A closer look at mean $E \times B$ shearing rate in negative triangularity tokamaks

(under review in NF 2023)

Rameswar Singh and P H Diamond
CASS, UCSD

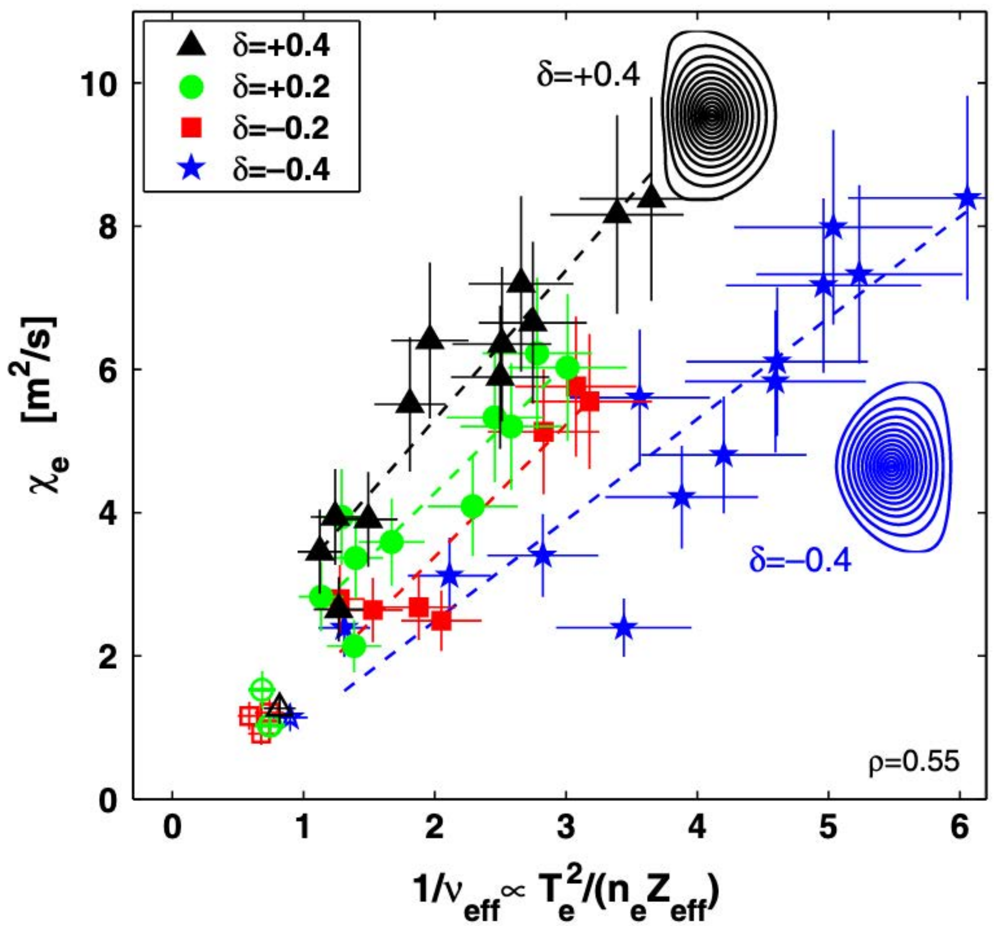
US TTF May 3 2023, Madison

Acknowledgement: T S Hahm for useful discussions. Supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, under Award Number DE-FG02-04ER54738.

Effect of triangularity on confinement, fluctuations and L-H transition

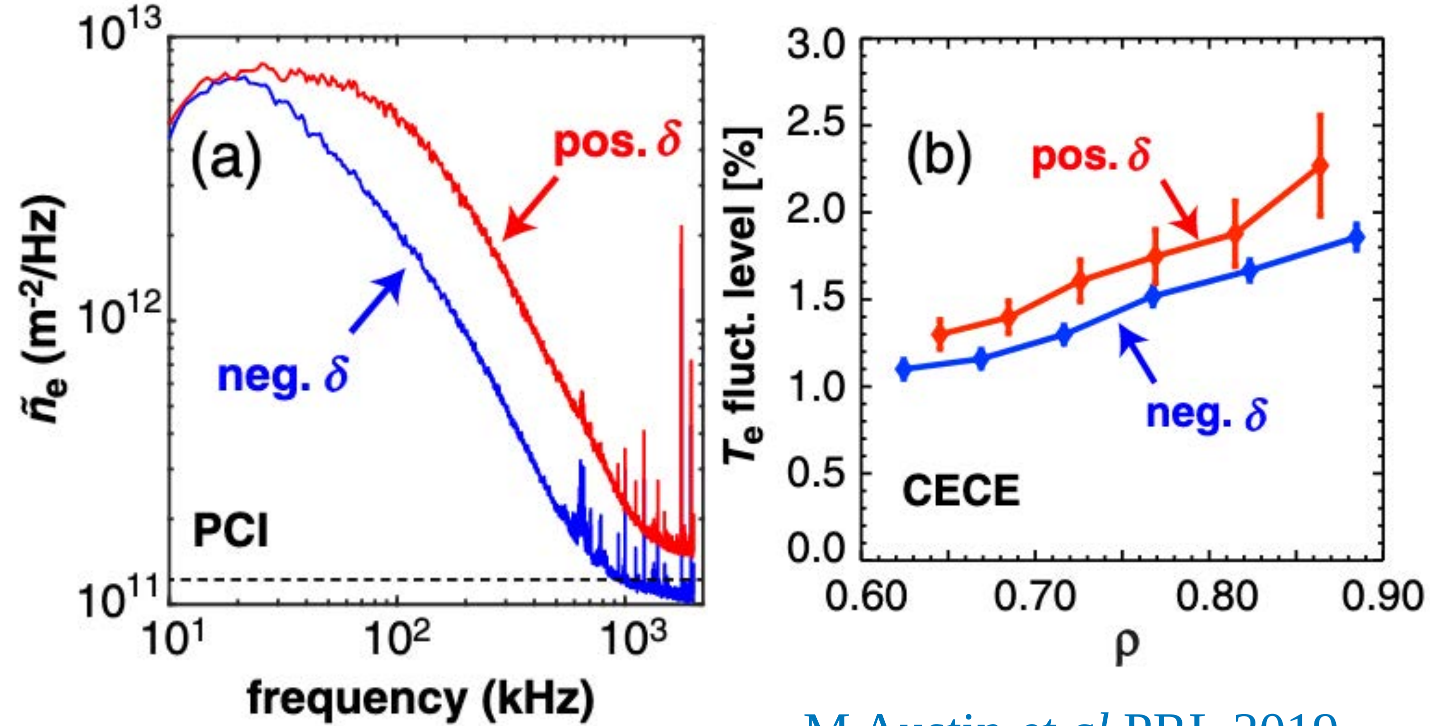
TCV experiments

EC Heated L-mode Plasmas Doubled Energy Confinement Time on TCV when $\delta \rightarrow -\delta$



Y. Camenen et *al* NF 2007

DIII-D experiments



M Austin *et al* PRL 2019

- High -ve δ shape is robust against L-H transition.

  $P_{LH,th} = 4\text{MW} \rightarrow >13\text{MW}$ when $\delta_u = -0.18 \rightarrow -0.36$

What about mean ExB shear?

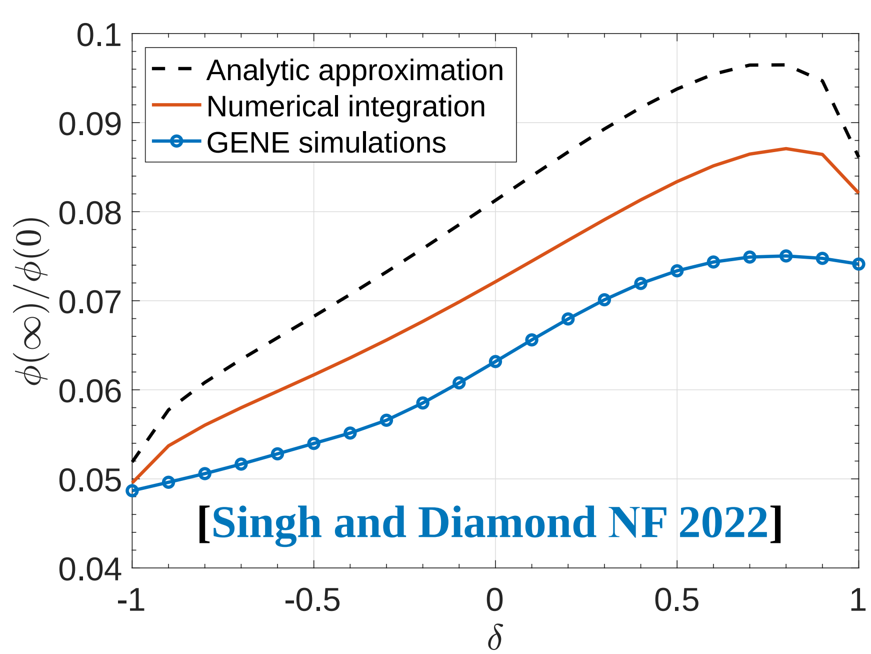
How to reconcile confinement improvement in NT L Mode with enhanced L-H power threshold?

- Understanding flux surface shaping effects on turbulence saturation mechanisms is the key.

Major players for turbulence saturation

Zonal flows → **Zonal flows reduced for NT due to enhanced neoclassical polarization, owing to increased trapped fraction. Consistent with increased $P_{L\rightarrow H,th}$!****

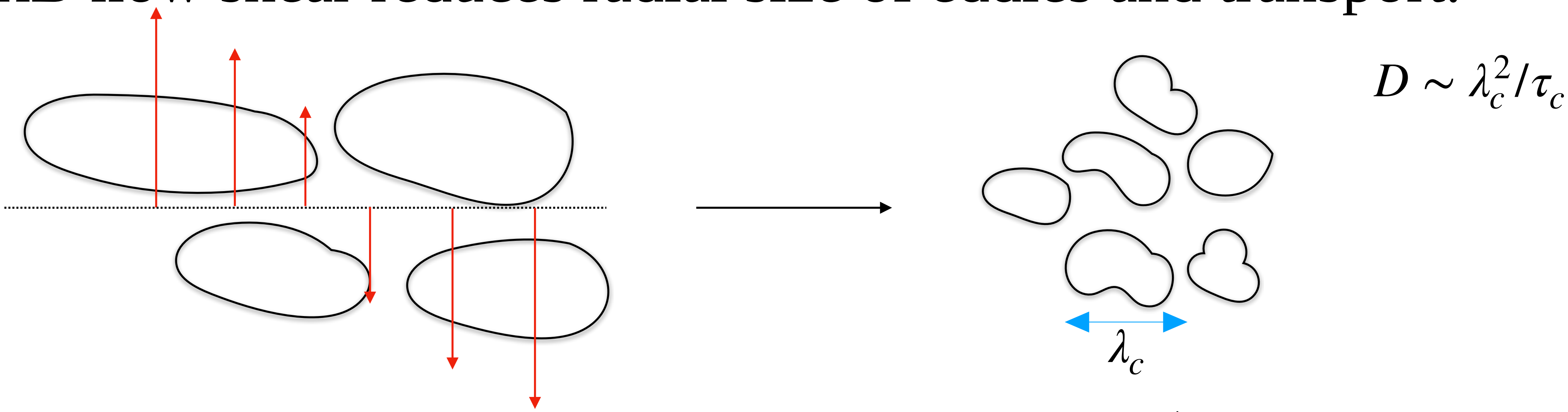
Mean ExB shear → **What happens to mean ExB shear for NT??**



** Increased $P_{L\rightarrow H,th}$ is also linked to loss of access to 2nd stability region of $n = \infty$ ideal MHD ballooning modes. [Saarelma *et al* PPCF 2021, Nelson *et al* NF 2022]

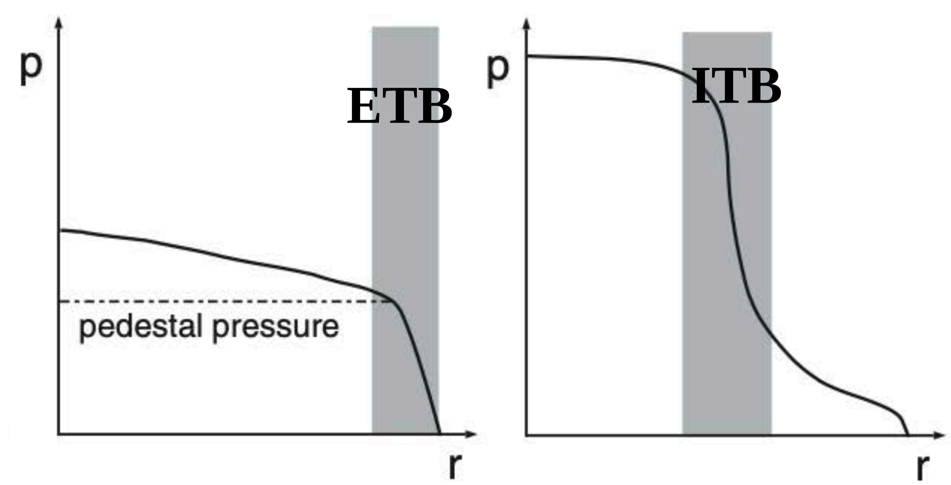
ExB shear suppression of turbulence is the holy grail of physics of transport barriers

- ExB flow shear reduces radial size of eddies and transport.



- Turbulence quenches when shearing rate $\omega_E > \Delta\omega \sim \gamma_{lin}$ [BDT 90]
- As a result transport is reduced and pressure gradient steepens.

- Transport bifurcation due to mean ExB shearing
  - ETB formation during L-H transition
  - ITB formation in high β_p reversed q discharges



Geometry dependence of mean ExB shearing rate ω_E

ExB shearing rate in general axisymmetric geometry: [Hahm & Burrell PoP 1995]

$$\omega_E = \left(\frac{\Delta\psi_0}{\Delta\zeta}\right)\frac{\partial^2}{\partial\psi^2}\Phi_0(\psi),$$

Geometry dependent

$$\frac{\Delta\psi_0}{\Delta\zeta} = \frac{\Delta r}{\Delta\theta}\frac{R^2\psi'^2}{I\mathcal{J}}$$

ψ:= poloidal flux

ζ:= toroidal angle

Φ_0:= Mean electrostatic potential

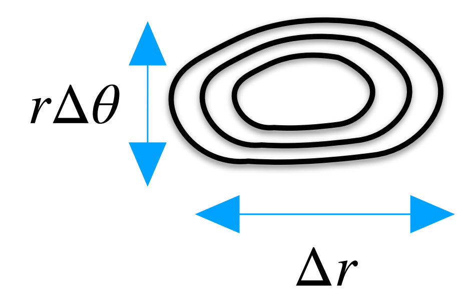
$\Delta\psi_0$:=Turbulence correlation length in ψ

$\Delta\zeta$:=Turbulence correlation length in ζ

$$\Delta\psi = \Delta r\frac{\partial\psi}{\partial r} = \Delta r\frac{RB_\theta}{\left|\vec{\nabla} r\right|} \quad \Delta\zeta = \nu\Delta\theta$$

$$\psi' = \frac{I(\psi)}{2\pi q(\psi)}\oint d\theta\frac{\mathcal{J}}{R^2} \quad \nu = \frac{I\mathcal{J}}{R^2\psi'}$$

Calculated for Miller's equilibrium for **fixed** $\frac{\Delta r}{\Delta\theta}$ and $\frac{\partial^2}{\partial\psi^2}\Phi_0(\psi)$.



$$R = R_0(r) + r\cos[\theta + (sin^{-1}\delta)sin\theta],\ Z = \kappa(r)r\sin\theta$$

$$\mathcal{J} = R\kappa r\left[R_0'\cos(\theta) + \cos\left(\sin^{-1}\delta\sin\theta\right) + \sin(\theta + \sin^{-1}\delta\sin\theta)\sin\theta\left\{S_\kappa - S_\delta\cos\theta + \left(1 + S_\kappa\right)\sin^{-1}\delta\cos\theta\right\}\right]$$

Shafranov shift gradient

Triangularity gradient

Elongation gradient

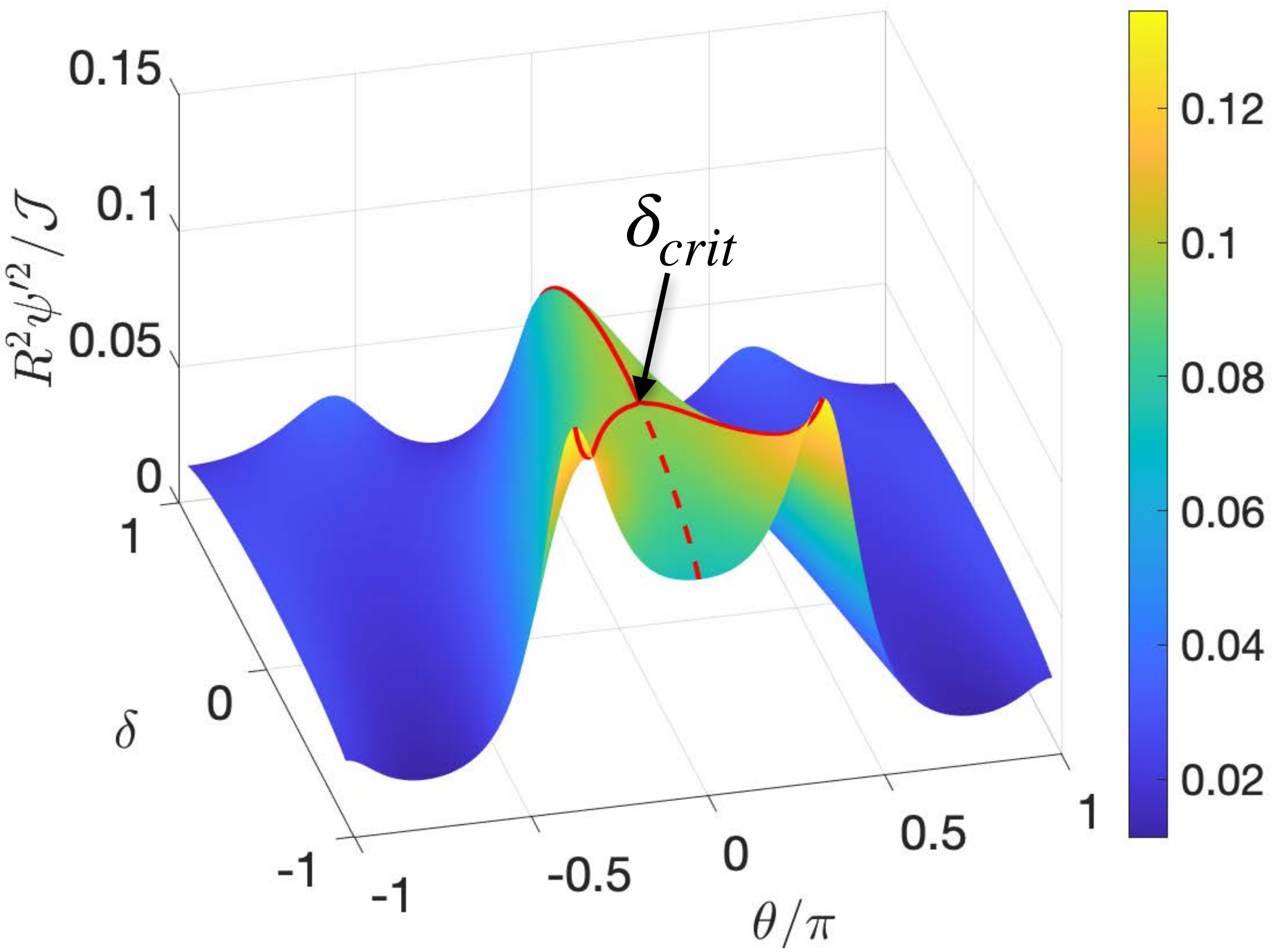
Shape dependence of shearing rate is inferred from shape dependence of $\frac{R^2\psi'^2}{\mathcal{J}}$.

Variation of mean ExB shearing rate with triangularity δ

- **Max shear off the outboard mid-plane for NT**$\rightarrow$ Shearing is less effective for $k_x = 0$ modes i.e, the modes ballooning at $\theta = 0$.

- **The peak shearing bifurcates at $\delta_{crit} \leq 0$.**
    - Why? The Jacobian is a nonlinear function of δ which exhibits spontaneous symmetry breaking.
    - Peak shears move toward good curvature region.

Geometric bifurcation of shearing rate

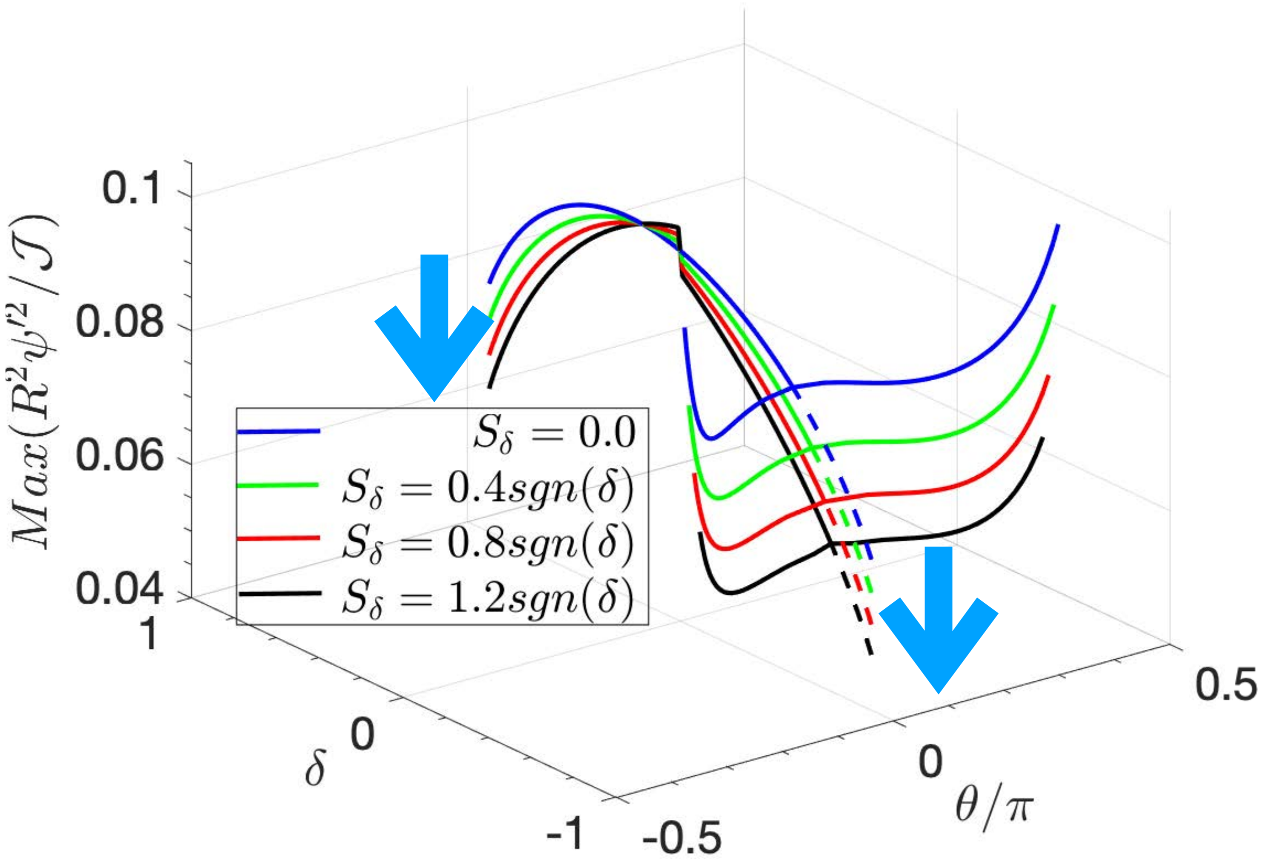


- **Shear at $\theta = 0$:** (for fixed $\Phi_0''(\psi)$)
    - $\downarrow$ with increasing NT.
    - Weaker for NT than for PT. Note that fluctuations balloon at $\theta = 0$. Thus shearing efficiency $\downarrow \implies P_{L\rightarrow H,th} \uparrow$.

Variation of mean ExB shearing rate with triangularity gradient S_δ

On increasing $|S_\delta|$:

- Shearing rate decreases.
- δ_{crit} moves along δ^-.

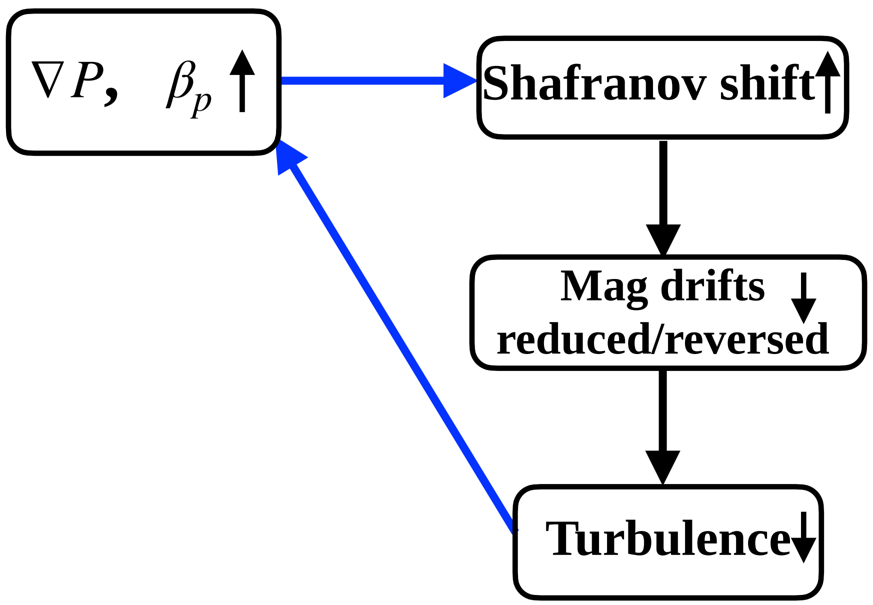


➡Radial profile of triangularity matters!

➡Can triangularity profile can be tailored to boost mean ExB shear?

Shafranov shift induced transport bifurcation

- ITB formation in high-β_p regime is often linked to transport bifurcation due to turbulence stabilization by shafranov shift due to mag drift reduction/reversal, *ignoring* the mean ExB shear effect. [S Ding et *al* PoP 2017, J McClenaghan et *al* PoP 2019, G M Staebler et *al* PoP 2017]



Feedback loop for Shafranov shift induced transport bifurcation

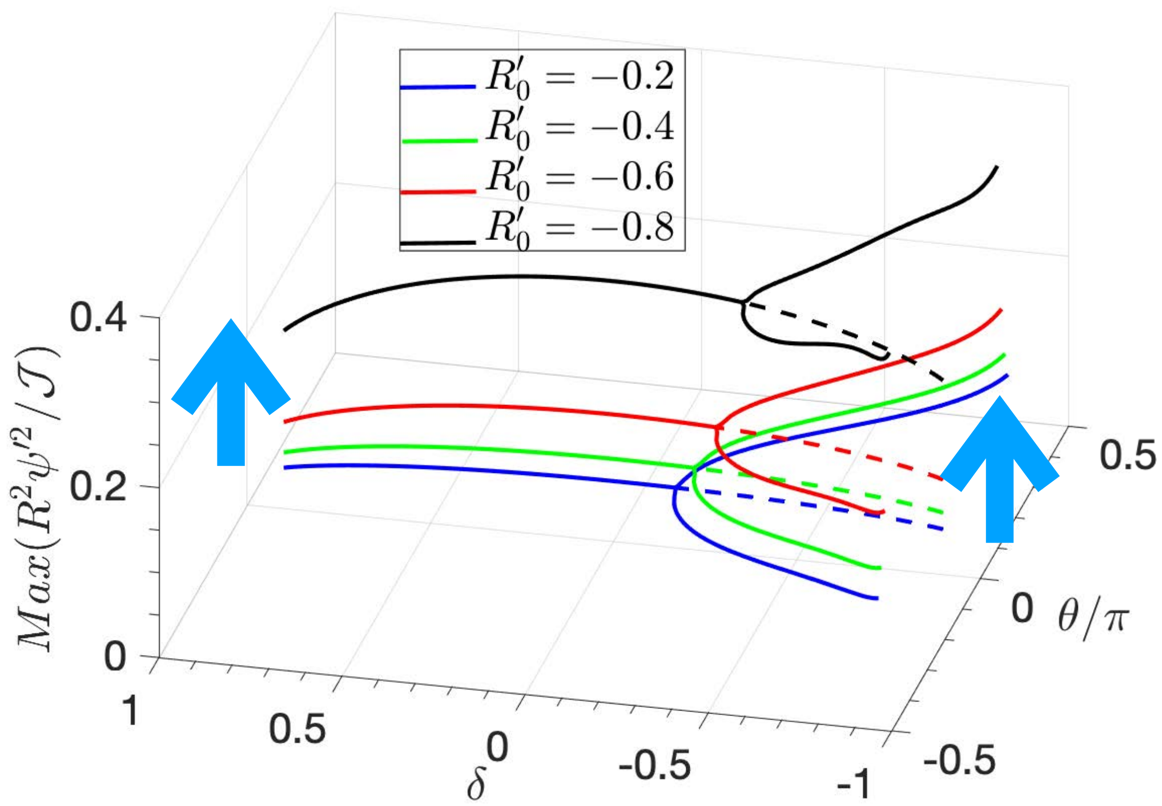
- But… like it or not - mean shear *exist* in high-β_p discharges!
- So how does mean shear and shafranov shift interact ?
- Interplay of mean ExB shear, Shafranov shift and NT?

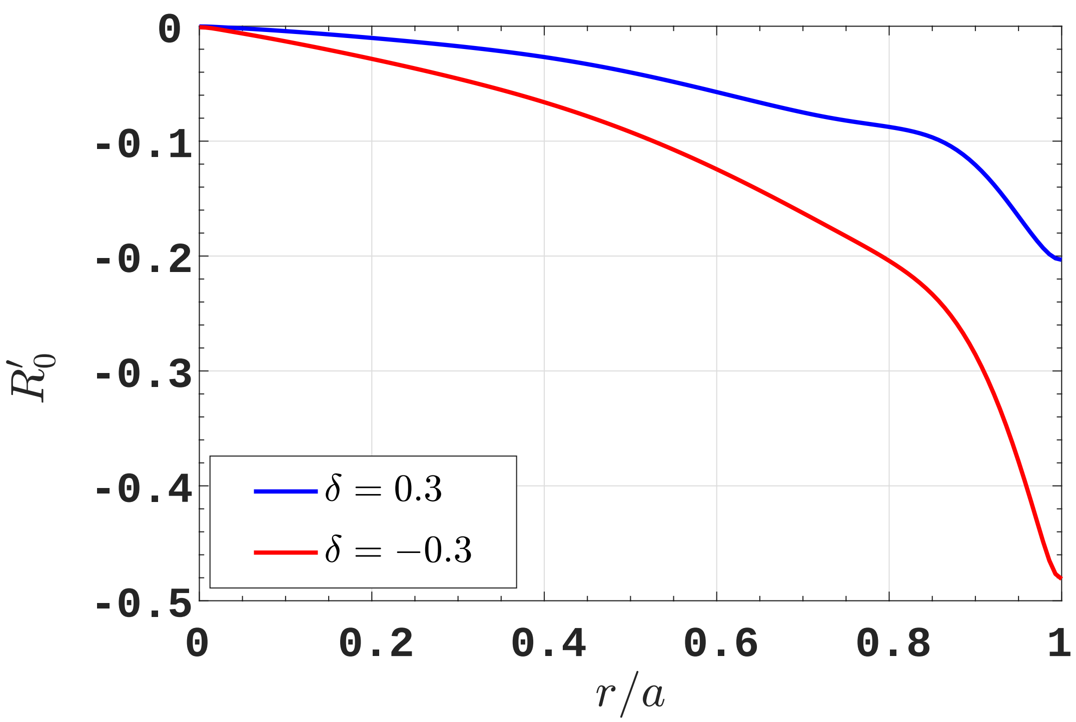
Variation of mean ExB shearing rate with Shafranov shift gradient R_0'

On increasing $-R_0'$:

- Shearing rate increases for all δ.
- δ_c moves toward δ^- on increasing $-R_0'$.

- Key reason→flux compression.





Shafranov shift gradient obtained using CHEASE code

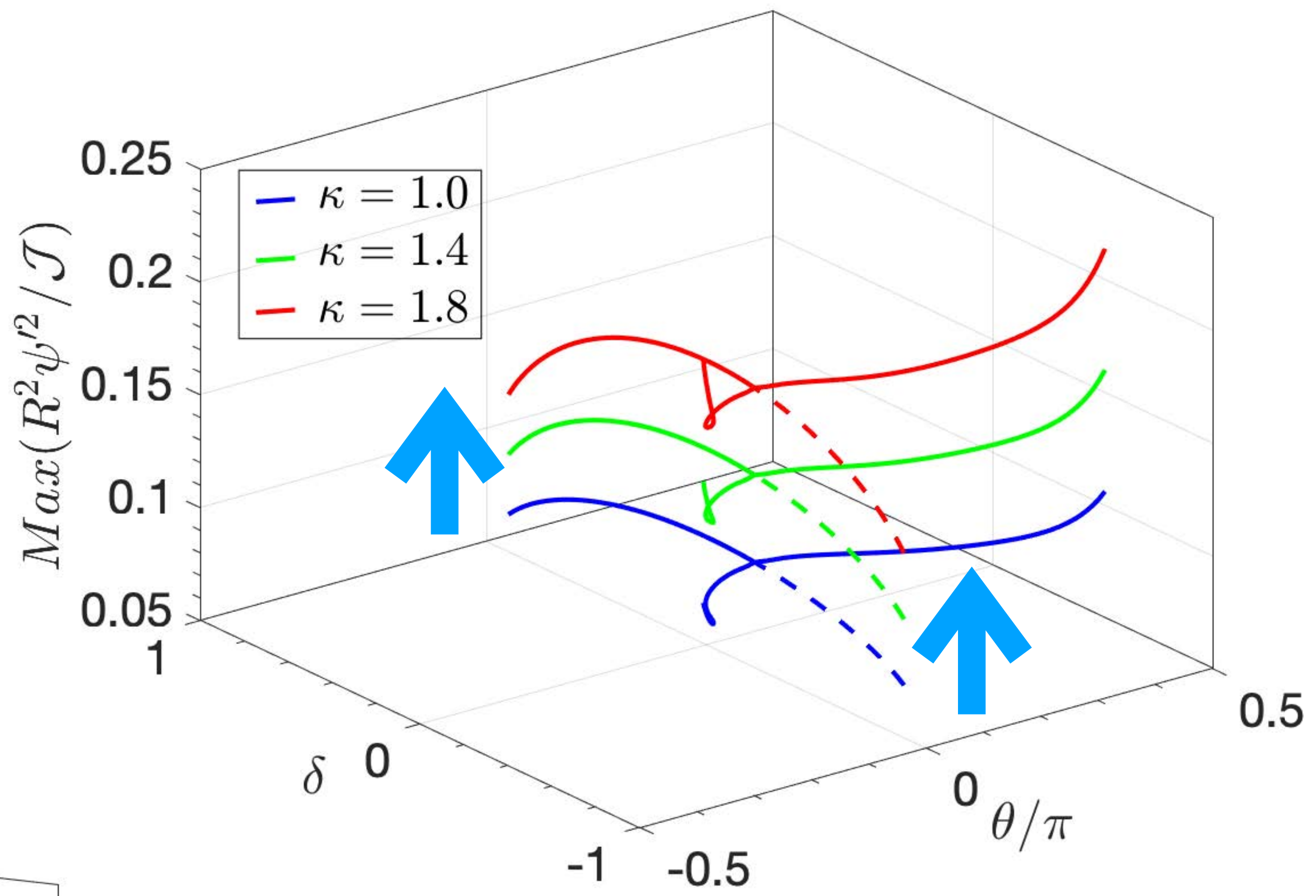
Significant for:

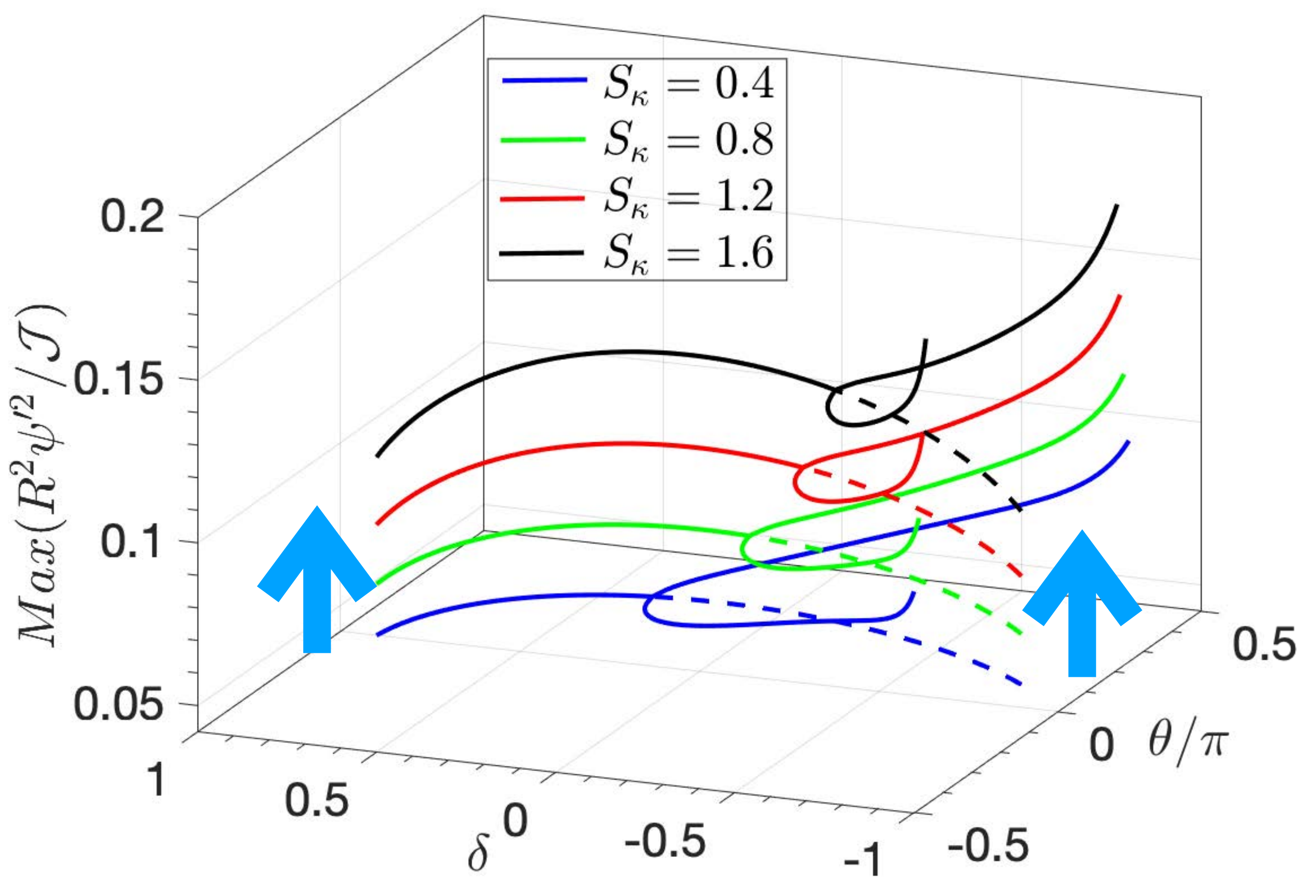
- high β_p regime (i.e, RS ITB) as $R_0' \propto \frac{r}{R_0}\beta_p$
- NT shapes
  - as $\beta_p(\delta^-) > \beta_p(\delta^+)$
  - Numerical MHD equilibrium study shows $R_0'(\delta^-) > R_0'(\delta^+)$ even for fixed β_p.

Mean ExB shearing rate increases with elongation κ and elongation gradient S_κ

On increasing κ:

- Shearing rate increases $\forall\, \theta$ and δ
- δ_{crit} is independent of κ.





On increasing S_κ:

- Shearing rate increases $\forall\, \delta$.
- δ_{crit} moves along δ^-.

Conclusions

Pure geometrical modulation to ExB shearing rate as PT → NT shapes

- **Max shear off the outboard mid-plane for NT as →** Shearing is more effective for $k_x \neq 0$ modes for NT.

- **The peak shearing bifurcates at** $\delta_{crit} \leq 0$. Peak shears move toward good curvature region and the shear at $\theta = 0$ decreases with increasing NT. Note that fluctuations balloon at $\theta = 0$. Thus shearing efficiency ↓ $\Longrightarrow P_{L\rightarrow H,th} \uparrow$.

- **Shearing rate decreases with increasing triangularity gradient S_δ and increases with increasing elongation κ, and elongation gradient S_κ.**

- **Direct effect of Shafranov shift gradient $-R_0'$ on shearing rate:** Shearing rate increases with increasing $-R_0'$ for all δ. Key reason→flux compression. Significant for high β_p regime and NT shapes.

These results has implications not just for L-H transition for NT but also for ITB discharges in PT and NT(proposed)!

Implications I

- Shafranov shift affects turbulence in 2 distinct ways:

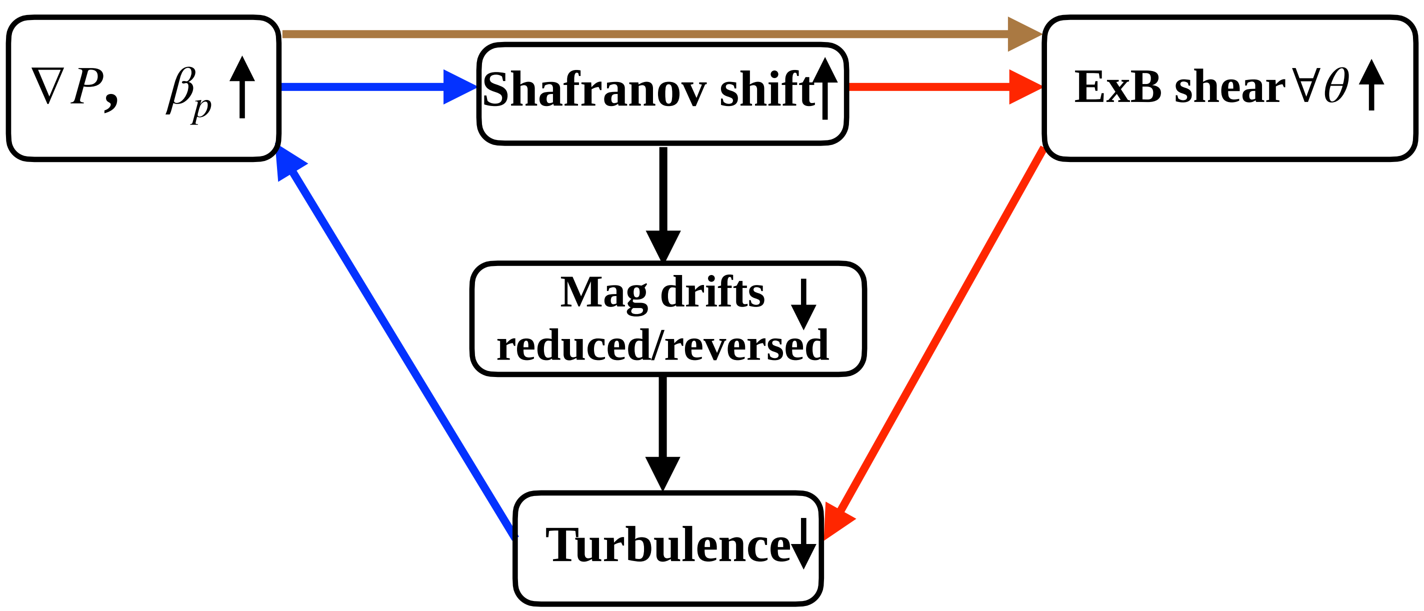
  (I) Stabilizes turbulence by reduction/reversal of magnetic drifts

  (II) Directly enhances the mean shear, → additional turbulence suppression

  Both can cause bifurcation to enhanced confinement state independently. Bifurcation by (I) is often invoked as a mechanism of confinement improvement in high-β_p regime, *ignoring* the mean shear effect.

Enhanced mean ExB shearing by Shafranov shift provides a +ve feedback on the feedback loop of the shafranov shift induced transport bifurcation.

Shafranov shift also has a +ve effect on the mean ExB shear induced transport bifurcation, not only through a reduction of the linear growth rate but also through the enhanced ExB sharing rate.

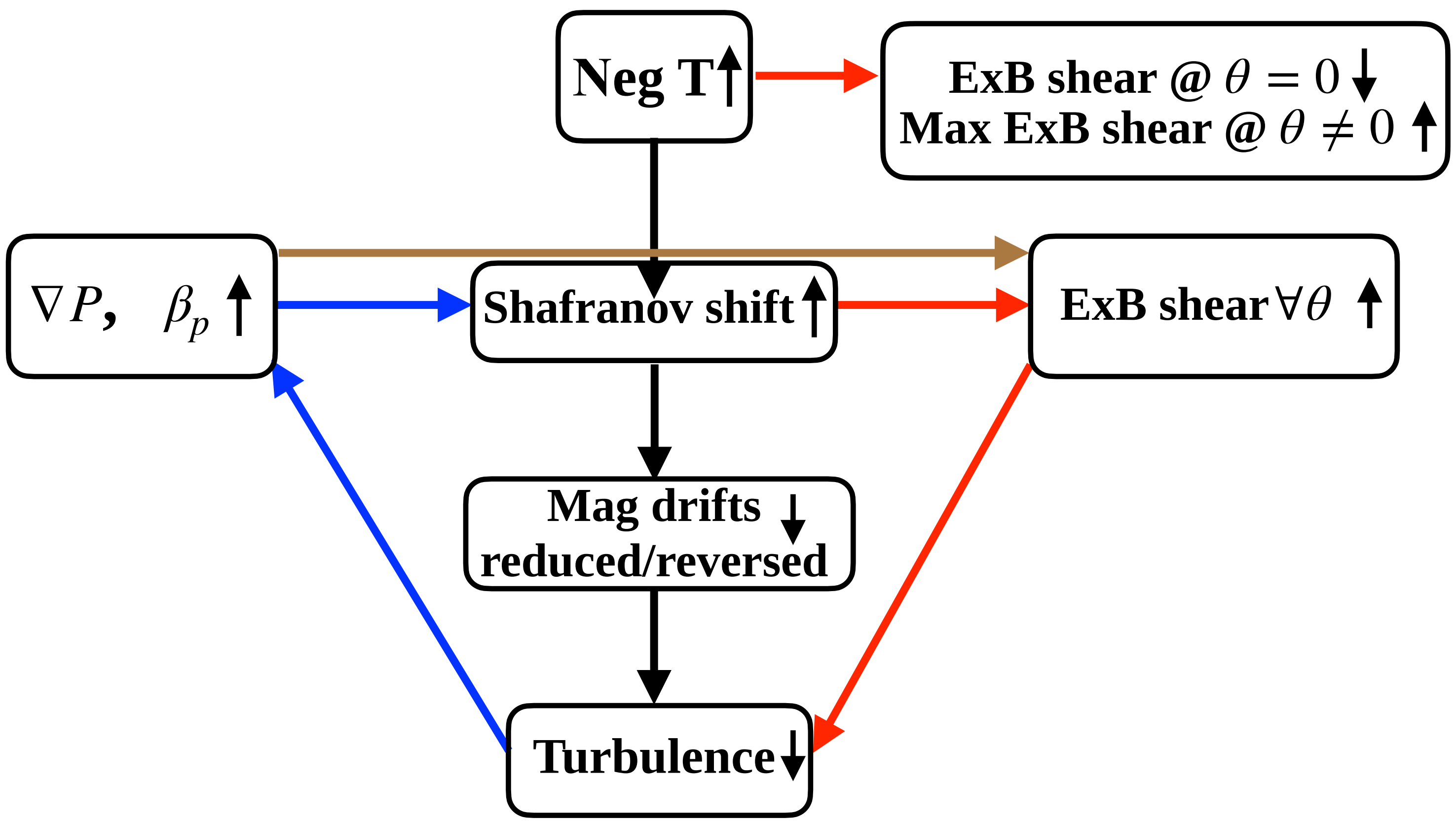


Both (I) and (II) can work in tandem to reduce the ∇P_{crit} for the onset of ITB in reversed shear PT shape

Implications II

- For realistic MHD equilibrium, Shafranov shift ↑ when PT→ NT

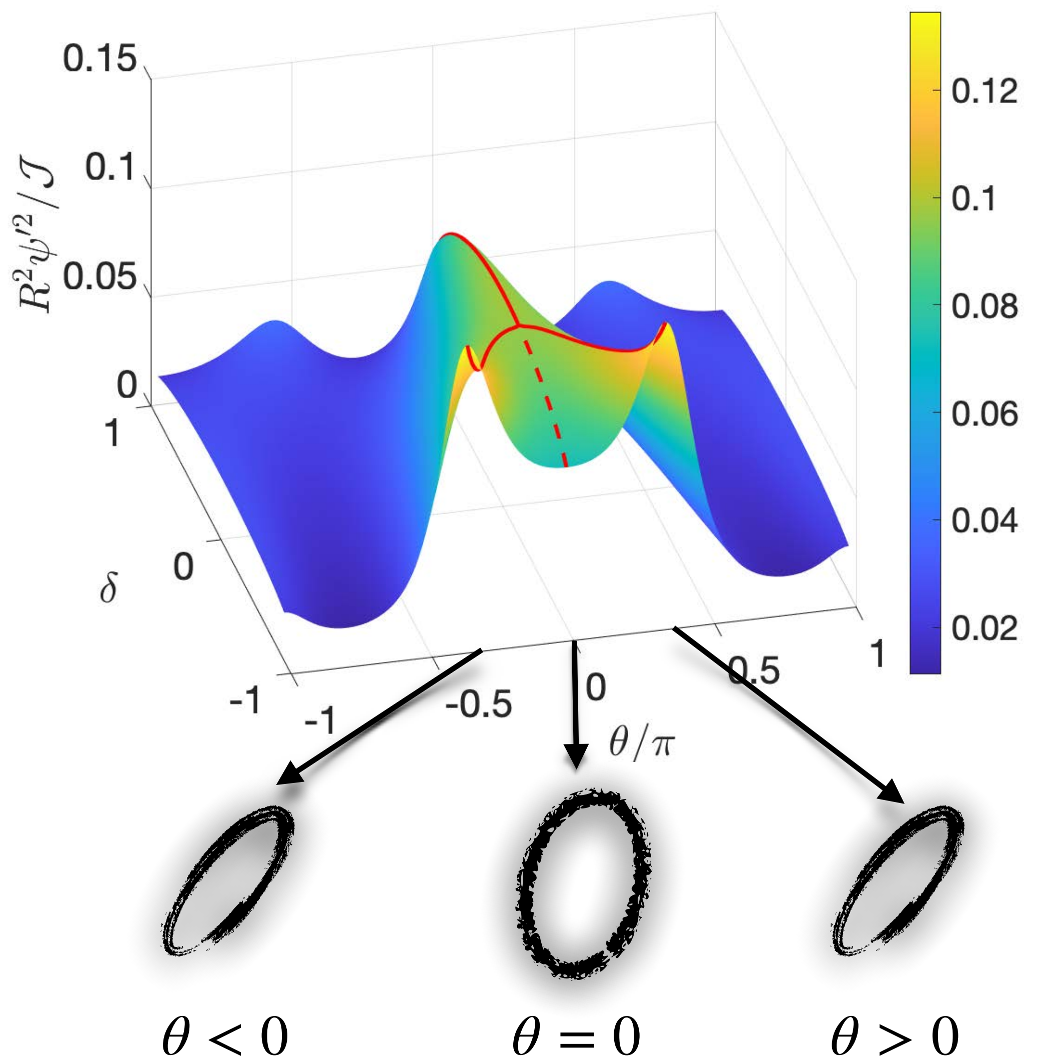
➡shear increase by enhanced shafranov shift competes with shear reduction at $\theta = 0$ when PT → NT

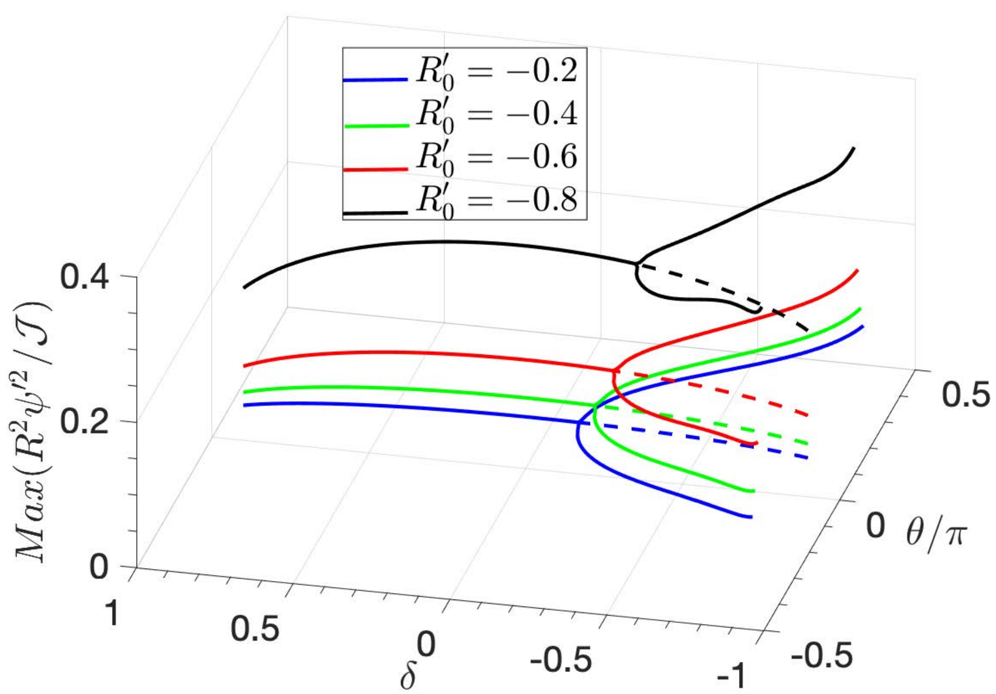


- For experimental equilibrium, parametric dependencies $R_0' \equiv R_0'(\delta)$, $\kappa \equiv \kappa(\delta)$, $S_\kappa \equiv S_\kappa(\delta)$, $S_\delta \equiv S_\delta(\delta)$ from numerical codes can help calculate shear accurately, - - -in progress!

For the experimentalists

- **Mean ExB Shearing is maximal off the mid-pane for NT:**
  $\Longrightarrow$Eddy tilting should be strongest off the mid-plane.
  - Direct imaging using gas-puffing?
  - Joint pdf of radial and poloidal velocity fluctuations (i.e., $\tilde{v}_r$ & $\tilde{v}_\theta$) should show max tilting (most-correlated) off the mid-plane for NT.





- **Shafranov shift gradient R_0' directly boosts the mean ExB shear:**
  - Re-assess the role of mean ExB shear in high-β_p reverse shear discharges.