

Shear flow instabilities in magnetized plasma and geostrophic fluids

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Zonal flows are large scale azimuthally symmetric plasma potential perturbations spontaneously generated from small scale drift-wave fluctuations via the action of Reynolds stresses. A positive feedback is provided due to the modulations of the wave packets by the shearing effect in the large scale flow. As a result, the propagation of small scale wave packets is accompanied by the instability of a low frequency, long wavelength component. There are two distinct regimes of this instability: resonant type shear flow instability, when the shear flow is in the resonance with the wave packet group velocity, and coherent, when the growth rate is large compared to the characteristic width of the wave packet so that different harmonics grow coherently. For plasma fluctuations with significant pressure fluctuations (such as ion temperature gradient driven modes) the Reynolds stress is modified with diamagnetic effects. In a finite β plasma the electromagnetic effects generally act to reduce the growth rate of the zonal flows instability due to partial compensation between of the electrostatic Reynolds stress and the electromagnetic Maxwell stress. In generic electromagnetic turbulence, the generation of the zonal flows can be accompanied by the generation of the large scale, poloidally symmetric, magnetic field -"zonal" field. Contrary to the case of the zonal flow instability, which does not require any dissipation, the generation of zonal fields is only possible when there is a finite dissipation such as that due to the wave-particle (Landau) interactions. There is a certain analogy between the structure of Reynolds stress in two dimensional magnetized plasmas and geostrophic fluids, so that a similar mechanism of the zonal flow instability could also be responsible for the generation of mean flows in the atmospheres of the rotating planets.

1 Introduction

The transfer of wave energy towards the long wavelength region and the formation of large scale structures (zonal flows and convective cells) is a result of the well known inverse cascade in two-dimensional and quasi two-dimensional fluids.¹⁻³ Such large scale structures are frequently observed in the turbulent motions of plasmas and geostrophic fluids⁴⁻⁶ (see also references in Ref. 4,5). The strongly sheared flow associated with such localized structures leads to turbulence suppression and enhancement of confinement in a tokamak that has been extensively studied, both theoretically and experimentally, in recent years.⁷⁻²⁰ It appears that zonal flows^{21,22} are an important element of drift wave dynamics regulation and may strongly affect anomalous transport in a tokamak. [Zonal flows are defined here as poloidal and toroidally symmetric ($q_z = q_\theta = 0$) perturbations with a finite radial scale q_r^{-1} larger than the scale of the underlying small scale turbulence, $q_r \ll k_r$, \mathbf{q} is the wave

vector for large scale motions, \mathbf{k} is the wave-vector of small scale turbulence, and r, θ , and z are axis of a straight cylindrical tokamak.] Recent advances in numerical simulations of tokamak plasmas²² have unambiguously demonstrated that a certain level of $\mathbf{E} \times \mathbf{B}$ flow (in the poloidal direction) triggers a transition to a state with greatly reduced anomalous transport. The suppression of the turbulence by the sheared $\mathbf{E} \times \mathbf{B}$ flow theoretically investigated in Refs. 11-13 has also been confirmed in experiment.¹⁴ There is a clear indication that zonal flows play a critical role in the dynamics of drift wave turbulence and its self-regulation. The general theory of zonal flows and the self-regulation of the drift-wave turbulence in a tokamak has been presented in Ref. 23 (see also earlier works on the generation of zonal flows in drift and Rossby wave turbulence²⁴⁻³³). Here, we review the theory of zonal flows with emphasis on toroidal ion temperature gradient (TITG) driven turbulence and electromagnetic effects. We also analyze the possibility of the generation of large scale, poloidally symmetric, magnetic field. Because of a similarity between equations for drift waves in plasma and Rossby waves in the rotating atmospheres,³⁴ development of the theory of zonal flows is also important in the geophysics context.⁴⁻⁶

2 Momentum deposition into the large scale flow from small scale fluctuations

To describe the dynamics of a large-scale plasma flow that varies on a longer time scale compared to the small-scale fluctuations we employ a multiple scale expansion thus assuming that there is a sufficient spectral gap separating large-scale and small-scale motions. The electrostatic potential is represented as a sum of fluctuating and mean quantities $\phi(\mathbf{X}, \mathbf{x}, T, t) = \bar{\phi}(\mathbf{X}, T) + \phi(\mathbf{X}, \mathbf{x}, T, t)$, where $\bar{\phi}(\mathbf{X}, T)$ is the mean flow potential. A similar representation is done for other plasma parameters such as pressure.

The generation of the mean flow is conveniently described by the total plasma momentum balance that can be written as³⁵

$$\frac{\partial}{\partial t} (n_e m_e \mathbf{v}_e + n_i m_i \mathbf{v}_i) + \nabla \cdot (n_e m_e \mathbf{v}_e \mathbf{v}_e + n_i m_i \mathbf{v}_i \mathbf{v}_i + p_e \mathbf{I} + \boldsymbol{\pi}_e + p_i \mathbf{I} + \boldsymbol{\pi}_i) = \rho \mathbf{E} + \frac{1}{c} \mathbf{J} \times \mathbf{B}. \quad (1)$$

The right hand side here describes the momentum exchange with the electromagnetic field

$$\frac{\partial}{\partial t} \frac{1}{4\pi c} \mathbf{E} \times \mathbf{B} + \nabla \cdot \left(\frac{1}{8\pi} E^2 \mathbf{I} - \frac{1}{4\pi} \mathbf{E} \mathbf{E} + \frac{1}{8\pi} B^2 \mathbf{I} - \frac{1}{4\pi} \mathbf{B} \mathbf{B} \right) = -\rho \mathbf{E} - \frac{1}{c} \mathbf{J} \times \mathbf{B}. \quad (2)$$

In the electrostatic case the exchange with electromagnetic field is not important, so the mean flow is generated as a result of the momentum and energy exchange between the zonal flow and small scale fluctuations.^{26,33} This has been confirmed also by direct calculations.³⁶

In the presence of the zonal flow the plasma momentum content is dominated by the slow $\mathbf{E} \times \mathbf{B}$ drift of the ion component

$$n_i m_i \mathbf{v}_i = n_i m_i \mathbf{v}_E \gg \left(n_e m_e \mathbf{v}_e, \frac{1}{4\pi c} \mathbf{E} \times \mathbf{B} \right). \quad (3)$$

When ions are magnetized the momentum drive is determined by the ion Reynolds stress tensor $n_i m_i \mathbf{v}_i \mathbf{v}_i \simeq n_i m_i \tilde{\mathbf{v}}_E \tilde{\mathbf{v}}_E$. Then

$$\frac{\partial}{\partial t} (n_i m_i \bar{\mathbf{v}}_E) + \nabla \cdot (n_i m_i \tilde{\mathbf{v}}_E \tilde{\mathbf{v}}_E) = 0 \quad (4)$$

Averaging (4) over the fast, small scales, we obtain the evolution equation for the mean flow

$$\frac{\partial}{\partial T} \nabla_{\perp}^2 \bar{\phi} = -\frac{c}{B_0} \overline{R\phi}, \quad (5)$$

$$\overline{R^\phi} = \overline{\mathbf{b} \cdot \nabla \tilde{\phi} \times \nabla \nabla_{\perp}^2 \tilde{\phi}}, \quad (6)$$

Here, B_0 is the equilibrium magnetic field, $\mathbf{b} = \mathbf{B}_0/B_0$. The electrostatic Reynolds stress (6) can be written in the form

$$R^\phi = (\partial_y^2 - \partial_x^2) (\partial_x \phi \partial_y \phi) + \partial_x \partial_y \left((\partial_x \phi)^2 - (\partial_y \phi)^2 \right). \quad (7)$$

After averaging over the fast scale we obtain in the leading order

$$\overline{R^\phi} = (\nabla_y^2 - \nabla_x^2) \left(\overline{\partial_x \phi \partial_y \phi} \right) + \nabla_x \nabla_y \left(\overline{(\partial_x \phi)^2} - \overline{(\partial_y \phi)^2} \right), \quad (8)$$

where we use ∇ for the derivatives over the slow (large scale) and ∂ for the derivatives over the fast (small scale) variables. This general expression is useful for investigation of zonal flow ($\nabla_x \neq 0$, $\nabla_y = 0$) and streamers ($\nabla_x = 0$, $\nabla_y \neq 0$).

The momentum exchange between plasma and electromagnetic waves becomes important for fluctuations with a significant magnetic component, e.g. Alfvén wave fluctuations. Then the contribution of the Maxwell stress becomes important

$$\frac{\partial}{\partial t} (n_i m_i \overline{\mathbf{v}_E}) + \nabla \cdot \left(n_i m_i \tilde{\mathbf{v}}_E \tilde{\mathbf{v}}_E + \frac{1}{8\pi} B^2 \mathbf{I} - \frac{1}{4\pi} \mathbf{B} \mathbf{B} \right) = 0. \quad (9)$$

In the ideal MHD ordering $\omega \simeq k_{\parallel} v_A$, the contribution of the Maxwell stress due to magnetic fluctuations is of the same order as the Reynolds stress

$$\frac{1}{8\pi} \tilde{B}^2 \mathbf{I} - \frac{1}{4\pi} \tilde{\mathbf{B}} \tilde{\mathbf{B}} \simeq n_i m_i \tilde{\mathbf{v}}_E \tilde{\mathbf{v}}_E. \quad (10)$$

There is a complete cancellation between two components in pure Alfvénic state $\omega = k_{\parallel} v_A$.

In a plasma of the finite temperature, the ion Larmor radius effects become important. The contribution of such effects is described by the viscosity tensor $\nabla \cdot (\boldsymbol{\pi}_i)^{37-39}$. In the limit of a small but finite Larmor radius, the viscosity tensor can be approximated by the standard gyro-viscosity tensor,^{40,41} see also Section 5. In general case, full kinetic calculations are required.³⁷⁻³⁹ The variations of plasma density in Eq.(9) may also contribute to the momentum drive which is essentially a mechanism of the Stringer spin-up.^{42,43} Such density variations can often be driven by externally injected electromagnetic waves.^{37,44} In toroidal geometry the effects of plasma pressure asymmetry may also contribute to the zonal flow drive.⁴⁵

3 Wave packet modulations and wave-action invariants

Modulations of the wave packets by the large scale structures are described by a WKB type wave kinetic equation for the quanta density of small scale fluctuations that is conserved along the rays. This method was originally proposed to describe the interaction of high frequency plasmons (Langmuir waves) with low frequency ion sound perturbations.⁴⁶ In studies of drift wave dynamics, it has been assumed^{47,48} that the relevant quantity that is conserved in the presence of slow variations is the drift-wave action density. It is well known that the standard wave action variables C_k associated with the number of wave quanta n_k , $n_k = |C_k|^2 = E_k/\omega_k$, where E_k is the wave energy, and ω_k is the wave frequency, is the basis for a Hamiltonian form of the wave-wave interaction equations. It has

been noted^{32,49} however that the normal variables used to describe self-interaction between small scale fluctuations without the shear flow are modified by the flow and may not be suitable for a system with a mean flow. Thus, in the presence of a shear flow, one can expect a new form of canonical variables and associated action invariants. It was also shown directly^{24,50} that for some models of drift waves the conserved action-like quantity (pseudo-action) is different from the usual definition of the wave action defined as the ratio of the wave energy to the wave frequency. The latter definition also fails when there are no oscillating eigen-modes, such as in ideal fluid, so that an alternative definition of the action-like integral is required.^{51,52}

A generic system of the drift wave fluctuations interacting with the mean flow can be written in the form

$$\frac{\partial \phi_k^>}{\partial t} + i\omega_k \phi_k^> + \int d^2 p L_{p,k-p} \phi_p^< \phi_{k-p}^> = 0, \quad (11)$$

where $\omega_k = \omega(k)$ is the frequency of the linear mode with a wave-vector k , and may include an imaginary part corresponding to the wave growth and decay. In the spirit of the scale separation we represent the field into the large-scale $\phi_k^<$ and small-scale $\phi_k^>$ components; $\phi_k^< = 0$ outside a shell $|\mathbf{k}| < \varepsilon \ll 1$, $\phi_k^> = 0$ for $|\mathbf{k}| < \varepsilon$. The self-interaction of small-scale fields is small compared to the interaction with the mean flow.⁴⁹ The equation for the evolution of the wave spectrum is obtained in the form

$$\frac{\partial}{\partial t} (\phi_k^> \phi_{k'}^>) + i(\omega_k + \omega_{k'}) \phi_k^> \phi_{k'}^> + \phi_{k'}^> \int d^2 p L_{p,k-p} \phi_p^< \phi_{k-p}^> + \phi_k^> \int d^2 p L_{p,k'-p} \phi_p^< \phi_{k'-p}^> = 0. \quad (12)$$

The small-scale turbulence is described by the spectral function (Wigner function) $I_k(\mathbf{x}, t)$, and defined as follows

$$\int d^2 q \langle \phi_{-k+q}^> \phi_k^> \rangle \exp(i\mathbf{q} \cdot \mathbf{x}) = I_k(\mathbf{x}, t). \quad (13)$$

The slow time and spatial dependence in $I_k(\mathbf{x}, t)$ corresponds to modulations with a ‘‘slow’’ wave vector, $\mathbf{q} \ll \mathbf{k}$, $\mathbf{k}' = -\mathbf{k} + \mathbf{q}$. Angle brackets in (13) stand for ensemble average, which is equivalent to a time average with appropriate ergodic assumptions. The equation for $I_k(\mathbf{x}, t)$ is derived from (12) by averaging it over fast scales and by taking the Fourier transform over the slow variable \mathbf{x} .⁵²

As an example, we consider two different models for drift waves in a magnetized plasma: the standard Hasegawa-Mima equation and a slab-like model for drift waves in a sheared magnetic field. The latter is similar to the standard Hasegawa-Mima equation with a modified plasma response to the slow modulations of the electrostatic potential. Such slow modes correspond to $k_{\parallel} \rightarrow 0$, so that the slow part of the potential does not follow Boltzmann distribution. [Note that zonal flows²³ ($m = n = 0$) are such slow modes with $k_{\parallel} = 0$.] As a result, the convective term appears in the lowest order, contrary to the case of the Hasegawa-Mima equation, where such term is due to the polarization drift. An appropriate equation for the drift wave dynamics in presence of a mean flow (neglecting the self-interaction) has the form^{47,53}

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla \right) \frac{e\tilde{\phi}}{T_e} + \mathbf{V}_* \cdot \nabla \frac{e\tilde{\phi}}{T_e} - \rho_s^2 \left(\frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla \right) \nabla_{\perp}^2 \frac{e\tilde{\phi}}{T_e} = 0, \quad (14)$$

where $\mathbf{V}_0 = c\mathbf{b} \times \nabla \bar{\phi} / B_0$ is the mean flow velocity. This equation can be written in the form (11) with $\omega_k = \mathbf{k} \cdot \mathbf{V}_* / (1 + k^2 \rho_s^2)$ and

$$L_{k_1, k_2} = -\frac{c}{B_0} \frac{\mathbf{b} \cdot \mathbf{k}_1 \times \mathbf{k}_2}{1 + (\mathbf{k}_1 + \mathbf{k}_2)^2 \rho_s^2} (1 + k_2^2 \rho_s^2). \quad (15)$$

From (12) we obtain a conservation law for the invariant $N_k = I_k(1 + k^2\rho^2)^2$,

$$\frac{\partial}{\partial t} N_k(\mathbf{x}, t) + \frac{\partial}{\partial \mathbf{k}} (\omega_k + \mathbf{k} \cdot \mathbf{V}_0) \cdot \frac{\partial N_k}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}} (\mathbf{k} \cdot \mathbf{V}_0) \cdot \frac{\partial}{\partial \mathbf{k}} N_k = 0. \quad (16)$$

A different expression for the action-like invariant is obtained for the standard Hasegawa-Mima (H.M.) model with a mean flow

$$\frac{\partial}{\partial t} \left(\frac{e\tilde{\phi}}{T_e} - \rho_s^2 \nabla_{\perp}^2 \frac{e\tilde{\phi}}{T_e} \right) + \mathbf{V}_* \cdot \nabla \frac{e\tilde{\phi}}{T_e} - \rho_s^2 (\mathbf{V}_0 \cdot \nabla) \nabla_{\perp}^2 \frac{e\tilde{\phi}}{T_e} = 0. \quad (17)$$

The appropriate interaction coefficient is

$$L_{k_1, k_2} = -\frac{c}{2B_0} \rho_s^2 \frac{\mathbf{b} \cdot \mathbf{k}_1 \times \mathbf{k}_2}{1 + (\mathbf{k}_1 + \mathbf{k}_2)^2 \rho_s^2} (k_2^2 - k_1^2). \quad (18)$$

In this case, the transport equation for I_k takes the form of the conservation law for the invariant $N_k = I_k k^2 \rho_s^2 (1 + k^2 \rho_s^2)$,^{24,26,54}

$$\frac{\partial}{\partial t} N_k + \frac{\partial}{\partial \mathbf{k}} \left(\omega_k + \frac{\mathbf{k} \cdot \mathbf{V}_0}{1 + k^2 \rho_s^2} k^2 \rho_s^2 \right) \cdot \frac{\partial N_k}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}} \left(\frac{\mathbf{k} \cdot \mathbf{V}_0}{(1 + k^2 \rho_s^2)} k^2 \rho_s^2 \right) \cdot \frac{\partial}{\partial \mathbf{k}} N_k = 0. \quad (19)$$

Similarly, this procedure can be used to derive the action-like invariant for the two-dimensional motion of an incompressible fluid. In the latter case, there are no oscillating modes so that the standard definition of the action as a ratio of the wave energy to wave frequency is not applicable. The 2-D Euler equation has a form

$$\partial \nabla_{\perp}^2 \phi + \mathbf{V}_0 \cdot \nabla \nabla_{\perp}^2 \phi = 0, \quad (20)$$

where \mathbf{V}_0 is the velocity due to the mean flow. In this case, the wave kinetic equation is

$$\frac{\partial}{\partial t} N_k(\mathbf{x}, t) + \frac{\partial}{\partial \mathbf{k}} (\mathbf{k} \cdot \mathbf{V}_0) \cdot \frac{\partial N_k}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}} (\mathbf{k} \cdot \mathbf{V}_0) \cdot \frac{\partial}{\partial \mathbf{k}} N_k = 0, \quad (21)$$

where the wave-action $N_k = k^4 I_k$.⁵¹

Note that both invariants for drift waves, (16) and (19), are different from the standard definition of the wave action defined as the ratio of the wave energy to the wave frequency is

$$n_k = |a_k|^2 = \frac{(1 + \rho_s^2 k_{\perp}^2)^2}{\omega_*} |\phi_k|^2 = \frac{E_k}{\omega_k}, \quad (22)$$

where $\omega_* = k_{\theta} V_*$. The difference between two forms of the action-like invariant is due to a different form of the coupling matrix, Eq. (15) and Eq.(18), describing the interaction of the small and large scale components.) New invariants, can be used to construct canonical variables in the presence of the shear flow.⁵²

4 Instability of the large scale shear flow

Coupled equations (5,16) can be solved to show that the modulations of the wave packets and zonal flow \mathbf{V}_0 are unstable.²³ We consider equations (5),(16) linearized for small perturbations $(\tilde{N}_k, \bar{\phi}) \sim \exp(-i\Omega T + iqr)$, where $q \equiv q_r = -i\partial/\partial r$ is the radial wave vector of the large scale perturbation. Then, Eq. (5) takes the form

$$-i\Omega \bar{\phi} = \frac{c}{B_0} \int k_r k_{\theta} |\phi_k|^2 d^2 k. \quad (23)$$

The modulation of \tilde{N}_k is calculated from (16)

$$\tilde{N}_k = -\frac{c}{B_0} q^2 \bar{\phi} k_\theta \frac{\partial N_k^0}{\partial k_r} \frac{i}{\Omega - qV_g}, \quad (24)$$

where $V_g = \partial\omega/\partial k_r$. Using (24) in (23) we obtain the following equation²³

$$-i\Omega = -q^2 c_s^2 \int d^2k \frac{k_\theta^2 \rho_s^2}{(1 + k_\perp^2 \rho_s^2)^2} k_r \frac{\partial N_k^0}{\partial k_r} \frac{i}{\Omega - qV_{gr}}. \quad (25)$$

For the case of the narrow resonant function approximated by a delta-function, the growth rate of the resonant instability is

$$\gamma_q = -q^2 c_s^2 \int d^2k \frac{k_\theta^2 \rho_s^2}{(1 + k_\perp^2 \rho_s^2)^2} k_r \frac{\partial N_k^0}{\partial k_r} \pi \delta(\Omega - qV_g). \quad (26)$$

The condition $\partial N_k^0/\partial k_r < 0$ is required for instability. This instability may be interpreted as a result of the resonant interaction of the wave packet with slow modulations of the mean flow. Note also that this instability has a character of the negative viscosity instability also investigated for the driven 2D hydrodynamic turbulence.^{55,56}

Equation (25) also describes another type of the instability that is not of the resonant type, but rather of the hydrodynamic variety. When the growth rate of the instability becomes large compared to the characteristic frequency spread for the background fluctuations, individual N_k components contribute to the instability coherently. Insight into this mechanism can be provided by a simple case of a monochromatic wave packet with $N_k^0 = N_0 \delta(\mathbf{k} - \mathbf{k}_0)$, with $\mathbf{k}_0 = (k_{r0}, k_{\theta0})$. Then we obtain⁵⁷

$$(\Omega - qV_{gr})^2 = q^2 c_s^2 k_\theta^2 \frac{N_k^0}{2k_\theta V_*} \frac{\partial V_g}{\partial k_r}. \quad (27)$$

Note the criterion for the instability:

$$\frac{N_k^0}{2k_\theta V_*} \frac{\partial V_g}{\partial k_r} < 0. \quad (28)$$

Calculating the derivative of the group velocity we obtain

$$\Omega = qV_{gr} - i|q|c_s \frac{|k_{\theta0}\rho_s|}{(1 + k_\perp^2 \rho_s^2)^{3/2}} N_0^{1/2} \sqrt{1 - 4k_{r0}^2 \rho_s^2 + k_{\perp0}^2 \rho_s^2}. \quad (29)$$

This equation describes a growth of the large scale zonal flow as a result of the instability. Note that the instability is stabilized for shorter wave lengths, provided that $1 - 3k_{r0}^2 \rho_s^2 + k_{\theta0}^2 \rho_s^2 < 0$. It can readily be seen that the coherent (hydrodynamic) instability has a larger growth rate $\text{Im } \Omega \sim N_0^{1/2}$ compared to that of the resonant instability (26); for the latter case $\text{Im } \Omega \sim N_0$. Parametric instability of the hydrodynamic type was also considered by a different approach in Refs. 31, 58, and 59.

We have considered a specific example of drift waves in plasmas, but, similar arguments can be made for Rossby-type waves in fluids. For the systems of interest (magnetized plasma and geostrophic fluids of rotating planets), the conservation of potential vorticity is an essential characteristic of wave dynamics. In all cases, nonlinear advection of the potential vorticity remains a source of large scale motion, though exact form for the potential vorticity conservation for different types of waves in plasma and rotating fluids may vary. One of the

most general form for the vorticity conservation is Hasegawa-Mima or Charney-Obukhov equation. In normalized form it can be written³⁴

$$\partial_t (\psi - \nabla_{\perp}^2 \psi) + \partial_{\theta} \psi - \{ \psi, \nabla_{\perp}^2 \psi \} = 0. \quad (30)$$

Here ψ is the stream-function for two-dimensional velocity in θ, r - plane (β -plane), and is as a sum of the mean flow and small scale fluctuations, $\psi = \bar{\psi} + \tilde{\psi}$. The system given by Eq. (30) has an adiabatic invariant^{24,26,51,52} $N_k = k_{\perp}^2 (1 + k_{\perp}^2) |\psi_k|^2$ and the wave frequency $\omega_k = k_{\theta} / (1 + k_{\perp}^2) + k_{\theta} V_0 k_{\perp}^2 / (1 + k_{\perp}^2)$. Then one obtains

$$(\Omega - qV_{gr})^2 = \frac{q^4}{1 + q^2} k_{\theta}^2 \frac{N_k^0}{2k_{\theta}} \frac{\partial V_{gr}}{\partial k_r}. \quad (31)$$

It is interesting to note that despite different definitions of the wave action and different contribution of the mean flow to the eigen-frequency, the criterion for the instability (28) remains the same.

Thus, the small scale wave packets in magnetized plasmas and geostrophic fluids are unstable with respect to the long wavelength perturbations. These perturbations are accompanied by the excitation of the long wavelength modes of the velocity, i.e. zonal flows. Two mechanisms of the instability constitute the ‘‘hydrodynamic’’ and ‘‘kinetic’’ regimes of the same process, similar to the case of plasma - beam instabilities. Relative importance of these two regimes will be determined by the relation between the nonlinear growth rate γ_q (given either Eqs. (26) or (29)) and the spectral width of the background turbulence, $\delta\omega_k$. The instability is of the resonant type, when the instability growth rate γ_q is smaller than the spectral width $\delta\omega_k$ of the small scale fluctuations. The instability becomes the coherent hydrodynamic type if $\gamma_q > \delta\omega_k$, so that all harmonics grow coherently. In the simplest case of weak wave-wave interaction, the spectral width $\delta\omega_k$ is merely the width of the wave packet of small scale fluctuations. The finite wave-wave interaction will further broaden the spectrum and the nonlinear broadening $\Delta\omega_k$ must be taken into account in the estimate for the spectrum width.

For the case of a monochromatic wave, $N_0 \sim \delta(\mathbf{k} - \mathbf{k}_0)$, the shear flow instability given by Eq.(29) does not exhibit an amplitude threshold. A more detailed analysis⁶¹ shows that there is an amplitude threshold for the zonal flow excitation of the order of ρ_s^2 / L_n^2 .

The shear flow instability was examined here in the linear approximation with respect to the amplitude of the large scale flow. The finite amplitude effects may lead to the wave trapping inside the shear flow features and formation of strongly nonlinear structures.^{32,53,60}

5 Role of pressure fluctuations in ion temperature gradient driven turbulence

Finite pressure fluctuations modify the momentum balance with additional contribution due to the gyro-viscosity tensor

$$\frac{\partial}{\partial t} (n_i m_i \bar{\mathbf{v}}_E) + \nabla \cdot (n_i m_i \tilde{\mathbf{v}}_E \tilde{\mathbf{v}}_E + n_i m_i \tilde{\mathbf{v}}_E \tilde{\mathbf{v}}_{pi}) = 0, \quad (32)$$

or

$$\frac{\partial}{\partial t} (\nabla_{\perp}^2 \bar{\phi} + \nabla_{\perp}^2 \bar{p}) = -\frac{c}{B_0} \overline{R^{\phi}} - \frac{c}{B_0} \overline{R^p}, \quad (33)$$

where the diamagnetic contribution to the Reynolds stress is given by

$$\overline{R^p} = \overline{\nabla_{\perp} \cdot [(\mathbf{b} \cdot \nabla \tilde{\phi} \times \nabla) \nabla_{\perp} \tilde{p}]}, \quad (34)$$

$$\overline{R^p} = \nabla_y^2 \left(\overline{\partial_y p \partial_x \phi} \right) - \nabla_x^2 \left(\overline{\partial_x p \partial_y \phi} \right) + \nabla_x \nabla_y \left(\overline{\partial_x p \partial_x \phi} - \overline{\partial_y p \partial_y \phi} \right). \quad (35)$$

In general, pressures fluctuations have a finite phase shift with respect to the electrostatic potential. It was shown⁴⁰ that both parts (in-phase and out-of-phase) contribute to the diamagnetic Reynolds stress tensor. The in-phase part provides a dominant contribution. The contribution of the out-of-phase component, however is important to de-couple zonal flow evolution from the slow evolution of ion pressure profile. It turns out that contribution of the out-of-phase component in the Reynolds stress tensor is similar to the contribution to the anomalous energy flux which cancel the slow pressure evolution from the vorticity equation (the second term on the left hand side of Eq. (33)). Thus, the only remaining contribution of pressures fluctuations to the Reynolds stress tensor is due to the in-phase component.

Basic dynamics of the toroidal ion temperature gradient driven mode is described by following fluid equations⁴⁰ :

$$\begin{aligned} & \frac{\partial n_i}{\partial t} \frac{1}{n_0} - V_{*i} \frac{\partial}{\partial y} \frac{e\phi}{T_i} + \mathbf{V}_E \cdot \nabla \frac{n_i}{n_0} + \mathbf{V}_{Di} \cdot \nabla \left(\frac{e\phi}{T_i} + \frac{p_i}{p_0} + \frac{T_i}{T_0} \right) \\ & - \rho_i^2 \nabla_{\perp} \cdot \frac{d_0}{dt} \nabla_{\perp} \left(\frac{e\phi}{T_i} + \frac{p_i}{p_0} \right) = 0, \end{aligned} \quad (36)$$

$$\begin{aligned} & \frac{3}{2} \left(\frac{\partial p_i}{\partial t} \frac{1}{p_0} - V_{*p} \frac{\partial}{\partial y} \frac{e\phi}{T_i} + \mathbf{V}_E \cdot \nabla \frac{p_i}{p_0} \right) + \frac{5}{2} \mathbf{V}_{Di} \cdot \nabla \left(\frac{e\phi}{T_i} + \frac{p_i}{p_0} + \frac{T_i}{T_0} \right) \\ & + \frac{5}{2} \rho_i^2 \nabla_{\perp} \cdot \frac{d_0}{dt} \nabla_{\perp} \left(\frac{e\phi}{T_i} + \frac{p_i}{p_0} + \frac{T_i}{T_0} \right) = 0. \end{aligned} \quad (37)$$

Here, $\omega_D = \mathbf{k} \cdot \mathbf{V}_D$, $\mathbf{V}_D = 2cT_i \mathbf{b} \times \nabla \ln B / eB_0$, $\omega_* = k_{\theta} c T_i / eB_0 L_n$, $\omega_{*p} = \omega_* (1 + \eta_i)$, $\eta_i = \partial \ln T_i / \partial \ln n$. We have also used Boltzmann electron density.

Equation (33) for the evolution of the mean flow can be written in a general two dimensional form

$$\frac{\partial}{\partial t} \nabla^2 \overline{\phi} = \frac{c}{B_0} (1 + \delta) \nabla_i \nabla_j \varepsilon_{jk} \overline{\partial_i \phi \partial_k \phi}, \quad (38)$$

where $\varepsilon_{12} = -\varepsilon_{21}$, $\varepsilon_{22} = \varepsilon_{11} = 0$, $i = (1, 2) = (r, \theta)$, and δ is the parameter that describes the diamagnetic enhancement of the Reynolds force due to temperature fluctuation. In the lowest order in the $k^2 \rho^2$ parameter, δ is independent of the wave vector and can be written⁴⁰ as

$$\delta = \frac{2 \omega_{*i} (1 + \eta_i) - 5 \omega_D (1 - \tau^{-1}) / 3}{\tau \omega_{*i} + \omega_D (10 \tau^{-1} / 3 - 1)}. \quad (39)$$

Substituting (39) into (38) one finds the growth rate $\gamma = q_r^2 (1 + \delta) D_{rr}$ for the zonal flow instability, where

$$D_{rr} = - \left(\frac{c}{B_0} \right)^2 \int R(\Omega - \mathbf{q} \cdot \mathbf{V}_g) k_{\theta}^2 k_r \frac{\partial}{\partial k_r} I_k^0 d^2 k. \quad (40)$$

In the opposite limit, $q_{\theta} \gg q_r \rightarrow 0$, equation (38) describes generation of streamers with the increment $\gamma = q_r^2 (1 + \delta) D_{\theta\theta}$,

$$D_{\theta\theta} = - \left(\frac{c}{B_0} \right)^2 \int R(\Omega - \mathbf{q} \cdot \mathbf{V}_g) k_r^2 k_{\theta} \frac{\partial}{\partial k_{\theta}} I_k^0 d^2 k. \quad (41)$$

The radially elongated and poloidally localized nonlinear structures were observed in numerical simulations of the ion temperature gradient driven turbulence.⁶³

6 Electromagnetic effects and the generation of large scale magnetic structures

For plasmas with a finite pressure (finite β), electromagnetic effects becomes important due to coupling of the drift waves to the Alfvén wave branch that described by the dispersion equation $(\omega - \omega_{*e}) \left(1 - \omega^2/k_{\parallel}^2 v_A^2\right) + k_{\perp}^2 \rho_s^2 = 0$.⁶² Fluctuating magnetic field tends to reduce the zonal flow drive due to the contribution of Maxwell stress tensor to Eq. (9). The electromagnetic effects on the zonal flow growth can be simply understood from the general expression for the instability growth rate⁶⁴

$$\gamma = q^2 c_s^2 \sum k_y^2 V_g R(\Omega - qV_g) \frac{\partial N}{\partial k_r}. \quad (42)$$

In the electrostatic case, this expression reduces to Eq. (26). The absence of the zonal drive at the shear Alfvén wave resonance can be seen in Eq. (42) due to vanishing radial component of the wave group velocity for pure Alfvén waves. For the kinetic shear Alfvén waves with finite $k_{\perp}^2 \rho_s^2$, expression (42) gives

$$\gamma = q^2 c_s^2 \sum \frac{k_r k_{\theta} \rho_s^2}{(1 + k_{\perp}^2 \rho_s^2)^{1/2}} R(\Omega - qV_g) \frac{\partial N}{\partial k_r}. \quad (43)$$

Clearly, the population inversion $\partial N/\partial k_r > 0$ is required for the instability. Note also, that contrary to the case of drift waves, the spectrum asymmetry is required for kinetic shear Alfvén waves because of the absence of the diamagnetic rotation.

Modulational instability of small scale electromagnetic fluctuations may also lead to the generation of large scale magnetic structures in a turbulent magnetized plasma. The large scale magnetic field is driven by the mean electromotive force term in Ohm's law, $\tilde{\mathbf{v}} \times \tilde{\mathbf{B}}$, a process somewhat similar to the current drive by the externally launched Alfvén wave.^{65,66} A finite phase shift between $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{B}}$ is required that can be provide by the dissipation such as Landau damping. In general case the generation of the zonal flow and magnetic field are coupled and exhibit complex dynamics.⁶⁴

As an example we consider a collisionless Alfvén wave turbulence in the presence of an ambient magnetic field $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$. A spontaneous excitation of large scale magnetic fields $\overline{\mathbf{B}} = \nabla \overline{A} \times \hat{\mathbf{z}}$, where $\overline{A} = \overline{A}(x)$, is a result of coupling of small scale turbulence and the initial perturbation of the mean field. Large scale random magnetic field refracts wave packets of the Alfvén waves and, thus, modulates spectrum of the turbulence. Modulated spectrum reacts back on the generated field via correlation between the perturbed small scale components of electrostatic and magnetic potentials. The latter provides electromotive force in the mean Ohm's law. As a result, under some conditions, the initial zonal magnetic field can be amplified. This instability can be classified as a fast dynamo process.

The parallel momentum balance for electrons (generalized Ohm's law) is

$$\frac{\partial}{\partial t} \overline{(nV_{\parallel})} + \overline{\tilde{\mathbf{V}}_E \cdot \nabla (nV_{\parallel})} + \frac{e}{m_e} \overline{\tilde{E}_{\parallel} \tilde{n}} + \frac{e}{m_e} \overline{\tilde{E}_{\parallel} n_0} + \overline{\tilde{\nabla} \cdot v_{\parallel}^2 \tilde{f}} = 0. \quad (44)$$

The electron response of the fast scale \tilde{f} is given by

$$\tilde{f} = \frac{ev_z}{T_e} f_0 \frac{\phi - \frac{\omega_k}{k_z c} A}{v_z - \frac{\omega_k}{k_z}}. \quad (45)$$

Neglecting inertia for slow motion we have

$$\frac{1}{c} \frac{\partial \bar{A}}{\partial t} = -\tilde{\nabla}_{\parallel} \bar{\phi} + \frac{1}{en_0} \tilde{\nabla}_{\parallel} \bar{p} + \frac{m_e}{e} \overline{\tilde{\mathbf{V}}_E \cdot \nabla \tilde{V}_{\parallel}} + \overline{\tilde{E}_{\parallel} \tilde{n}}. \quad (46)$$

The right hand side of this equation describes four different sources for the mean magnetic field. The first and second terms are contributions to the mean field growth due to the field line bending. The third term is the radial transport of the momentum, and the last term is the helicity injection. We do not consider here the helicity term that require certain asymmetry in the spectrum (helicity). The contribution of the first two terms is conveniently calculated by using the relation

$$\phi - \frac{\tilde{p}}{en_0} = -\omega \frac{m_e}{e} \tilde{V}_{\parallel} + \frac{\omega}{k_z c} \tilde{A} = \frac{\omega}{k_z c} \left(1 + \frac{k_{\perp}^2 c^2}{\omega_{pe}^2} \right) \tilde{A} \simeq \frac{\omega}{k_z c} \tilde{A}. \quad (47)$$

We assume $\beta > m_e/m_i$ and $\omega < k_z v_A$, so that $k_{\perp}^2 c^2 / \omega_{pe}^2 < 1$. Then we have

$$-\overline{\tilde{\nabla}_{\parallel} \tilde{\phi}} + \frac{1}{en_0} \overline{\tilde{\nabla}_{\parallel} \tilde{p}} = \frac{1}{B_0} \frac{\partial}{\partial X} \left(\overline{\tilde{A} \frac{\partial}{\partial y} \frac{\omega}{k_z c} \tilde{A}} \right). \quad (48)$$

The structure of this term clearly shows that a finite phase shift (wave damping) is required for this term to be finite. A phase shift between ϕ and A is conditioned by the Landau resonance leading to wave damping and can be found from equation

$$\phi = \frac{k_{\parallel} c}{\omega} \frac{v_A^2}{c^2} A, \quad (49)$$

and

$$\omega = k_{\parallel} v_A \left(1 + \frac{1}{2} k_{\perp}^2 \rho_s^2 - \frac{i}{2\sqrt{\pi}} k_{\perp}^2 \rho_s^2 s \right). \quad (50)$$

Here $s \equiv \omega / k_{\parallel} v_{Te} < 1$; we also assumed $k_{\perp}^2 \rho_s^2 < 1$ to simplify the expressions. The third term in (46) is small compared to the first two as $k_{\perp}^2 c^2 / \omega_{pe}^2 < 1$. From equations (46), (48), (49) and (50) we find

$$\frac{1}{c} \frac{\partial \bar{A}}{\partial t} = \frac{\sqrt{\pi} v_a}{2B_0 c} \frac{\partial}{\partial X} \left(\sum |A_k|^2 k_y s k_{\perp}^2 \rho_s^2 \right). \quad (51)$$

In the case of Alfvén turbulence, the wave packet are modulated by the refraction due to modulations of the Alfvén wave frequency $\delta\omega = \delta k_{\parallel} v_A$, where

$$\delta k_{\parallel} = -\frac{1}{B_0} \frac{\partial \bar{A}}{\partial X} k_y v_A. \quad (52)$$

The modulations of wave packet then are described by

$$\frac{\partial \tilde{N}}{\partial t} + v_g \frac{\partial \tilde{N}}{\partial x} - \frac{\partial \delta\omega}{\partial x} \frac{\partial N_0}{\partial k_x} = 0. \quad (53)$$

In (52) and (53) we consider the perturbations $(\bar{A}, \tilde{N}) \sim \exp(-i\Omega t + iqX)$. The generalized action density N_k here is given by $N_k = k_{\perp}^2 \rho_s^2 (1 + k_{\perp}^2 \rho_s^2) |e\phi/T_e|^2 / \omega_k$. Combining the Eqs. (52) and (53) we obtain the dispersion equation for the growth rate of the large scale magnetic field

$$\Omega = iq^2 c_s^2 \frac{v_A}{v_{te}} \sum k_y^2 \frac{\partial N_0}{\partial k_x^2}. \quad (54)$$

We have assumed $qV_g > \Omega$ for the large scale magnetic field.

7 Summary

We have considered mechanisms for the zonal flow generation in drift-wave turbulence. It is shown that the shear flow may develop as a result of modulational type instabilities of the saturated turbulence. The instability develops due to positive feedback of modulations of small scale fluctuations that refracted in the shear flow. We examined this instability for the basic drift wave fluctuations as well as for the ion temperature gradient driven modes and electromagnetic drift Alfvén waves. Similar instabilities may occur in isomorphic to drift waves geophysical systems. It is shown that the generation of large scale magnetic structures is also possible in drift Alfvén wave turbulence. The spontaneous excitation of the large scale magnetic field is mediated by the dissipation processes such as Landau wave-particle interaction. In general case, the generation of the large scale magnetic field and shear flows are coupled. The complex intermittent dynamics of flows and magnetic structures may be important for the understanding of electron transport due to the small scale electromagnetic turbulence, e.g. electron temperature gradient driven modes.^{67,68}

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