Staircases and Transport Barriers Near the Density Limit in Hasegawa-Wakatani Channel Flows

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Abstract

- Edge shear layers generated by drift waves collapse when the adiabaticity $\alpha = k_{\parallel}^2 V_{\text{th}}^2/\omega_{ci}\eta \lesssim \alpha_{\text{crit}} \sim 1$. We have investigated the role of α and the initial density profiles in the formation and dynamics of transport barriers, using the Hasegawa-Wakatani (HW) model.
- This study is relevant to the fate of shear layer and transport barriers near the density limit.
- Steep density profile are formed after, e.g., a pellet- or supersonic neutral beam injection. In simulations, no shear flow is initially imposed but created by the drift waves generated by the density gradient itself. The density profile then relaxes under the competition between the turbulent transport and its suppression by the shear flow.
- Density relaxation generically goes through a staircase phase. The shear flow is formed and the turbulent transport is suppressed. The final breadth of the transport barrier depends on the adiabaticity α and the initial density contrast, Δn. Turbulent fronts propagate in both directions away from the initial localization of the transport barrier.

Overview

The HW model offers a vital link between the experiments or full-scale 3D simulations and reduced 1- or 0-D physical models. The reduced models often provide a crucial insight into transport bifurcations. We have narrowly tailored the HW model to this end. Apart from the traditional doubly-periodic box setting, we use a channel setting. It accommodates the density/temperature contrasts across the channel, thus elucidating the net-flux-driven turbulence. After benchmarking and comparing our code with the doubly-periodic HW simulations and investigating the zonal flow (ZF) dependence on the model parameters (e.g. number and strengths of ZF jets in the channel vs the adiabaticity $\alpha = k_{\parallel}^2 V_{\rm th}^2/\omega_{ci}\eta$ and instability driver κ, as reported earlier) we are now focused on the following two objectives:

Overview cont'd

1. Collapse of edge shear layers on a low-temperature side of the channel, where the adiabaticity α (flow bifurcation parameter) drops below $\alpha_{\rm crit} \lesssim 1$.
   $\nabla\alpha$ triggers the formation of a barrier shear layer, which separates the region of isotropic turbulence from ZF-dominated. The barrier is pinned to the location of $\alpha_{\rm crit}$ and does not propagate. We observe that this spontaneously generated shear layer forms where $\alpha = \alpha_{\rm crit}(\kappa)$ and disappears where $\alpha < \alpha_{\rm cr}$ inside of the domain. This behavior is consistent with that observed at the density limit when high edge density also forces a drop in the edge layer value of α.
2. Decay and relaxation of a step-like density profile formed after, e.g., a pellet- or supersonic neutral beam injection. No shear flow is initially imposed but created by the ∇n -driven DWs. Both internal- and edge barriers are formed if $\alpha \sim 1$ and $\Delta n \gg n_{\rm min}$, where $n_{\rm min}$ is the density at the lower-density wall of the channel.

Modified Hasegawa-Wakatani Model

- two equations: for density and vorticity

$$\frac{\partial \zeta}{\partial t} + \{\phi, \zeta\} = \alpha(y)\left(\tilde{\phi} - \tilde{n}\right) - D\nabla^4 \zeta$$
$$\frac{\partial n}{\partial t} + \{\phi, n\} = \alpha(y)\left(\tilde{\phi} - \tilde{n}\right) - \kappa \frac{\partial \phi}{\partial x} - D\nabla^4 n$$

- $\{f, g\} \equiv (\partial_x f)\,\partial_y g - (\partial_x g)\,\partial_y f$ – Poisson bracket
- $\zeta \equiv \Delta\phi$ – flow vorticity
- DW instability driver $\kappa = n_0^{-1}\partial n_0/\partial y$, $n_0(y)$ – equilibrium density
- n – deviation from equilibrium normalized to n_0
- adiabaticity $\alpha = k_{\|}^2 V_{\mathrm{Te}}^2/\eta\,\omega_{\mathrm{ci}}$, resistivity η

$$\bar{n} \equiv \int n dx, \quad \bar{\phi} \equiv \int \phi\, dx, \quad \tilde{n} \equiv n - \bar{n}, \quad \tilde{\phi} \equiv \phi - \bar{\phi}$$

Why Channel Flow instead of Doubly-Periodic?

Neither is perfect, BUT...

Doubly-Periodic Box

$f(x+L_x,y,t) = f(x,y+L_y,t) = f(x,y,t)$,

Pros:

- simple
- some of the channel settings can still be implemented (with limited capacity)
- if rigid boundary is a bad choice, physically

Channel

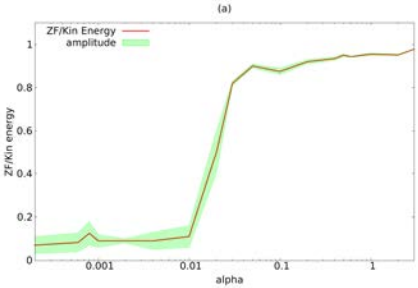
- $f(x+L_x,y,t) = f(x,y,t)$
  $f(x,0,t) \neq f(x,L_y,t)$ $(= C)$

Pros:

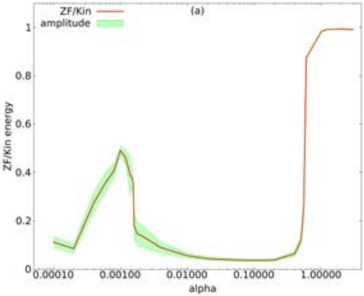
- meaningfully apply density, temperature, etc., contrast across the channel
- **impose** $\alpha(y)$, average shear
- explore geometry (aspect ratio)
- better connect to physical boundary (edge physics)
- wall recycling, fueling, drag,...

Flows with constant α and κ : box - channel ZF generation efficiency

$$\varepsilon = E_{ZF}/E_k \equiv \int \left(\frac{\partial \phi}{\partial y}\right)^2 dxdy \Big/ \int |\nabla \phi|^2 dxdy$$

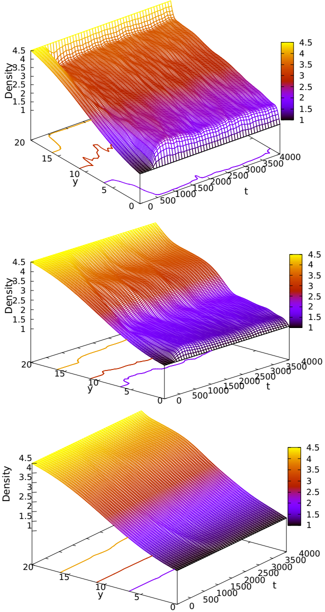


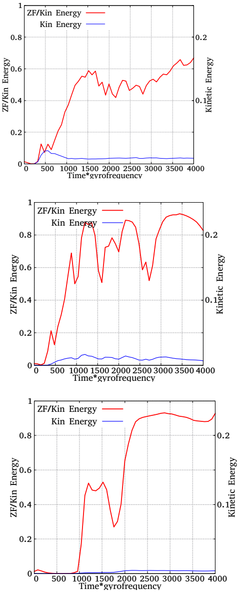
- ε vs α scan for flows in a periodic box
  - $\kappa = 0.3$



- ε vs α scan for flows in a channel
  - $\kappa = 0.3$
    — **NB**: back transition from a 50% ZF state at $\alpha \ll 1$

Density Relaxation In a Channel with Variable α

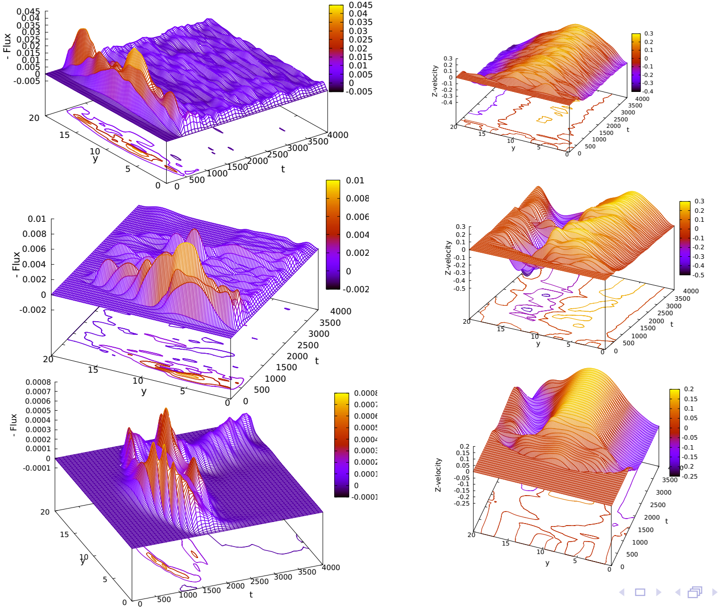




1st col: Density surface plots vs coordinate across the channel (y) and time (zonally averaged)
2nd col: ZF generation efficiency in time (spatially-averaged)

- 1st row: $\alpha \approx 0.01$ -subcritical: strong turbulent flux, wall to wall
- 2nd row: $\alpha \approx 0.01$ for $y \in (0, 10)$, $\alpha \approx 1$ for $y \in (10, 20)$ (supercritical): strong turbulent flux only on subcritical part
- 3rd row: $\alpha \approx 1$ -supercrit.: strong flux suppression by ZF shearing everywhere

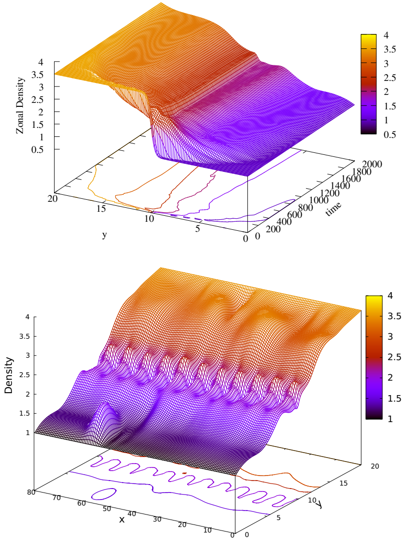
ZF + Flux (cont'd from prev. slide)



1st col: Flux across channel (zon-averaged)
2nd col: ZF

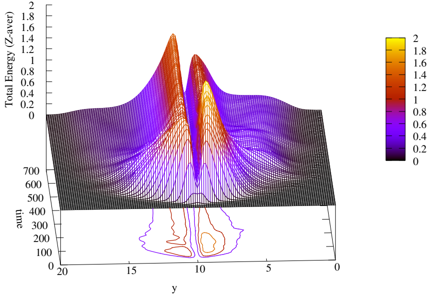
- 1st row: $\alpha \approx 0.01$ - subcritical
- 2nd row: $\alpha \approx 0.01$ for $y \in (0, 10)$, $\alpha \approx 1$ for $y \in (10, 20)$ (supercr.)
- 3rd row: $\alpha \approx 1$- supercr.

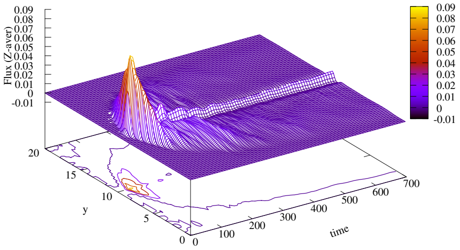
Density-Step Decay

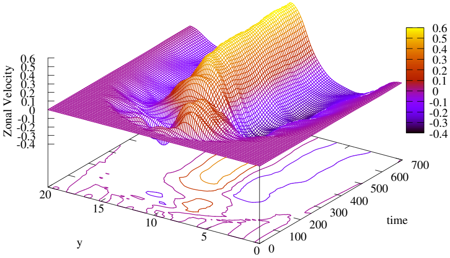


- **Top panel:** Decay of a density step into a staircase. Full density profile is shown as a function of time and radial coordinate, y, zonally averaged.
- **Bottom panel:** Snapshot of the density profile as a function of x and y, taken at the final time
- **Parameters: Density Contrast** $\Delta n = 2.5$, $\alpha = 0.5$, initial density gradient scale $\delta = 0.02$

Density-Step Decay cont'd







- Turbulence energy, zonal flow velocity, and particle flux, all zonally averaged, as functions of time and coordinate across the channel

Conclusions

- Density limit phenomenology studied, using modified HW system with constant and variable adiabaticity parameter $\alpha = k_{\|}^2 V_{\mathrm{th}}^2 / \omega\eta$
  - Both periodic box and channel flows simulated and compared
  - Somewhat stronger and coherent channel flows documented
- **Transport barrier forms in the channel where $\alpha(y) \sim \alpha_{\mathrm{cr}}(\kappa)$**
- ZF shear flattens where α decreases below α_{cr}
- Initially sharp density gradient decays into a staircase structure