On Fluctuating Layered States: A Reduced Model of Near-Marginal Systems

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Outline

1) **Background**

2) **Fixed Cellular Array (FCA) Problem**

3) **Relaxing FCA with Fluctuating Vortex Array**

4) **Passive Scalar Dynamics**

   * Summary of results

5) **Active Scalar Dynamics**

   * Ongoing work

Near-Marginal Systems

<u>Near-Marginal</u>

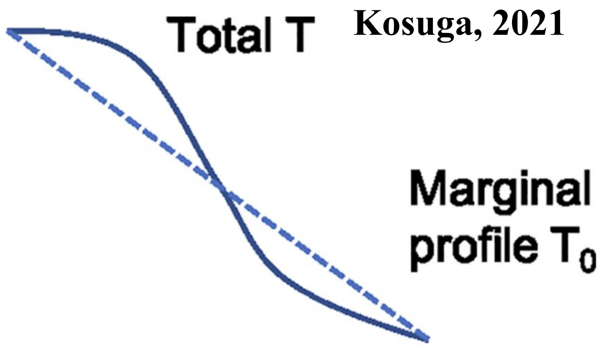
- Weak turbulence
  - ***E* × *B*** convective cells and magnetic islands excited but not strongly overlapping.

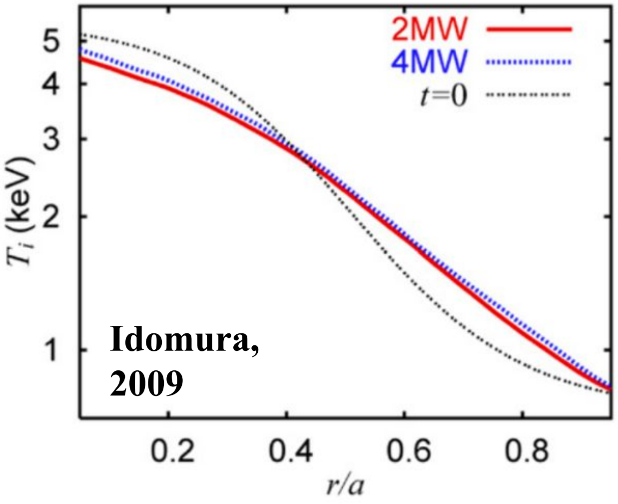
→ Instabilities are excited but not so strong as to produce large transport.

Characteristic of **<u>Stiff profiles</u>**

- I.e., Profiles that adopt roughly the same shape regardless of the applied heating and fueling profiles

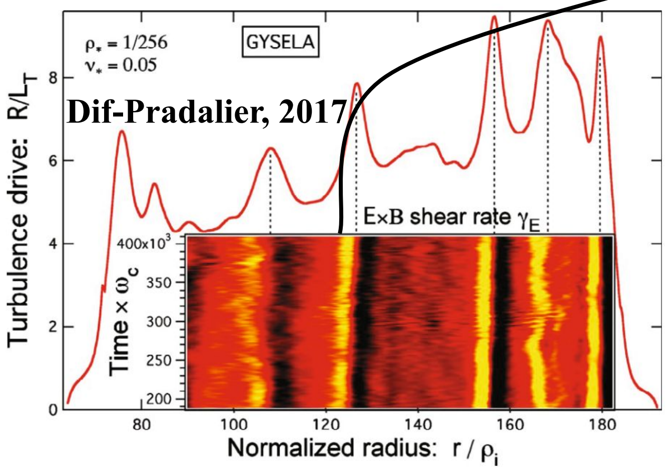
Near-marginal plasmas can sometimes naturally evolve towards a **globally organized** critical state of micro-barriers and strong avalanche-like transport.





$E \times B$ Staircase

***E* × *B* <u>staircase</u> current subject in M.F.E**

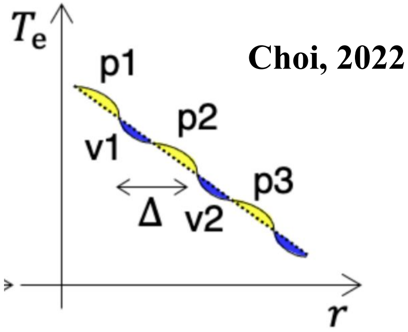


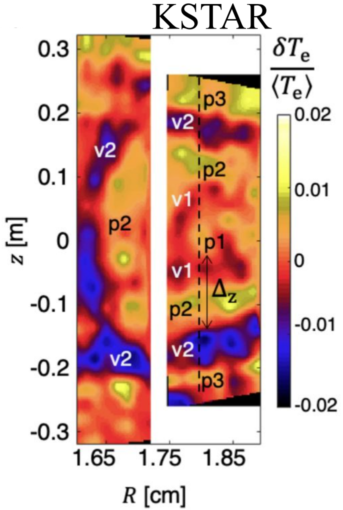
Yellow and black colors are a rapid transition of the direction of flows around peaks in turbulence drive.

<u>Context</u>: Flat spots of high transport and nearly vertical layers acting as mini-barriers coexist. In plasmas, avalanches happen in flat spots and shear layers due to zonal flows occur in the areas of mini-barriers.

Suggested ideas (from self-organization):

- ***E* × *B*** shear feedback, predator-prey
  - Zonal flows (**predator**) and turbulence intensity (**prey**)
- Jams (time-delay between temperature modulations and local heat flux)

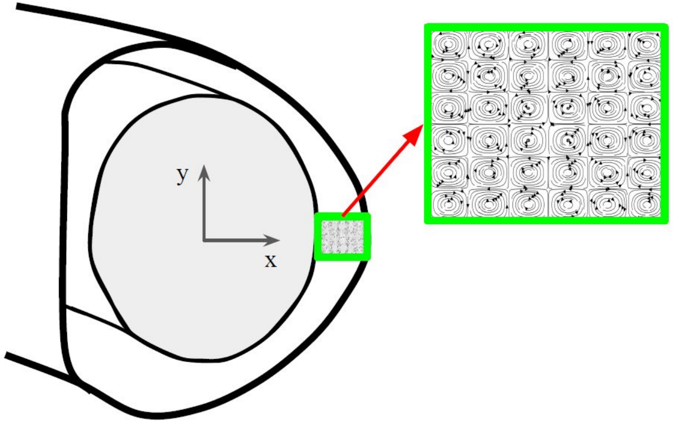




Jet like patterns in δT/<T> image correspond to staircase corrugations

Some Questions

- How does staircase beat homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?



Fixed Cellular Array Problem

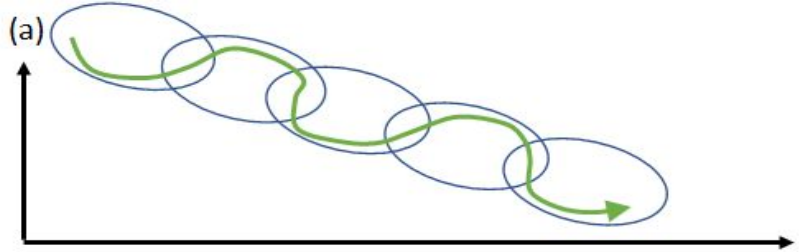
(another way to get a Staircase)

<u>But</u>… is there an even **simpler** physical mechanism that can produce **layering?**
Answer: Yes (e.g., pattern of cells)

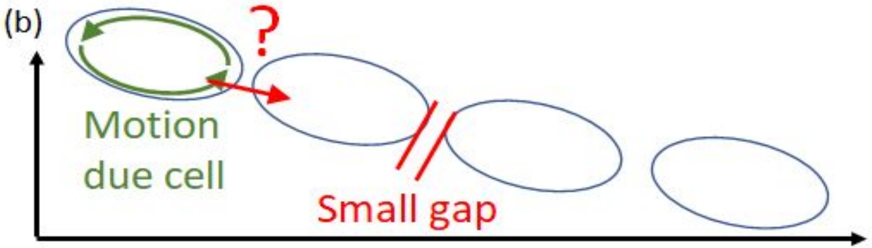
Marginally Overlapping Cells

Transport of particles between marginally overlapping cells (**characteristic of near marginal**) is an important topic in fusion plasma. Two different transport mechanisms:

Overlapping case: particles can transport directly from cell to cell, wandering along streamlines



Nearly-overlapping case (cells sit at near overlap): transport is a synergy of motion due to cells and random kicks (Collisional diffusion, ambient scattering) thru gap regions.



→ Characteristic of **near marginal**.

→ The transport over gap is random kicks (ambient diffusion): collisions, micro-turbulence.

→ **Coexistence** of:
~ **Fast transport** - Mixing in cell
~ **Slow transport** - Kicks between cells

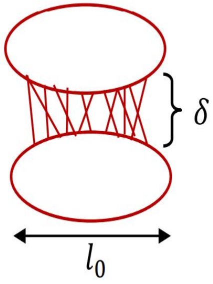
N.B.: "Profile stiffness" → Cells near overlap
→ Rapid increase in transport prevents strong overlap

(Nearly-Overlapping Cells) What of Interest?

- **Relevant to key question of "near marginal stability"** **Features**

→ Representative of state in <u>marginal stability</u>.
- Stiff systems hovering near threshold (relevant question)

→ **Natural candidate to near marginal stability!**
- Zonal (mean) flows
- similarities SOC (fronts, spreading,...)
- **<u>Staircases</u>**



δ → BL width
l_0 → cell size

<u>Back-of-Envelope Calculation</u>

$D^* \approx f_{active}((\Delta x)^2 / \Delta t)$;

$f_{active} \equiv$ active fraction $\sim \delta / \ell_o$

$\Delta t \sim \ell_o / v_o \rightarrow$ cell circulation time

So, $\delta^2 \sim D\, \Delta t \sim D\, \ell_o / v_o$

$D^* \sim [(D\, \ell_o / v_o)^{1/2} 1 / \ell_o]\, (\ell_o^2 / \ell_o)\, v_o \sim [D\, D_{cell}]^{1/2}$

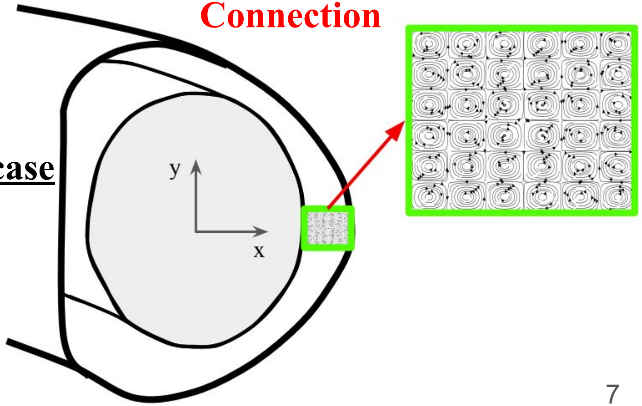
Transport? <u>Answer</u>: $Deff \sim [D\, D_{cell}]^{1/2}$ {**<u>Not a simple addition of process!</u>**}

→ Two time rates: $\tau_H = d / v$ (fast), $\tau_D = d^2 / D$ (slow)

→ $Pe = v\, d / D >> 1$

Consider a **<u>general</u>** case of a system of eddies not overlapping but tangent → **<u>Staircase</u>**

Connection



Profile?

Consider concentration of injected dye (passive scalar transport in eddys) → profile

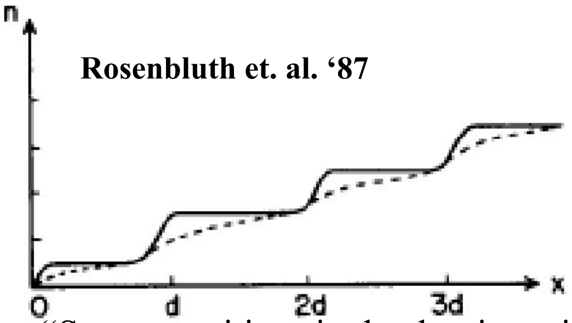
$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D \nabla^2 n,$$

FCA → Staircase!

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D\nabla^2 n,$$

Profile?

Consider concentration of injected dye (passive scalar transport in eddys) → profile



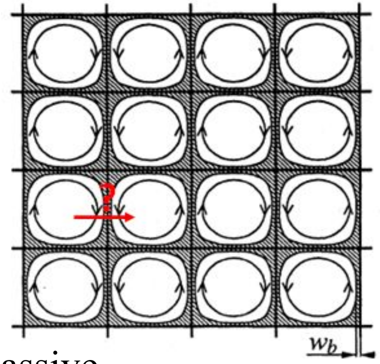
"Steep transitions in the density exist between each cell."

Relevant to key question of "near marginal stability"

→ Layering!

→ **Simple** consequence of **two rates**

→ $Pe = \tau_H / \tau_D >> 1$ (Necessary criteria)



Important:

- **Staircase** arises in stationary array of passive eddies (Note that there is no **FEEDBACK**)
- Global transport hybrid:
  - → fast rotation in cell
  - → slow diffusion in boundary layer
- Irreversibility localized to inter-cell boundary.

BUT, this setup is contrived, NOT self-organized!!! Cellular array is severely constrained!

Staircase arises in an array of stationary eddies!

What about the dynamics of a **less constrained** cell array (i.e., vortex array with fluctuations) ?

Relaxing Fixed Cellular Array with Fluctuating Vortex Array

Consider a Broader Approach

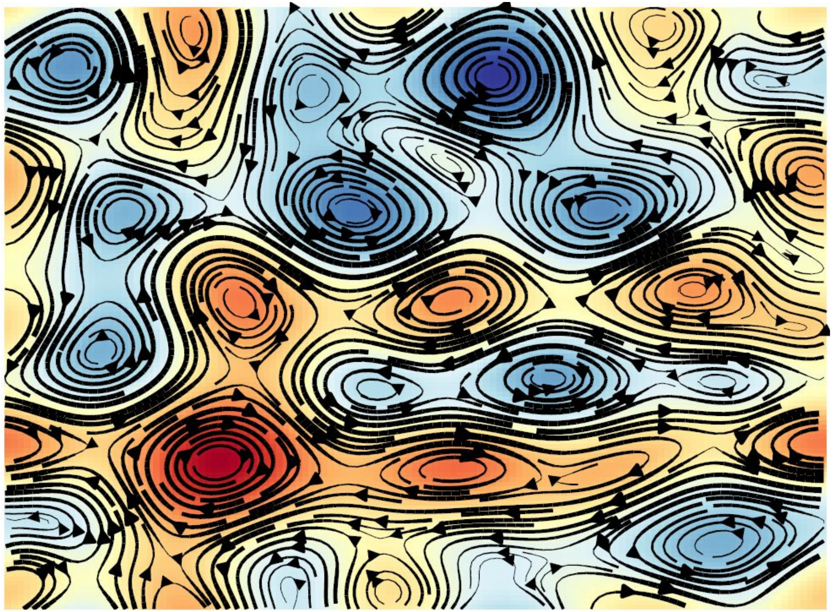
- We want to study a much more **general** and **less constrained** version of the cell array.
  - Consider a vortex array with fluctuations; jitters.
- How **resilient** is the staircase in the presence of these small variations to a fixed vortex array?

In the process of studying the **resilience** of the staircase, we aim to answer the following:

1. What happens to interspersed regions of strong scalar concentration mixing as cells relax? What about general cell interactions/behavior?
2. What is the behavior of the scalar trajectory through the vortex array?
3. How does the increase of scattering in the vortex array affect the transport of scalar concentration?

To answer these questions, we use the idea of a **Melting Vortex Crystal**…

Example of **less constrained** cell array



Fluctuating Vortex Array

Why are we doing this? We know that a system with two disparate time scales forms a staircase!

- Now consider fluctuations… → Will staircase survive?

Vortex array is an alternative way to view convection cells!

→ We begin with the 2D NS equation that can be written in nondimensional form (Perlekar and Pandit 2010),

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

→ The "vortex array" is simply the array of cells and "fluctuation" is related to turbulence induced variability in the structure. The fluctuating vortex array (FVA) allows us to study a **less constrained** version of the array! **Improved model of cells near marginality.**

→ The fluctuating flow structure is created by **slowly increasing the Reynolds number** in the NS equation

$$\Omega = \frac{\tau_\nu}{\tau_H}$$

→ By increasing the Reynolds number this modifies the forcing and drag term, thus, **scattering** the vortex array. The **resilience** of the staircase is studied by **increasing disorder** in the vortex crystal through F_ω

$$F_\omega \equiv -n^3\left[\cos(nx) + \cos(ny)\right]/\Omega$$

The streamfunction, ψ, at different evolutionary stages of the "fluctuating" vortex array is inserted into the passive scalar equation to study the resilience of the staircase structure.

Comparison of Vortex Array model to Drift-wave Turbulence in fusion devices

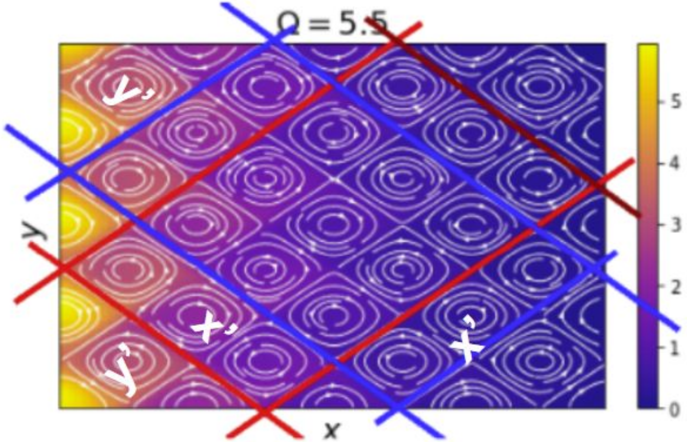
	Vortex Field	Drift-Wave Turbulence (tokamak)
Inhomogeneity (free energy source)	∇n	B_0, ∇n, and ∇T
Reynolds number	$\Omega = 0 - 40$	$Re = 10^1 - 10^2$ (Landau Damping)
Flux	Scalar	Heat
Zonal Flow	Boundary layer between cells	$\mathbf{E} \times \mathbf{B}$ shear flow (poloidal)

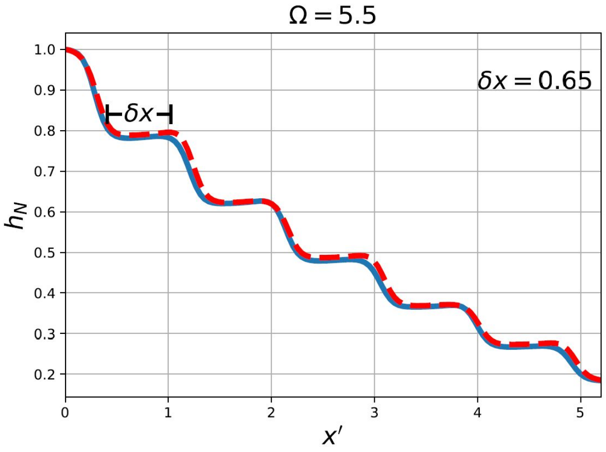
What Happens to the Staircase? (Passive Scalar Dynamics)

$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = D\nabla^2 n,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right)\omega = \frac{1}{\Omega}\nabla^2\omega + F_\omega - \alpha\omega, \qquad \nabla^2\psi = \omega.$$

Baseline staircase structure

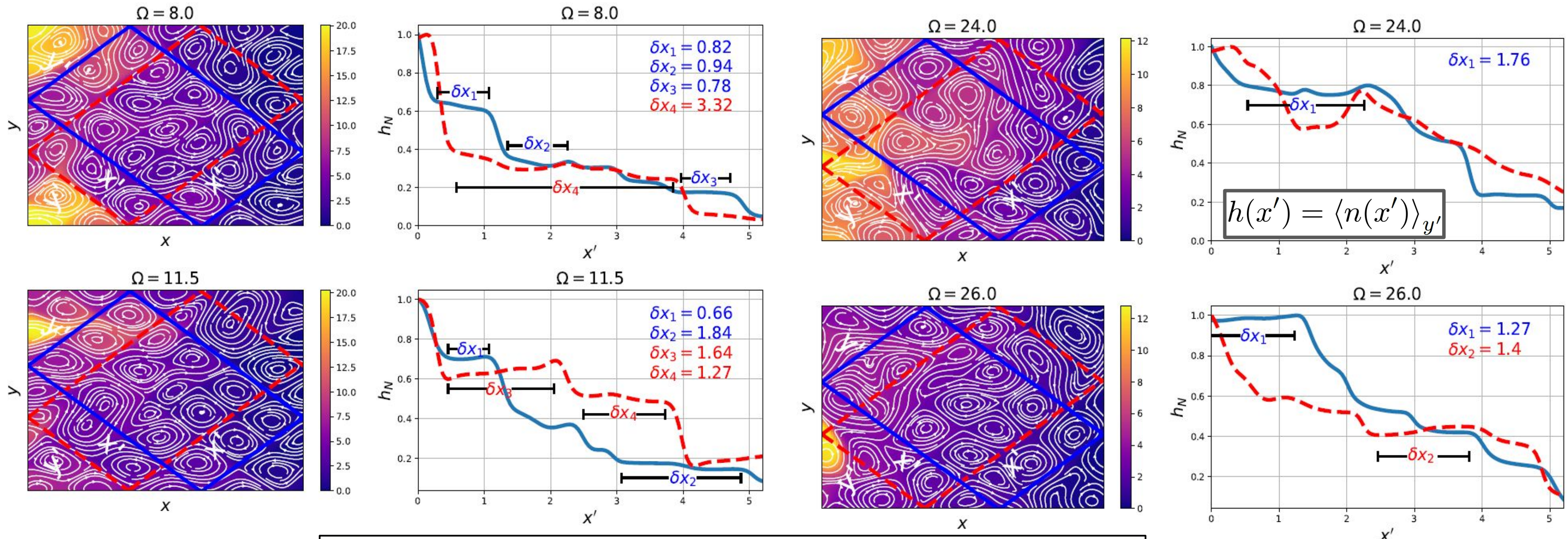




$$\langle n(x')\rangle_{y'} = \int_0^{L'} n(x', y')dy'$$

$$h(x') = \langle n(x')\rangle_{y'}$$

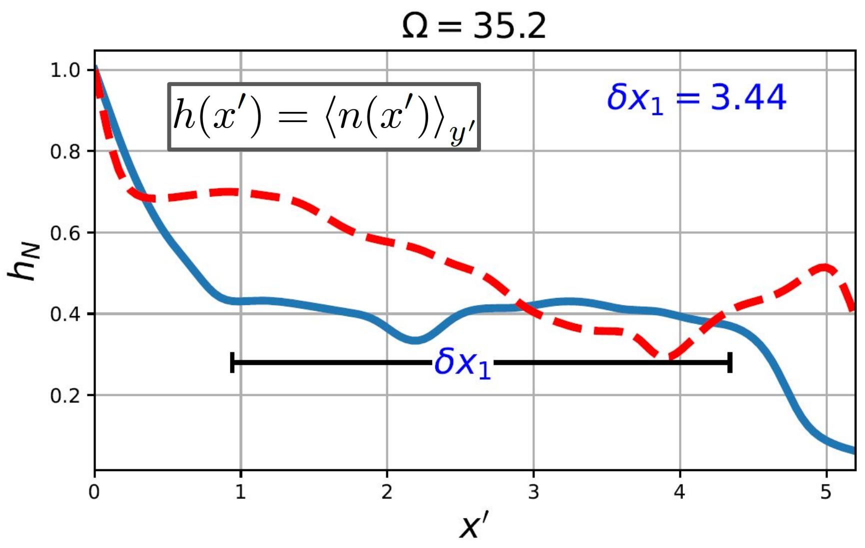
Staircase Resiliency to Fluctuations

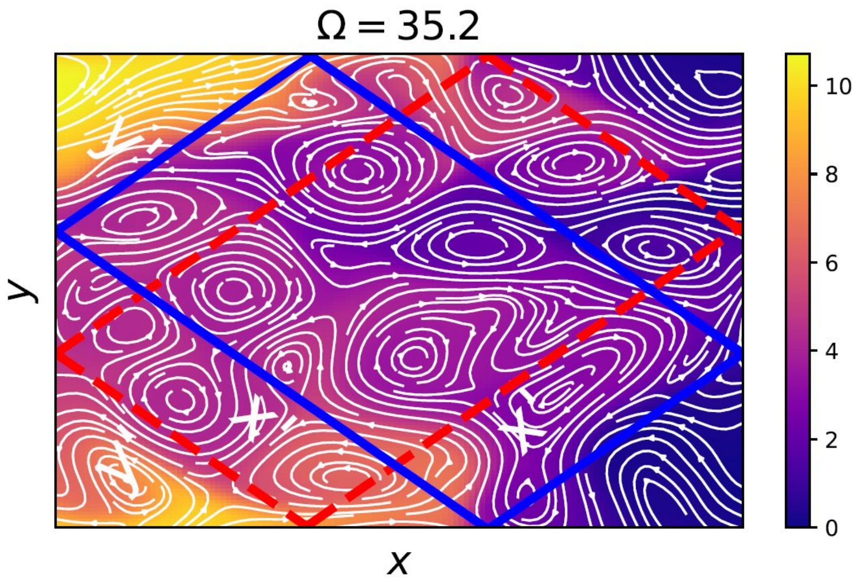


We indicate steps as regions where $dh/dx' \sim 0$.

- As we **increase fluctuations in vortex array** through *Ω*, we can see **merger/connections** of vortex structures in the flow.
- These **vortex mergers** are shown in the scalar profile plot as **mergers in steps**.
  → As we increase jittering, staircase steps merge together.

Staircase Resiliency to Fluctuations (cont.d)



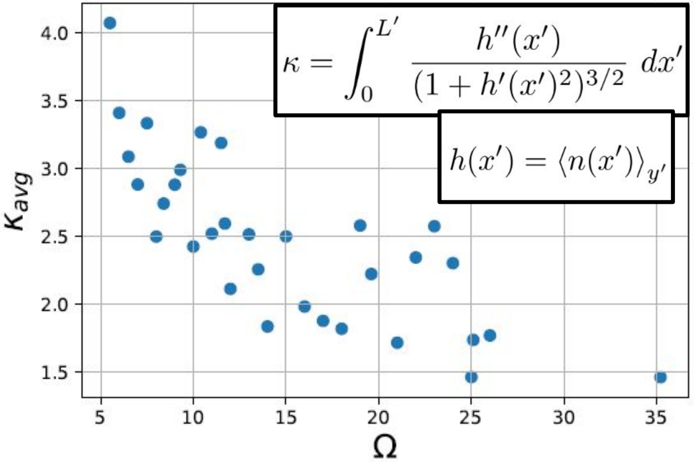


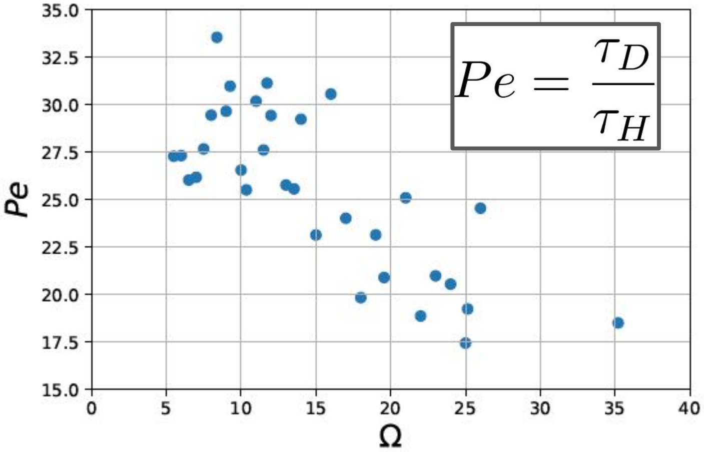
<u>Main Point</u>: Despite that vortex array becoming more turbulent, the staircase structure does not collapse.

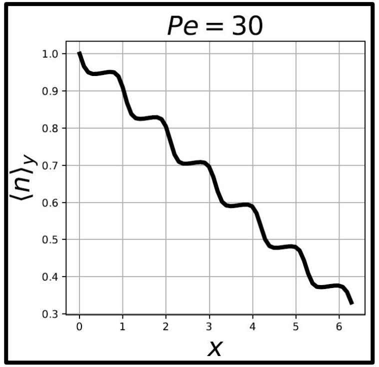
- Staircase steps become **less regular**. They merge into longer steps.

→ **Okay, but how to quantify?**

Criteria for Staircase Resiliency





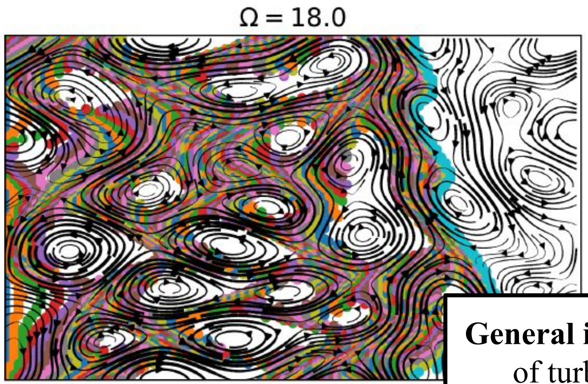


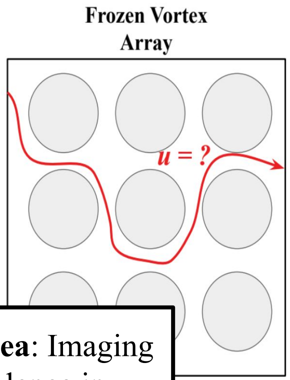
We establish a **set of criteria** to give a precise meaning to the statement of "**resiliency**":

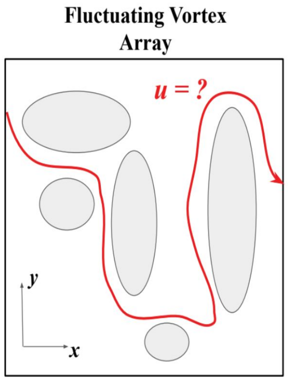
1) $Pe \gg 1$ is a **necessary** condition for the **formation of transport barriers** in the process of scalar mixing (**First principles**). $Pe \gg 1$ criterion is satisfied for the range of $0 < \Omega < 40$.
2) A staircase should **maintain a sufficiently high curvature** (equivalent to sustaining a sufficient number of steps). Our studies suggest that $\kappa \gtrsim 1.5$ is an adequate value for a staircase.

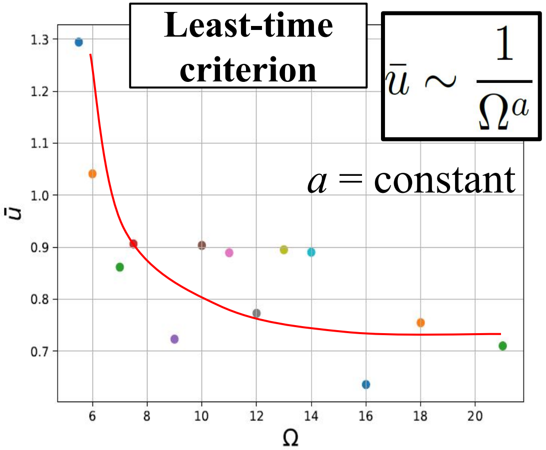
Transport in the Fluctuating Vortex Array

Trajectory in Scattered VA → How Avalanches Propagate









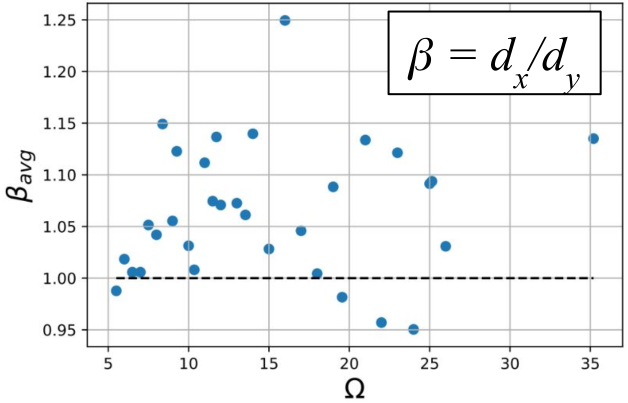
General idea: Imaging of turbulence in near-marginal state

Before the staircase structure forms, scalar flows **quickly in regions of strong shear** and around vortices!

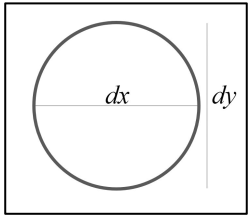
- Staircase **barriers form first!** Scalar travels along cell boundaries.
- Overtime, vortex **entrains** scalar by a kind of "**homogenization**" process via the synergy of differential rotation and diffusion.

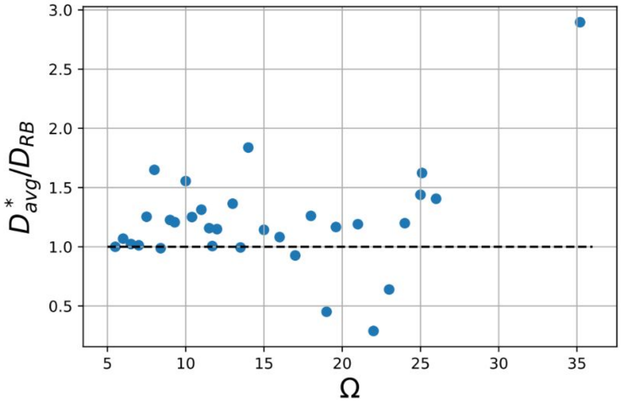
The **scattering of vortices** leads to an overall **decrease** in scalar concentration **velocity**! Agrees with least time criterion (similar idea to scattered path of light in atmosphere).

D^* in Fluctuating Vortex Array $D^* \approx \sqrt{D D_{\text{cell}}}$



$$D^* \propto \sqrt{u_{\text{cell}}\, d_x \beta}$$





As cells fluctuate, the **effective diffusivity** deviates but **remains close** to the FCA effective diffusivity (D_{RB}).

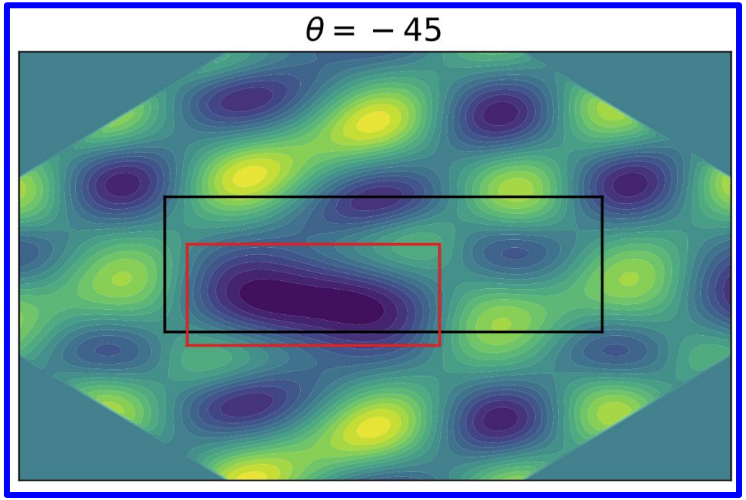
- **<u>Note:</u>** Only **dimensions** and **turn-over velocity** of the cells **affect transport**.

This **suggests** that the fixed array effective diffusivity is a **good approximation** even if **<u>cells are irregular</u>**!

We find that as long as the **boundaries** of the cells are **maintained**, the effective diffusivity and transport **does not change significantly**.

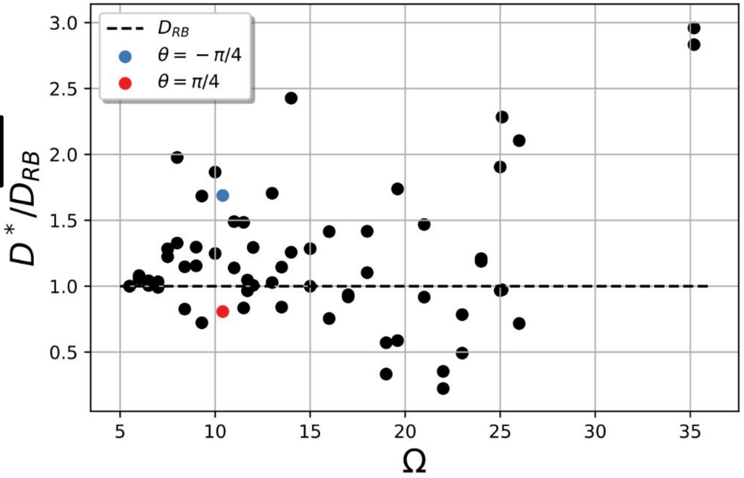
- Here, we examine the effects of d_x and d_y, as our emphasis is on the **<u>impact</u>** of cell geometry on pattern formation. (β approximates trend)

Closer look at cell geometric properties (β)



$$D^* \propto \sqrt{u_{\text{cell}}\ d_x \beta}$$

$$\beta = d_x/d_y$$



Effective diffusivity **increases/decreases** if the cells length along the gradient (d_x) increases/decreases compared to the length perpendicular to the gradient (d_y).

- Cells on average remain around $\beta \sim 1$, but there are cells that are larger in size due to cell mergers which cause the deviation of the effective diffusivity.

Summary

- Staircase form and are **resilient** and **persistent** to increasing Reynolds number (i.e., fluctuating vortex array).

- Scalar concentration **travels along** regions of **strong shear**.
    - **IMPORTANT**: Staircase barriers form first! Vortex "homogenizes" scalar at a later time!

- The scattering of vortices leads to an overall decrease in scalar concentration velocity.
    - Agrees with **least time criterion**.

- If background diffusion is kept fixed, **cell geometric properties** can qualitatively approximate the trend of the effective diffusivity!
    - Effective diffusivity of the perturbed vortex array **does not deviate** significantly from that of the fixed cellular array!

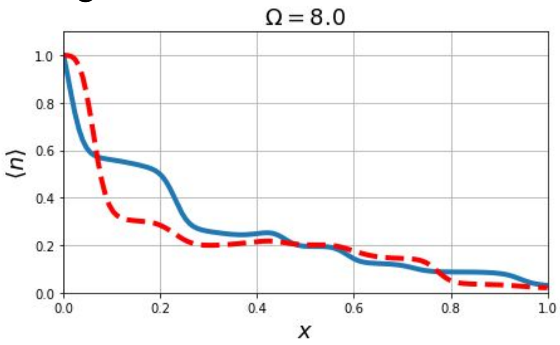
Why would a fusion experimentalist care about this?

These results have interesting implications for experiment and theory:

1. Effective diffusivity for fixed cellular array is a suitable approximation for the fluctuating cellular array (**not simple addition**: $D^* = D_0 + D_{cell}$).
    - Relevant to cells touching (similar to what we find near-marginal stability).

2. Staircase structure is resilient in the regime of low-modest Reynolds numbers (this regime is relevant to drift-wave turbulence).
    - Structures/Profiles are not exotic.
        - Staircase profile structure does not require special tuning.

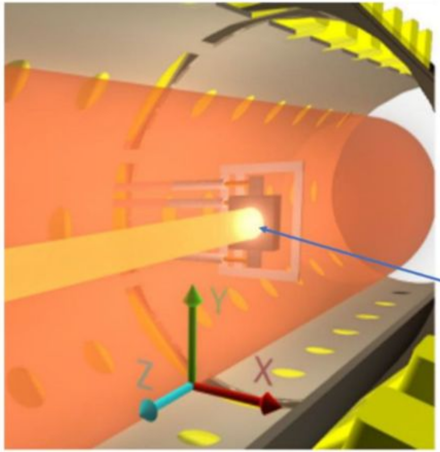
3. Geometry of streamlines is important. If more saddles than close vortices, Heat avalanches will first form the staircase barrier.
    - Fluctuating cellular flow hinders avalanche propagation.

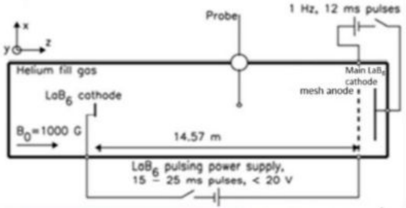


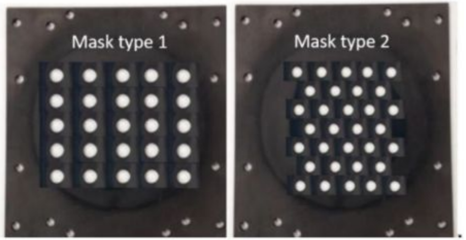
IMPORTANT: We can test the theory presented here with actual experimental data.

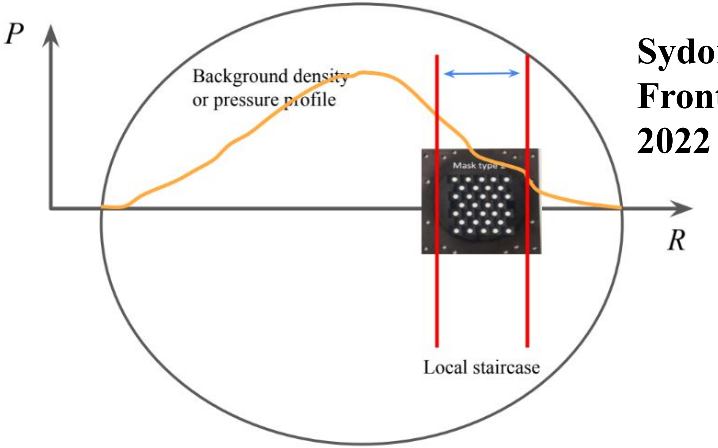
LAPD Experiment

Work in progress!









Sydora, Frontiers Proposal 2022

A vortex array can be created in the large linear magnetized plasma device (LAPD)

- Modification of a cathode plasma source with designer masks that form multiple current channels in a cellular pattern → form staircase!
  - Experiment will be conducted in the afterglow phase of the main discharge.
- Staircase structure can be subject to controllable amount of of low frequency density fluctuations, which act as a noise source.
  - Allow us to test hypotheses and models of staircase resiliency!

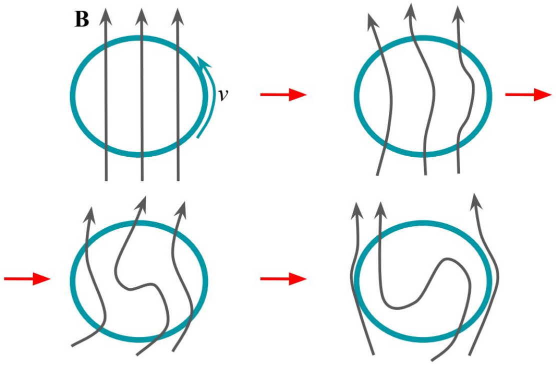
Results of experiment will yield a unique set of observations that can be used to test staircase models.

Active Scalar Dynamics

(Current work)

Active Scalar

$$M = \frac{v_A}{U_0}$$



A logical next step to explore is the effects than an *active* scalar has on the cellular array and inhomogenous mixing.

- Converting passive to active will result in effects such as flux expulsion
  - Flux expulsion is simplest dynamic problem in non-ideal MHD.

Why this model?

- ***B*** expelled to boundaries, thus holds cells together! → Rigid staircase.

We turn passive scalar into an active scalar, creating a feedback between magnetic field and vortices:

Flux expulsion:

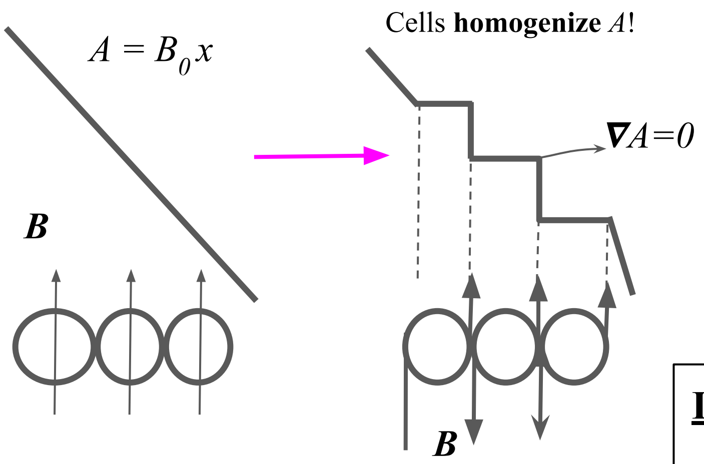
- Background ***B*** is wind up and folded by an eddy → field inside eddy drops → expelled to boundary layer of eddy.
- Time scale for flux expulsion is, $\tau_{fe} = R_m^{1/3}\,\tau_H$
- **Note:** Larger R_m results in greater expulsion (weaker field in interior).
- Here **emergent scale** is the ratio of layer width and cell size ($l/L \propto R_m^{-1/3}$).

$$\frac{\partial n}{\partial t} + \mathbf{u}\cdot\nabla n - D\nabla^2 n = 0 \longrightarrow \left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)A = \frac{1}{R_m}\nabla^2 A + F_A$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\omega = \frac{1}{\Omega}\nabla^2\omega + M^2\left(\mathbf{B}\cdot\nabla\nabla^2 A\right) + F_\omega$$

Note: Strength of B_o plays an important role!

Kinematic/Dynamic Regime



Consider a **linear** magnetic potential profile:

- We expect that the vortex array will homogenize ($\nabla A=0$) the profile in areas of vortices.
- Expect that magnetic field will maintain or restore the cell array structure when fluctuations are present (i.e., B_0 will elasticise the cell array).

$$M = \frac{v_A}{U_0}$$

$M^2 R_m < 1$ (Flux expulsion)

$M^2 R_m \geq 1$ (Vortex bursting)

Mak et. al. 2017

Important: Flux expulsion only occurs in the **kinematic** regime

- Useful to explore **dynamic** regime (aka Vortex bursting).

Since $v_A \propto B_0$, the strength of the magnetic field will play a role in the dynamics of the cellular array.

- If B_0 is sufficiently small, we get cell strengthening.
- If B_0 is large, vortices will not be allowed to form.

Through scans of B_0, we will address what **occurs** to expulsion of **neighbor cells** and their **interaction**…

To be clear, staircase forms in the flux expulsion regime.

- Now does layering occur in vortex bursting regime?

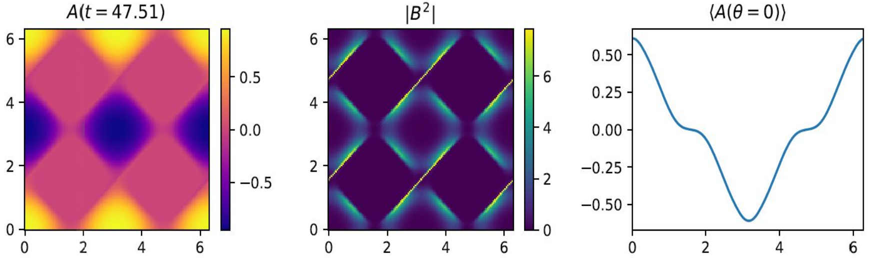
Magnetic Staircase

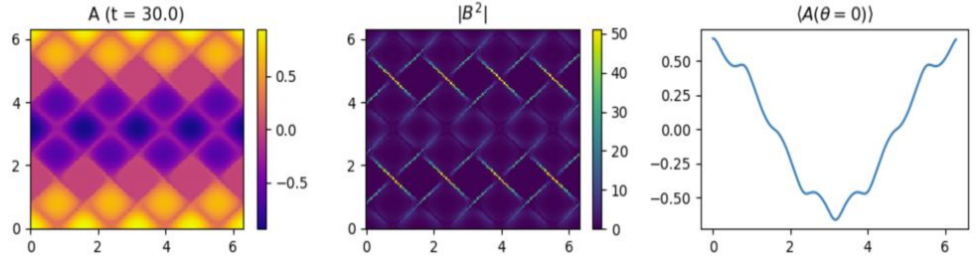
$\Sigma = M^2 R_m$

We study the process of inhomogeneous mixing by first initializing the active scalar as

$$A(x, y, t = 0) = A_0 \cos x$$

For all simulations, we fix R_m and only vary the values of B_0 and Ω. As a baseline case, we first study the evolution of the active scalar for $\Omega = 1$ and $\Sigma = 1$ $(n=2$ & $n=4)$.

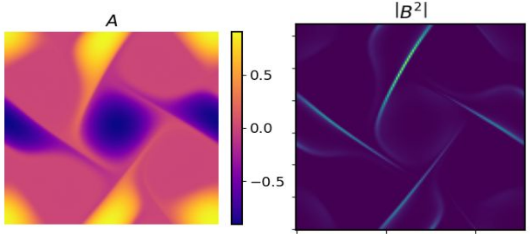




A **homogenizes** within regions of **vortices**, thus producing **steps** in the profile. Magnetic field lines are expelled at boundaries and hold cell structure together.

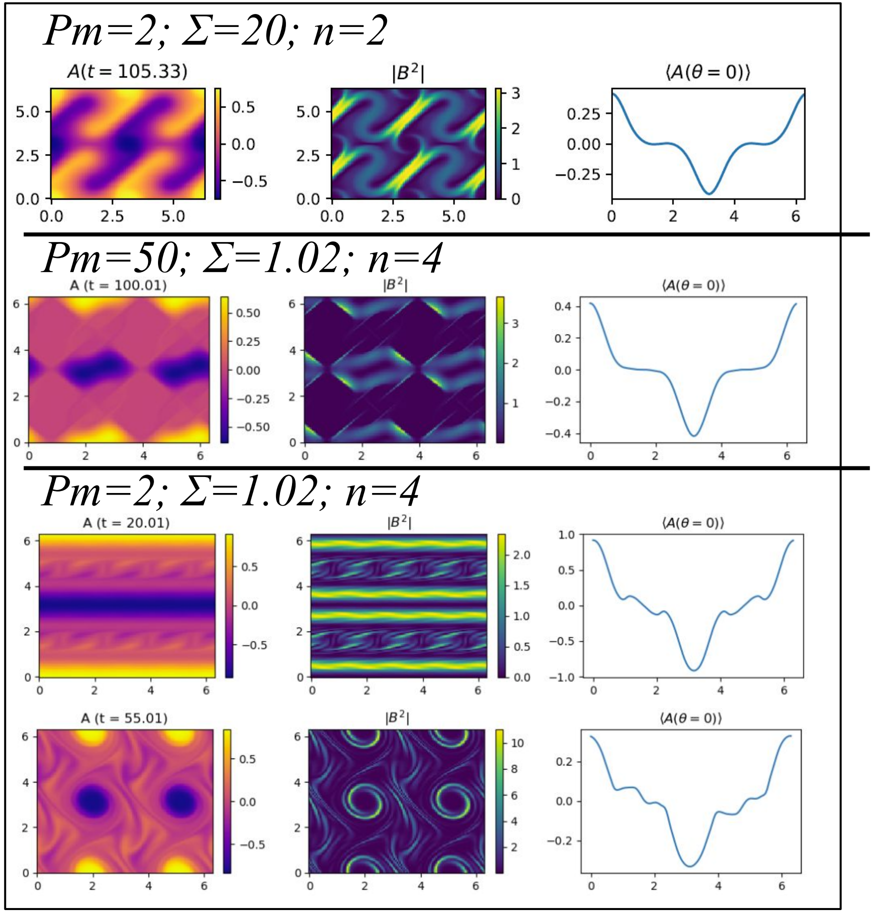
- NOTE: ***B*** eventually **decays** in 2D, so the **structure** is only **temporary**!

At a later time…



Cowling's anti-dynamo theorem

Patterns of Layering in the Fluctuating Vortex Array



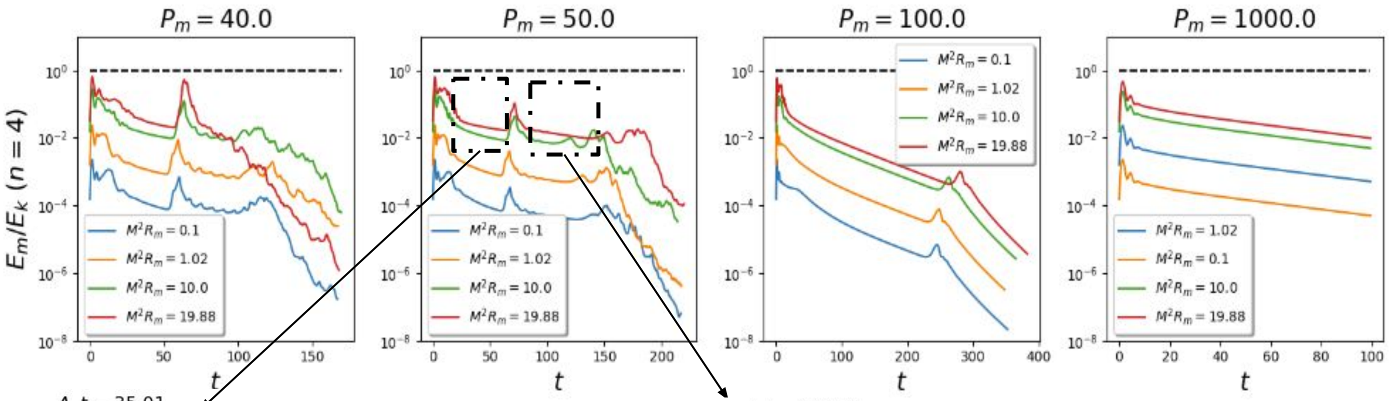
As $P_m \to 1$, a **variety** of **layering patterns** appear.

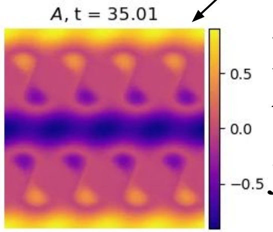
- And also observe **transitions** between different **layering patterns**.

To **understand** the **dynamics** of active scalar inhomogeneous mixing, we study the **evolution** of the following quantities:

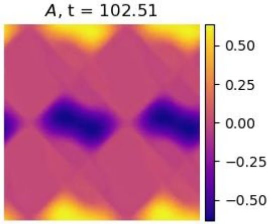
1. Energy (Both Kinetic and Magnetic)
2. Mean-squared magnetic potential ($<A^2>$)
3. Disruption parameter (Δ)
4. Profile curvature (κ)
5. Energy dissipation rate (ε)

Evolution of Energy





In between these two transitions, there is a jump in E_M/E_k



NOTE: In $<A^2>$ there is change in timescales.

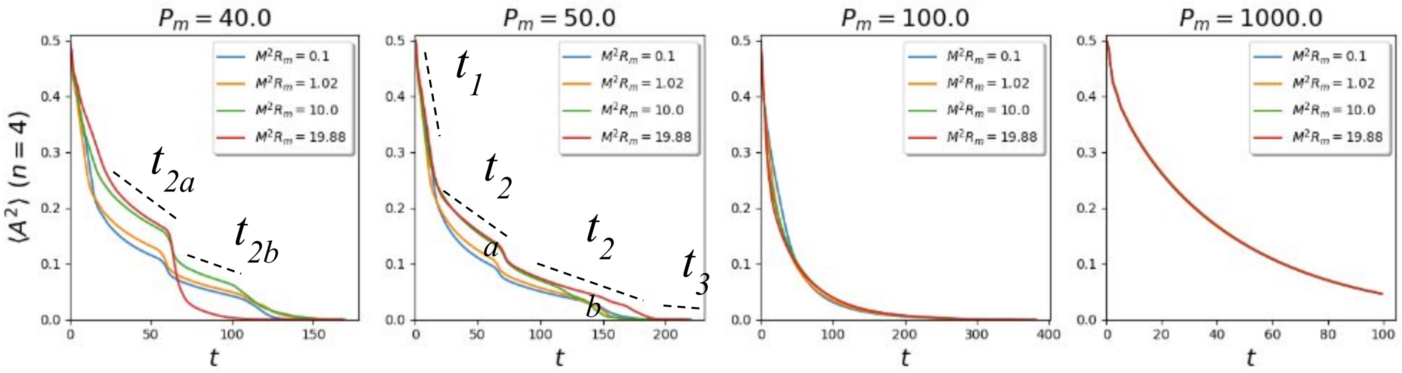
Note that layering occurs when $d/dt(E_m/E_k) \sim 0$.

Magnetic field lines hold the structure together.

Time-trace of energy shows an initial **suppression stage** followed by a **kinematic decay stage**.

For *Pm >100*, flow is frozen into vortex array structure. For *Pm <100*, cell jittering induces current which generates magnetic fields → suppression stage.

Evolution of <A^2>



NOTE: t_2 begins to form around P_m~50.

There are at least **three time scales**:

1. Flux expulsion (initial)
2. Magnetic diffusion (intermediate)
3. Viscous (final)

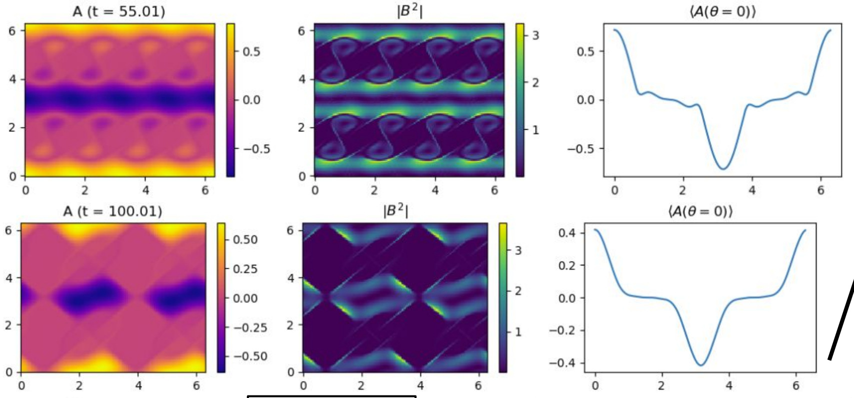
(1) occurs early on (magnetic field wraps around vortices), (2) occurs while magnetic field lines hold array together with jittering present, and (3) finally occurs once the magnetic field has dissipated.

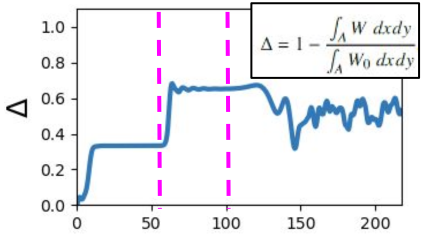
Recall that for P_m=50 there is a layering pattern transition. Here, we observe that t_2 breaks up. Layering is present in both t_{2a} and t_{2b}.

Dynamics of the Magnetic Staircase (*n=4*)

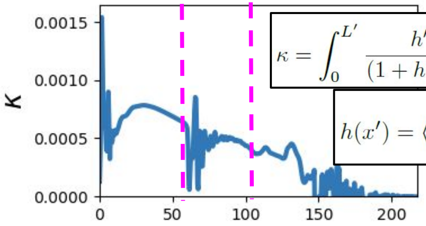
$$\Sigma = M^2 R_m$$

Pm=50; Σ=1.02; n=4





$$\Delta = 1 - \frac{\int_A W\,dxdy}{\int_A W_0\,dxdy}$$



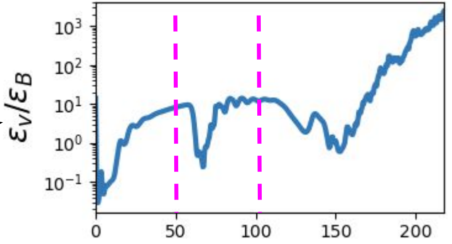
$$\kappa = \int_0^{L'} \frac{h''(x')}{(1+h'(x')^2)^{3/2}}\,dx'$$

$$h(x') = \langle n(x') \rangle_{y'}$$

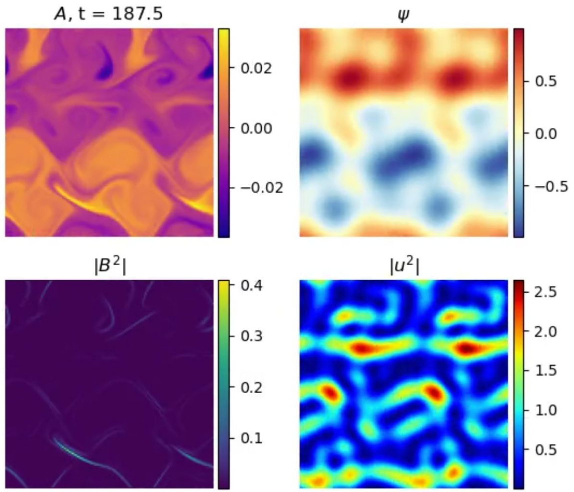
$W \equiv$ Okubo–Weiss Field

$$\varepsilon_v = (1/\Omega) \int d^2x\, (\nabla^2\psi)^2$$

$$\varepsilon_B = (1/R_m) \int d^2x\, (\nabla^2 A)^2$$



Transitions in layering patterns are visible in both evolution of the disruption parameter and profile curvature.
There is a balance between magnetic and kinetic dissipation once layered structure appears.



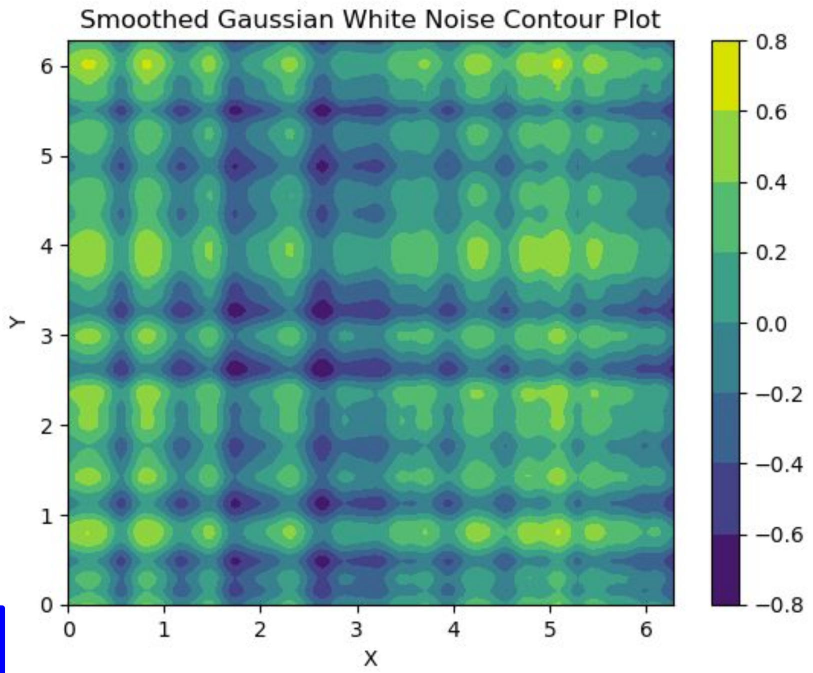
Again, magnetic field dissipates overtime and layered structure dissolves.

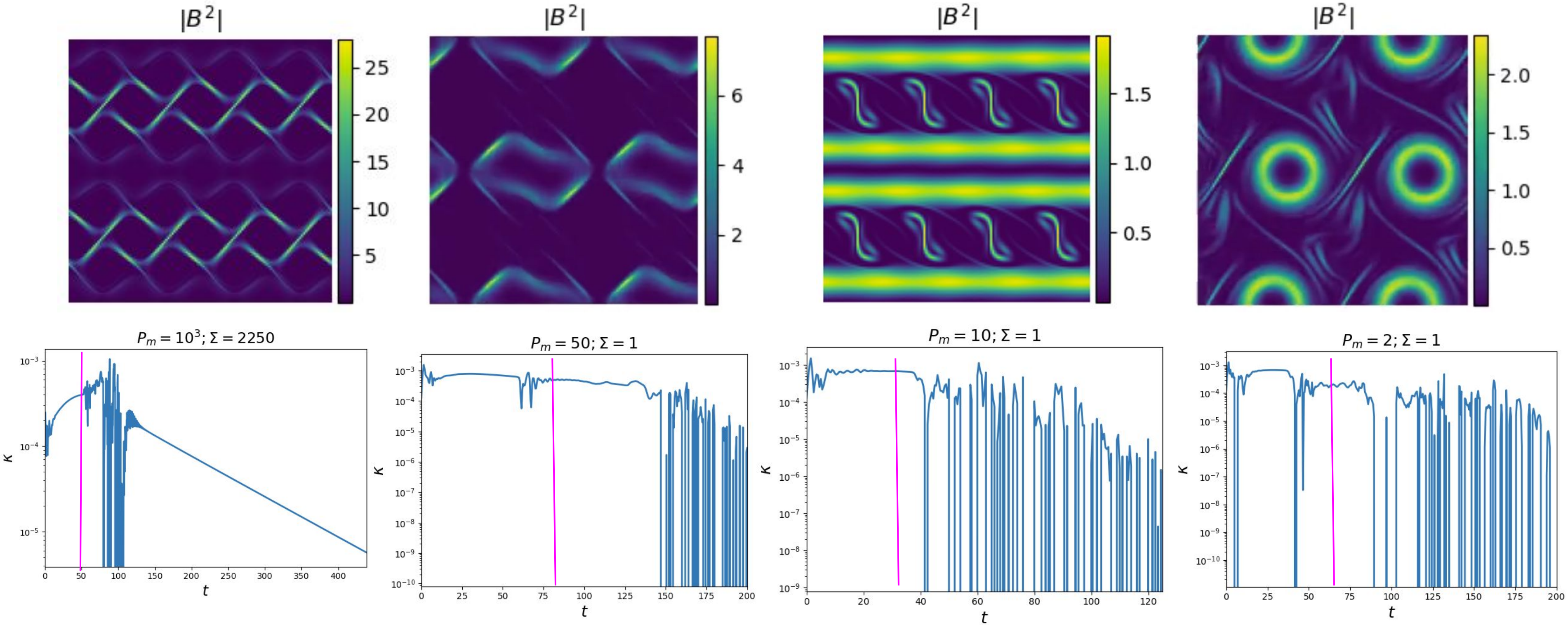
Summary & Future Work

Summary:

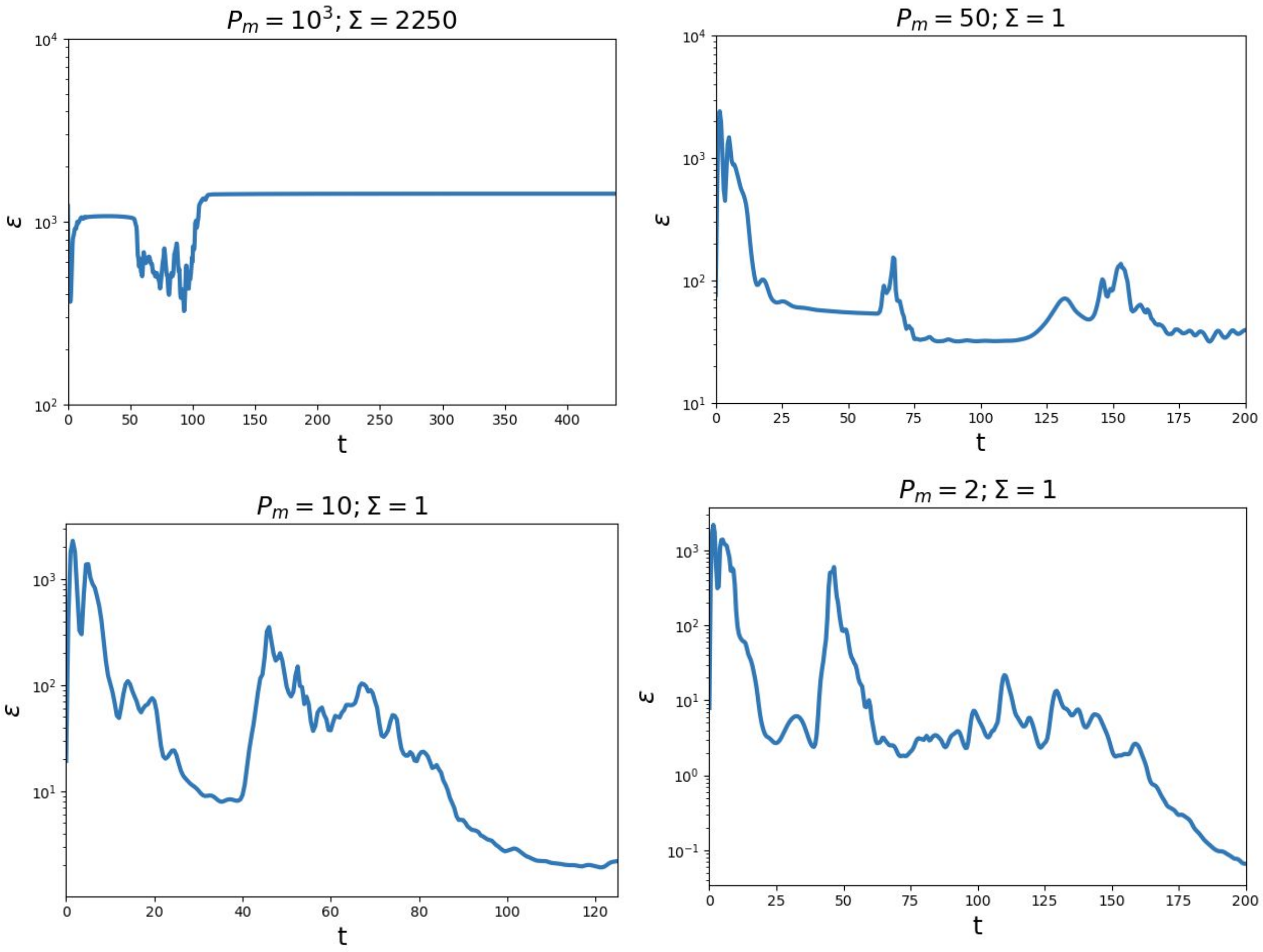
1. Demonstrate that **layering occurs** (both in flux expulsion and vortex bursting regime).
   - Different patterns appear.
   - Transitions between different forms of layering.
2. **Three time scales present** (flux expulsion, magnetic diffusivity, viscous) and appear in $\langle A^2 \rangle$ time-trace.
3. When layering occurs, there is a balance between *Em* and *Ek*, which is also reflected in the dissipation rate.
   - When layering occurs profile curvature stabilizes. **<u>NOTE:</u>** This steady state is transient, lasting only for a brief period due to magnetic field decay.

Next step is to introduce stochastic magnetic potential forcing. **Question**: Will stochastic forcing reinforce layering? Will layering spontaneously reappear (when cells are fully disrupted - i.e., $\Delta \sim 1$)?





$|B^2|$
$|B^2|$
$|B^2|$
$|B^2|$
$P_m = 10^3; \Sigma = 2250$
$P_m = 50; \Sigma = 1$
$P_m = 10; \Sigma = 1$
$P_m = 2; \Sigma = 1$
κ
t


$P_m = 10^3; \Sigma = 2250$
ε
t
$P_m = 50; \Sigma = 1$
ε
t
$P_m = 10; \Sigma = 1$
ε
t
$P_m = 2; \Sigma = 1$
ε
t

Thank you!

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