L-H Transition Threshold Physics for Weakly Collisional Plasmas

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Abstract

H-mode operation is the regime of choice for good confinement. This renders the questions of access to, and remaining in, H-mode to be critical. Foremost of these issues is the L→H transition power threshold and the related problem of hysteresis. To predict ITER transitions, one must understand the threshold in low collisionality, electron heated regimes where the physics may differ significantly from present day discharges. In this paper, we discuss new transition scenarios, characterized by the sensitivity of transition evolution to pre-existing L-mode profiles.
Ongoing studies are concerned with understanding the transition in the *collisionless regime*. This challenging regime presents at least two problems:

1. the electon-ion coupling is now *anomalous,* due to $\langle \mathbf{E} \cdot \mathbf{J} \rangle$ work
2. the shear flow damping is turbulent, and not due to collisional drag.

To address 1.) we have utilized a model of collisionless power coupling between electrons and ions. Most notable in this model is that the power coupling is not simply proportional to $T_e - T_i$, but depends on the turbulence intensity. To address 2.) we have extended a recently developed theory of minimum enstrophy relation which predicts that the flow damping should have the form of a turbulent *hyper-viscosity*. In addition, the nonlinear flow damping leads to additional nonlinear viscous heating of the ions.

Preliminary results of studies of collisionless regimes suggest that L$\rightarrow$H transition occurs as the endstate of an *anomalous electron-ion thermal coupling front.* This front is attached to a propagating turbulence intensity front. The transition occurs when the front arrives at the edge and impulsively raises T_i there, thus raising ∇P_i, and the diamagnetic electric field shear, so triggering the transition. Further studies of this interesting and relevant phenomenon are ongoing. This study highlights the importance of collisionless energy transfer process to transitions in regimes of ITER relevance.

Finally, we are exploring new transition scenarios. Recent studies have revealed that a spontaneous transition can occur *in the absence of turbulence driven shear flow.* The key point here is the sensitivity of the transition to the pre-existing L-mode density profile. Ongoing work is focused on elucidating this sensitivity and understanding how to exploit it to optimize the access to H-mode. Our aim is to map the basins of attraction for different transitions.

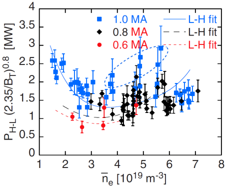
Motivation: Physics of L→H Threshold

Present

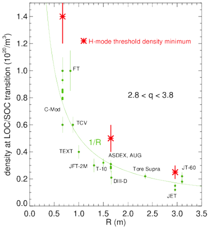
- Origin of $P_{th}(n)$ minimum?
  - collisional electron-ion coupling (Ryter) especially electron heating
  - P_{th} set only by edge local physics? $a^2/\chi\tau_{equil} \gtrsim 1$? coupling $\Longleftrightarrow$global dependence?
  - connection to LOC-SOC transition

Looking Ahead

- Collisionless, electron heated plasmas ?
  - coupling anomalous $\langle \mathbf{E} \cdot \mathbf{J}_{e,i} \rangle$ transfer via fluctuations
  - shear flow regulation – no collisional drag ?
- How $\nabla P_i|_{edge}$ rise?



Ryter et al 2012



Rice et al 2012

Model

- Previous version
  - "$k - \epsilon$" for evolution of intensity, shear flow field, n, T_i,$\langle V_\vartheta \rangle$
  - shear flow damped by drag
- Novel Features:
  - Separated Species:

$$\partial T_e/\partial t + \text{transport} = Q_e - \text{collisional transfer} \ - \text{collisionless coupling}$$

$$\partial T_i/\partial t + \text{transport} \quad = \quad Q_i + \text{collisional transfer}$$
$$+\text{collisionless heating} \ \ (\text{due shear flow})$$
$$-\text{collisionless shear flow} \ \ \text{damping}$$

$$\gamma_{SF} = \gamma_{visc} \left(\frac{\partial \sqrt{E_0}}{\partial r} \right)^2 + \gamma_{Hvisc} \left(\frac{\partial^2 \sqrt{E_0}}{\partial r^2} \right)^2$$

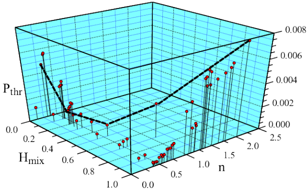
Results with Collisional Transfer

- critical parameter:
  Heating mix

$$H_{i/i+e} = Q_i/(Q_i+Q_e) \equiv H_{mix}$$

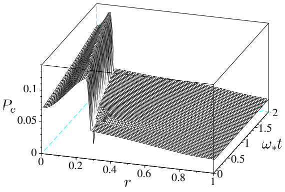
- Relating H_{mix} and n by monotonic $H_{mix}(n)$ recovers $P_{thr}(n)$ min$\nearrow$

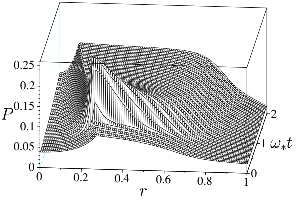
- $P_{thr}(n)$ minimum recovered *only* when *both* n and H_{mix} evolved $\rightarrow$



Collisionless, Electron Heated Regimes– Preliminary Results

- Coupling anomalous $\langle \mathbf{J}_{i,e} \cdot \mathbf{E} \rangle$, not $\propto T_e - T_i$
- Flow damping: turbulent hyper-viscosity (c.f. P.C. Hsu, et al. PoP 2015)
- Transition mechanism: anomalous $e \to i$ *thermal equilibration front*





- Transition occurs when P_i–front approaches edge
  $\Rightarrow$triggers increase in ion ∇P_i
- New Scenario!, *Prediction.*

Conclusions from 6-field model

- $P_{th}(n)$ minimum recovered with collisional coupling
- Threshold physics not limited to edge
  $a^2/\chi\tau_{eq} > 1$ required for electron heated transitions
  $\Longrightarrow$some global dependence

Predictions: Collisionless, Electron Heated Regime

- Anomalous heat exchange and shear flow damping initiated in collisionless, electron heated regimes (ITER).
- Transition manifested as propagating thermal equilibration front; triggers ∇P_i increase at edge.

Four-Field Model

Motivation: Understand L$\rightarrow$H at low collisionality

- Understanding of thermal front propagation is difficult within full 6-field model
- Indispensable parts:
  - DW energy, I
  - ZF velocity, W
  - electron and ion temperature: T_e, T_i
- Equations (density frozen, mean flow ignored)

$$
\begin{aligned}
\frac{\partial I}{\partial t} &= \left(\gamma_L - \Delta\omega I - \alpha_0 W^2/2 - \alpha_v R\right) I + \chi_N \left(I'^2 + I \cdot I''\right) \\
\frac{\partial W}{\partial t} &= \frac{\alpha_0 I W}{1 + \zeta_0 R} - \gamma_c W + \gamma_v \left(I' W' + I W''\right)
\end{aligned}
$$

Equation's continued

$$\text{Here,} \qquad R = \left[\kappa_n \left(\kappa_n T + T'\right)\right]^2, \qquad T = T_i + T_e$$

$$\gamma_L = \sqrt{T_e}\left[\gamma_{L0}\Re\sqrt{-T_i'/T_i - T_{i0}'} - \gamma_{e0}\left(\kappa_n + \sigma T_e'/T_e\right)\right]$$

- Temperature transport

$$\frac{\partial T_e}{\partial t} = \frac{\partial}{\partial x}\left(\frac{I}{1+\alpha_t R} + \chi_{neo}^e\right) T_e' - \frac{1}{\tau}\left(T_e - T_i\right) + Q_e + \gamma_{e0} I\left(\kappa_n + \sigma T_e'/T_e\right)$$

$$\frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x}\left(\frac{I}{1+\alpha_t R} + \chi_{neo}^i\right) T_i' + \frac{1}{\tau}\left(T_e - T_i\right) + Q_i - \gamma_{e0} I\left(\kappa_n + \sigma T_e'/T_e\right) + \gamma_v I W'^2$$

$$\gamma_c = n\gamma_{c0}/T_i^{3/2}, \tau = \tau_0 T_e^{3/2}/n, n = n_0\left(1 - \beta x^2\right), \kappa_n = d\ln n/dx$$

Steady state solutions

For $\tau \ll 1$, $T_e = T_i + \mathcal{O}(\tau) \approx T/2$.
We thus obtain

$$\left(\frac{\zeta_0 - \alpha_t}{1 + \alpha_t R} R + \chi\right) T' + \frac{\alpha_0}{2\gamma_c} (\chi_i - \chi_e) \Delta T' - S = const$$

where $\Delta T = T_i - T_e$, $\partial S / \partial x = -\alpha_0 (Q_e + Q_i) / \gamma_c$

$$\chi = 1 + \frac{\chi^i_{neo} + \chi^e_{neo}}{2} \frac{\alpha_0}{\gamma_c}$$

Assuming $|\Delta T| \ll T$, we have

$$\left[\chi - \frac{\alpha (\kappa_n T + T')^2}{1 + \omega (\kappa_n T + T')^2}\right] T' = S(x) \qquad (1)$$

where

$$\alpha = (\alpha_t - \zeta_0) \kappa_n^2$$

Flux-driven transitions

- Assume the sources $Q_{e,i}(x)$ are localized near the origin ($x = 0$, core plasma). Then, $S \simeq const$, for all $x > 0$.
- solutions $T = T(x)$ depend on five parameters: χ, α, ω, κ_n and S
- need understand bifurcations of these solutions
  - somewhat easier after rescaling:
    - $\kappa_n x \to x$, $S/\kappa_n \to S$, $\kappa_n^2 \alpha \to \alpha$,$\kappa_n^2 \omega \to \omega$

$$\left[\chi - \frac{\alpha(T+T')^2}{1+\omega(T+T')^2}\right] T' = S = const$$

solve for $T(T')$:

$$T = c\sqrt{\frac{T'-a}{b-T'}} - T' \qquad (2)$$

where

$$a = S/\chi, \quad b = S/(\chi - \alpha/\omega), \quad c = (\omega - \alpha/\chi)^{-1/2}$$

Flux-driven cont'd

- $a < T' < b$, transport bifurcation problem in three parameters, (a, b, c). Bifurcation point

$$\frac{\partial T}{\partial T'} = \frac{\partial^2 T}{\partial^2 T'} = 0$$

$$b = a + \frac{8c}{3\sqrt{3}}, \quad T' = a + \frac{2c}{3\sqrt{3}}, \quad T = -a + \frac{c}{3\sqrt{3}}$$

- need to find the profile $T(x)$ (easier parametrically, as $x(T')$) :

$$\begin{aligned} x(T') &= x_0 - \ln|T'| + \frac{c}{b}\left\{\sqrt{\frac{T'-a}{b-T'}}\right\} + \frac{b-a}{2\sqrt{ab}} \\ &\times \left[\tan^{-1}\frac{T'-\sqrt{ab}}{\sqrt{(b-T')(T'-a)}}\right] - \left[\tan^{-1}\frac{T'+\sqrt{ab}}{\sqrt{(b-T')(T'-a)}}\right] \end{aligned}$$

Flux-driven cont'd

- $T(T')$ and $x(T')$ determine temperature profile $T(x)$ parametrically, via parameter T'
- via BC, the temperature is given at the edge $x = -\kappa_n$
- use the variables

$$\xi = \left(T' + T\right)/c, \quad \delta = (a + T)/c, \quad \beta = (b + T)/c \tag{3}$$

so

$$\xi = \sqrt{\frac{\xi - \delta}{\beta - \xi}} \tag{4}$$

Upon solving this equation for ξ, T' explicitely, as a function of T, a, b, c.

$$\xi_n = \frac{2}{3}\sqrt{\beta^2 - 3}\sin\left[\frac{1}{3}\sin^{-1}(P) + \frac{2}{3}\pi n\right] + \frac{1}{3}\beta, \quad n = 0, \pm 1, \tag{5}$$

where

$$P = -\frac{\beta\left(2\beta^2 - 9\right) + 27\delta}{2\left(\beta^2 - 3\right)^{3/2}}$$

On the (β, δ) plane where $|P| \leq 1$ all three roots in eq.(5) are real (phase coexistence). For $|P| > 1$, only one root is real, no phase coexistence. The phase coexistence condition ($|P| \leq 1$)

$$\frac{2}{27}\max\left[0, \beta\left(\frac{9}{2} - \beta^2\right) - \left(\beta^2 - 3\right)^{3/2}\right] \leq \delta$$
$$\leq \frac{2}{27}\left[\beta\left(\frac{9}{2} - \beta^2\right) + \left(\beta^2 - 3\right)^{3/2}\right], \quad \beta \geq \sqrt{3}$$

If (β, δ) lie outside of the phase-coexistence domain, only one solution is real. For $\beta > \sqrt{3}$ and $|P| > 1$:

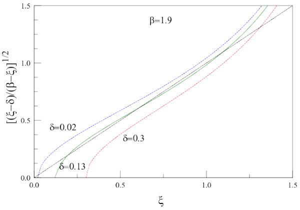
$$\xi = \frac{2}{3}\sqrt{\beta^2 - 3}\cosh\left(\frac{1}{3}\cosh^{-1}|P|\right) + \frac{1}{3}\beta$$

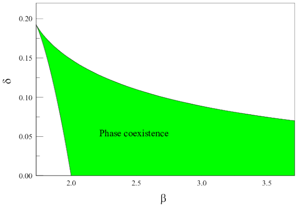
Flux-driven cont'd

This is H-mode solution. For $\beta \leq \sqrt{3}$, we obtain

$$\xi = \frac{2}{3}\sqrt{3-\beta^2}\sinh\left[\frac{1}{3}\sinh^{-1}(iP)\right] + \frac{1}{3}\beta$$

This is L-mode solution, as it corresponds to the lowest real T'





Graphic illustration of the bifurcaton diagram. The three curves correspond to the r.h.s. of eq.(4) drawn for three values of δ that represent, respectively, an L-mode solution, LH coexistence phase, and the H-mode solution. These values of δ are taken from the region below the shaded (phase-coexistence zone), from that zone, and from the one above it.

Spontaneous Transition

- new transition scenario: occurs *in the absence of turbulence driven shear flow.*
- sensitive to pre-existing L-mode density profile
- need to understand how to optimize the access to H-mode

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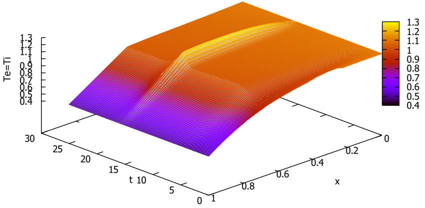

Figure: Spontaneous LH transition with suppressed shear flow. The heat pulse applied mid-time of integration with no effect on the H-state established spontaneously.