res}}{\chi_\phi}$	$\langle v_\phi\rangle' = \dfrac{R * \tau_{ext} + \Pi^{Res}}{\chi_\phi(\nabla n_0, \nabla\langle v_\parallel\rangle) - \lvert\chi_\phi^{Inc}\rvert}$

Comparison (cont'd)

- Feedback Loop:



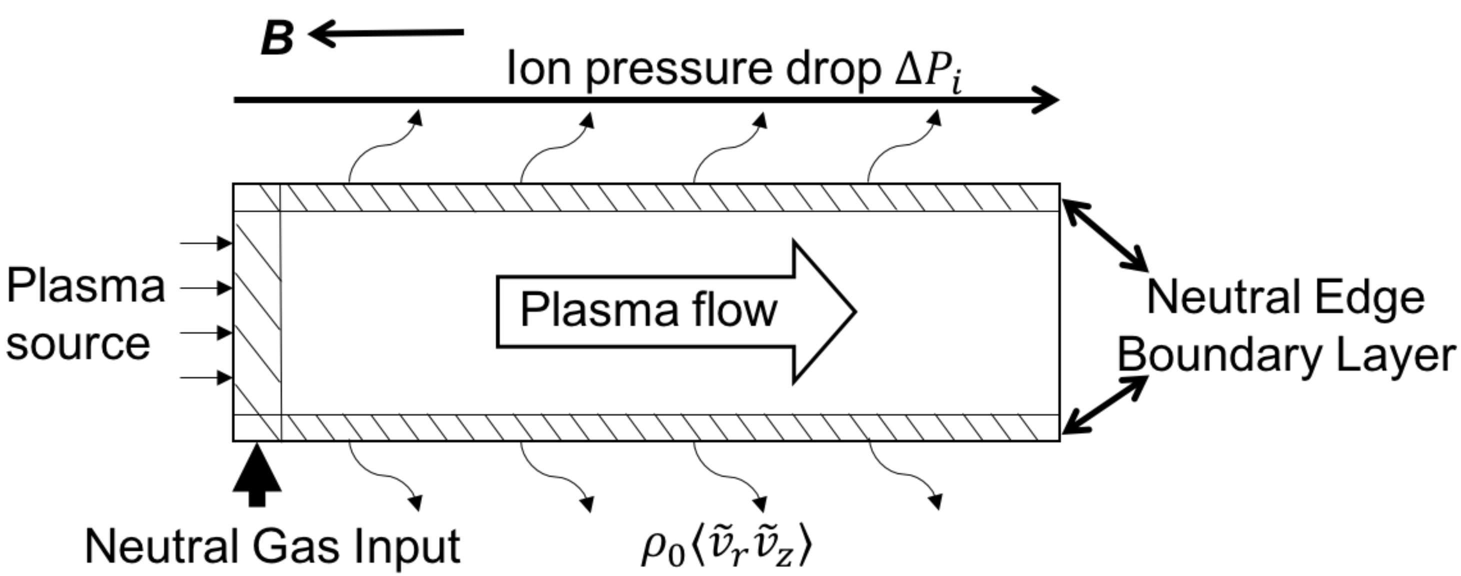
Also relevant to Tokamaks

- $\nabla\langle v_\phi\rangle$ steepening and Π^{Res} can act in synergy
- Rotation profile gradient enhanced by negative viscosity effect

- $\langle v_\phi\rangle' \sim \dfrac{\text{Drive}}{\chi_\phi^{DW}+\chi_\phi^{PSFI}\Theta(\langle v_\parallel\rangle'-\langle v_\parallel\rangle'_{crit})-\left|\chi_\phi^{Inc}\right|}$
- Drive can be external (τ_{NBI}) or intrinsic (Π^{Res})

Future Work

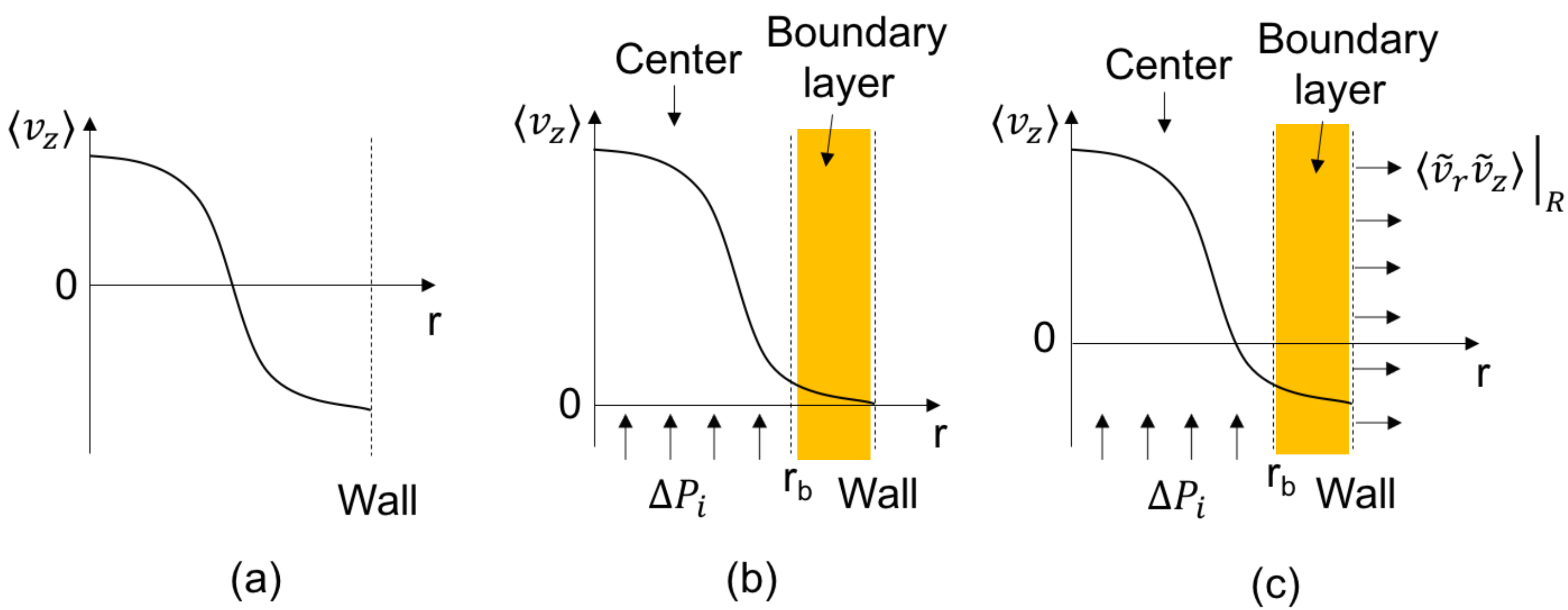
- Boundary condition matter
- Flow boundary controlled by neutral particles
- For simple analysis: No-slip boundary condition due to friction
- In tokamaks: Interaction with SOL



Boundary Dynamics Impacts Flow Profile

- Evolution of net axial ion flow:

$$\frac{\partial}{\partial t}\int_0^R dr\langle v_{i,z}\rangle = \int_0^R dr\frac{\Delta P_i}{\rho_0 L} - \langle \tilde{v}_r\tilde{v}_z\rangle\Big|_R - \int_{r_b}^R dr\nu_{ni}\big(\langle v_{i,z}\rangle - \langle v_{n,z}\rangle\big)$$



(a) No external source/sink. Net flow = 0.

(b) No-slip at wall → $v_z \cong 0$, $\langle \tilde{v}_r\tilde{v}_z\rangle \cong 0$. Net flow > 0.

(c) ΔP_i drive at center, outflux at wall.

Boundary Layer

- Partially ionized → Neutral flow within BL
- Neutral flow dynamics (can be solved numerically by BOUT++):

$$\rho_{\mathrm{n}}\left(\frac{\partial \mathbf{v}_{\mathrm{n}}}{\partial t}+\mathbf{v}_{\mathrm{n}}\cdot\nabla\mathbf{v}_{\mathrm{n}}\right)=-\nabla P_{\mathrm{n}}+\rho_{\mathrm{n}}\nu_{\mathrm{ni}}\left(\mathbf{v}_{\mathrm{i}}-\mathbf{v}_{\mathrm{n}}\right)$$

- Within neutral layer: $\langle v_{\mathrm{i},z}\rangle \cong \langle v_{\mathrm{n},z}\rangle$
- Neutral flow within BL sets boundary condition for plasma flows

[Z.H. Wang et al, NF, 2014]

Summary

- We know:
  - Weak shear + rotation → beneficial for confinement
  - Intrinsic toroidal rotation exists in weak shear regions
- Question:
  - Intrinsic rotation generation for weak shear → $q' \rightarrow 0$?
- New mechanism for intrinsic axial flow generation
  - CSDX: a test bed for intrinsic flows with unsheared magnetic fields
  - No requirement on magnetic shear → Broader lesson

Summary (cont'd)

- Results:
  - Dynamical symmetry breaking
  - Negative viscosity increment induced by Π^{Res}
    - $\delta\Pi^{Res} = \left|\chi_\phi^{Inc}\right| \delta\langle v_z\rangle'$
    - Total viscosity: $\chi_\phi^{tot} = \chi_\phi - \left|\chi_\phi^{Inc}\right|$
    - $\chi_\phi^{tot} < 0$ → Modulational growth of $\delta\langle v_z\rangle'$
  - Flow profile: pipe flow analogy
    - Balance between drive and viscosity
    - In CSDX: $\langle v_z\rangle' \sim \Delta P_i / \chi_\phi^{tot}$

Summary (cont'd)

- Broader lesson for tokamaks
  - Synergy of $\langle v_\phi \rangle'$ self-amplification and Π^{Res}
  - $\langle v_\phi \rangle'$ driven by τ_{NBI}, $\Pi^{Res}(\nabla n_0, \nabla T)$
  - $\langle v_\phi \rangle'$ enhanced by $-|\chi_\phi^{Inc}|$
- Future work: flow boundary condition
  - Ion-neutral coupling within boundary layer

List of Related Talks/Posters

- April 1st:
  - A. Ashourvon, “On the Structure of the Zonal Shear Layer Field and its Implication for Multi-scale Interactions”
- March 31st:
  - R. Hajjar, “Modelling Transport Bifurcations in the CSDX Linear Device”
  - P. Vaezi, “Nonlinear Simulation of CSDX Including Sheath Physics”
- March 30th:
  - S.C. Thakur, “Spontaneous self-organization from drift wave plasmas to a mixed ITG-drift wave-shear flow system via a transport bifurcation in a linear magnetized plasma device”
  - R. Hong, “Effects of Density Gradient on Axial Flow Structures in a Helicon Linear Plasma Device ”

Back-up

Heat Engine

- Efficiency of intrinsic rotation generation by turbulence
- Entropy evolution:

$$\partial_t S_0 = \int d^3x \left[n\chi_i \left(\frac{\nabla T}{T} \right)^2 - nK \frac{\langle V_E \rangle'^2}{v_{\mathrm{thi}}^2} + n\chi_\phi \frac{\langle V_\parallel \rangle'^2}{v_{\mathrm{thi}}^2} - n \frac{\Pi_{r\parallel}^{\mathrm{res}2}}{v_{\mathrm{thi}}^2 \chi_\phi} \right].$$

- Efficiency ~ $\frac{\text{Entropy destruction due to flow generation}}{\text{Net production due to thermal relaxation}}$

$$e \equiv \frac{\left| \int d^3x \mathcal{P}_{\mathrm{flow}} \right|}{\int d^3x \mathcal{P}_{\mathrm{net}}} \quad \Rightarrow \quad e_{\mathrm{IR}} \cong \frac{\int d^3x n (\Pi_{r\parallel}^{\mathrm{res}})^2 / (v_{\mathrm{thi}}^2 \chi_\phi)}{\int d^3x n \chi_i (\nabla T/T)^2}.$$

Non-resonant mode structure

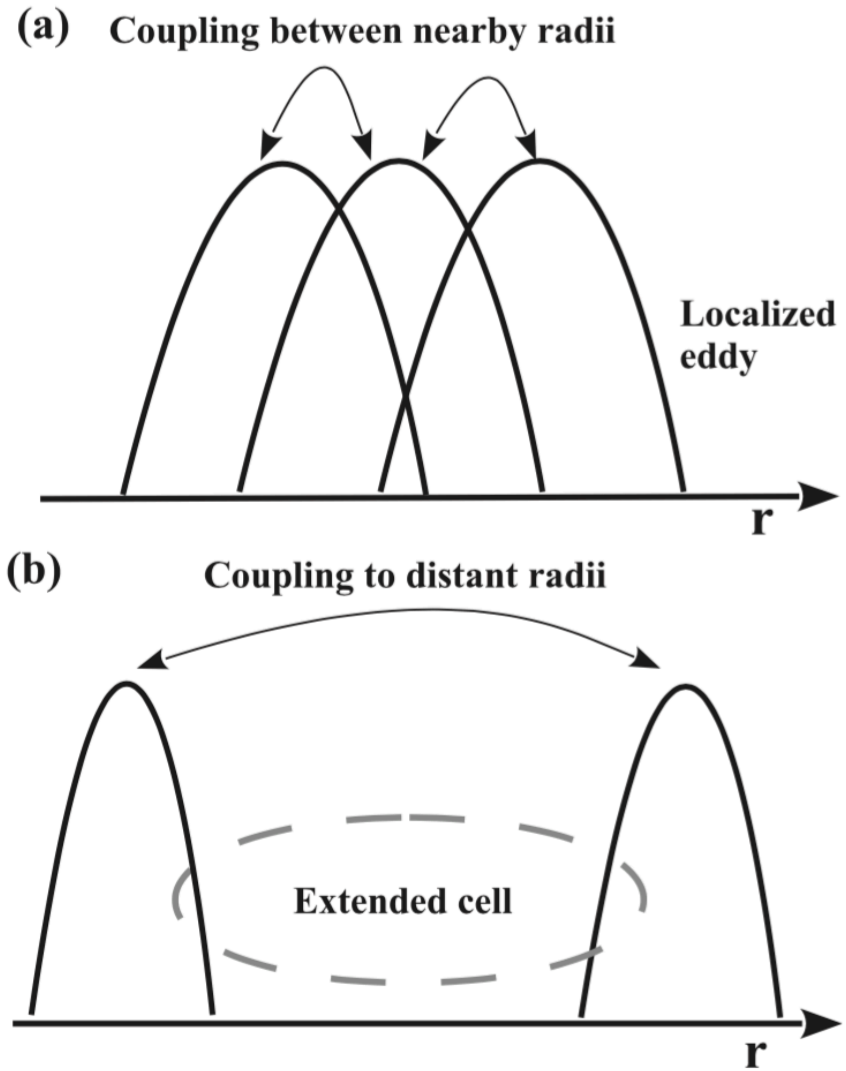
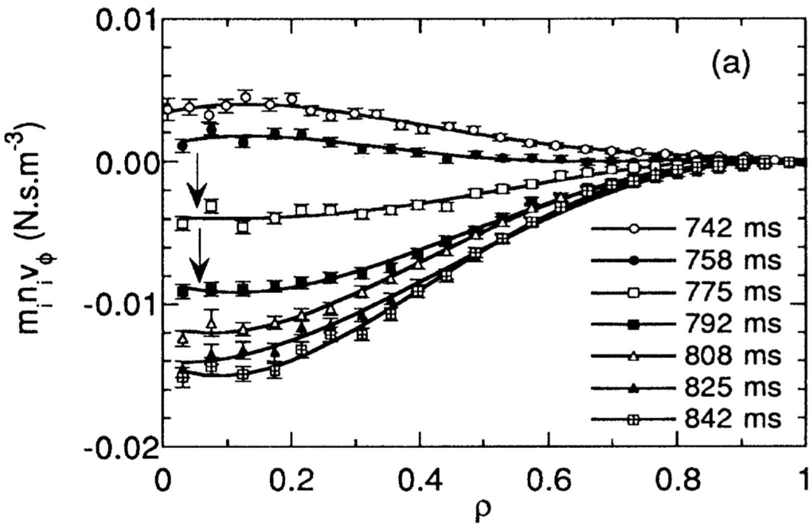


FIG. 1. Cartoon of the spatial extent of mode-mode interactions (a) among resonant modes only and (b) involving non-resonant modes.

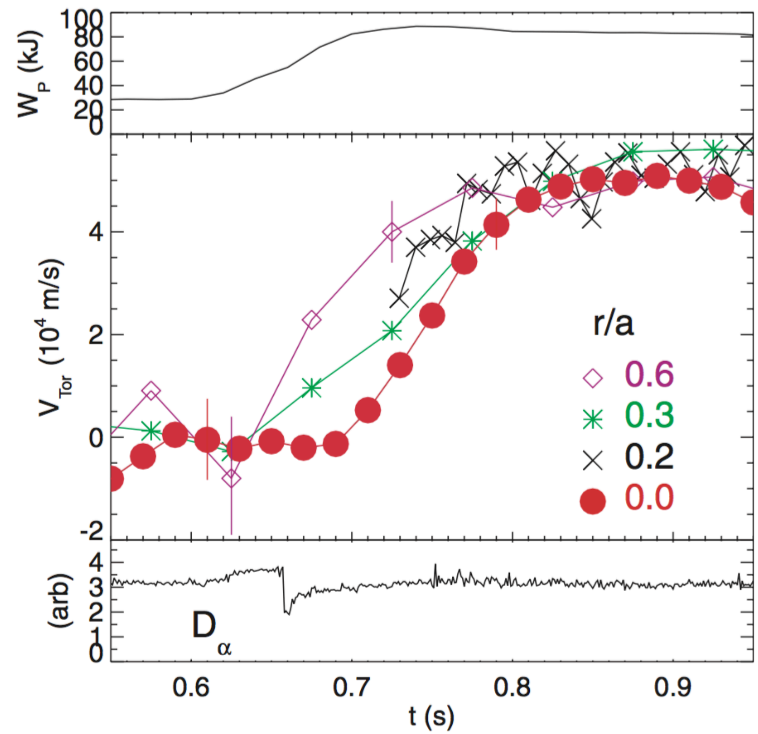
- Non-resonant mode structure [S. Yi, PoP, 2012]

Intrinsic rotation

- Discovered in JFT-2M, Alcator C-Mod plasmas



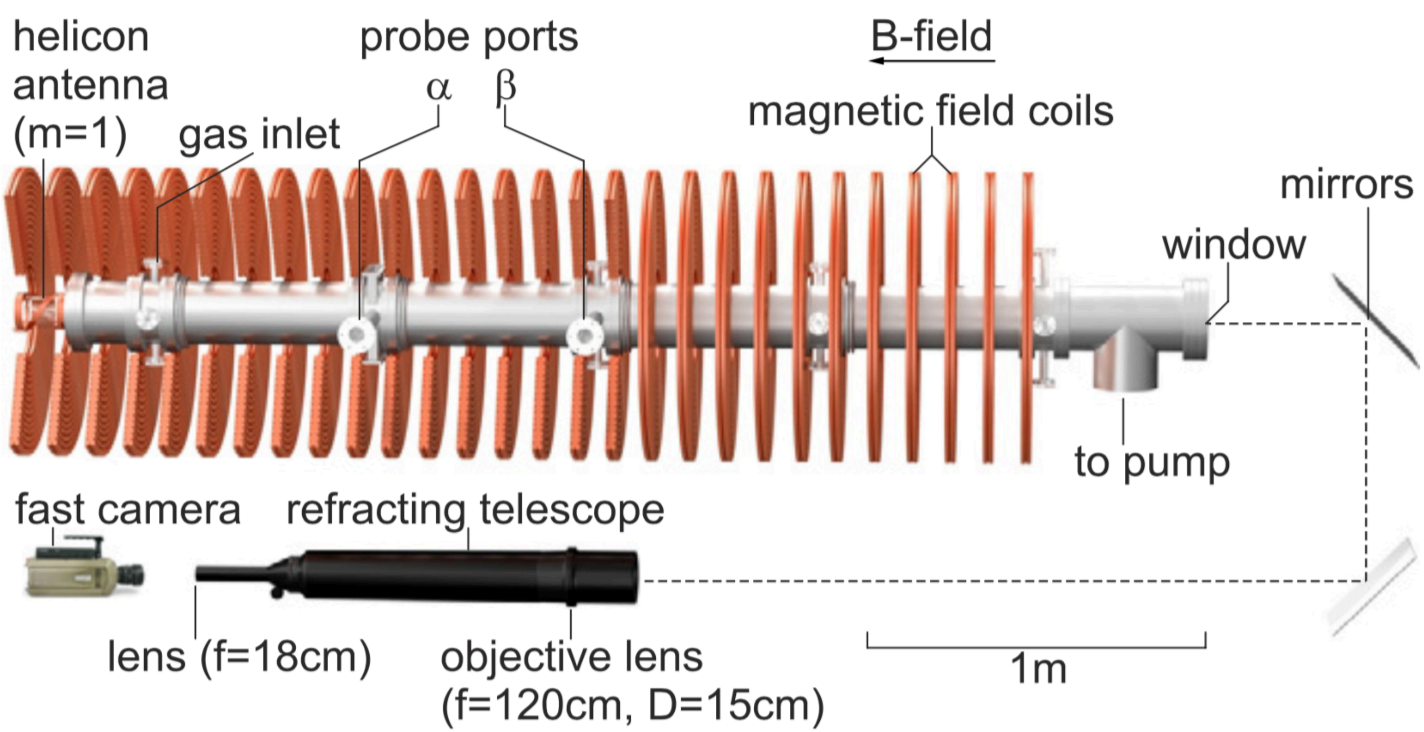
Steady state toroidal momentum in counter-current injection of NBI is two to three times larger than that in co-current injection. [Ida, JFT-2M, 1995]



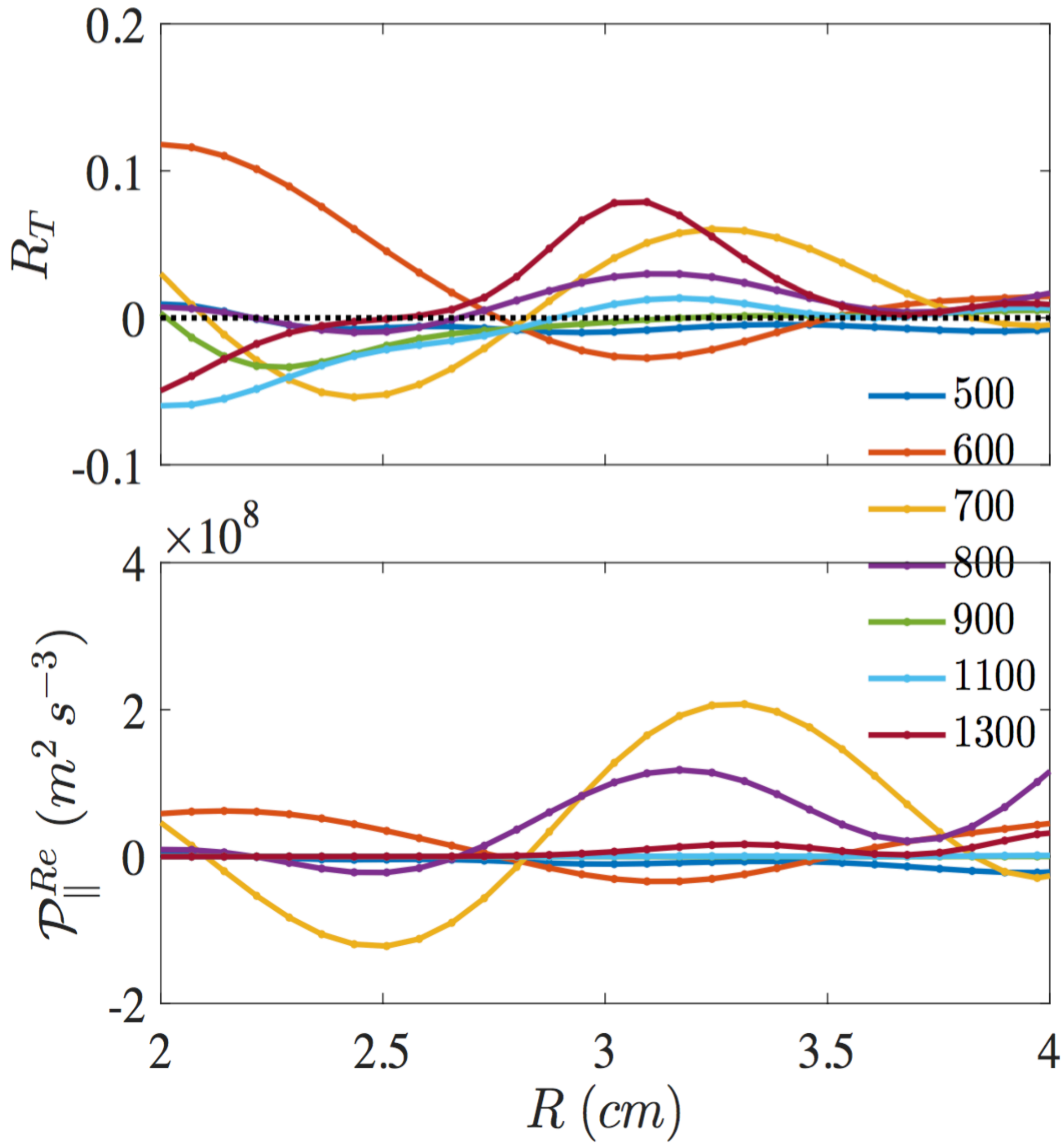
Time histories of (impurity) toroidal rotation for a 2.0 MW **ICRF** heated EDA H-mode plasma [Rice, C-Mod, 2004]

CSDX Parameters

- These experiments were carried out in the Controlled Shear Decorrelation Experiment, which is a 2.8 m long linear helicon plasma device with a source radius 7.5 cm, 1.6 kW RF power input (reflected power less than 30 W), and a gas fill pressure of 3.2 mTorr.



Energy Transfer Ratio



- Axial transfer ratio:
- $(R_T)_A = \frac{P^{Res}}{\langle \tilde{v}_z^2 \rangle} \tau_{ac}$
- $(R_T)_A$ decreases when $B > B_{crit}$
- $B_{crit} \cong 900G$

Residual Stress

- Reynolds stress: $\langle \tilde{v}_r \tilde{v}_z \rangle = -\chi_\phi \langle v_z \rangle' + \Pi_{rz}^{\mathrm{Res}}$
- Turbulent diffusivity:

$$\chi_\phi = \sum_k \frac{\nu_{\mathrm{ei}}}{k_z^2 v_{\mathrm{The}}^2} \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} k_\theta^2 \rho_s^2 |\phi_k|^2$$

⇔ Driven by ambient background turbulence

- Residual stress

$$\Pi_{rz}^{\mathrm{Res}} = \mathrm{Sign}(\delta \langle v_z \rangle') \sum_{\{k+\}} \frac{\nu}{k_\parallel^2 v_{\mathrm{The}}^2} (2 + k_\perp^2 \rho_s^2) \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2} |k_y k_\parallel| \rho_s c_s \Delta I_k(\delta \langle v_z \rangle')$$

$$\Delta I_k(\delta \langle v_z \rangle') \equiv |\phi_k|^2 \big|_{\{k+\}} - |\phi_k|^2 \big|_{\{k-\}}$$

⟷ Spectral imbalance $\sim \delta \langle v_Z \rangle'$

Parallel Shear Flow Instability

- PSFI: recall dispersion relation of the model with adiabatic electrons:

$$\left(1+k_\perp^2\rho_s^2\right)\omega^2-\omega_*\omega+k_\theta k_z\rho_s c_s\langle v_z\rangle'-k_z^2c_s^2=0$$

- Unstable ↔ discriminant = $\omega_*^2-4(1+k_\perp^2\rho_s^2)(k_\theta k_z\rho_s c_s\langle v_z\rangle'-k_z^2c_s^2)<0$
- → $\langle v_z\rangle'>\langle v_z\rangle'_{\rm crit}\equiv\frac{1}{k_\theta k_z\rho_s c_s}\left[\frac{\omega_*^2}{4(1+k_\perp^2\rho_s^2)}+k_z^2c_s^2\right]$ → PSFI
- With non-adiabatic electrons

$$\langle v_z\rangle'_{\rm crit}=\frac{1}{k_\theta k_z\rho_{\rm s}c_{\rm s}}\left[\frac{\omega_*^2(1+k_\perp^2\rho_{\rm s}^2)}{4[(1+k_\perp^2\rho_{\rm s}^2)^2+\delta^2]}+k_z^2c_{\rm s}^2\right]$$

Parallel Shear Flow Instability

- Negative compressibility

$$\left(1+k_\perp^2\rho_s^2\right)\omega^2 - \omega_*\omega + k_\theta k_z \rho_s c_s \langle v_z\rangle' - k_z^2 c_s^2 = 0$$

$$\omega_*^2 - 4(1+k_\perp^2\rho_s^2)(k_\theta k_z \rho_s c_s \langle v_z\rangle' - k_z^2 c_s^2) < 0$$

$$k_\theta k_z \langle v_z\rangle' > 0$$

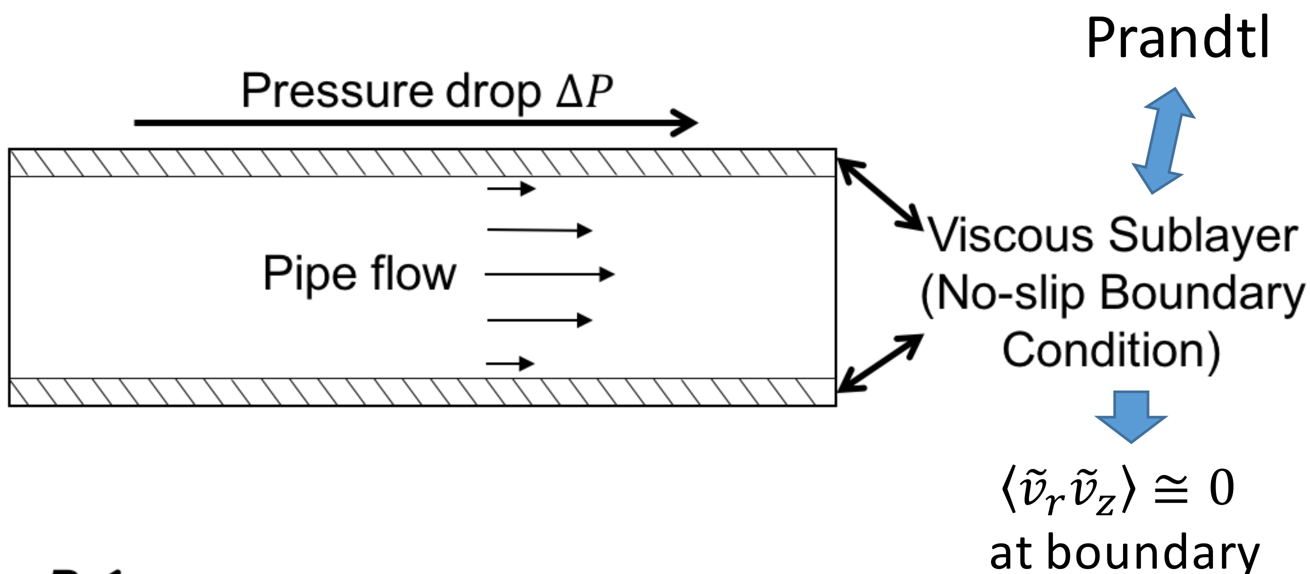
$$\omega^2 \sim \left(1 - \frac{k_\theta \rho_s}{k_z c_s}\langle v_z\rangle'\right) k_z^2 c_s^2$$

Negative compressibility

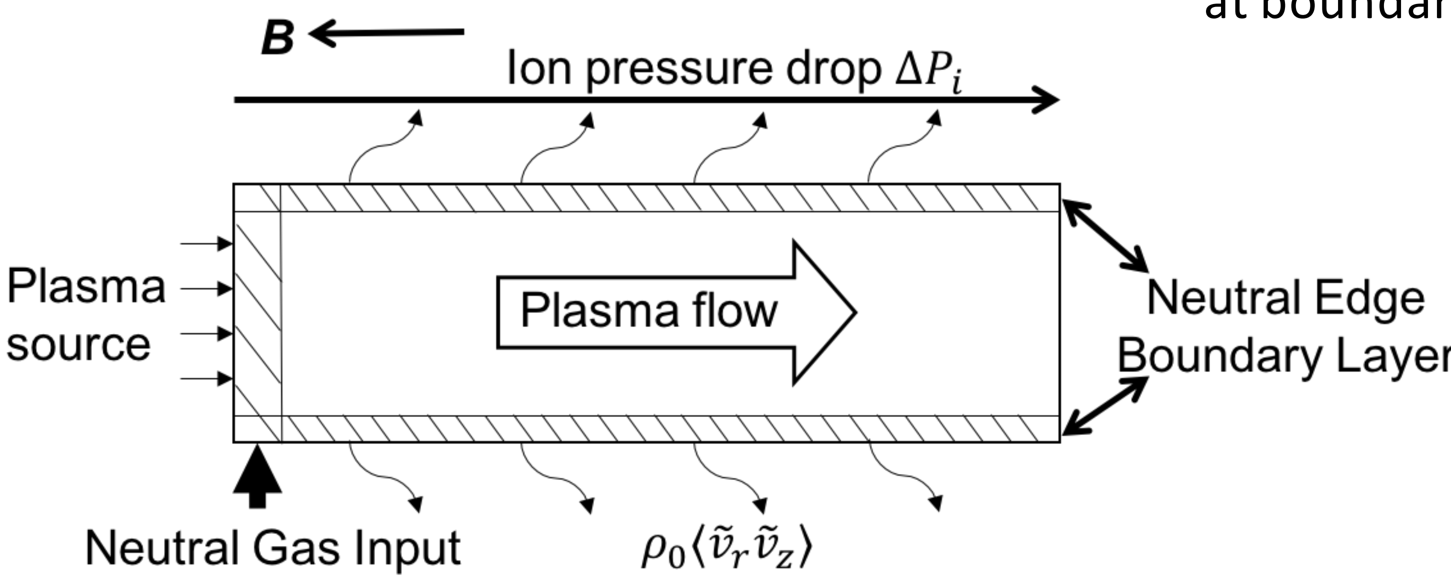
Flow Structure

- Pipe flow analogy

Pipe flow:



CSDX:



Flow Structure

- With external drive ΔP_i
  - Don't need modulational growth to generate intrinsic flow
  - $-\left|\chi_\phi^{Inc}\right|$ enhances flow gradient
- Momentum balance:
- $\langle v_z \rangle' \sim \frac{\Delta P_i}{\chi_\phi^{DW} + \chi_\phi^{PSFI}\Theta(\langle v_z \rangle' - \langle v_z \rangle'_{crit}) - \left|\chi_\phi^{Inc}\right|}$
- $\langle v_z \rangle'$ is kept at or below $\langle v_z \rangle'_{crit}$ due to χ_ϕ^{PSFI}

Applications to Tokamaks

- Weak shear
- Initial flow shear (seed)

  → Π^{Res} by dynamical symmetry breaking

  → Accelerate rotation
- Need modulational growth of $\delta\langle v_\phi\rangle'$
- OR weak net NBI torque (external) + negative viscosity increment → $\langle v_\phi\rangle'$ enhanced by $-\left|\chi_\phi^{Inc}\right|$