Cross Phase Evolution and Its Role in ELM Bursts

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Collaborators

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  See: P.W. Xi, Xu, P.D. – PRL, PoP 2014

- Related work:

  T. Rhee – NFRI

  J.M. Kwon – NFRI

  W.W. Xiao - UCSD, NFRI, SWIP

  See: T. Rhee, et. al. PoP 2012

Acknowledgements

- Xiao-Lan Zou, R. Singh, Todd Evans, Rajesh Mainji, Sang-Hee Hahn, Michael Leconte, Guilhem Dif-Pradalier

Caveat Emptor

- Not a professional ELM-ologist
- Perspective is theoretical, and focus is on issues in understanding dynamics
- Perspective is that of a transport theorist
- Aim is to distill elements critical to model building
- Unresolved issues are discussed

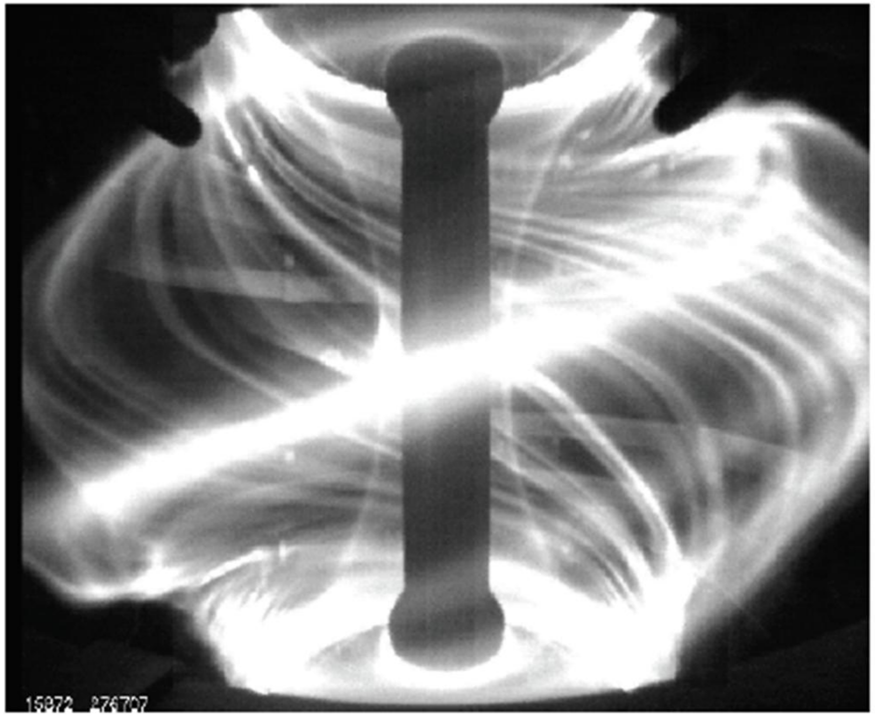
Outline

- ELMs
  - Conventional wisdom: A Quick Look
  - Some physics questions
- Recent Progress:
  - cross phase coherence and the origin of bursts
  - phase coherence as leverage for ELM mitigation
  - a deeper – but incomplete – look at phase dynamics
- Conclusions and Discussion

Terra Firma: Conventional Wisdom of ELMs

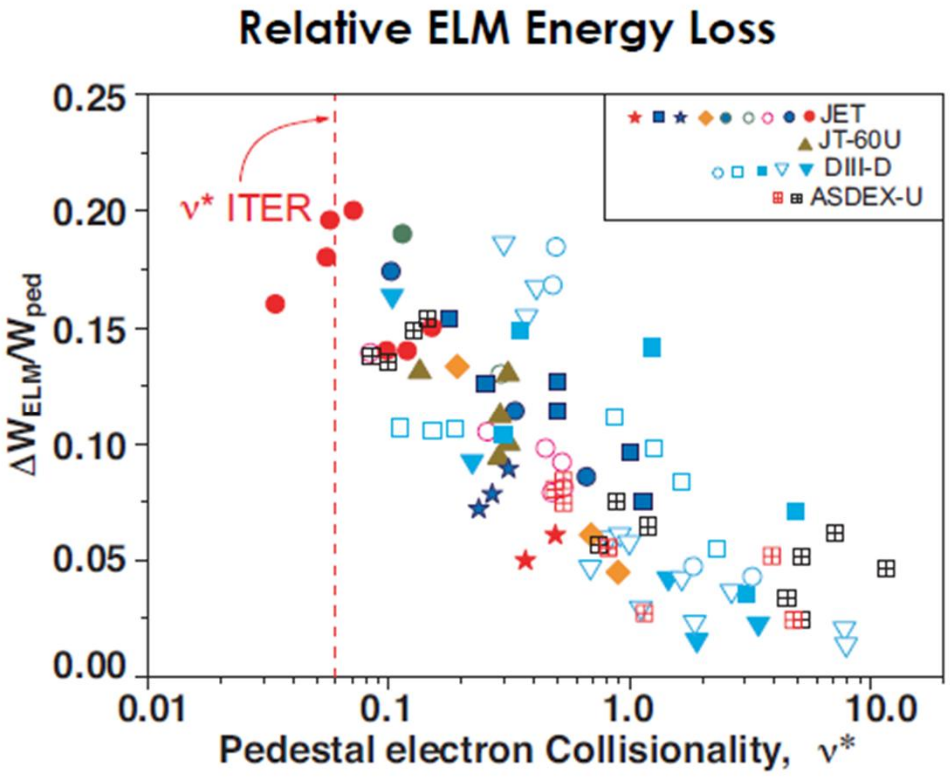
- ELMs are ~ quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma
- ELMs
  - Limit edge pedestal –
  - Expel impurities +
  - Damage PFC –
- ELMs → a serious concern for ITER
  - $\Delta W_{ELM} \sim 20\%\ W_{ped} \sim 20\ \mathrm{MJ}$
  - $W_{ELM}\ /\ A \sim 10 \times$ limit for damage
  - $\tau_{rise} \sim 200\ \mu sec$

MAST



Terra Firma: Conventional Wisdom of ELMs

- ELM Types
  - I, II: ω_{ELM} ↑ as P ↑, greatest concern, related to ideal stability
  - III: ω_{ELM} ↓ as P ↑, closer to P_{Th}, unknown → resistive ??



[A. Loarte, *Plasma Phys. Control. Fusion* 45 1549 (2003)]

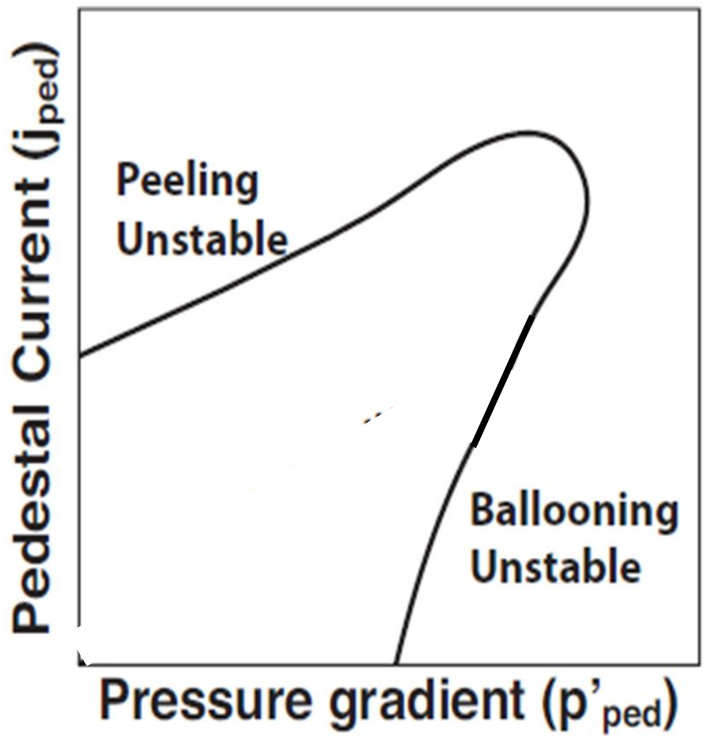
- Physics
  - Type I, II ELM onset → ideal stability limit
  - i.e. peeling + ballooning

Peeling + ballooning ⇠ mode ⇢ Edge kink / Curvature vs bending

δW + Pedestal, geometry

Terra Firma: Conventional Wisdom of ELMs

- Some relation of ELM character to collisionality is observed
  - Low collisionality → peeling ~ "more conductive"
  - High collisionality → ballooning ~ "more convective"
- Many basic features of ELMs consistent with ideal MHD peeling-ballooning theory
- Pedestal perturbation structure resembles P-B eigen-function structure (?!)



Some Physics Questions

- What IS the ELM? Why is the ELM?
  - ELMs single helicity or multi-helicity phenomena?

    Relaxation event ↔ pedestal avalanche?, turbulence spreading?
  - How and why do actual bursts occur?

    Why doesn't turbulence force $\nabla P \sim \nabla P_{crt}$ oscillations?
  - Pedestal turbulence develops during ELM. Thus, how do P-B modes interact with turbulence? – either ambient or as part of MH interaction?
  - Does, or even should, the linear instability boundary define the actual ELM threshold?

I) Basic Notions of ELMs:

ELM Bursts and Thresholds as

Consequence of Stochastic Phase Dynamics

Simulation model and equilibrium in BOUT++

- **3-field model for nonlinear ELM simulations**
  - ✓ **Including essential physics for the onset of ELMs**

Peeling-ballooning instability
Resistivity
Hyper-resistivity
Ion diamagnetic effect

$$\frac{d\varpi}{dt} = B\nabla_{\parallel}J_{\parallel} + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla\widetilde{P} + \mu_{i,\parallel}\partial_{\parallel}^2\varpi$$

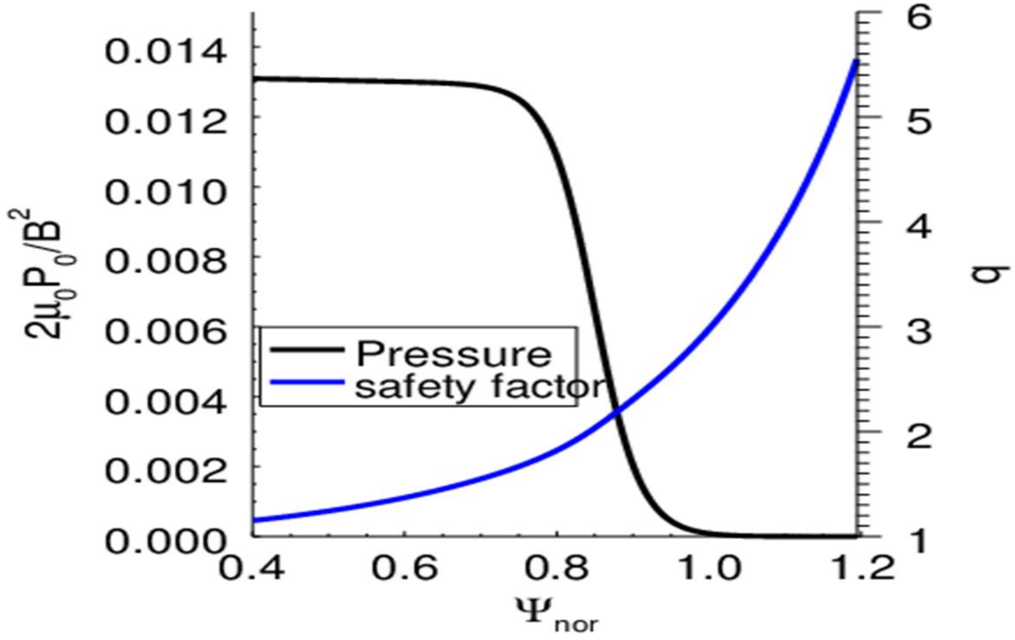
$$\frac{d\widetilde{P}}{dt} + \mathbf{V}_E \cdot \nabla P_0 = 0$$

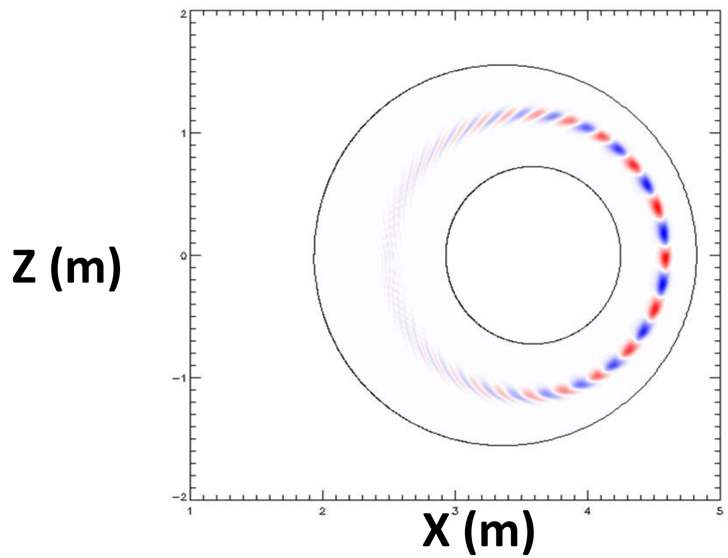
$$\frac{\partial A_{\parallel}}{\partial t} + \partial_{\parallel}\phi_T = \frac{\eta}{\mu_0}\nabla_{\perp}^2 A_{\parallel} - \frac{\eta_H}{\mu_0}\nabla_{\perp}^4 A_{\parallel}$$

hyper resistivity

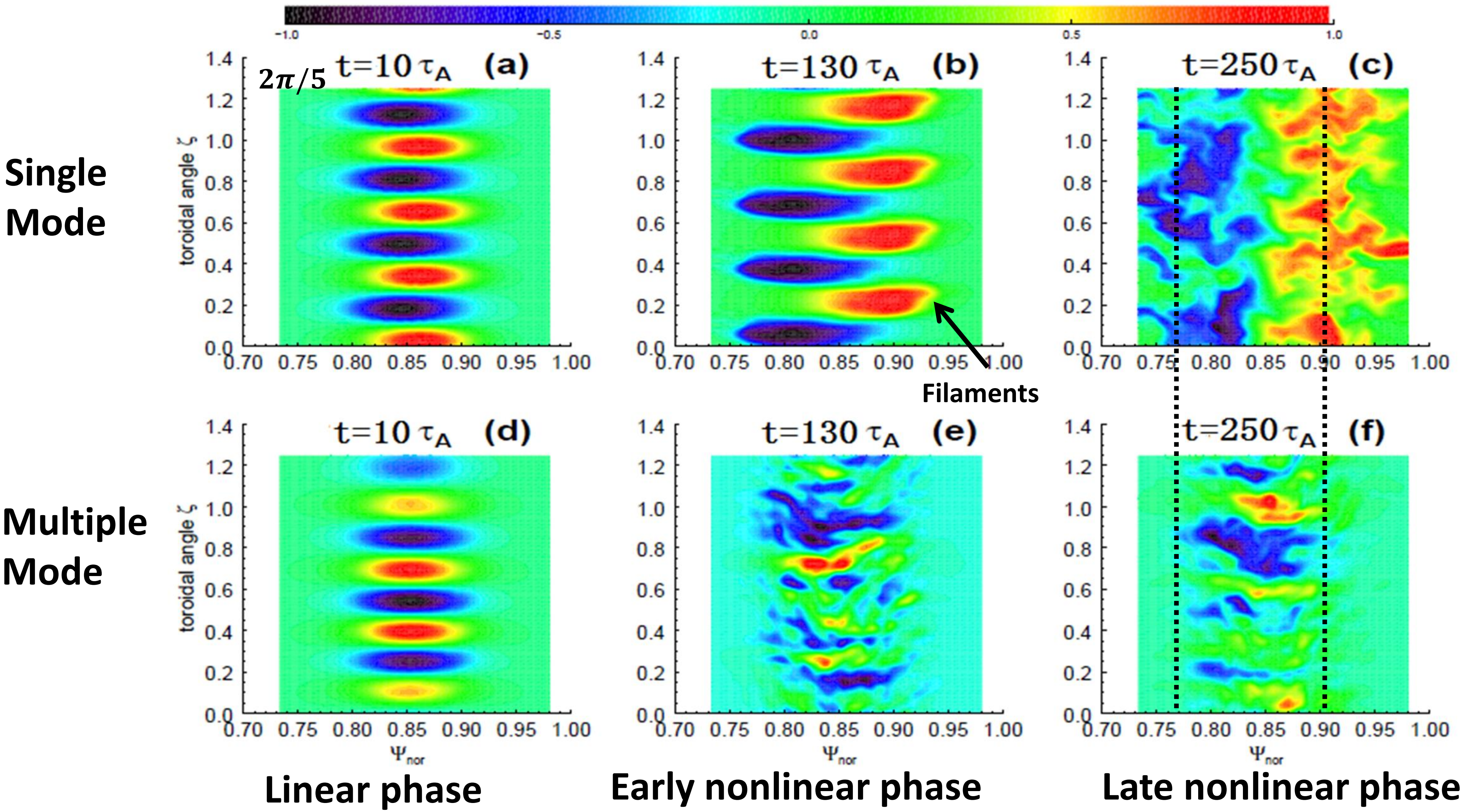
$$\varpi = \frac{m_i n_0}{B}\left(\nabla_{\perp}^2\phi + \frac{1}{e n_0}\nabla_{\perp}^2 \widetilde{P}_i\right)$$

$$d/dt = \partial/\partial t + \mathbf{V}_{ET}\cdot\nabla,\ \mathbf{V}_{ET} = \frac{1}{B}\mathbf{b}_0 \times \nabla\phi_T,\ \phi_T = \phi_0 + \phi,\ \nabla_{\parallel} f = B\partial_{\parallel}\frac{f}{B},\ \partial_{\parallel} = \partial_{\parallel}^0 + \delta\mathbf{b}\cdot\nabla,\ \delta\mathbf{b} = \frac{1}{B}\nabla A_{\parallel} \times \mathbf{b}_0,\ J_{\parallel} = J_{\parallel 0} + \widetilde{J}_{\parallel},\ \widetilde{J}_{\parallel} = -\nabla_{\perp}^2 A_{\parallel}/\mu_0$$



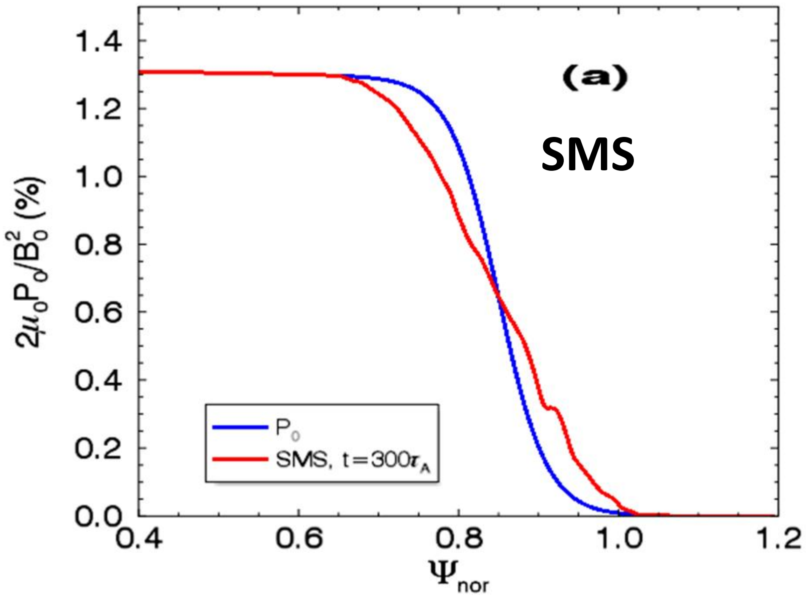


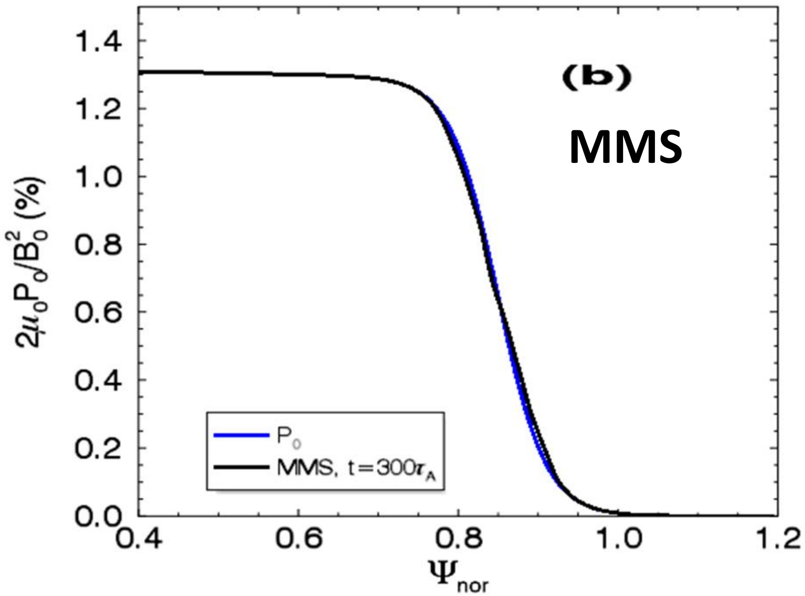
Contrast of perturbation evolution (1/5 of the torus)

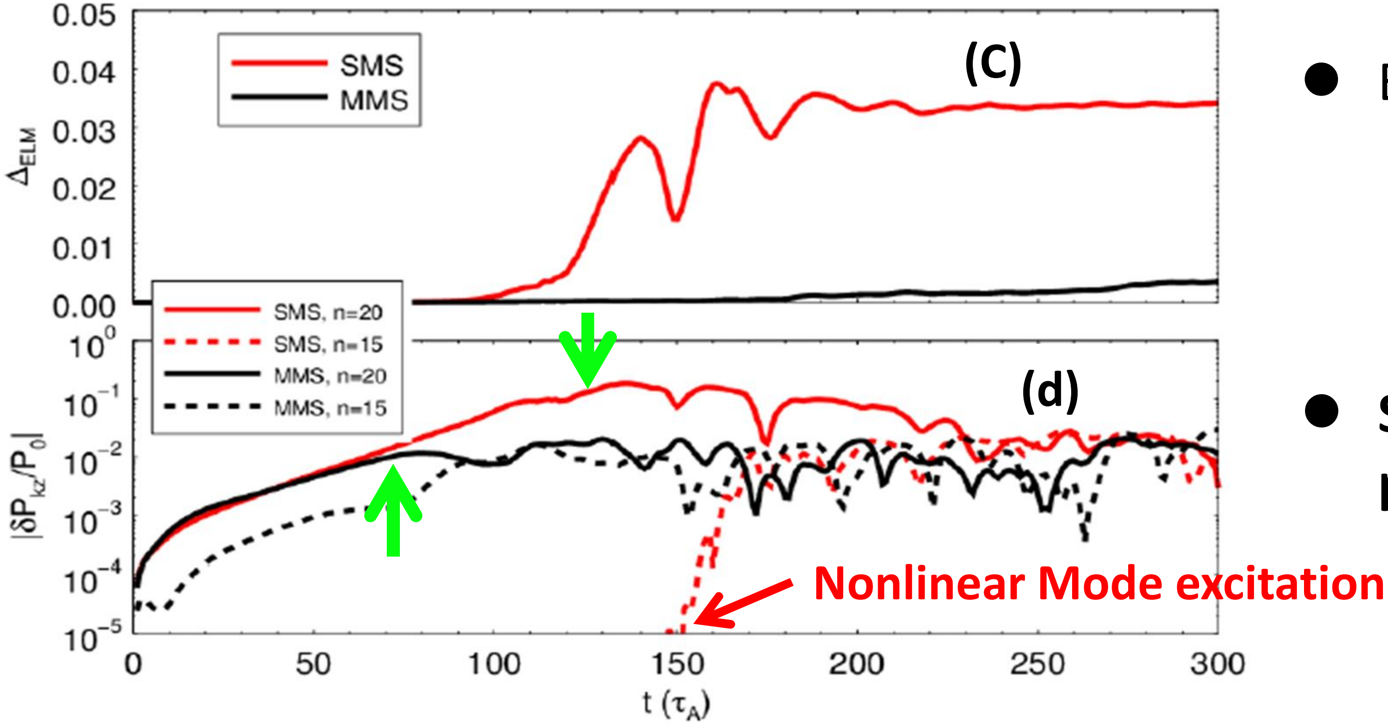


- Single mode: Filamentary structure is generated by linear instability;
- Multiple modes: Linear mode structure is disrupted by nonlinear mode interaction and no filamentary structure appears

Single mode: ELM crash || Multiple modes: P-B turbulence







- ELM size larger for SMS

$$\Delta_{ELM} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\int dx^3 (P_0 - \langle P \rangle_\zeta)}{\int dx^3 P_0}$$

- **SMS has longer duration linear phase than MMS**

Relative Phase (Cross Phase) Dynamics and Peeling-Ballooning Amplification

Peeling-Ballooning Perturbation Amplification is set by Coherence of Cross-Phase

i.e. schematic P.B. energy equation:

$$\frac{\partial}{\partial t}E_k = \langle\tilde{\phi}2\hat{b}_0 \times \vec{k}\cdot\nabla\tilde{P}\rangle_{\vec{k}}$$

← $\sim\langle\tilde{v}_r\tilde{P}\rangle$ → energy release from $\nabla\langle P\rangle$
→ quadratic

$$+\sum_{\vec{k}',\vec{k}''}\tau_{c\vec{k}}C(\vec{k}',\vec{k}'')E_{\vec{k}'}E_{\vec{k}''} - \sum_{\vec{k}'}\tau_{c\vec{k}+\vec{k}'}C(\vec{k}',\vec{k})E_{\vec{k}'}E_{\vec{k}}$$ - dissipation

nonlinear mode-mode coupling → quartic

NL effects
- energy couplings to transfer energy (weak)
- response scattering to de-correlate $\tilde{\phi}$, $\tilde{P}$ → regulate drive

Growth Regulated by Phase Scattering

τ_c → phase coherence time

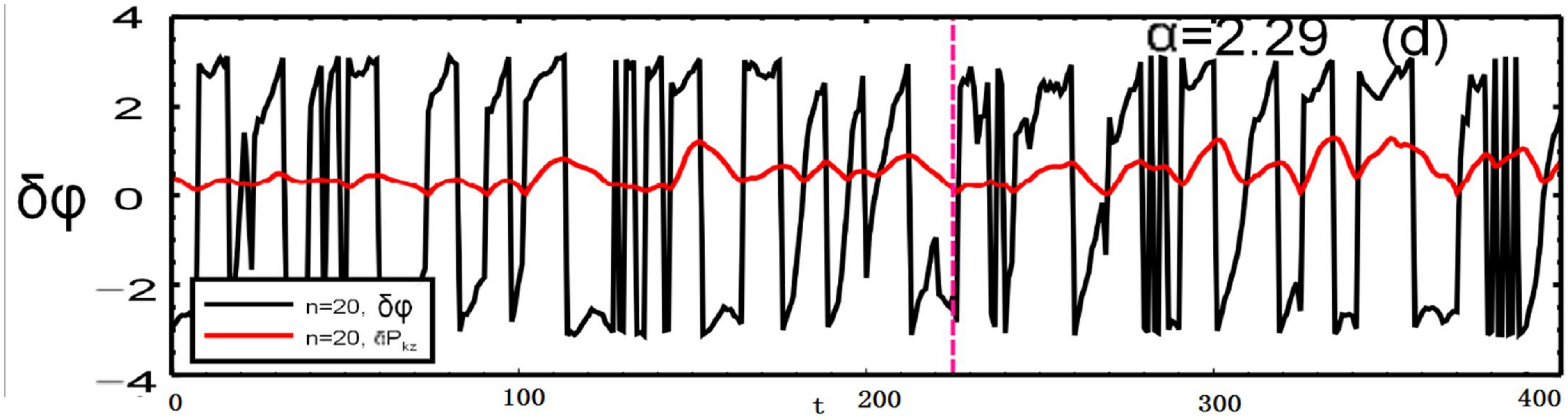
$\nabla P_o \xrightarrow{k} \tilde{P}$ ↗ $\langle \tilde{v}_r \tilde{P} \rangle$ → net growth → intensity field → crash?

↑ phase scattering

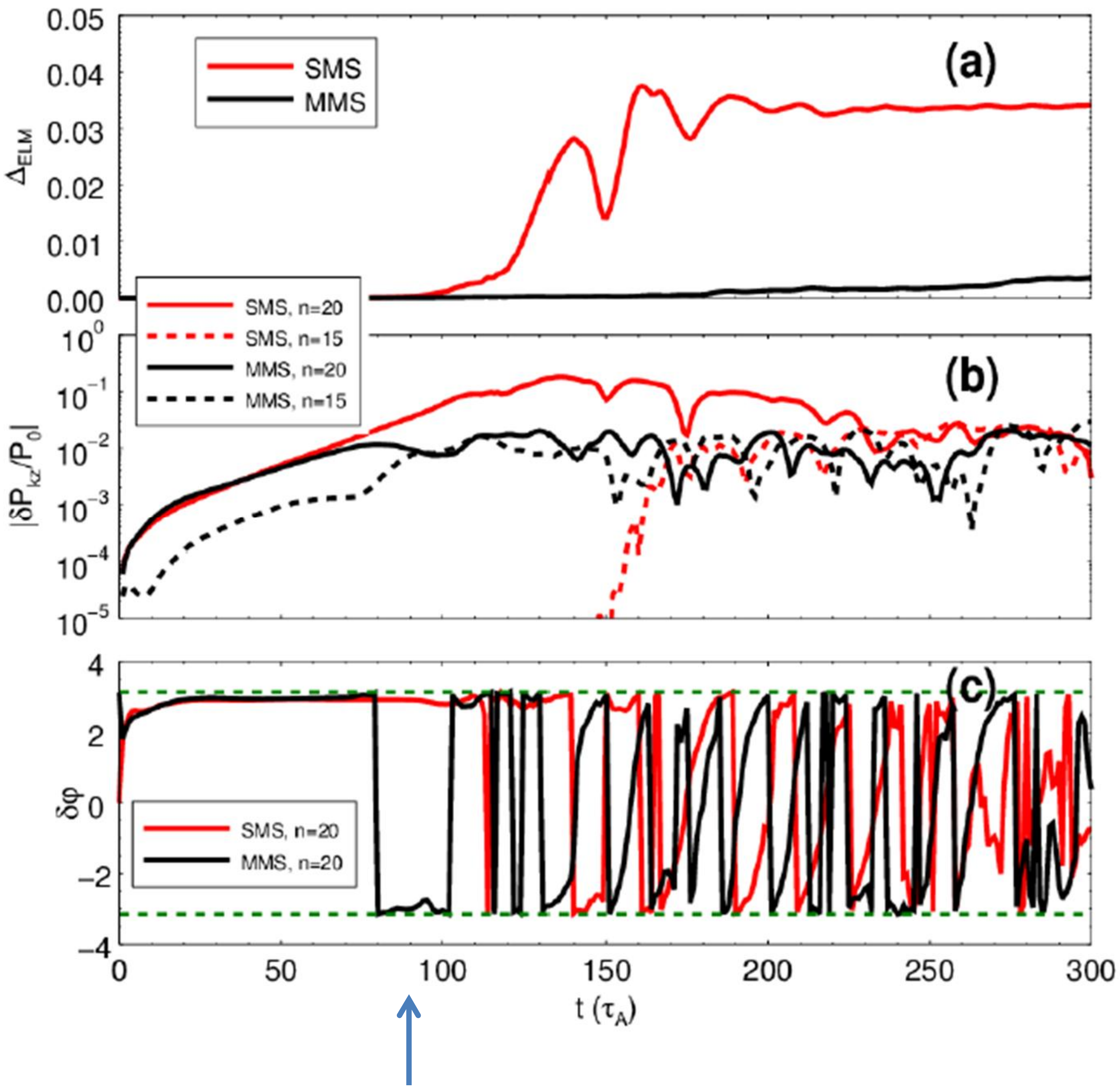
$\nabla P_o \xrightarrow{k} \tilde{P}$ ↘ transfer → dissipation (weak)

Critical element: relative phase
$\delta\varphi = arg\ [\hat{p}_n\ /\ \hat{\phi}_n]$

Phase coherence time sets growth



Cross Phase Exhibits Rapid Variation in Multi-Mode Case



- Single mode case → coherent phase set by linear growth → rapid growth to ‘burst’
- Multi-mode case → phase de-correlated by mode-mode scattering → slow growth to turbulent state

Key Quantity: Phase Correlation Time

- Ala' resonance broadening (Dupree '66):

$$\frac{\partial}{\partial t}\hat{P} + \tilde{v}\cdot\nabla\tilde{P} + \langle v\rangle\cdot\nabla\hat{P} - D\nabla^2\hat{P} = -\tilde{v}_r\frac{d}{dr}\langle P\rangle$$

Nonlinear scattering — Linear streaming (i.e. shear flow) — Ambient diffusion

$$\hat{P} = Ae^{i\phi}$$

Relative phase ↔ cross-phase (ϕ)

Amplitude (A)

$$\hat{v} = B$$

Velocity amplitude (B)

➔ $$\partial_t\tilde{\phi} + \tilde{v}\cdot\nabla\tilde{\phi} + \langle v(r)\rangle\cdot\nabla\tilde{\phi} - D\nabla^2\tilde{\phi} - \frac{2D}{A}\nabla A\cdot\nabla\tilde{\phi} = 0$$

NL scattering — shearing

$$\partial_t A + \tilde{v}\cdot\nabla A + \langle v(r)\rangle\cdot\nabla A + D\left(\nabla\tilde{\phi}\right)^2 A - D\nabla^2 A = -B\frac{d}{dr}\langle P\rangle$$

Damping by phase fluctuations

Phase Correlation Time

- Stochastic advection:

$$\frac{1}{\tau_{ck}} = \vec{k} \cdot D_\phi \cdot \vec{k} + k^2 D$$

$$D_\phi = \sum_{k'} \tau_{ck'} |\tilde{v}'_{\perp k}|^2$$

- Stochastic advection + sheared flow:

$$\frac{1}{\tau_{ck}} \approx \left(k_\perp^2 (D_\phi + D) \langle v_\perp \rangle'^{\,2}\right)^{1/3}$$

➔ Coupling of radial scattering and Shearing shortens phase correlation

- Parallel conduction + diffusion:

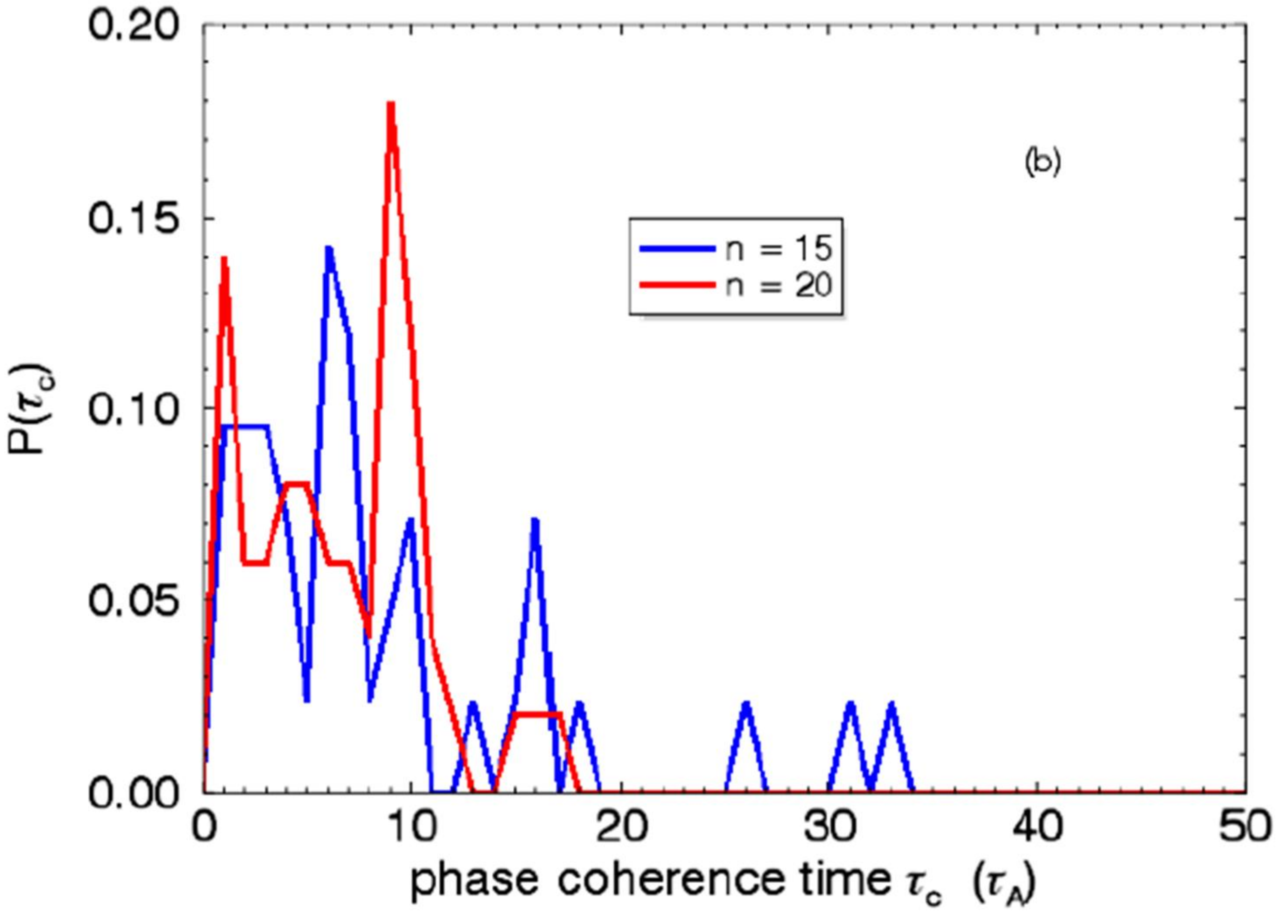
$$\frac{1}{\tau_{ck}} \approx \left[\frac{\hat{s}^2 k_\perp^2}{(Rq)^2} \chi_\parallel \left(D_\phi + D\right)\right]^{1/2}$$

➔ Coupling of radial diffusion and conduction shortens phase correlation

What is actually known about fluctuations in relative phase?

- For case of P.-B. turbulence, a broad PDF of phase correlation times is observed

pdf of τ_c



Implications for: i) Bursts vs Turbulence

ii) Threshold

Bursts, Thresholds

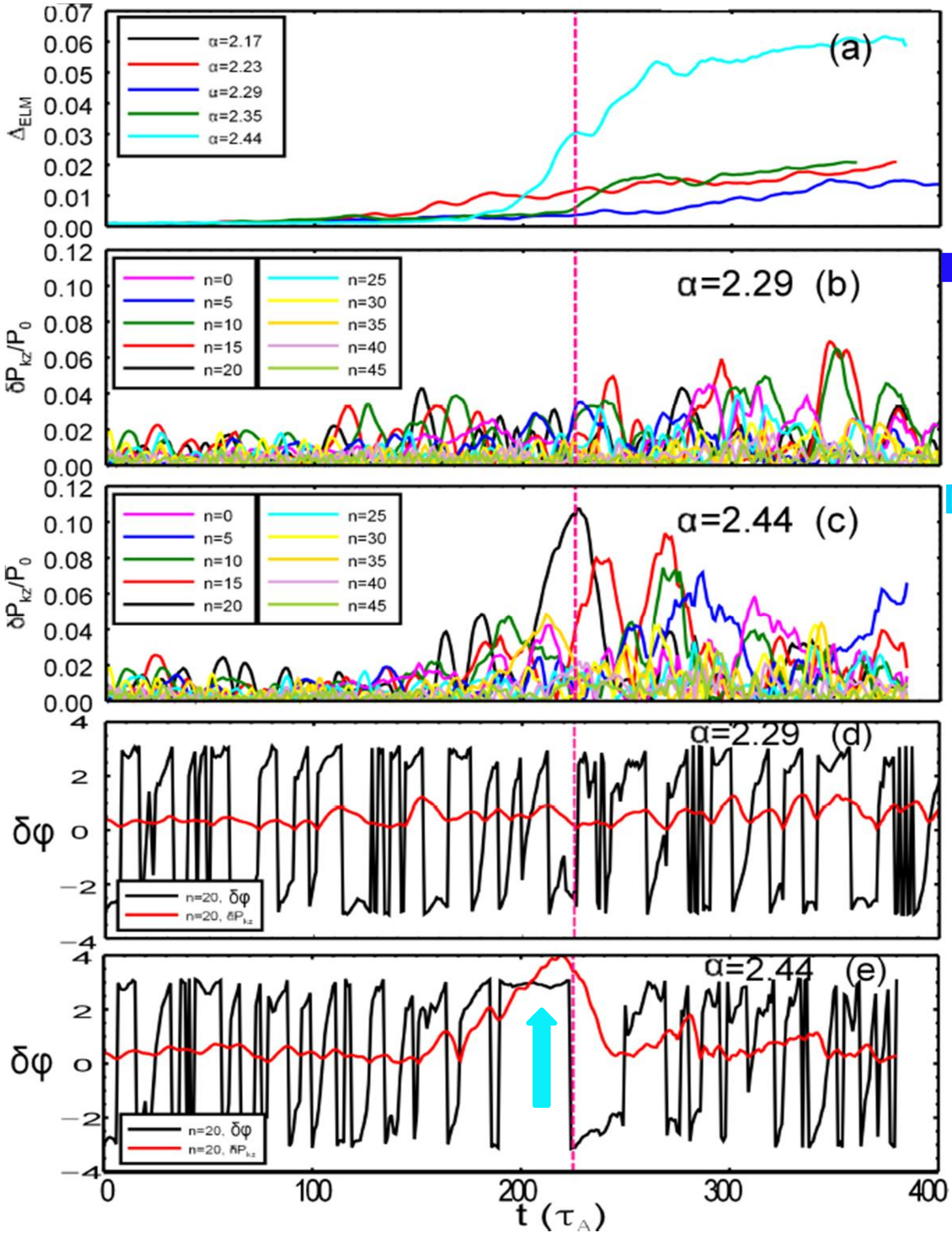
- P.-B. turbulence can scatter relative phase and so reduce/limit growth of P.-B. mode to large amplitude
- Relevant comparison may be:

γ_k^L (linear growth) vs $\frac{1}{\tau_{ck}}$ (phase de-correlation rate)

- Key point: Phase scattering for mode $\vec{k}$ set by 'background modes $\vec{k}'$' i.e. other P.-B.'s or micro-turbulence

➔ is the background strong enough??

The shape of growth rate spectrum determines burst or turbulence

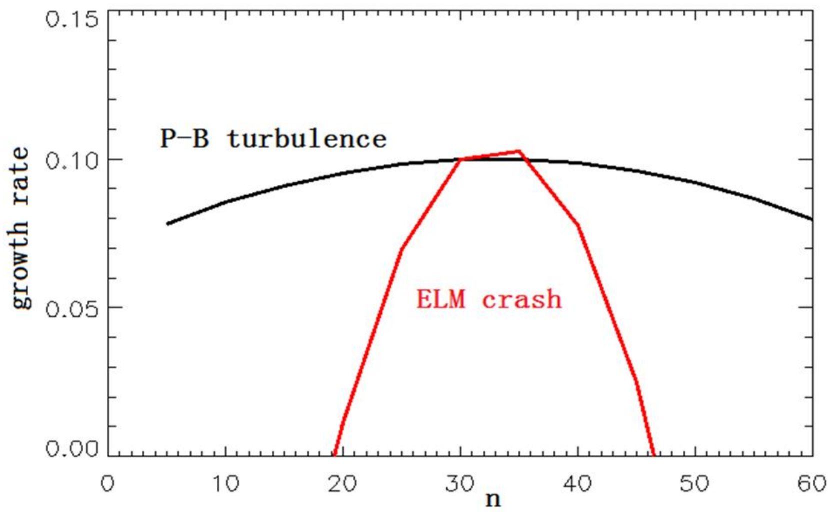


P-B turbulence

$$\gamma(n)\tau_c(n) < \ln 10$$

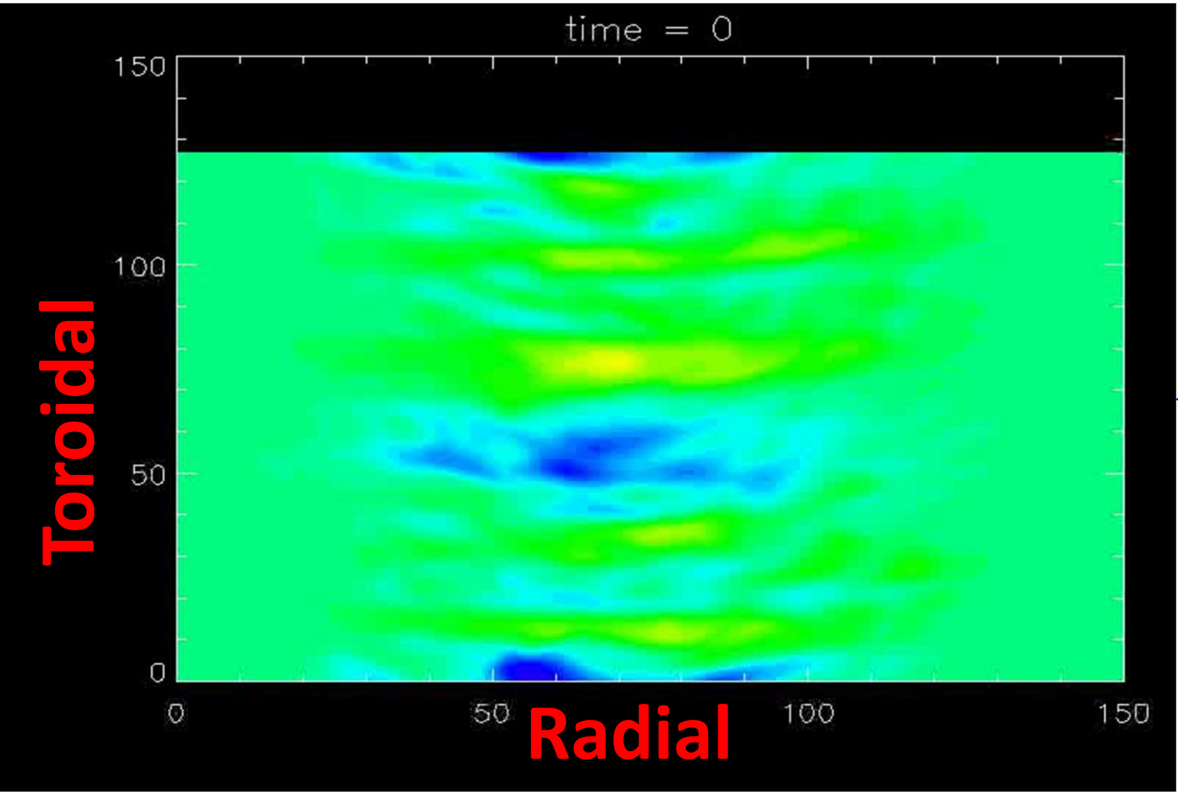
Isolated ELM crash

$$\begin{cases} \gamma(n)\tau_c(n) > \ln 10, n = n_{dom} \\ \gamma(n)\tau_c(n) < \ln 10, n \neq n_{dom} \end{cases}$$

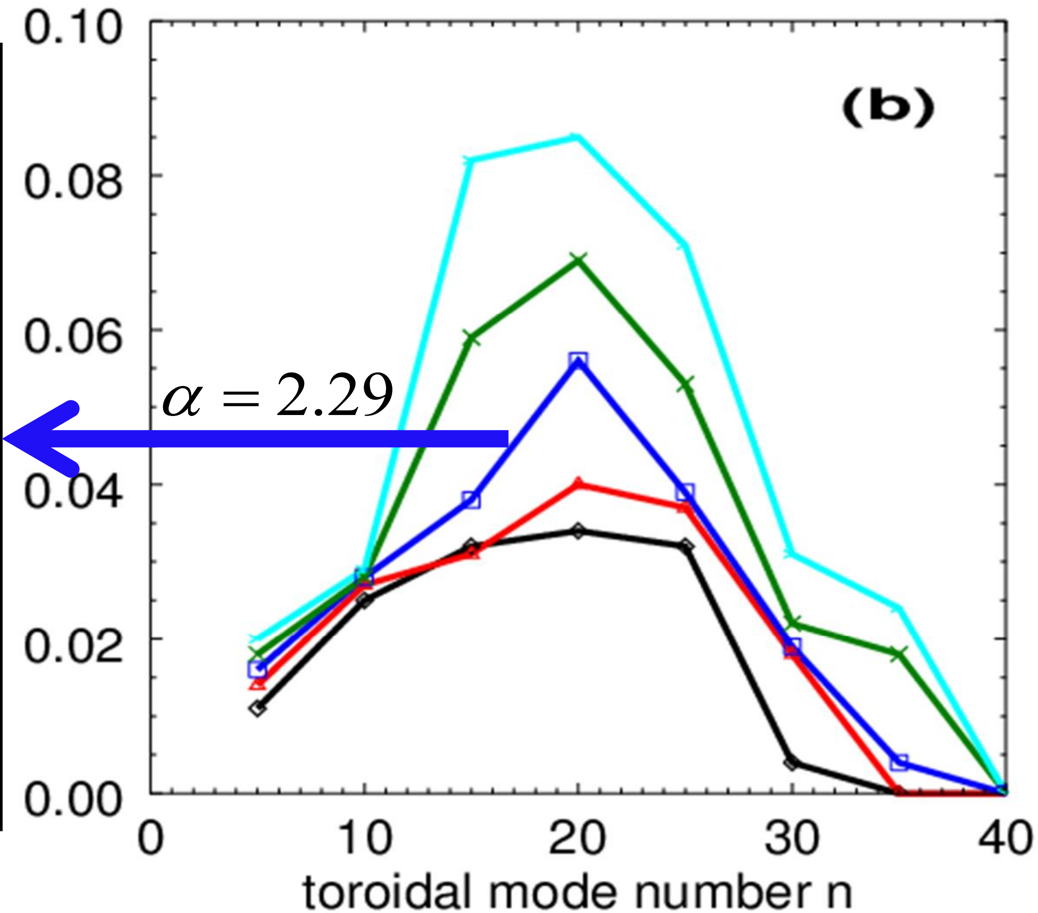


So When Does it Crash?

Modest $\gamma(n)$ Peaking ➔ P.-B. turbulence



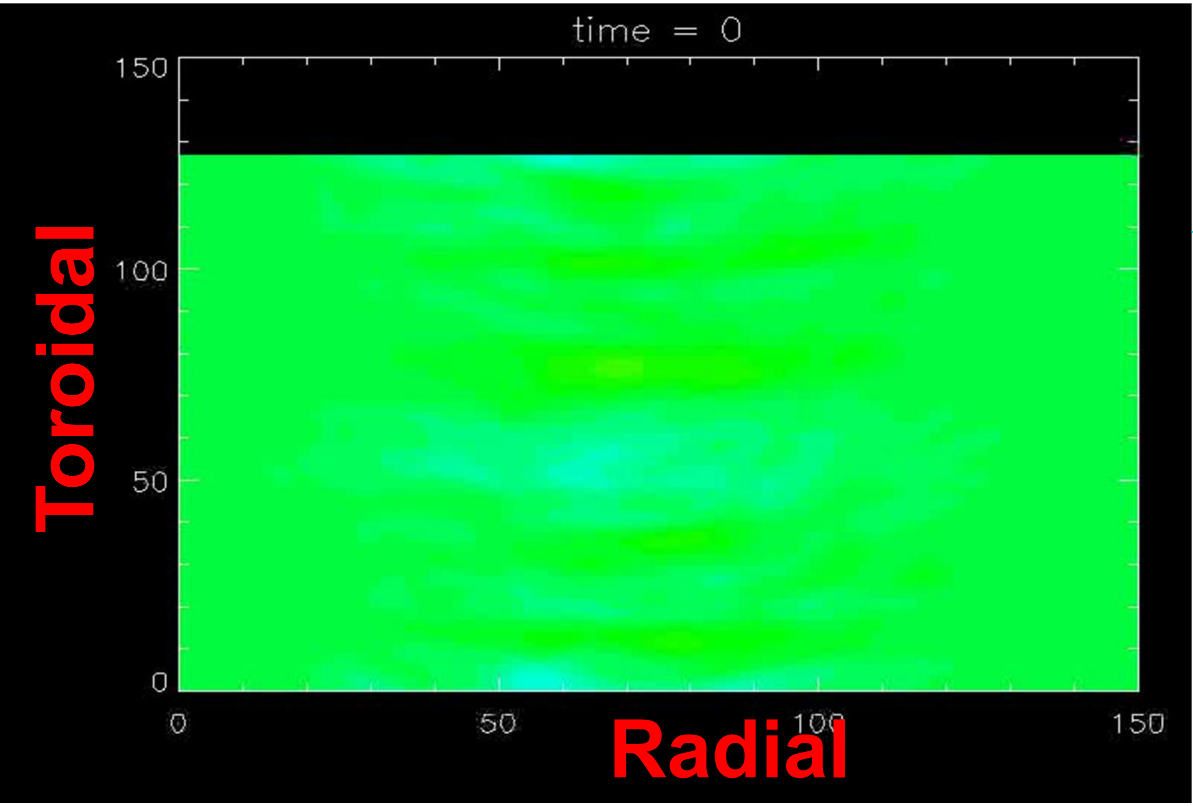
$\gamma(n)$

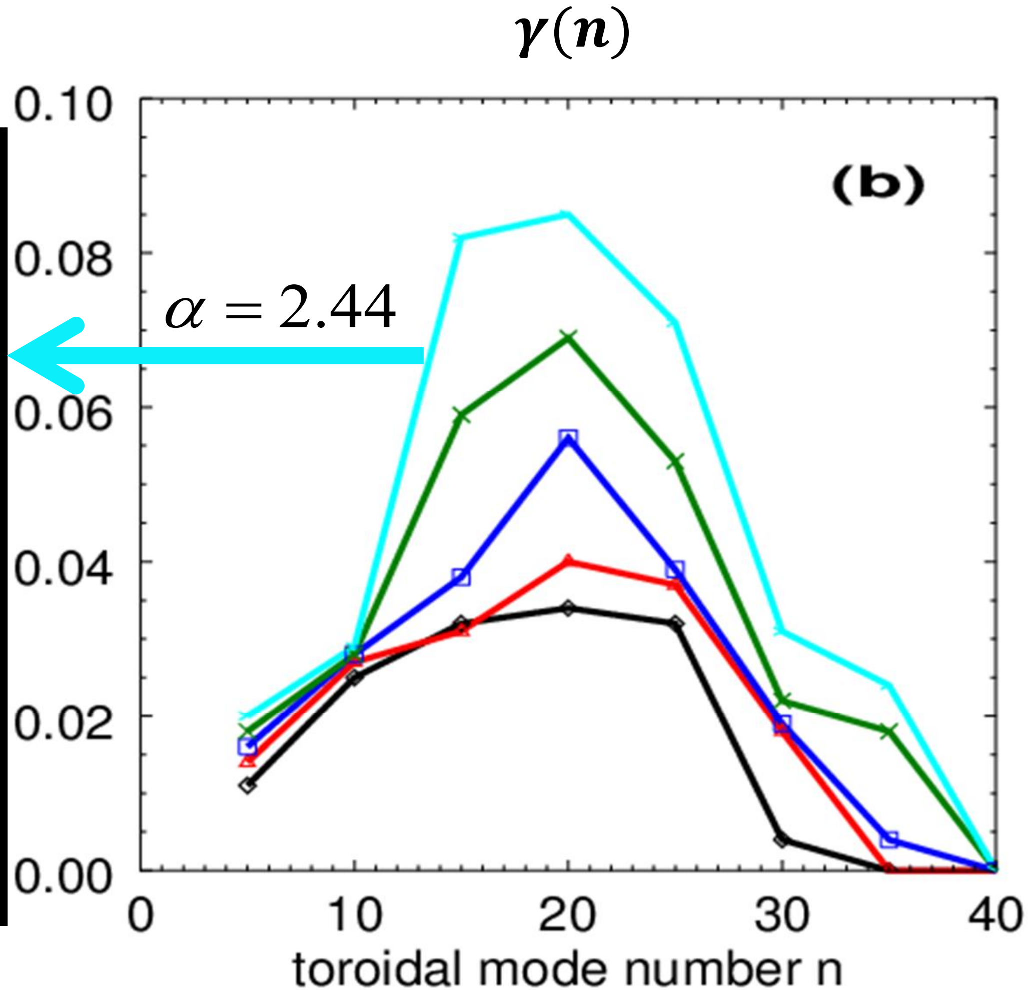


$$\alpha = -2\mu_0 R P_0' q^2 / B^2$$

- Evolution of P-B turbulence
  - No filaments
  - Weak radial extent

Stronger Peaking $\gamma(n)$ ➔ ELM Crash



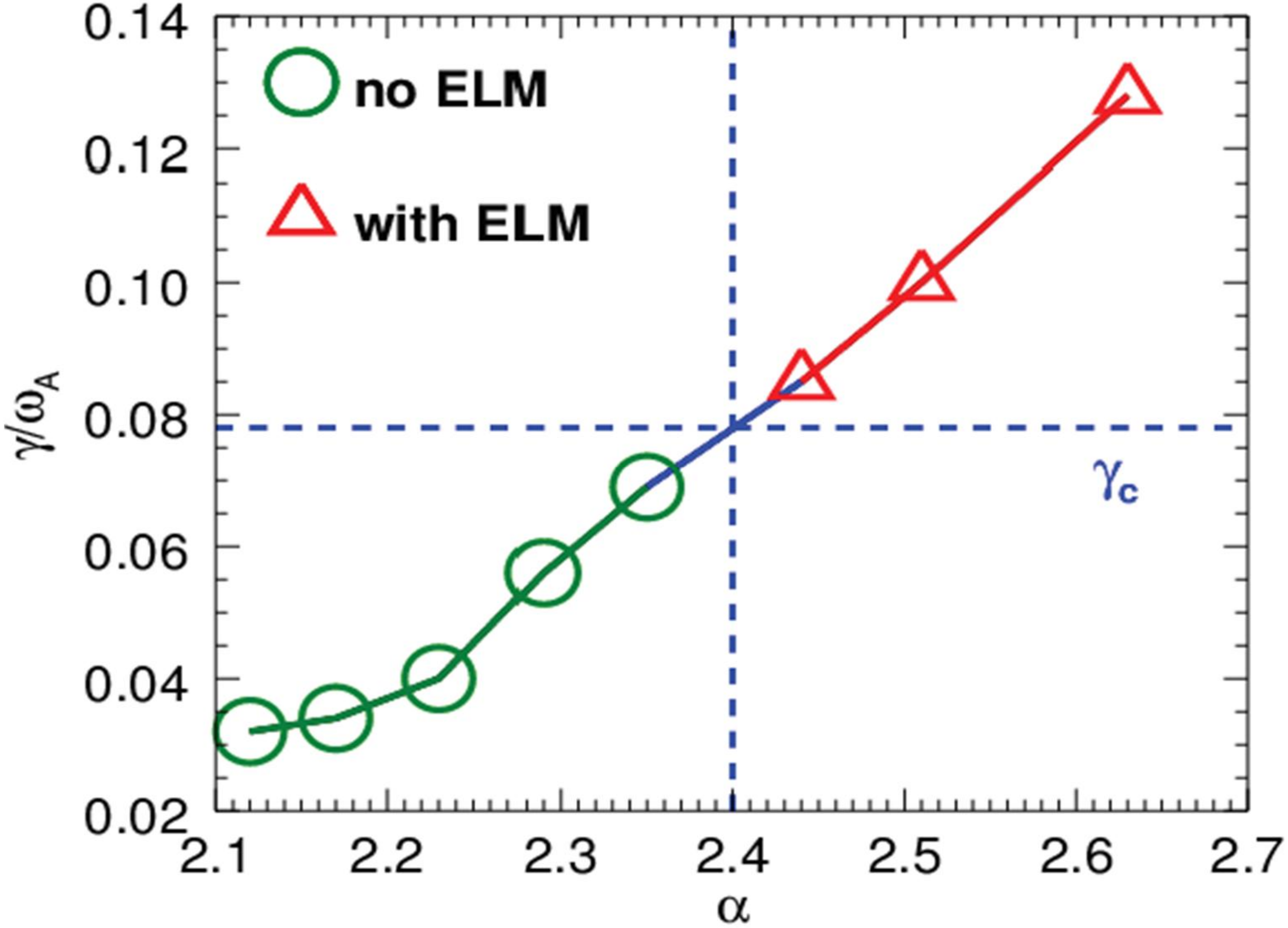


- ELM crash is triggered
- Wide radial extension

$$\alpha = -2\mu_0 R P_0' q^2 / B^2$$

Linear criterion for the onset of ELMs $\gamma > 0$ is replaced by the nonlinear criterion

$$\gamma > \gamma_c \sim 1/\,\tau_c$$



- **Criterion for the onset of ELMs**

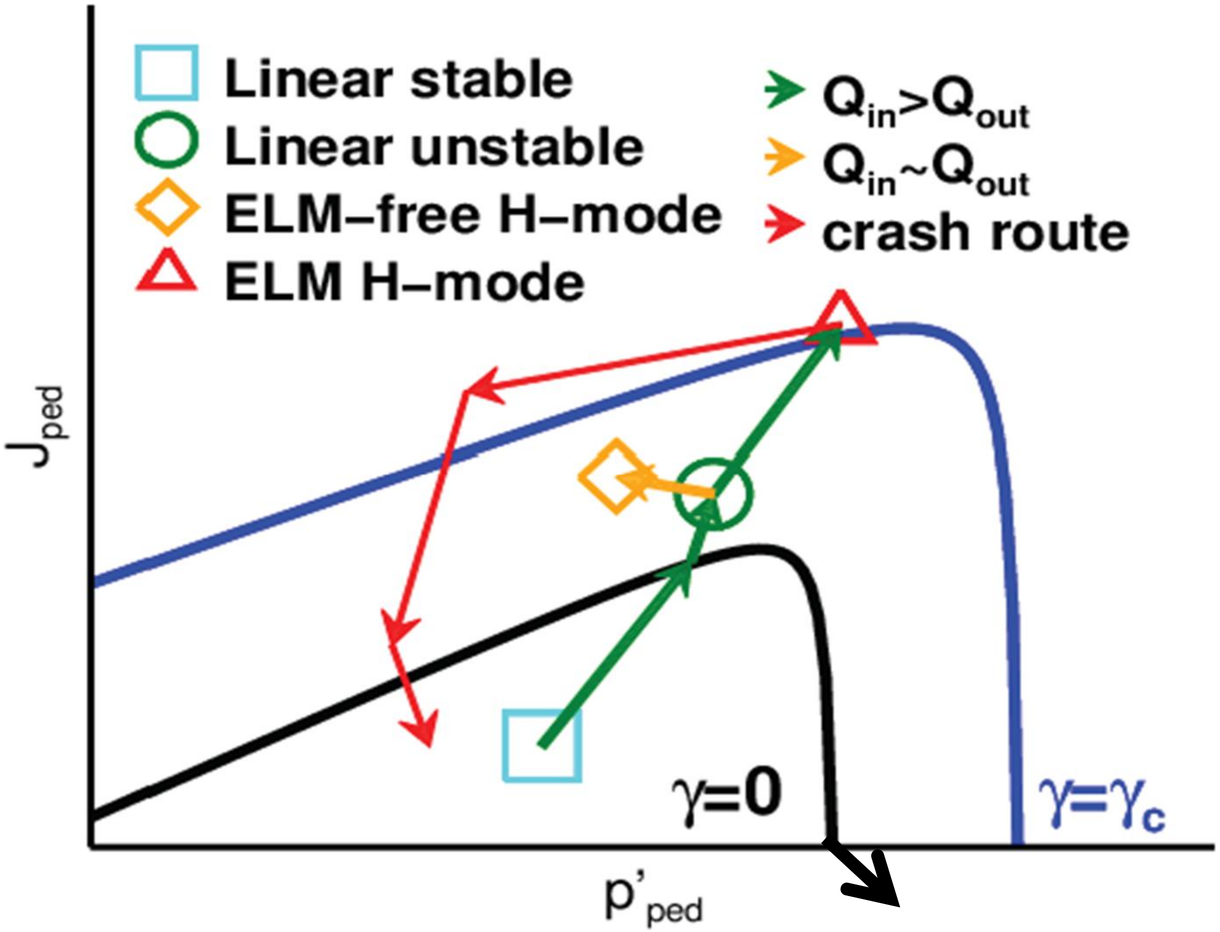
$$\gamma\tau_c > \ln 10 \Rightarrow \gamma > \frac{\ln 10}{\tau_c} \equiv \gamma_c$$

- **Linear limit**

$$\lim_{\tau_c \to \infty} \Rightarrow \gamma > 0$$

- **γ_c is the critical growth rate which is determined by nonlinear interaction in the background turbulence**
- **N.B. $1\,/\,\tau_c$ - and thus γ_{crit} - are functionals of $\gamma_L(n)$ peakedness**

Nonlinear Peeling-ballooning model for ELM:



- $\gamma < 0$:
  Linear stable region
- $0 < \gamma < \gamma_c$: **Turbulent region**
  Possible ELM-free regime→
  Special state: EHO, QCM (?!)
- $\gamma > \gamma_c$:
  ELMy region

✓ **Different regimes depend on both linear instability and the turbulence in the pedestal.**

Including relevant linear physics
(not only ideal P-B with ω_*)
Resistivity / Electron inertia /…

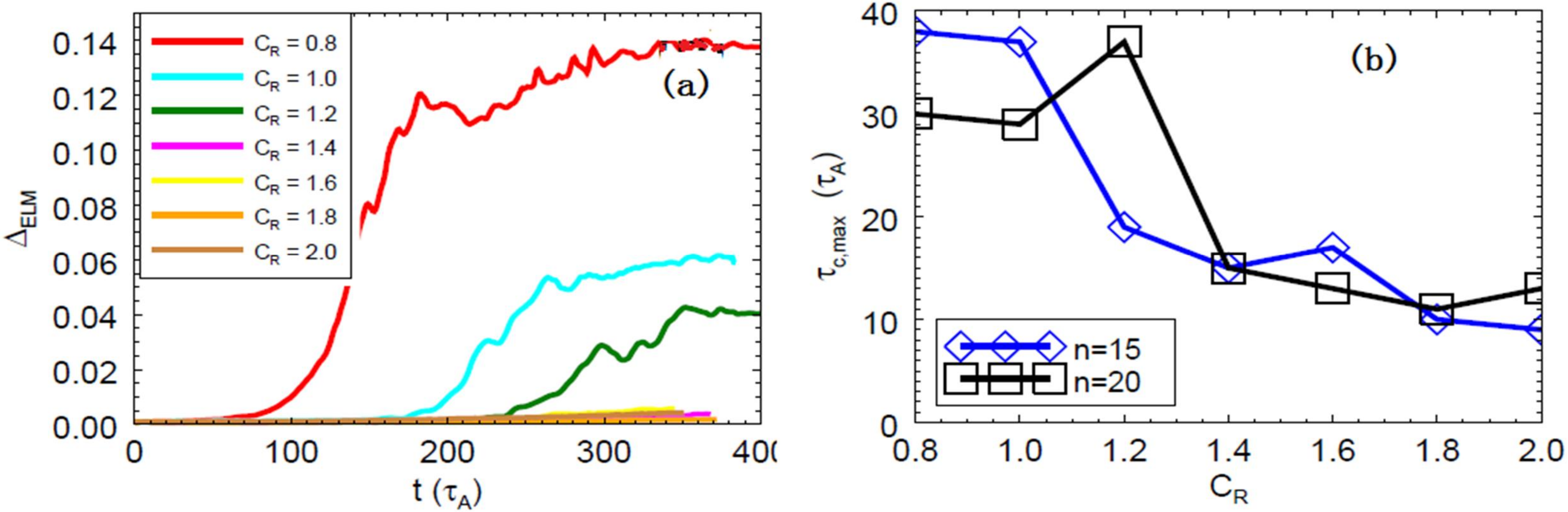
→ **Turbulence can maintain ELM-free states**

How can these ideas be exploited for ELM mitigation and control?

ELMs can be controlled by reducing phase coherence time

$$\frac{\partial \varpi}{\partial t} + C_R \frac{\mathbf{b} \times \nabla \phi}{B} \cdot \nabla \varpi = RHS$$

i.e. scan C_R for fixed profiles



- ELMs are determined by the product $\gamma(n)\tau_c(n)$;
- Reducing the phase coherence time can limit the growth of instability;

- **Different turbulence states lead to different phase coherence times and, thus different ELM outcomes**

Keys to τ_c

- Scattering field
- ‘differential rotation’ in $\hat{P}$ response to $\hat{v}_r$

→ enhanced phase de-correlation

Knobs:

- ExB shear
- Shaping
- Ambient diffusion
- Collisionality

Mitigation States:

- QH mode, EHO
- RMP
- SMBI
- …

Scenarios

- QH-mode
  - enhanced ExB shear → $\frac{1}{\tau_c} \rightarrow \left(k_\perp^2 D \, \langle V_E \rangle'^2\right)^{1/3}$
  - Triangularity strengthens shear via flux compression (Hahm, KHB)
  - Enhanced de-correlation restricts growth time

Also:

- Is EHO peeling/kink + reduced τ_c? How maintained?
- $\langle V_E \rangle'$ works via γ_L and τ_c

N.B. See Bin Gui, Xu; for more on shearing effects

Scenarios

- RMP
  - $\frac{1}{\tau_c} = \left(\frac{k_\perp^2 \hat{s}^2}{(Rq)^2} \chi_\parallel D\right)^{1/2}$ $\quad D = D_\phi + D_{am\ b}$
  - RMP → $D_{am\ b} \uparrow$ → enhanced de-correlation

  or

  - Enhanced flow damping → enhanced turbulence → increased D_ϕ

    (Leconte, P.D., Y. Xu)
- SMBI
  - enhanced D_ϕ → reduced τ_c ?

  and/or

  - Disruption of pedestal avalanches?

Phase Dynamic → A Deeper Look

Phase Dynamics in P.-B. Turbulence is INTERESTING

- i.e. usually $\tau_{ac} \sim \left|\left(\frac{\omega}{k} - \frac{d\omega}{dk}\right)\Delta k\right|^{-1}$ sets phase coherence time by ~ linear processes → wave propagation and dispersion
- P.-B. turbulence in strong coupling regime $\omega \rightarrow 0$,

  if insist on ω_*, non-dispersive

  $\therefore$ τ_c set by nonlinear dynamics

Phase Dynamics in P.-B. Turbulence is HARD

- Recall:

$$\partial_t \tilde{\phi} + \tilde{v} \cdot \nabla \tilde{\phi} + \langle v(r) \rangle \cdot \nabla \tilde{\phi} - D \nabla^2 \tilde{\phi} - \frac{2D}{A} \nabla A \cdot \nabla \tilde{\phi} = 0$$

$$\partial_t A + \tilde{v} \cdot \nabla A + \langle v(r) \rangle \cdot \nabla A + D (\nabla \phi)^2 A - D \nabla^2 A = -B \frac{d\langle P \rangle}{dr}$$

  – Turbulent $\tilde{v}$, self-consistency?

  – ϕ, A coupling?

  – Vorticity equation, Ohm's law?

Phase Dynamics in P-B Turbulence is HARD

- For hard problems recall advice of G. Polya in "How to Solve It"

  ~ "If you didn't know how to solve a problem, convert it to an easier problem you do understand."

- What familiar paradigm(s) does the phase dynamics problem resemble?

Paradigms

- There are at least 2; both involving phase dynamics:

a) if ignore $2D\nabla A \cdot \nabla\phi \,/\, A$:

$$\partial_t\tilde{\phi} + \tilde{v}\cdot\nabla\tilde{\phi} + \langle v(r)\rangle\cdot\tilde{\phi} - D\nabla^2\tilde{\phi} = Noise$$

→ scalar evolution with noise

→ if ignore feedback on $\tilde{v}$ → passive scalar

∴ considerable body of insight into pdf[$\tilde{\phi}$].

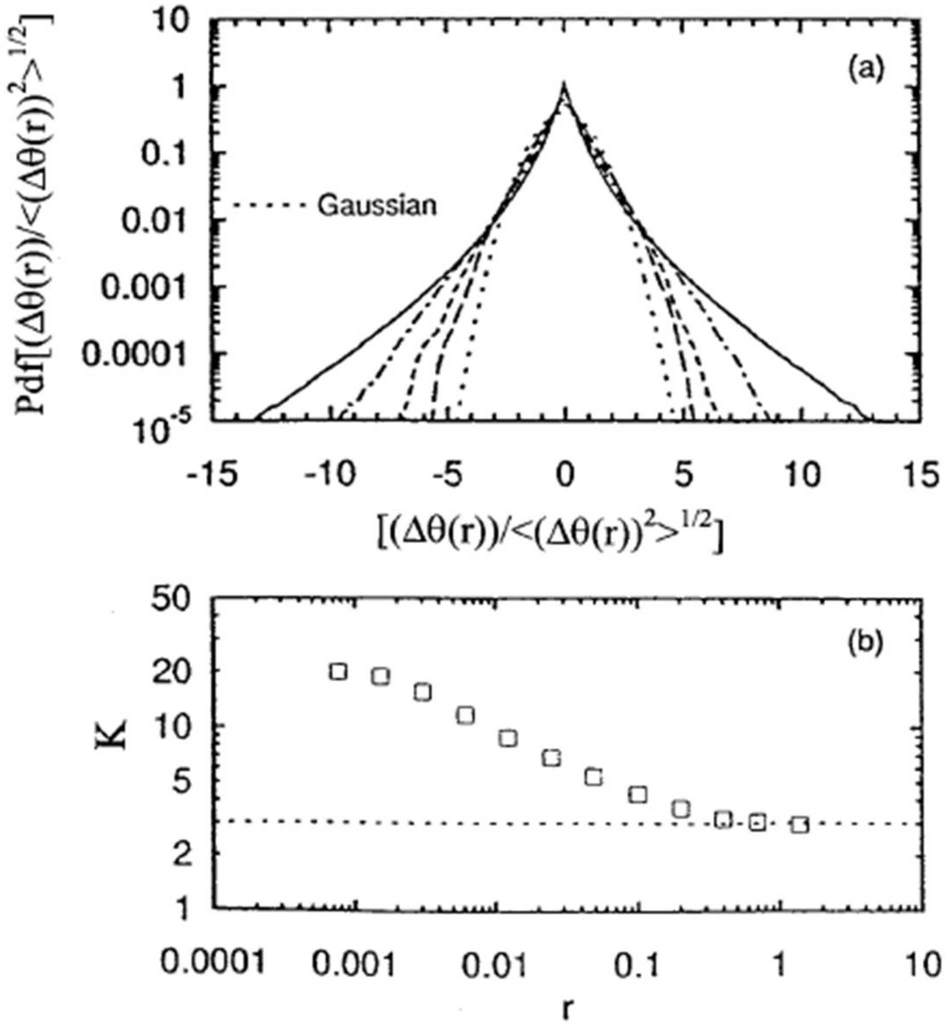
N.B. Little or no "physics" analysis of passive scalar statistics with shear flow

A) Scalar

If $\langle v \rangle' \to 0$

$$\partial_t \tilde{\phi} + \tilde{v} \cdot \nabla \tilde{\phi} - D\nabla^2 \tilde{\phi} = 0$$

- PDF $\Delta\theta$ as function r
- Significant deviation from Gaussian for smaller scales
- Approaches Gaussian at large scales
- See also large kurtosis



Expect strongly non-Gaussian phase statistics

b) If ignore or simplify spatial structure, have synchronization problem ←→ Kuramoto

i.e.

i) Single oscillator

$$\frac{d\psi}{dt} = -\gamma + \epsilon\, q(\psi) + \varepsilon(t)$$

ii) Oscillator lattice, continuum →

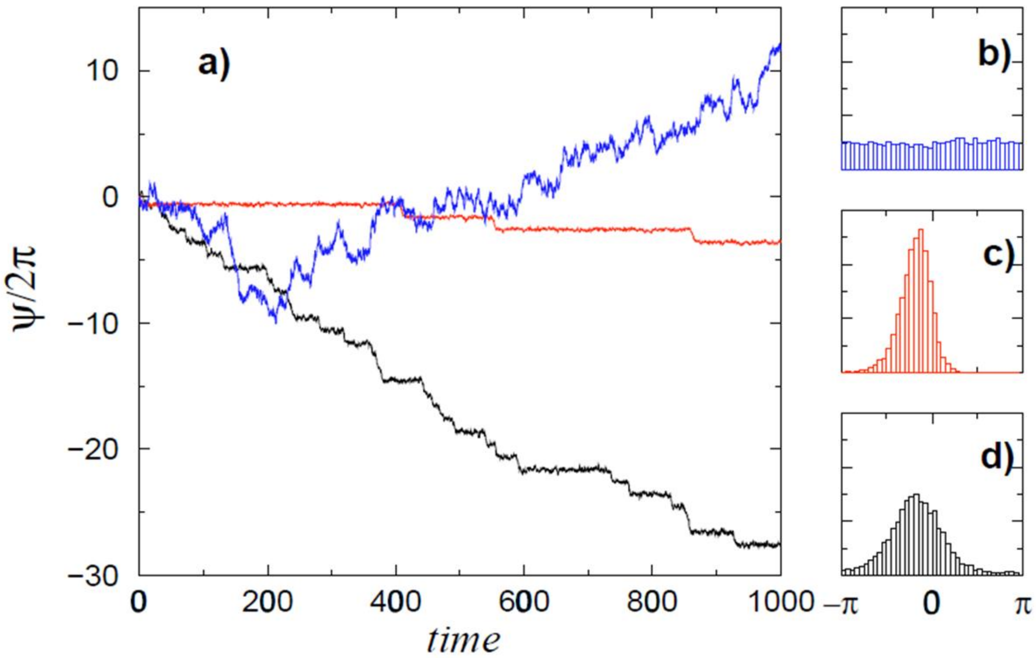
roughening —— synchronization

↕ ↕

de-coherence coherence

$$\frac{\partial \phi(x,t)}{\partial t} = \omega(x) + \alpha \nabla^2 \phi(x,t) + \beta (\nabla \phi)^2 + \varepsilon \quad \textit{(extended KPZ)}$$

B) Noisy Phase Dynamics ←→ Single Oscillator



- Phase slips occur
- Phase slips resemble cross phase jumps

- Observe
  - No noise (blue) → phase diffusive, distribution flat
  - Weak noise (red) → phase slips occur, distribution narrow
  - Strong noise (black) → more phase slips, broader distribution

Note:

→ Qualitative consistency of noisy oscillator phase

and P.-B. turbulence cross phase

i.e. slips/jumps occurrence increase with

noise/turbulence level

→ in P.-B. turbulence,

- noise multiplicative
- profile evolution introduces another interaction channel.

→ Phase Dynamics Problem

Promising but Challenging

Conclusions – Coarse Grained

Conclusions

- ELM phenomena are intrinsically multi-mode and involve turbulence
- P.-B. growth regulated by phase correlation

  → determines crash + filament vs turbulence
- Phase coherence can be exploited for ELM mitigation

Where to Next?

- Simulations MUST move away from IVP – even if motivated by experiment – and to dynamic profile evolution, with:
  - sources, sinks i.e. flux drive and particle source essential
  - pedestal transport model
  - anomalous electron dissipation

  i.e. → - what profiles are actually achieved?

  - how evolve near P.-B. marginality?

- Should characterize:
  - pdf of phase fluctuations, correlation time
  - Dependence on τ_c control parameters
  - Threshold for burst
- Need understand feedback of P.-B. growth on turbulent hyper-resistivity
- Continue to develop and extend reduced models.

Some Inflammatory Questions

- Why bother with RMP?
  – Focus on ‘self-kinked state’ of QH with EHO.
    Is this relevant and reproducible?
- Is SMBI ELM mitigation due:
  – phase de-correlation?
  – avalanche fragmentation?
  → stability of SMBI-mitigated EAST profiles?