L-H Transition Threshold Physics at Low Collisionality

M.A. Malkov[1], P.H. Diamond[1], P.-C. Hsu, J. Rice[2], K. Miki[3] and G.R. Tynan[1]

[1]CMTFO and CASS, UCSD, USA; *e-mail*: mmalkov@ucsd.edu
[2]PCFS, MIT, USA; [3]JAEA, Rokkasho, Japan

Supported by US DoE, Award Number DE-FG02-04ER54738

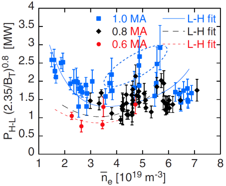
Abstract

An L$\rightarrow$H power threshold scaling including the minimum in $P_{th}(n)$ is discussed, elucidating the impact of inter-species energy transfer on threshold physics. Using a new four-field LH transition model, we study transitions in collisionless, electron heated regimes where the electron-ion coupling is allowed to be completely *anomalous*, mediated by the fluctuation of $\langle \mathbf{E} \cdot \mathbf{J} \rangle$ work on electrons and ions. New transition scenarios, characterized by the sensitivity of transition evolution to pre-existing L-mode profiles are also considered, using the new model.

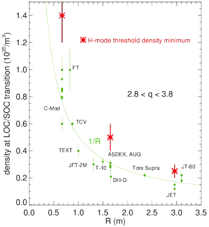
Motivation: Physics of L→H Threshold

Present

- Origin of $P_{th}(n)$ minimum?
- collisional electron-ion coupling (Ryter) especially electron heating
  - P_{th} set only by edge local physics? $a^2/\chi\tau_{equil} \gtrsim 1$? coupling $\Longleftrightarrow$global dependence?
  - connection to LOC-SOC transition



Ryter [1]



Rice[2]

Looking Ahead

- Collisionless, electron heated plasmas ?
  - coupling anomalous $\langle \mathbf{E} \cdot \mathbf{J}_{e,i} \rangle$ transfer via fluctuations
  - shear flow regulation – no collisional drag ?
- How does $\nabla P_i|_{edge}$ rise?

Introduction

- H-mode operation [3, 4, 5, 6] is the regime of choice for good confinement.
- Issues:
  - optimum access to, and efficient sustainability, of H-mode [7, 8]
  - L→H transition power threshold, understanding its minimum
  - related problem of hysteresis
- ITER-specific transitions:
  - understand threshold in low collisionality, electron heated regimes
  - (deep) relation to $P_{th\ min}$
- low-collisionality transitions requires model extension beyond collisional $e - i$ coupling
- discussed in this paper:
  - L→H *power threshold* scaling and the origin of $P_{\mathrm{th}}(n)$ -min
  - transitions in collisionless, electron heated regimes
  - new transition scenarios, characterized by sensitivity to pre-existing L-mode profiles

Previous Model

- $k - \epsilon$ for evolution of intensity, shear flow field, n, T_i,$\langle V_\vartheta \rangle$
- shear flow damped by drag
- Separated Species:

$$\partial T_e / \partial t + \text{transport} = Q_e - \text{collisional transfer} \; - \text{collisionless coupling}$$

$$\begin{aligned} \partial T_i / \partial t + \text{transport} = Q_i &+ \text{collisional transfer} \; + \text{collisionless coupling} \\ &- \text{collisionless shear flow} \;\; \text{damping} \\ &+ \text{collisionless heating} \;\; \text{(due shear flow)} \end{aligned}$$

$$\gamma_{SF} = \gamma_{visc} \left(\frac{\partial \sqrt{E_0}}{\partial r} \right)^2 + \gamma_{Hvisc} \left(\frac{\partial^2 \sqrt{E_0}}{\partial r^2} \right)^2$$

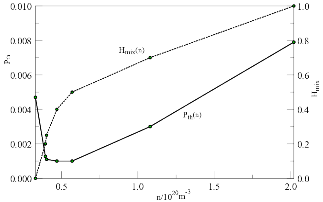
- $\sqrt{E_0}$ stands for ZF velocity: two contributions come from viscose and hyperviscose ZF damping and corresponding ion heating

Results with Collisional Transfer

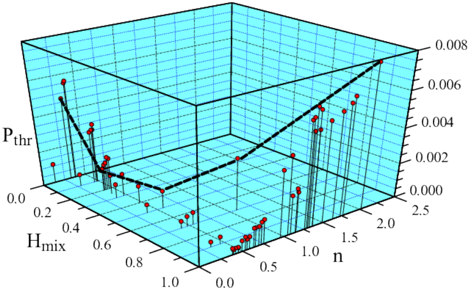
- critical parameter:
  Heating mix

$$H_{i/i+e} = Q_i / (Q_i + Q_e) \equiv H_{mix}$$

- Relating H_{mix} and n by monotonic $H_{mix}(n)$ recovers $P_{thr}(n)$ min $\nearrow$

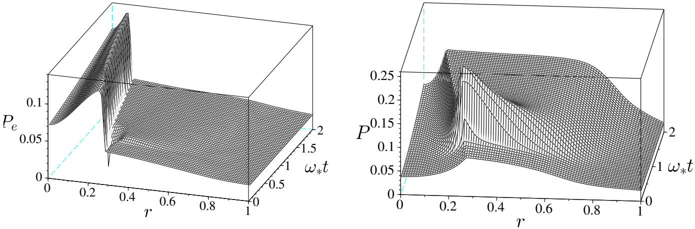


- $P_{thr}(n)$ minimum recovered *only* when *both* n and H_{mix} evolved $\rightarrow$
- 3D curve $P_{thr}(n, H_{mix})$, projected on (n, P_{thr})-plane (top plot)u has a minimum



Collisionless, Electron Heated Regimes– Predictions

- Coupling anomalous $\langle \mathbf{J}_{i,e} \cdot \mathbf{E} \rangle$, not $\propto T_e - T_i$
- Flow damping: turbulent hyper-viscosity (c.f. P.C. Hsu, et al. PoP 2015)
- Transition mechanism: anomalous $e \to i$ *thermal equilibration front* ($e-$cooling front, left Figure)

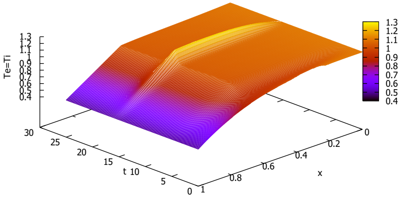


- Transition occurs when P_i-front (i-heating front, right Figure) approaches edge $\Rightarrow$ triggers increase in ion ∇P_i
- New Scenario!, *Prediction.*

New Model

- Motivated by:
  - complexity of transitions studied within the previous 6-field model [9, 10]
  - strong requirements for capturing sharp fronts
- Features:
  - 4-field model ($T_{e,i}$, DW, ZF) allows to explore new transition scenarios
  - adaptive mesh refinement, high-fidelity collocation scheme
- Aimed at:
  - understanding low-collisionality transition
  - spontaneous transition *in the absence of turbulence driven shear flow*
  - sensitivity of the transition to the pre-existing L-mode *density profile*
  - optimizing access to H-mode
  - mapping basins of attraction for different transitions

Key Physical Elements of 4-field model.



Spontaneous LH transition with suppressed shear flow

- A new analytical and numerical 4-field model for describing $L \to H$ transitions in weakly collisional ITER-related regimes
- transitions in collisionless, electron heated regimes where the electron-ion coupling is allowed to be completely *anomalous*, due to the fluctuation of $\langle \mathbf{E} \cdot \mathbf{J} \rangle$ work on electrons and ions
- New transition scenarios, characterized by the sensitivity of transition evolution to pre-existing L-mode profiles are considered (Figure, upper-left corner).

Model New Capabilities and Equations

- the shear flow damping is turbulent, and not just due to collisional drag
- nonlinear flow damping leads to additional turbulent viscous heating of the ions

Equations evolve $T_{e,i}$, DW intensity, I , and ZF velocity W :

$$\frac{\partial T_e}{\partial t} = \frac{\partial}{\partial x}\left(\frac{I}{1+\alpha_t R} + \chi^e_{neo}\right) T'_e - \frac{1}{\tau}\left(T_e - T_i\right) + S'_e + \gamma_{e0} I \left(\kappa_n + \sigma T'_e / T_e\right)$$

$$\frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x}\left(\frac{I}{1+\alpha_t R} + \chi^i_{neo}\right) T'_i + \frac{1}{\tau}\left(T_e - T_i\right) + S'_i - \gamma_{e0} I \left(\kappa_n + \sigma T'_e / T_e\right) + \gamma_v I W'^2$$

$$\frac{\partial I}{\partial t} = \left(\gamma_L - \Delta\omega I - \alpha_0 W^2/2 - \alpha_v R\right) I + \chi_N \left(I'^2 + I \cdot I''\right)$$

$$\frac{\partial W}{\partial t} = \frac{\alpha_0 I W}{1+\zeta_0 R} - \gamma_c W + \gamma_v \left(I' W' + I W''\right)$$

Definitions and notations

- **initial study:** suppression factor R obtained for strong thermal $e-i$ coupling ($T_e \approx T_i$) anomalous and collisional (will relax in the next phase)

$$R = \left[\kappa_n \left(\kappa_n T + T'\right)\right]^2, \quad \text{with} \quad T \equiv T_i + T_e.$$

  - notation/units: $T' = \nabla T$, T/MC_s^2, $\kappa_n = L_n^{-1}$, length in min. rad., a

- heat sources for electrons and ions (at $x \simeq a_{e,i}$)

$$S'_{e,i} = \frac{2S'_{0e,i}}{\sqrt{\pi} D_{e,i} \left[\text{erf}\left(\left(1 - a_{e,i}\right)/D_{e,i}\right) + \text{erf}\left(a_{e,i}/D_{e,i}\right)\right]} \exp\left[-\left(\frac{x - a_{e,i}}{D_{e,i}}\right)^2\right]$$

- ITG and CTEM contributions to growth of DW:

$$\gamma_L = \sqrt{T_e}\left[\gamma_{L0}\Re\sqrt{-T'_i/T_i - T'_{i0}} - \gamma_{e0}\left(\kappa_n + \sigma T'_e/T_e\right)\right]$$

- shear flow velocity in suppression factor

$$V_E = (c/eBn)\, p', \quad p = n\left(T_e + T_i\right), \quad \langle V_E \rangle' \approx -(c/eB)\, \kappa_n \left(\kappa_n T + \kappa_n\right)$$

Stationary Analytic Solutions

- limit of small $\tau \to 0$, $T_e = T_i + \mathcal{O}(\tau) \approx T/2$
- turbulent components sit at their thresholds: $\gamma_v = \chi_N = 0$
- steady state solution for $T_{e,i}(x)$, I and W, obtained from Eqs. on p.10 by setting $\partial_t = 0$ (saturated instabilities for I and W)

$$\left(\frac{\zeta_0 - \alpha_t}{1 + \alpha_t R} R + \chi\right) T' + \frac{\alpha_0}{2\gamma_c} (\chi_i - \chi_e) \Delta T' - S = const$$

where $\Delta T = T_i - T_e$, $\partial S/\partial x = -\alpha_0 (S_e' + S_i') / \gamma_c$ and $\chi = 1 + \left(\chi_{neo}^i + \chi_{neo}^e\right) \alpha_0 / 2\gamma_c$

- Assuming $|\Delta T| \ll T$

$$\left[\chi - \frac{\alpha (\kappa_n T + T')^2}{1 + \omega (\kappa_n T + T')^2}\right] T' = S(x) \qquad (1)$$

here $\alpha = (\alpha_t - \zeta_0) \kappa_n^2$ and $\omega = \alpha_t \kappa_n^2$, $S(0) = 0$.

Flux-driven transitions

- sources $S'_{e,i}(x)$ are localized near the origin ($x = 0$, core plasma)
- $S = const$, for all $x > 0$, flux-driven transitions
- solutions $T = T_e + T_i = T(x)$ depends on five parameters
  - $\chi, \ \ \alpha, \ \ \omega, \ \ \kappa_n$ and S.
  - after rescaling: $\kappa_n x \to x$, $S/\kappa_n \to S$, $\kappa_n^2\alpha \to \alpha$,$\kappa_n^2\omega \to \omega$, obtain simplified bifurcation problem

$$\left[\chi - \frac{\alpha\left(T + T'\right)^2}{1 + \omega\left(T + T'\right)^2}\right] T' = S = const \qquad (2)$$

solve for T as a function of T' and three parameters, a, b, c

$$T = c\sqrt{\frac{T' - a}{b - T'}} - T' \qquad (3)$$

$$a = S/\chi, \quad b = S/\left(\chi - \alpha/\omega\right), \quad c = \left(\omega - \alpha/\chi\right)^{-1/2}$$

- resolving above eq. for $T(x) \Longrightarrow$ solution multiplicity, bifurcation in (a, b, c) parameter space

Phase coexistence and bifurcation diagram

- solution is easily obtained in terms of $x\,(T')$

$$x\,(T') = x_0 - \ln|T'| + \frac{c}{b}\sqrt{\frac{T'-a}{b-T'}}$$

$$+\frac{c\,(b-a)}{2b\sqrt{ab}}\left[\tan^{-1}\frac{T'-\sqrt{ab}}{\sqrt{(b-T')\,(T'-a)}} - \tan^{-1}\frac{T'+\sqrt{ab}}{\sqrt{(b-T')\,(T'-a)}}\right]$$

- using new variables

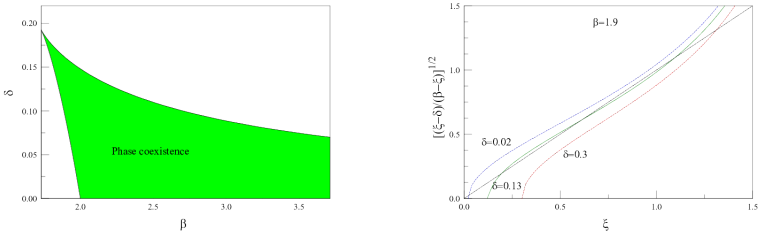
$$\xi = \left(T' + T\right)/c, \quad \delta = (a+T)/c, \quad \beta = (b+T)/c \qquad (4)$$

the phase coexistence domain (green zone, left panel) is bound by two inequalities

$$\frac{2}{27}\max\left[0, \beta\left(\frac{9}{2}-\beta^2\right) - \left(\beta^2-3\right)^{3/2}\right] \le \delta \le \frac{2}{27}\left[\beta\left(\frac{9}{2}-\beta^2\right) + \left(\beta^2-3\right)^{3/2}\right],$$

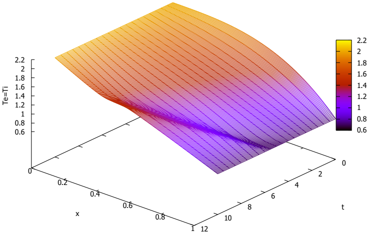
$$\beta \ge \sqrt{3}$$

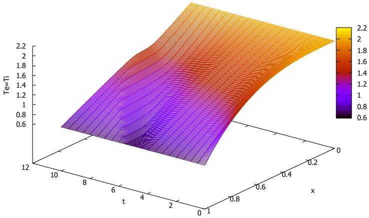
Phase coexistence and bifurcation

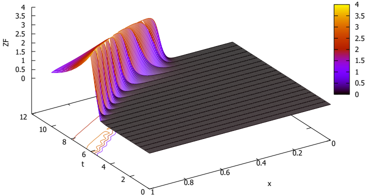


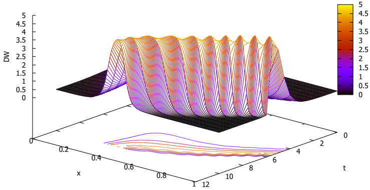
- if (β, δ) are outside of the phase-coexistence domain (green) only one solution out of the three possible is real
- for $\beta > \sqrt{3}$ it corresponds to an H-mode solution (right panel, lower dashed curve
- for $\beta \leq \sqrt{3}$ it corresponds to an L-mode solution (lowest real T' value out of the three solutions with the other two roots becoming complex)

Example of Spontaneous Transition





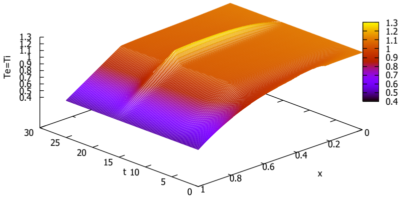


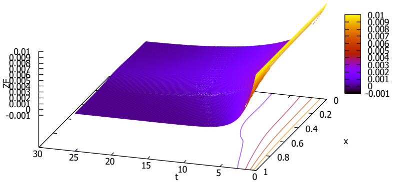


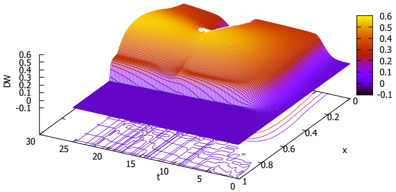
Spontaneous Transition: Description

- simulation starts from a parabolic temperature profile in L-mode (top two panels, same surface viewed at different angles)
- T relaxes to a linear profile but DW is generated at the edge (where ∇T was initially the strongest) and propagates inward
- T-profile flattens in the region of active DW but ZF also grows at the edge
- both DW and ZF fronts continue to propagate inward but the DW has also a rear, cancelling front
- it leaves an H-mode state behind with a residual ZF and zero DW

No-Flow Spontaneous Transition

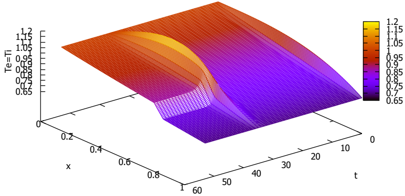


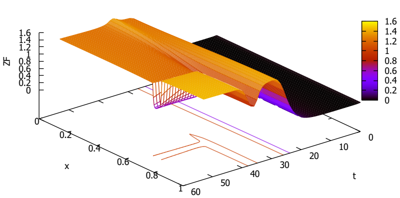


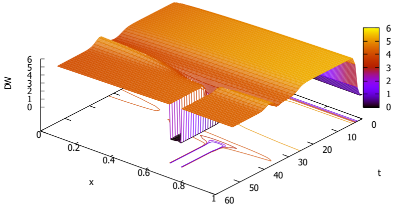


- preexisting temperature and density profiles allow for spontaneous LH transition
- ⟸DW suppressed over a broad edge region
- ⇑ZF dies out everywhere
- established H-mode is resilient to a heat pulse↖

Pulse-Triggered ITB (Work in progress, preliminary results)







- run starts from an L-mode which quickly relaxes to a linear temperature profile

- strong DW turbulence and slowly rising ZF
- heat pulse is applied at $t \simeq 40$
- it triggers rapid ZF growth which suppresses the DW in a narrow region
- strong temperature gradient builds up in this region, ITB$\nwarrow$

Conclusions

- new analytical and numerical 4-field model for describing $L \to H$ transitions in weakly collisional ITER-related regimes is developed
- new type of transition scenario, which is more sensitive to the pre-existing L-mode structure than to the power variation near the threshold is identified
- dynamical realization of such transitions became possible after an accurate analytic determination of the phase coexistence domain and transition criteria in a multi-dimensional parameter space of the system
- stationary solutions of the model, obtained analytically for that purpose, are also crucial for the code verification
- work studying dynamical evolution of $L \to H$ transitions numerically is ongoing

References

- F. Ryter *et al.*, Nucl. Fusion **53**, 113003 (2013).
- J. E. Rice *et al.*, Physics of Plasmas **19**, 056106 (2012).
- F. Wagner *et al.*, Physical Review Letters **49**, 1408 (1982).
- K. H. Burrell *et al.*, Physics of Plasmas **12**, 056121 (2005).
- L. Schmitz *et al.*, Phys. Rev. Lett. **108**, 155002 (2012).
- G. R. Tynan *et al.*, Plasma Physics and Controlled Fusion **58**, 044003 (2016).
- J. W. Hughes *et al.*, APS Meeting Abstracts (2015).
- Y. Ma *et al.*, Nuclear Fusion **52**, 023010 (2012).
- M. A. Malkov *et al.*, Physics of Plasmas **22**, 032506 (2015).
- K. Miki *et al.*, Phys. Plasmas **19**, 092306 (2012).